Opportunities in Diquark Physics

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JAEA/ASRC Reimei Workshop

New exotic hadron matter at J-PARC

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Diquarks — where?

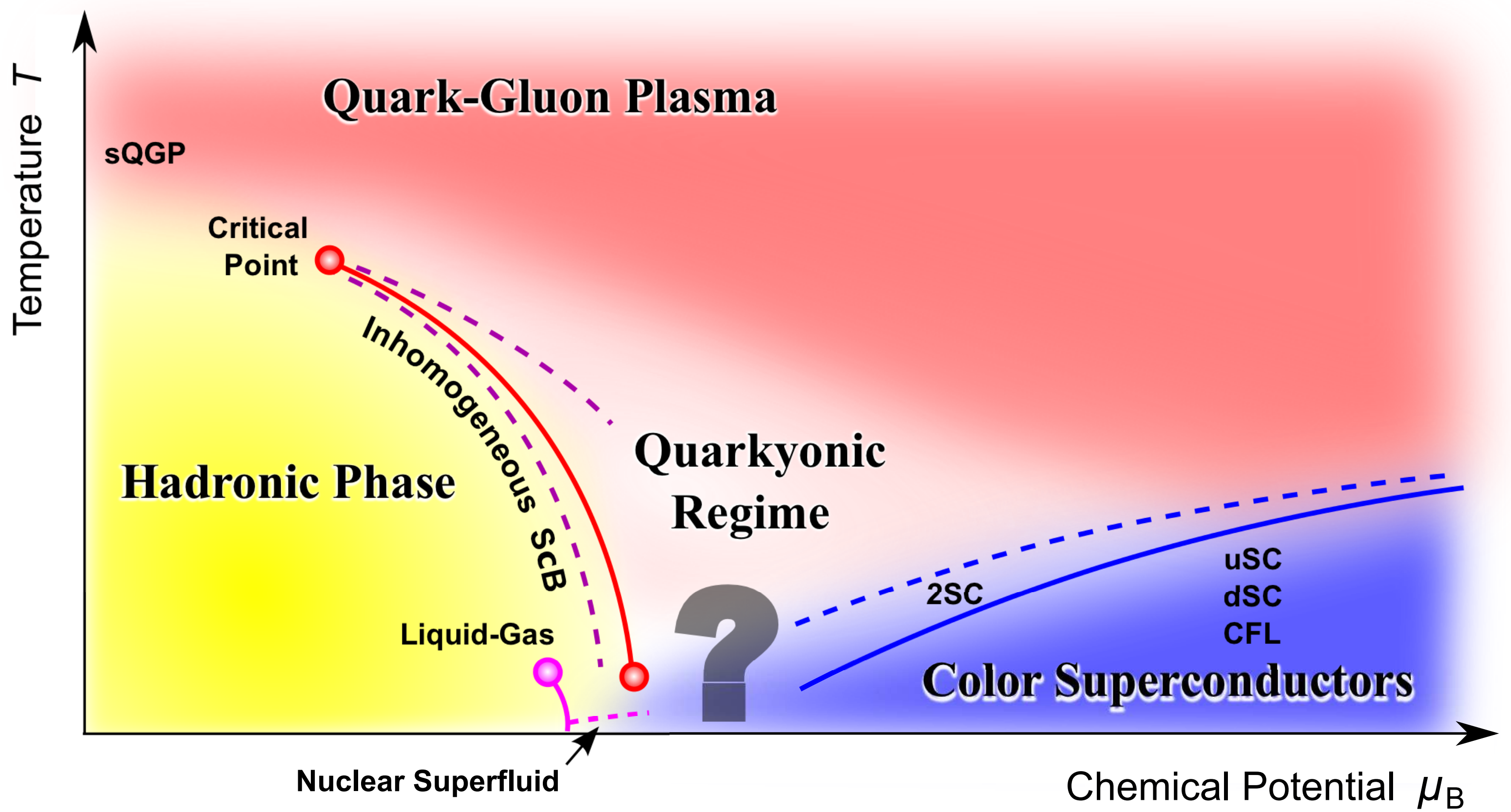
Diquarks in Matter

- □ Phases of QCD and diquarks
- □ Color superconductivity — theoretical illusion?
- □ Diquark condensate in nuclear matter

Diquarks in Hadrons

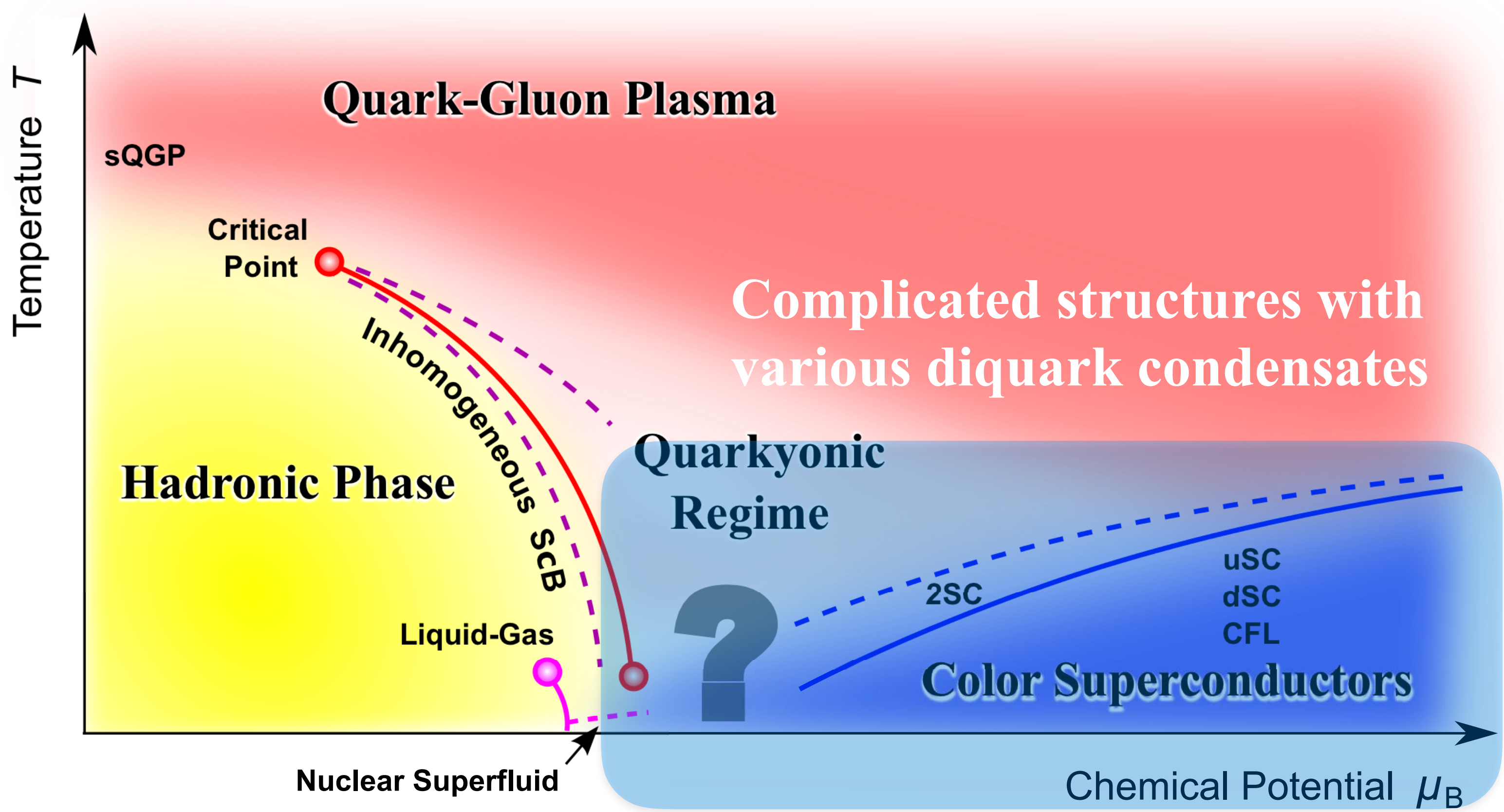
- □ Diquark correlation in baryons
- □ Phenomenological mass formula and diquarks
- □ Implication from hadrons to matter

Phases of QCD



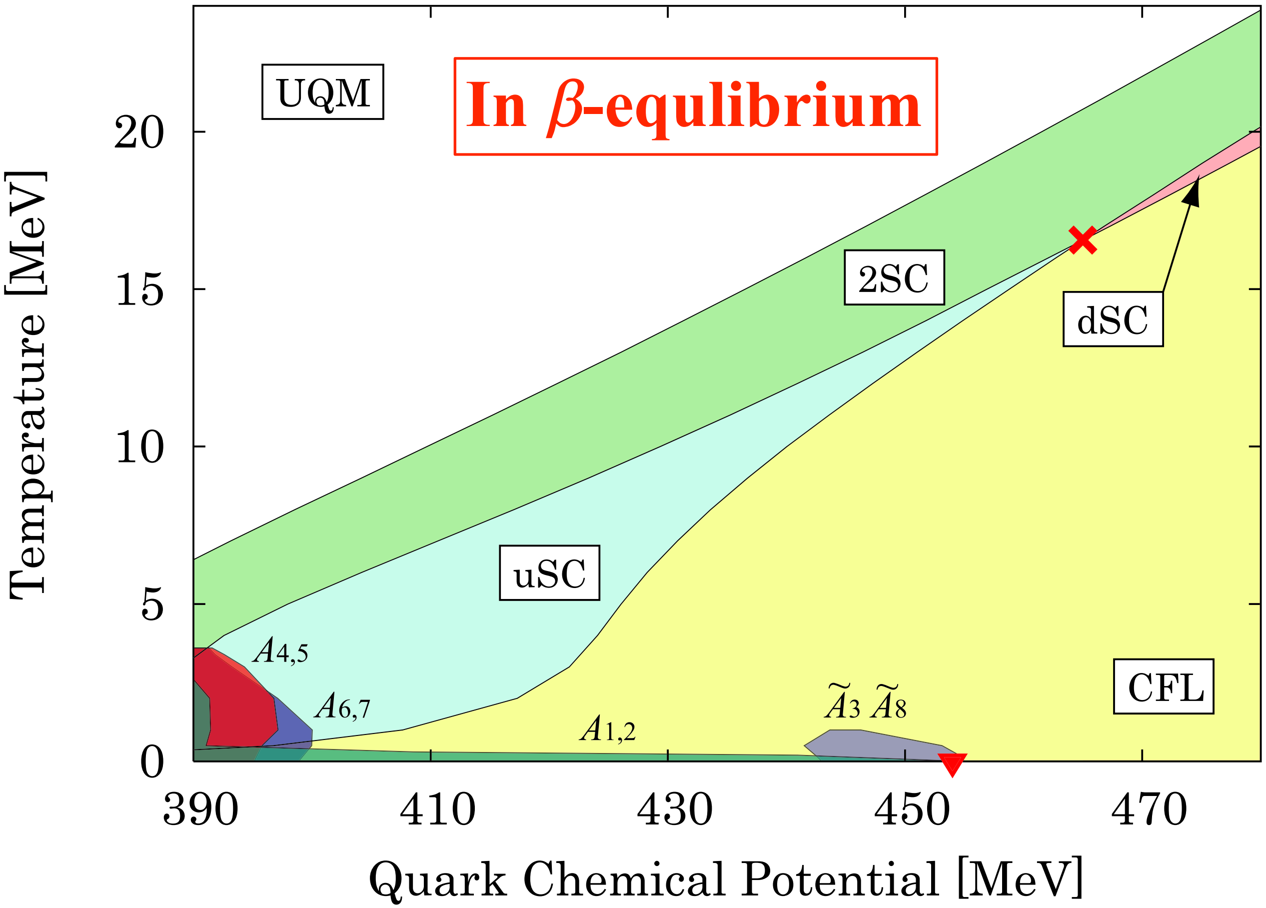
Fukushima-Hatsuda (2010) / Fukushima-Sasaki (2013)

Phases of QCD



Fukushima-Hatsuda (2010) / Fukushima-Sasaki (2013)

Diquark Condensation Phases



Fukushima (2005)

Many diquarks — many phases

Instabilities (Crystallized)

All complications by *s*-quarks

Caution!

Your phase diagram may not be the phase diagram for neutron stars or for heavy-ion collision (HIC)

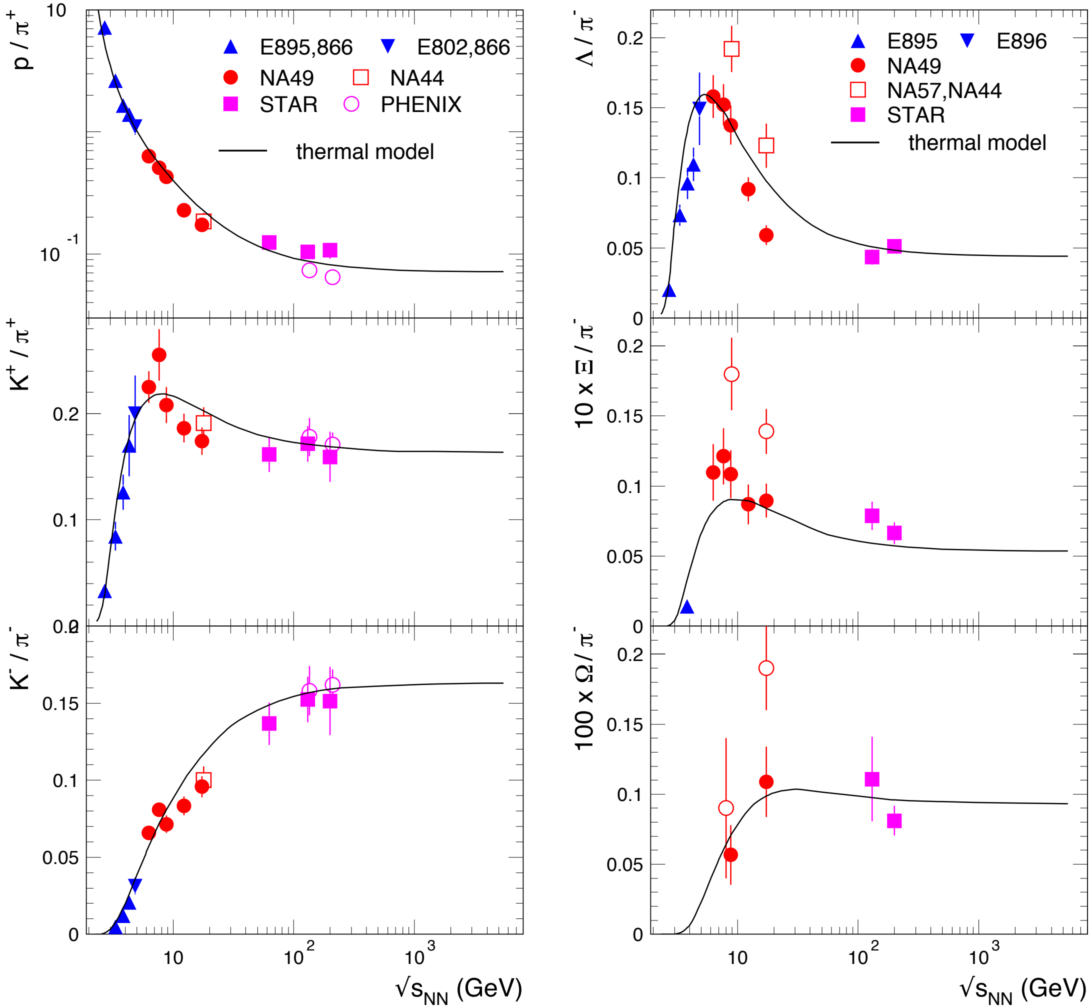
Neutron Stars

Electric neutrality condition (n-rich; electron density)
β-equilibrium (hyperons to lower the Fermi surface)

HIC

Electric charge conservation ($Q/(p+n)$ fixed)
Zero net strangeness (Λ, Σ compensated by K)

Particle Abundances in HIC



Around HIC energies of J-PARC, NICA, FAIR,… the baryon density is maximized and so the strangeness pair production is also maximized (but the NET *s* is still zero)

Classification of Diquarks

Color Interaction

$$(t^a)_{ij}(t^a)_{kl} = -\frac{N_c+1}{4N_c}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}) + \frac{N_c-1}{4N_c}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})$$

Color Triplet (antisymmetric)
Attractive

Dominant

Only this channel considered
(**flavor**) (**spin**) (~~**orbital**~~)
should be symmetric

Color Sextet (symmetric)
Repulsive

Always mixed with triplet
No new physics brought in
Harmlessly neglected

Classification of Diquarks

Spin-dependent Part **Breit Interaction**

$$H_{\text{color-spin}} = \alpha_s \sum_{i \neq j} M_{ij} \underbrace{(\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j)}_{\text{color}} \underbrace{(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)}_{\text{spin}}$$

> spin-singlet (antisymmetric) + flavor triplet (antisymmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{0}\rangle = -(3/4)|\mathbf{0}\rangle$$

Good Diquark

> spin-triplet (symmetric) + flavor sextet (symmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{1}\rangle = +(1/4)|\mathbf{1}\rangle$$

Bad Diquark

Remarks on "Bad Diquark"

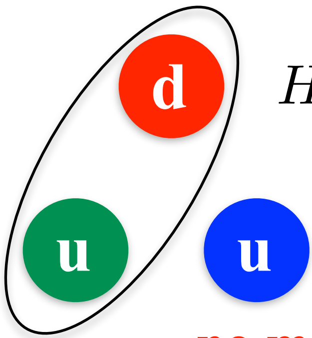
Bad diquarks play a role in hadron spectroscopy
but their importance is minor in bulk properties

Spin-1 color-superconducting phase
spontaneously breaking rotational symmetry
(*color-spin locked phase*)
Gap is more than one order of magnitude smaller

If Fermi surfaces of different flavors are mismatched
(due to neutrality (neutron stars) or conservation (HIC))
single-flavor diquarks may be favored (flavor-symmetric)

Remarks on "Bad Diquark"

$N : S = 1/2$



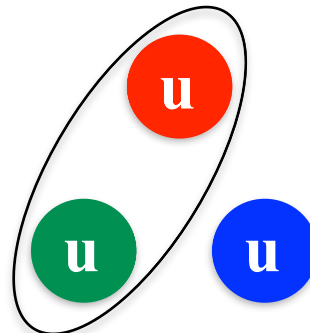
$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$S = 0$

no more s-s int.

$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$\Delta : S = 3/2$



$$H_{\text{color-spin}} = +\frac{1}{4}C$$

$S = 1$

$\boldsymbol{S} \cdot \boldsymbol{s} = 1/2$

$$H_{\text{color-spin}} = +\frac{3}{4}C$$

$$m_{\text{bad}} - m_{\text{good}} \approx \frac{2}{3}(M_{\Delta} - M_N)$$ confirmed in lattice QCD

Color Superconductivity (CSC)

Three triplet diquark condensates

$$\langle ud \rangle \quad \langle ds \rangle \quad \langle su \rangle$$

break which symmetry of QCD???

2-flavor case

$\langle ud \rangle$ has electric charge and baryonic charge

Baryon-U(1) is NOT broken mixed with QED-U(1)
QED-U(1) is NOT broken mixed with Color

5 out of 8 gluons are Meissner screened

No change in global symmetry

Color Superconductivity (CSC)

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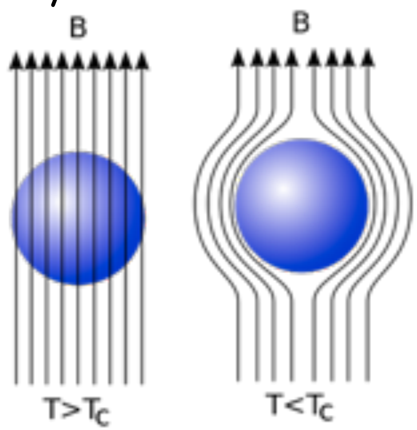


No Meissner Effect even if an external *B* is imposed

Color Superconductivity (CSC)

Three triplet diquark condensates

$$\langle ud \rangle \quad \langle ds \rangle \quad \langle su \rangle$$

break which symmetry of QCD???

3-flavor case

$\langle ud \rangle \langle ds \rangle \langle su \rangle$ have electric charge and baryonic charge

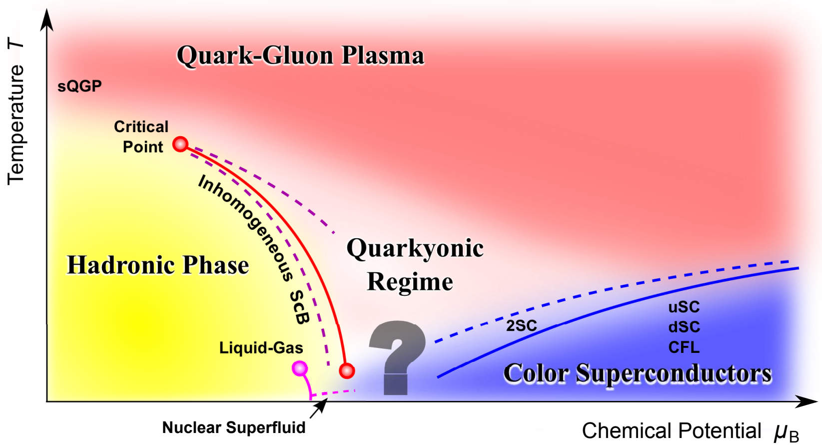
Baryon-U(1) is broken
QED-U(1) is NOT broken mixed with Color

8 gluons are all Meissner screened

Chiral symmetry broken and superfluid

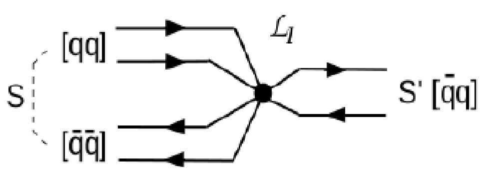
CSC : Illusion? Reality?

No phase transition between Higgs and confining phases

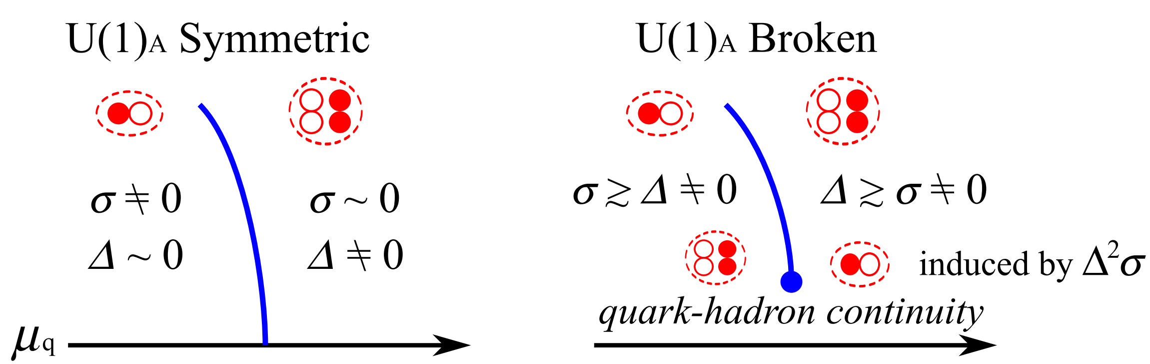


CSC **Hadronic**

3-flavor symmetric NM is always mixed with CSC
Diquark IS there!



't Hooft et al. (2008)



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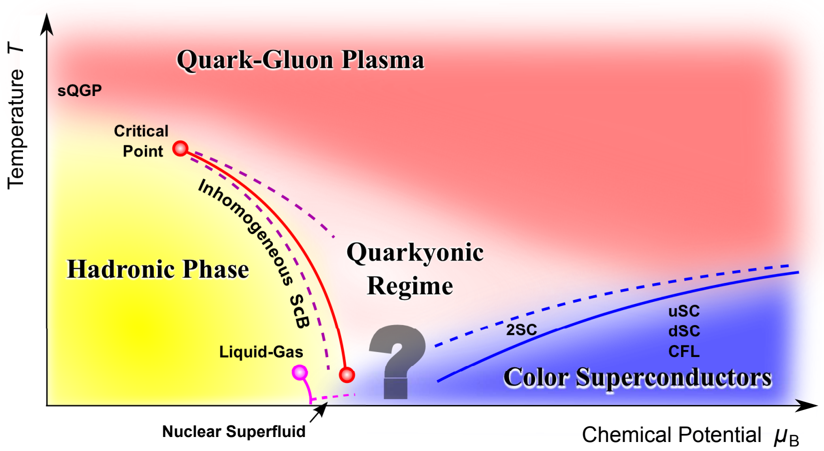


CSC Hadronic

2-flavor NM can be also mixed with CSC with $\langle \bar{q}q \rangle$
Diquark could be there!

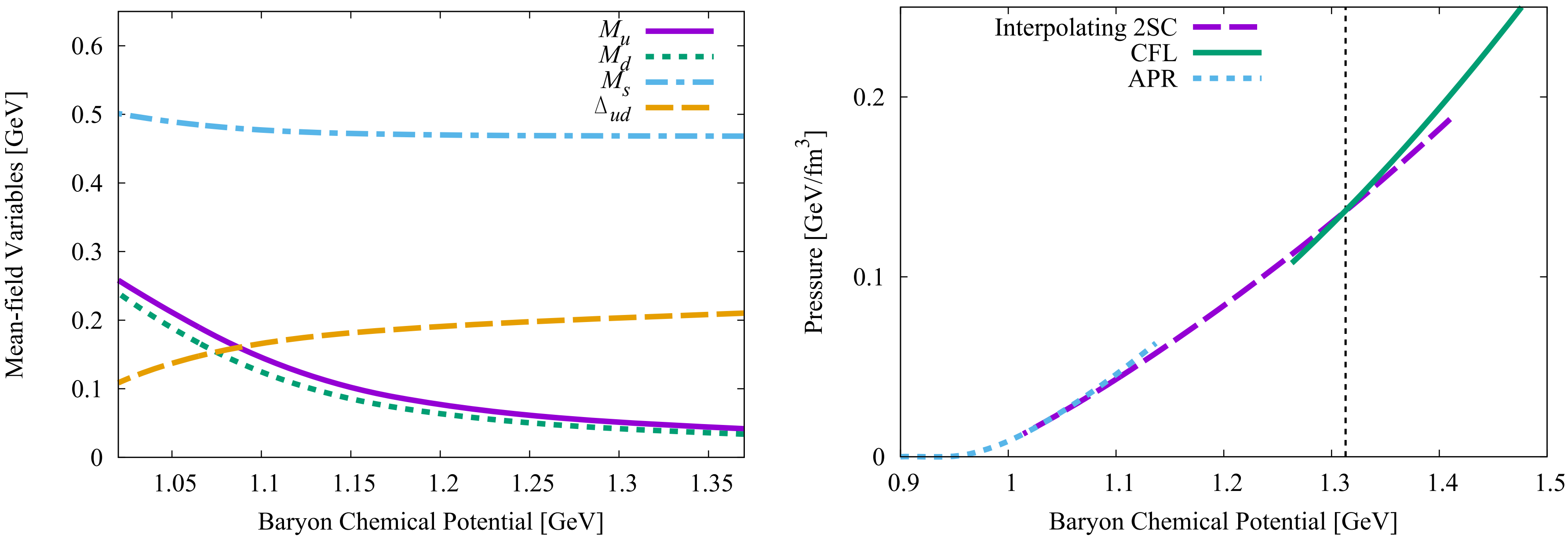
2-flavor CSC does NOT break any global symmetry but it can co-exist with chiral condensate as in NM

NM EoS could be described by QM EoS with diquarks

Quarkyonic Star

Quark model (NJL) constrained to reproduce NM EoS

Fukushima-Kojo (2015)



No sharp chiral phase transition
1st-order transition between 2- and 3-flavor CSCs

Diquarks in NM

Can we "exclude" the presence of diquarks in NM?

Suppose $\langle ud \rangle \neq 0$

Physical observables must be gauge invariant

$$\langle (\bar{u}\bar{d})(ud) \rangle \sim \langle (\bar{u}u)(\bar{d}d) \rangle \neq 0$$

$\langle pp \rangle$ $\underline{\langle pn \rangle}$ $\langle nn \rangle$ ⟷ $\underline{\langle uu \rangle}$ $\langle ud \rangle$ $\underline{\langle dd \rangle}$

Relation?

Fermi surface mismatched
(*N*=*Z* nuclei?)

Antisymmetric color makes a difference

Summary (for Matter)

Nothing has excluded a mixture of diquark condensate in nuclear matter (even at normal nuclear density!)

Theoretical subtlety comes from "gauge invariance"
What observable can tell us anything about diquarks?

Ordinary Nuclear Matter without CSC

Exotic Nuclear Matter with CSC

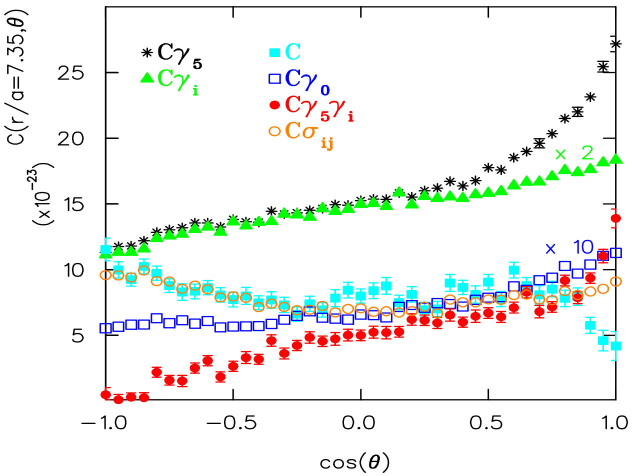
If anything could be experimentally measurable,
how can we "clearly" distinguish one from the other?

Some hints may come from diquarks in exotic hadrons

Diquarks in Baryons

Diquark correlations measured in lattice-QCD

Alexandrou-de Forcrand-Lucini (2006)



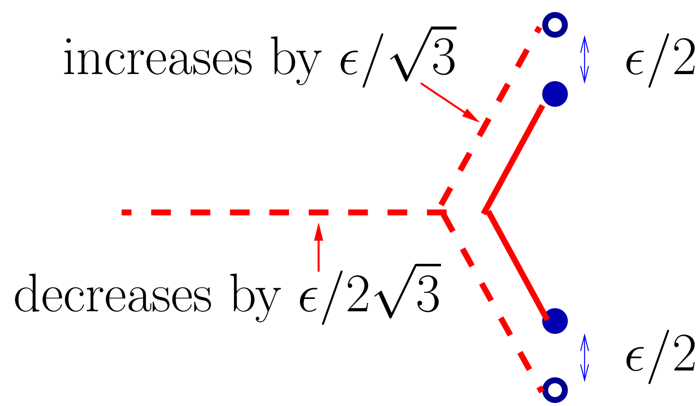
Quarks are heavier than physical
Volume is as small as proton size

Density-correlation must be re-analyzed with better data

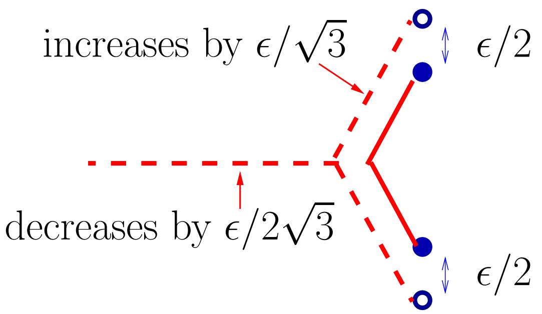
An interesting observation is:

$$\sigma_{qq} = (\sqrt{3}/2)\sigma_{\bar{q}q}$$

qq* int not suppressed by 1/*Nc



Remarks



String tension is "**screened**" by q (?)
qq int is "**strengthened**" by Y junction

Diquark in a baryon can be bound as tightly as $\bar{q}q$ in mesons

This already indicates that in-medium diquarks can be very different from isolated diquarks (no "diquark gas")

Force to split a diquark up is (only) 15% smaller than that to separate a diquark and a quark apart.
In lattice QCD it should be possible to extract also the spin-dependent part (projected into S=0 channel).

Phenomenological Mass Formula

$$M = \sum_i m_i + C'|\psi(0)|^2 \sum_{i \neq j} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{\boldsymbol{s}_i \cdot \boldsymbol{s}_j}{m_i m_j}$$

$$\sim \sum_i m_i + C \sum_{i \neq j} \frac{\boldsymbol{s}_i \cdot \boldsymbol{s}_j}{m_i m_j}$$

$$C_{\bar{q}q} > C_{qq}$$

$$C_M \qquad C_B$$

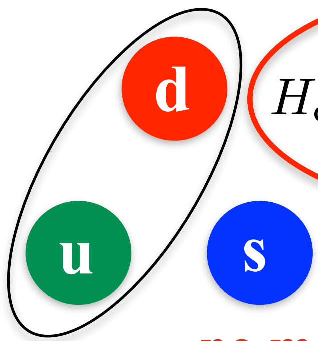
Lee-Yasui
Lee-Yasui-Liu-Ko (2007)

Hadron masses and stability can be easily studied once the wave-function is assumed (diquark model)

Σ-Λ Mass Splitting

$\Lambda : S = 1/2$

Favors diquark

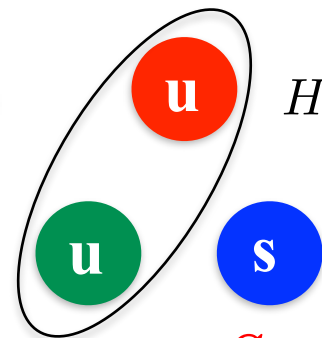


$$H_{\text{color-spin}} = -\frac{3}{4}\frac{C_B}{m_u^2}$$

$S = 0$

no more s-s int.

$\Sigma : S = 1/2$



$$H_{\text{color-spin}} = +\frac{1}{4}\frac{C_B}{m_u^2}$$

$S = 1$

$\boldsymbol{S} \cdot \boldsymbol{s} = -1$

Different from *Δ*

$$M_\Sigma - M_\Lambda = C_B\left(\frac{1}{m_u^2} - \frac{1}{m_u m_s}\right)$$

$$\to m_{\text{bad}} - m_{\text{good}} \quad (m_s \to \infty)$$

(~ 200MeV)

Implication to diquark onset?

$$M_N = m_D + m_u \quad (m_D = 2m_u - (3C_B/4m_u^2))$$

$$M_\Delta = m_{D'} + m_u + \frac{1}{2}\frac{C_{Dq}}{m_{D'} m_u} \quad (m'_D = 2m_u + C_B/(4m_u^2), \;\; C_{Dq} \simeq 2C_B)$$

etc etc…

implying to matter properties:

$$\mu_B(\text{threshold for } N) = M_N(\cancel{-\delta_B})$$

$$\mu_B(\text{threshold for } D) = \frac{3}{2} m_D = M_N - \frac{3}{8}\frac{C_B}{m_u^2} < M_N \;\; \text{(surprise!)}$$

Support for the presence of CSC in normal NM!

Summary (for Hadrons)

Lattice QCD needs updates with lighter quark mass and larger volume

— Electric charge distribution function (charge radius)

— Density-density correlation

— qq string tension is well-defined only with another q

Establishment of "constituent diquark model" may have a big impact to the high-density phases of QCD

— Residual (spin-indep.) interaction of diquarks and quarks

— Diquark properties with more surrounding quarks

$$q\ \text{(baryon)}\ ,\bar{q}\bar{q}\ \text{(tetraquark)}\ , D\bar{q}\ \text{(pentaquark)}\ , \ldots , \rightarrow\ \text{(matter)}$$