MADX Studies. Coupling

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Coupling 1-turn map

- For coupled motion the matrix transformation can be written in a form

$$M = VUV^{-1}$$

where

- M is a symplectic 1-turn map

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

- U is a symplectic decoupled block-diagonal matrix

$$U = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}$$

- V is a symplectic "rotation" matrix in a form

$$V = g \begin{pmatrix} I & \bar{R}_M \\ -R_M & I \end{pmatrix}$$

where $g \equiv cos(\phi)$ (ϕ-rotation angle about the reference trajectory) and $\bar{R}_M$ is a symplectic conjugate of R_M, i.e.

$$\bar{R}_M = SR_M^T S^T, \text{ where } S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Coupling element-by-element

- Given that M_1 is the one-turn matrix at point 1 (known), M_2 is the one-turn matrix at point 2 (unknown), and we know M_{12}, the transfer matrix between points 1 and 2, we can express the change of the one-turn map from point 1 to point 2 by $M_2 = M_{12} M_1 M_{12}^{-1}$
- Then

$$U_2 = V_2^{-1} M_2 V_2 = V_2^{-1} M_{12} M_1 M_{12}^{-1} V_2 = (V_2^{-1} M_{12} V_1) U_1 (V_1^{-1} M_{12}^{-1} V_2)$$

So,

$$U_2 = W_{12} U_1 W_{12}^{-1}, \text{ where } W_{12} \equiv V_2^{-1} M_{12} V_1$$

- Knowing V_1 and M_{12} (transfer matrix of the element), we can calculate

$$V_2 W_{12} = M_{12} V_1,$$

where we use V_2 for computing the coupling parameters and W_{12} to propagate to the end of the element.
- One can find V_2 such that W_{12} is block-diagonal or off block-diagonal

$$W_{12} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix} \text{ or } W_{12} = \begin{pmatrix} 0 & F \\ E & 0 \end{pmatrix}$$

Modes flip

- For weakly coupled lattice we always use **block-diagonal** solution, i.e. large tune (mode1) is associated with x-plane, and smaller tune (mode2) is associated with y-plane by convention.
- For a highly coupled lattice, it may happen at some places in the lattice that mode1 is associated with y-plane, and mode2 with x-plane.
- Switching the association of mode1 and mode2 with x and y-planes as one propagates the twiss parameters through the lattice is called "modes flip" (use **off-block diagonal** solution for W_{12}).
- Modes flip is triggered only in the presence of coupling AND negative determinants of "uncoupled" sub-matrices E and F, which would lead to negative β-functions.

Mode determination

- No coupling ($B = C = 0$): M_{12} is already block-diagonal, i.e. $W_{12} = M_{12}$.
- Coupling: find V_2 that makes W_{12} block-diagonal.
  - If determinants of E and/or F of W_{12} are negative, **apply modes flip** and find V_2 that makes W_{12} off block-diagonal.
  - Check rotation of V_2 and stability condition of W_{12}.

    $$g \leq 1 \text{ (rotation) and } Tr(E, F) \leq 2 \text{ (stability)}.$$

    Emit an error message "twiss failed" if these conditions are not fulfilled.
  - Check that number of modes flip is even at the end.
    Emit a warning message "odd number of modes flips" if not even.
- Number of modes flip will be soon stored in the Twiss summary table.

Examples

HL LHC lattice with a "special" seed produces unexpectedly large misalignment:

- Forced to modes flip at the element **mqxfb.b2r1** (negative determinant).
- Comes out of the modes flip at the element **mqsxf.3r1**, with #flips $= 2$.
- In **mqxfb.b2r1** $g > 1$ and $Tr(F) > 2$ trig the error message (twiss):
  g = 0.102976E+01; trace of decoupled matrix is 0.260941E+03
- Misalignment of **mqxfb.b2r1** is dpsi $= -0.0035$.
  Adjusting misalignment to -0.0018 brings W_{12} back to the block-diagonal form, allowing proper calculation of coupling parameters.

Check of tunes fractional part

- The phase advance (μ) in one period is related to the reference particle tune by $Q = \frac{\mu_{total}}{2\pi}$.
- Calculate $\cos(\mu)$ based on eigenvalues of the one-turn map:

$$\lambda + \lambda^{-1} = e^{i\mu} + e^{-i\mu} = 2\cos(\mu)$$

- Calculate $\cos(\mu)$ based on decoupled one-turn map U:

$$\cos(\mu_1) = trace(E)/2 \quad ; \quad \cos(\mu_2) = trace(F)/2$$

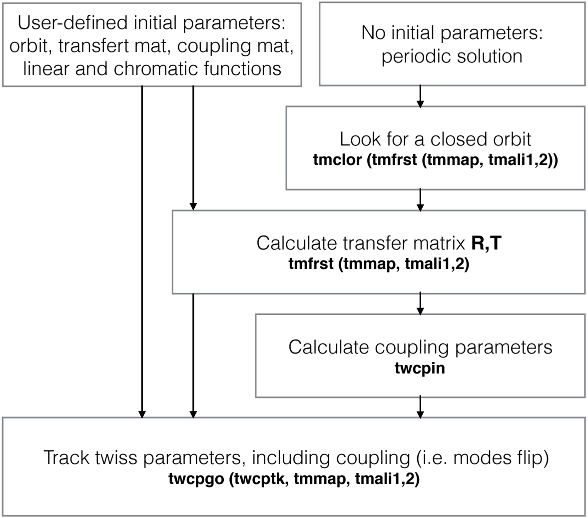
- Calculate $cos(\mu_{total})$ based on twiss parameters:

$$\mu_n = \mu_{n-1} + atan\left(\frac{E_{12}|F_{12}}{E_{11}|F_{11} \times \beta_{n-1} - E_{12}|F_{12} \times \alpha_{n-1}}\right),$$

$$\mu_{total} = \mu_0 + \sum_{n=1}^{N} \mu_n$$

- Compare $cos(\mu)$, which represents a comparison of a fractional part of a tune, calculated by different methods, and emit a warning message if there is a disagreement.

Twiss scheme



Summary

- Added symplecticity check of the intial one-turn map and for each element matrices as the coupling calculation requires this condition.
- Added modes flip for highly coupled optics where "block-diagonal" solution is invalid (e.g. negative determinant).
- Added stability checks $g > 1$ and $Tr(E, F) > 2$ after coupling decomposition.
- Added check of even number of modes flips, and save it in the summary table.
- Added a check on a fractional part of tune.