

# Few-body approach for structure of light kaonic nuclei

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In collaboration with

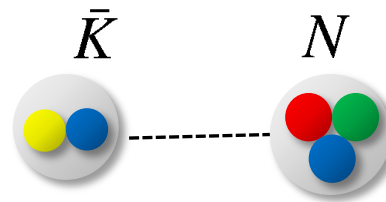
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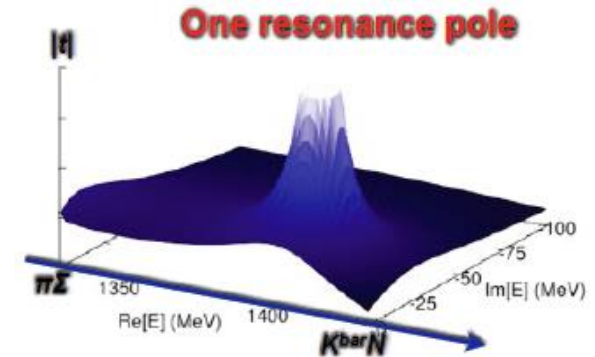
# $K^{\text{bar}}N$ interaction



Dalitz, Wong, Tajasekaran,  
PR 153(1967)1617.

- Phenomenological  $K^{\text{bar}}N$  interactions

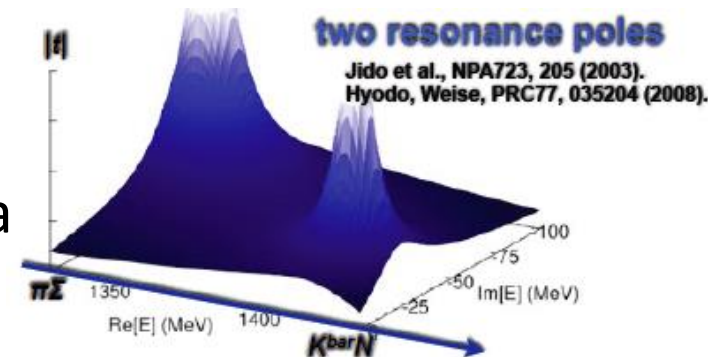
- Scattering length and flavor SU(3)
- Strongly attractive
- produce quasi-bound state of  $K^{\text{bar}}N$ , so-called  $\Lambda(1405)$



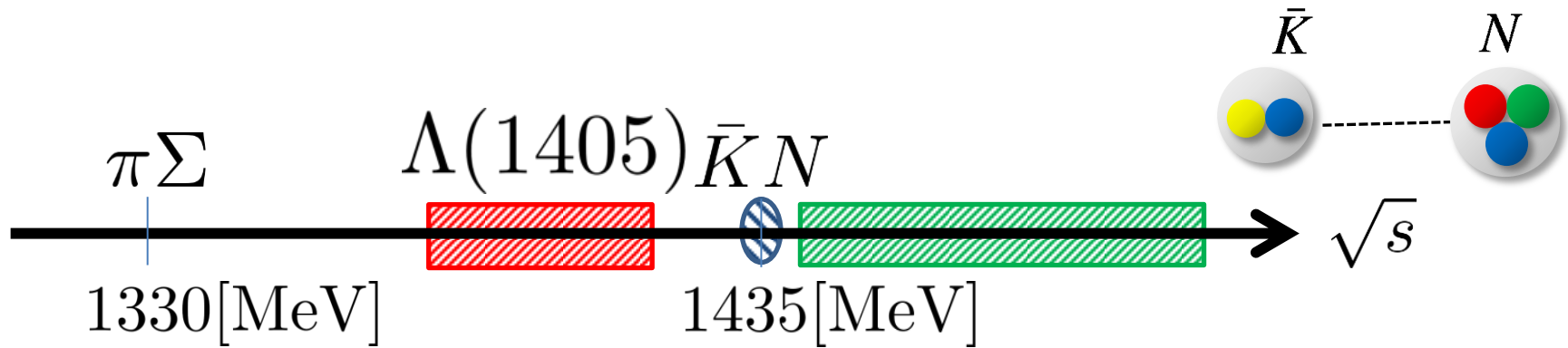
- Chiral SU(3) symmetry

Kaiser, Siegel, Weise,  
NPA594(1995).

- NG boson associated with spontaneous breaking of chiral SU(3) symmetry
- Strongly attractive
- Consistent with  $K^{\text{bar}}N$  scattering data
- Two poles,  
 $K^{\text{bar}}N$  quasi-bound state  $\rightarrow$  1420MeV



# Experimental constraint on $K^{\text{bar}}N$ interaction



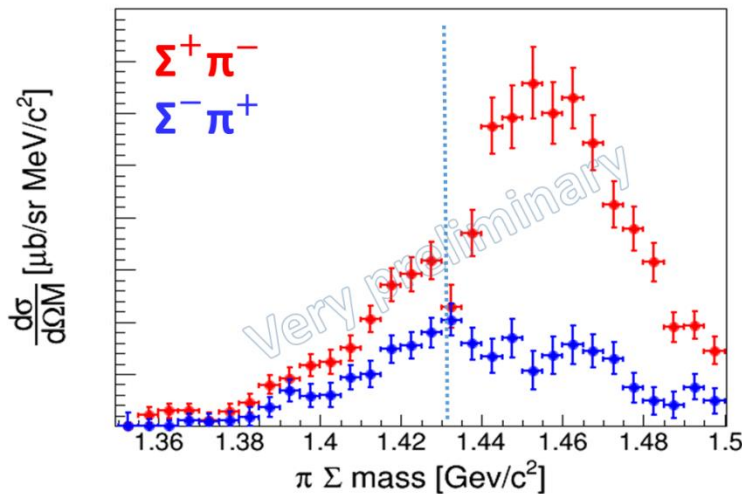
- above  $K^{\text{bar}}N$  threshold energy: -  $K^-p$  cross section
- at/just-below  $KN$  threshold energy: - Branching ratio  
- kaonic atom(new data by SIDDHARTA)
- below the  $K^{\text{bar}}N$  threshold energy:
- So far, cannot perform  $\pi\Sigma$  elastic scattering experimentally  
→ large ambiguity still remains
  - Few-body reaction ( $K^-d \rightarrow \pi\Sigma n$ )
  - Few-body K-bound system ( $KNN, KNNNN, KNNNNN.....$ )

# J-PARC E31 experiment

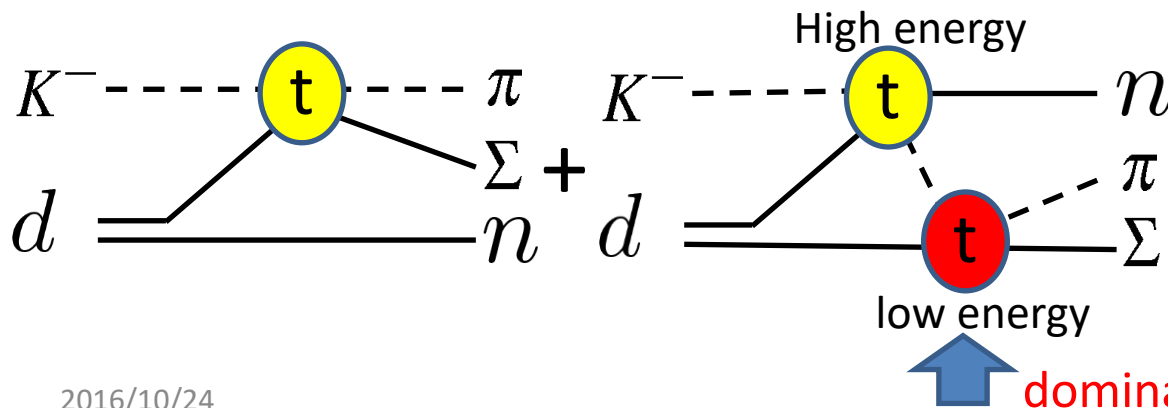
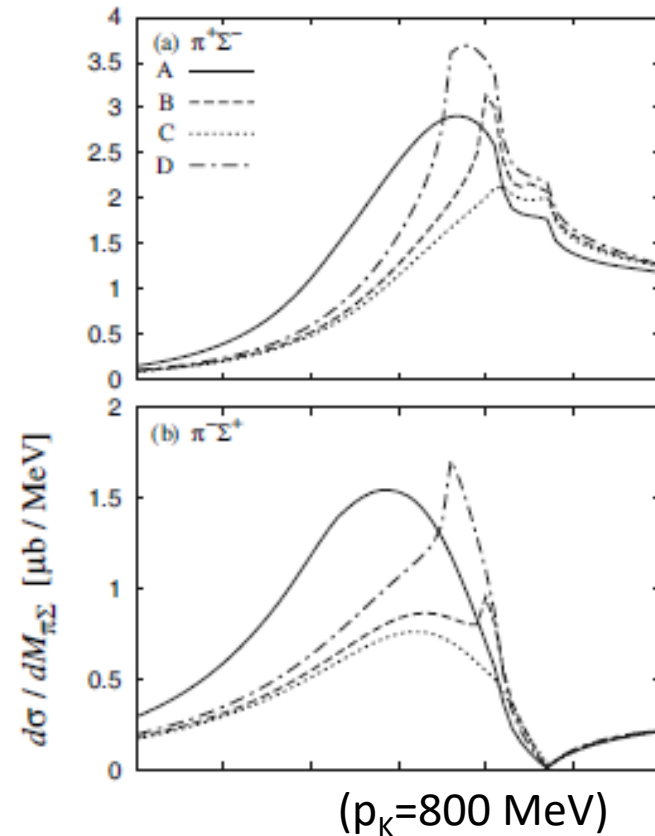
[http://j-parc.jp/researcher/Hadron/en/pac\\_0907/pdf/Noumi.pdf](http://j-parc.jp/researcher/Hadron/en/pac_0907/pdf/Noumi.pdf)

- ✓  $\Lambda(1405)$  production via the  $K^-d \rightarrow \pi\Sigma n$ . ( $p_K=1$  GeV)
- ✓ We can access below the  $K^{bar}N$  threshold.

Kawasaki's slide in MENU2016

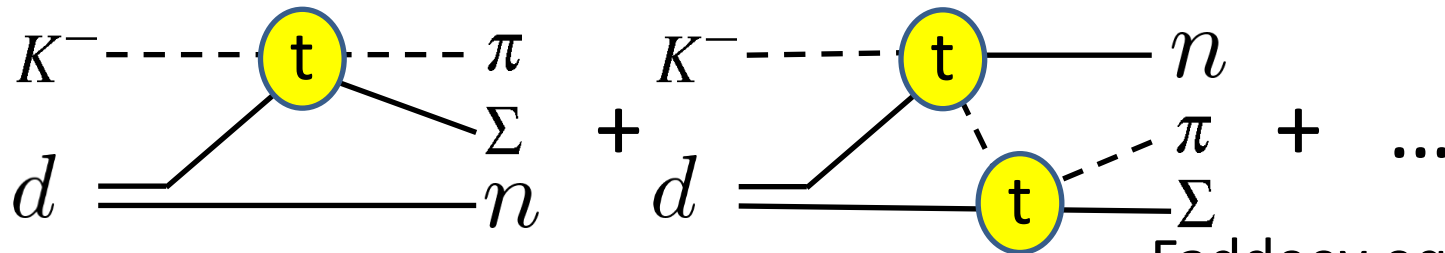


Jido, Oset, Sekihara, EPJA42, 257(2009).

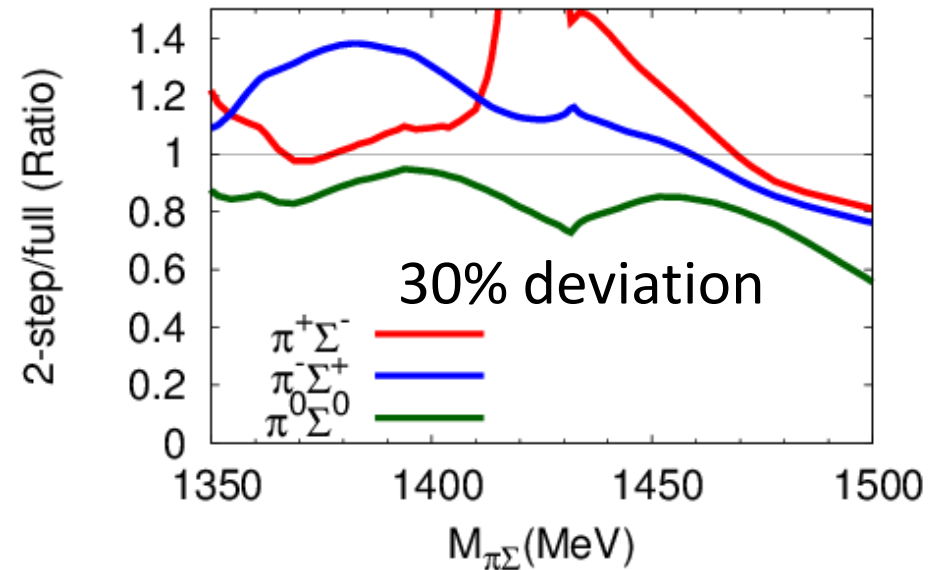
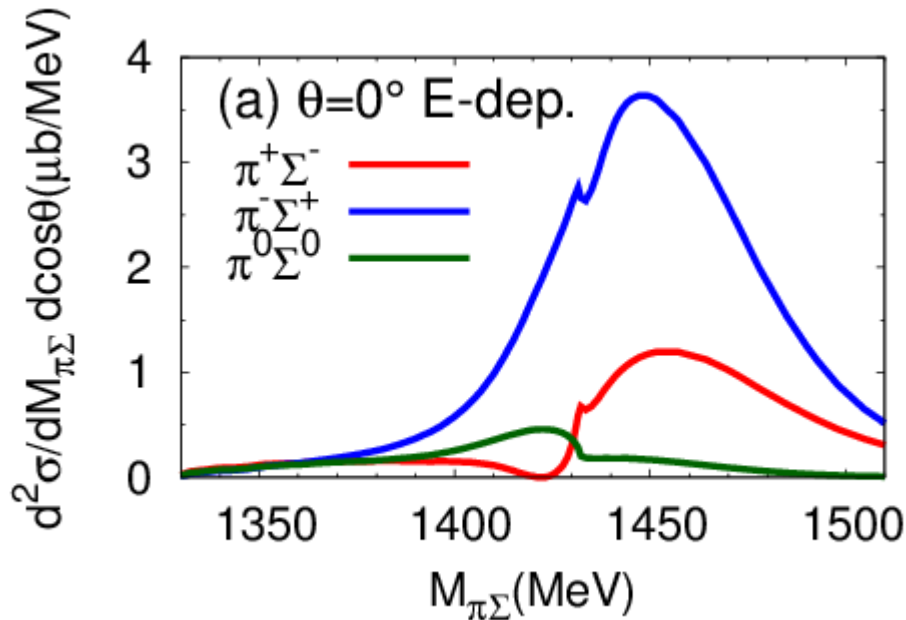


# Full multiple scattering

SO, Y. Ikeda, T. Hyodo, W. Weise, Phys. Rev. C93, 025207 (2016).



Faddeev equations

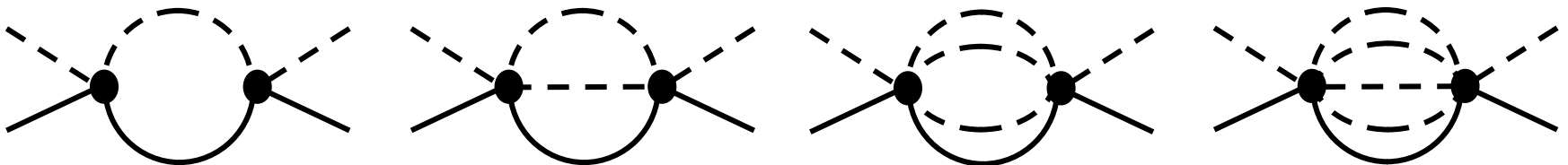
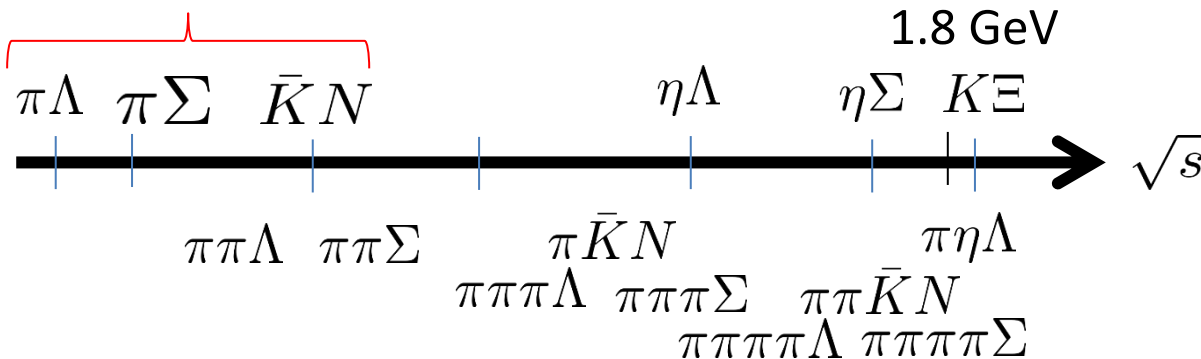
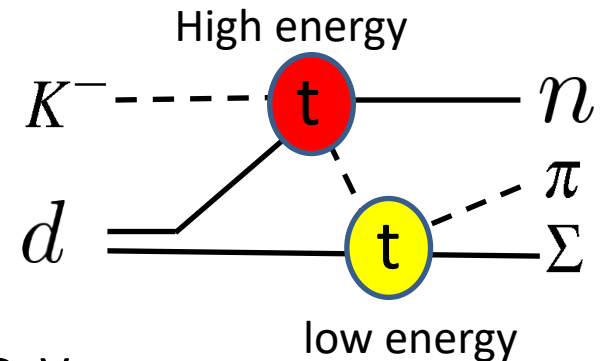


- 2-step calculation qualitatively reproduce the cross section obtained by full calculation
- Quantitative (Resonance parameters)
  - Faddeev calculation is necessary

# Magnitude of cross section

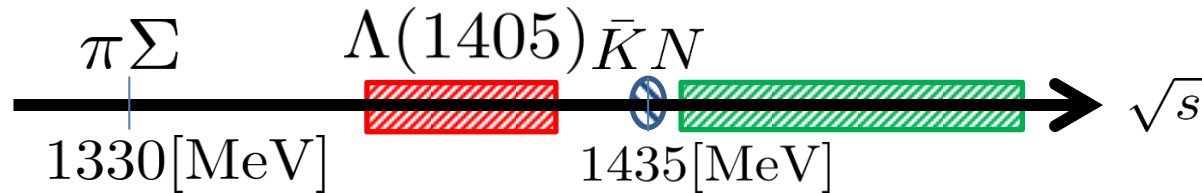
For the magnitude of the cross section,  
high energy amplitude is important

H. Kamano and T.-S. H. Lee, arXiv:1608.03470[nucl-th].

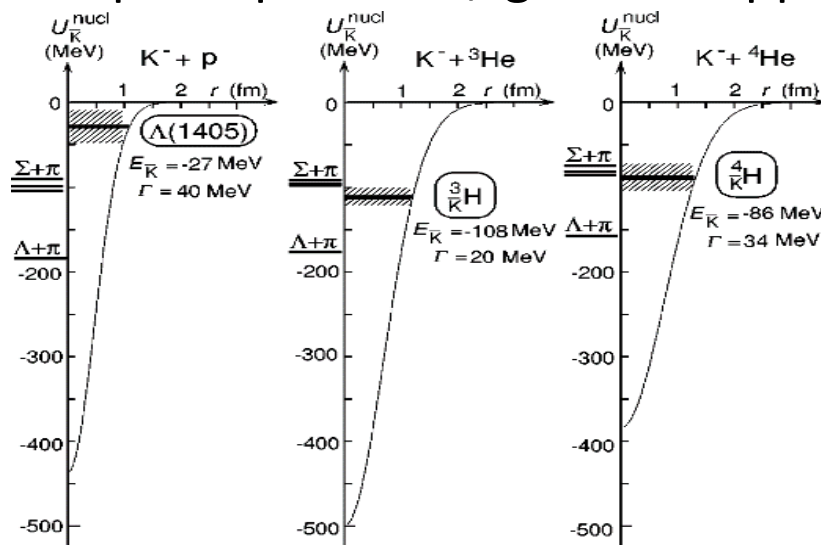


# Kaonic nuclei

- Few-body K-bound system ( $KNN$ ,  $KNNNN$ ,  $KNNNNN$ ....) is useful to study subthreshold  $K^{bar}N$  interaction



- deeply bound and high density systems are proposed
  - ✓ phenomenological  $\bar{K}N$  potential which reproduce the  $\Lambda(1405)$  as  $\bar{K}N$  quasi-bound state (strongly attractive in  $I=0, L=0$ )
  - ✓ optical potential/ g-matrix approach



Y. Akaishi, T. Yamazaki, PRC 65, 044005 (2002).

A. Dote, et. al., PLB590, 51(2004).

# Strategy of this work

Y. Akaishi, T. Yamazaki, *PRC* 65, 044005 (2002).  
Dote, et. al., *PLB* 590, 51(2004).

## AY-potential

- Phenomenological
- Energy independent



## Many-body approximation

- Optical potential
- g-matrix interaction



Deeply binding and compressed systems



This works

## SIDDHARTA pot.

- Chiral SU(3) dynamics
- Energy dependent

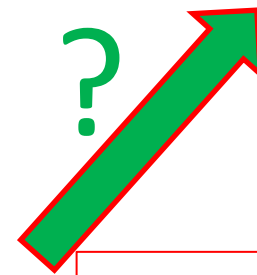
Miyahara, Hyodo,  
*PRC* 93 (2016) 1, 015201.



## Few-body approach

- Correlated Gaussian basis
- Stochastic variational method
- Three- to seven-body calc.

Varga, Suzuki,  
*Phys. Rev C* 52 (1995) 2885.



How structure of light nuclei is changed by injected kaon?



# $K^{\text{bar}}N$ interactions

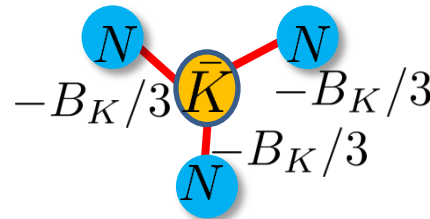
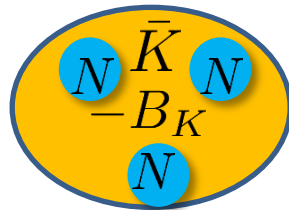
## SIDDHARTA potential

*K.Miyahara, T.Hyodo, PRC 93 (2016) 1, 015201.*

- Energy-dependent  $K^{\text{bar}}N$  single-channel potential
- Chiral SU(3) dynamics using driving interaction at NLO
- Pole energy: 1424 - 26i and 1381 - 81i MeV *Y.Ikeda, T.Hyodo, W.Weise, NPA881 (2012) 98.*
- $K^{\text{bar}}N$  two-body energy in N-body systems are determined as:

$$\sqrt{s} = m_N + m_{\bar{K}} + \delta\sqrt{s} \quad , \quad -B_K \equiv \langle \Psi | H | \Psi \rangle - \langle \Psi | H_N | \Psi \rangle \quad ,$$

$$\text{Type I: } \delta\sqrt{s} = -B_K \quad , \quad \text{Type II: } \delta\sqrt{s} = -B_K / (N - 1) \quad , \quad \text{for } N\text{-body}$$



*A. Dote, T. Hyodo, W. Weise, NPA804, 197 (2008).*

## Akaishi-Yamazaki (AY) potential

*Akaishi, Yamazaki, PRC65, 04400(2002).*

- Energy-independent
- Reproduce  $\Lambda(1405)$  as  $K^{\text{bar}}N$  quasi-bound state

# Correlated Gaussian basis

Varga, Suzuki, Phys. Rev C52 (1995) 2885.

Varga, Suzuki, Comp. Phys. Com. 106 (1997) 157.

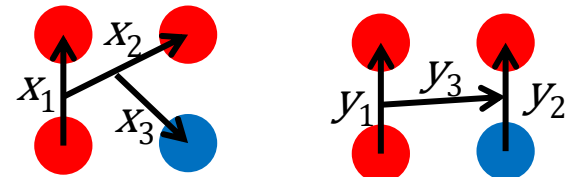
$$\Psi = \sum_{i=1}^K c_i \phi_i, \quad \phi_i = \mathcal{A} \{ e^{-\frac{1}{2} \tilde{\mathbf{x}} A_i \mathbf{x}} \chi_{iJM} \eta_{iTM_t} \}$$

$A_i$ :  $(N-1) \times (N-1)$  matrix (parameters of coordinates) for  $N$ -body

$\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}\}$ ,  $\chi_{iJM}$ : spin function,  $\eta_{iTM_t}$ : isospin function

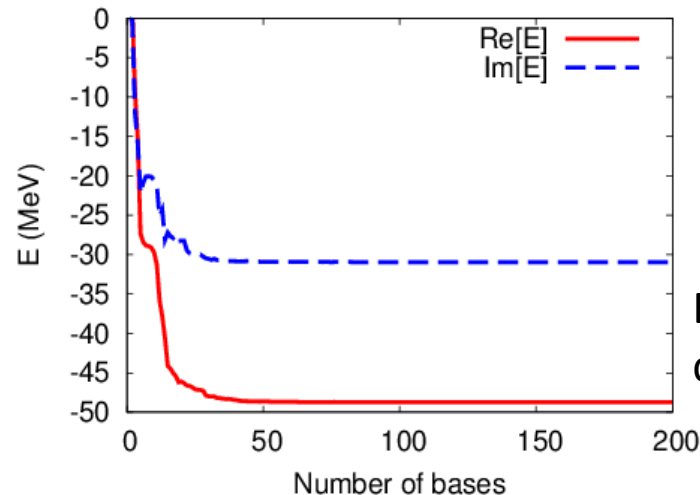
- Higher partial wave for each  $\mathbf{x}_i$  are included by off-diagonal component of  $A_i$ .
- Matrix elements are analytically calculable for  $N$ -body systems
- Functional form of the correlated Gaussian remains unchanged under the coordinate transformation

$$\mathbf{y} = T\mathbf{x} \Rightarrow \tilde{\mathbf{y}} B \mathbf{y} = \tilde{\mathbf{x}} \tilde{T} B T \mathbf{x}$$



## Stochastic variational method

- To obtain the well variational basis, we increase the basis size one-by-one by searching for the best variational parameter  $A_i$  among many random trials



Energy convergence curve for KNN

# Structure of kaonic nuclei ( $N=3-5$ )

Model	SIDDHARTA		AY
	Type I	Type II	
$K^-pp\text{-}\bar{K}^0pn\ (J^\pi = 0^-)$			
$B[\text{MeV}]$	27.9	26.1	48.7
$\Gamma[\text{MeV}]$	30.9	59.3	61.9
$^3\text{He}K^--^3\text{H}\bar{K}^0\ (J^\pi = 1/2^-)$			
$B[\text{MeV}]$	45.3	49.7	72.6
$\Gamma[\text{MeV}]$	25.5	69.4	78.6
$^4\text{He}K^--^4\text{H}\bar{K}^0\ (J^\pi = 0^-)$			
$B[\text{MeV}]$	69.6	75.5	87.4
$\Gamma[\text{MeV}]$	28.0	74.5	87.2

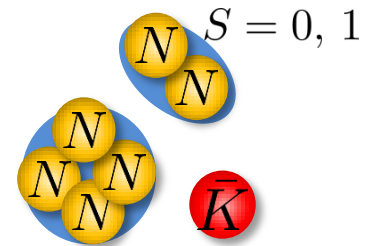
- Binding energies are similar values for Type I, II
- Binding energies for SIDDHARTA pot. are 20 MeV smaller than AY pot.
- Width of Type II is 2-3 times larger than Type I
- Binding energy for AY-potential is less than 100 MeV

# Structure of $K^{bar}NNNNNN$ with $J^\pi=0^-$ and $1^-$

${}^6\text{Li}K^- - {}^6\text{He}\bar{K}^0 (J^\pi = 0^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	68.7	77.0	102
$\Gamma[\text{MeV}]$	24.0	73.2	86.4

${}^6\text{Li}K^- - {}^6\text{He}\bar{K}^0 (J^\pi = 1^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	71.5	78.8	93.7
$\Gamma[\text{MeV}]$	26.3	74.0	86.7



	$B[\text{MeV}]$
${}^6\text{He} (0)$	32.22
${}^6\text{Li} (1)$	35.81

- $1^-$  state are ground state for SIDDHARTA potential, but the  $0^-$  state is ground state for AY potential
- From the energy spectra of seven-body system, we may extract the information of KN interaction

# Structure of $K^{bar}NNNNNN$ with $J^\pi=0^-$ and $1^-$

A=2

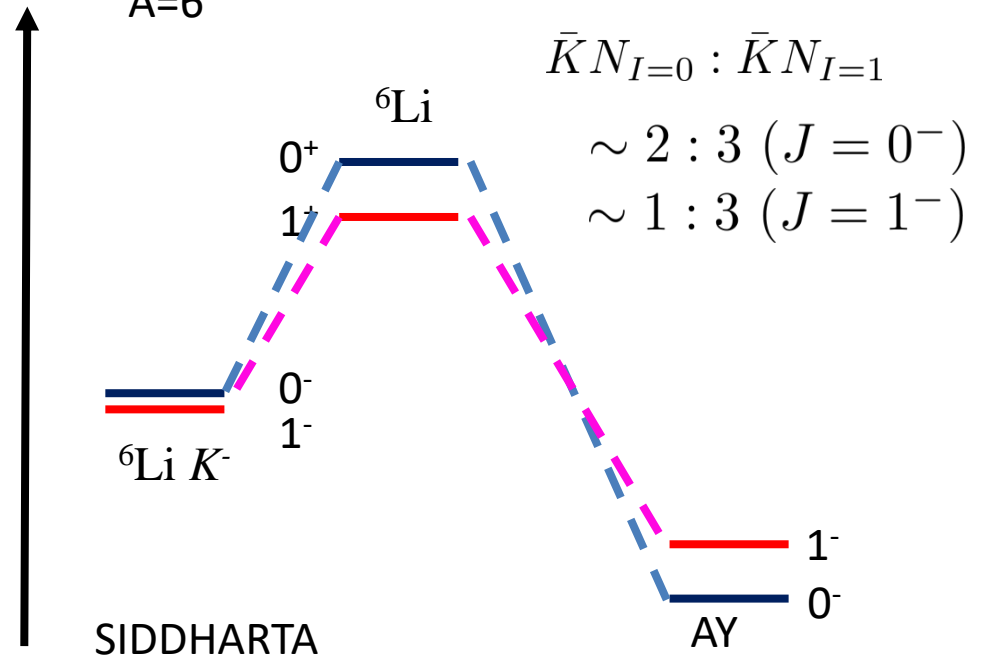
	NN	KNN
J=0	unbound	bound
J=1	bound (d)	unbound

$\bar{K}N_{I=0} : \bar{K}N_{I=1}$

$\sim 3 : 1 (J = 0^-)$

$\sim 1 : 3 (J = 1^-)$

A=6



- $1^-$  state are ground state for SIDDHARTA potential, but the  $0^-$  state is ground state for AY potential
- From the energy spectra of seven-body system, we may extract the information of KN interaction

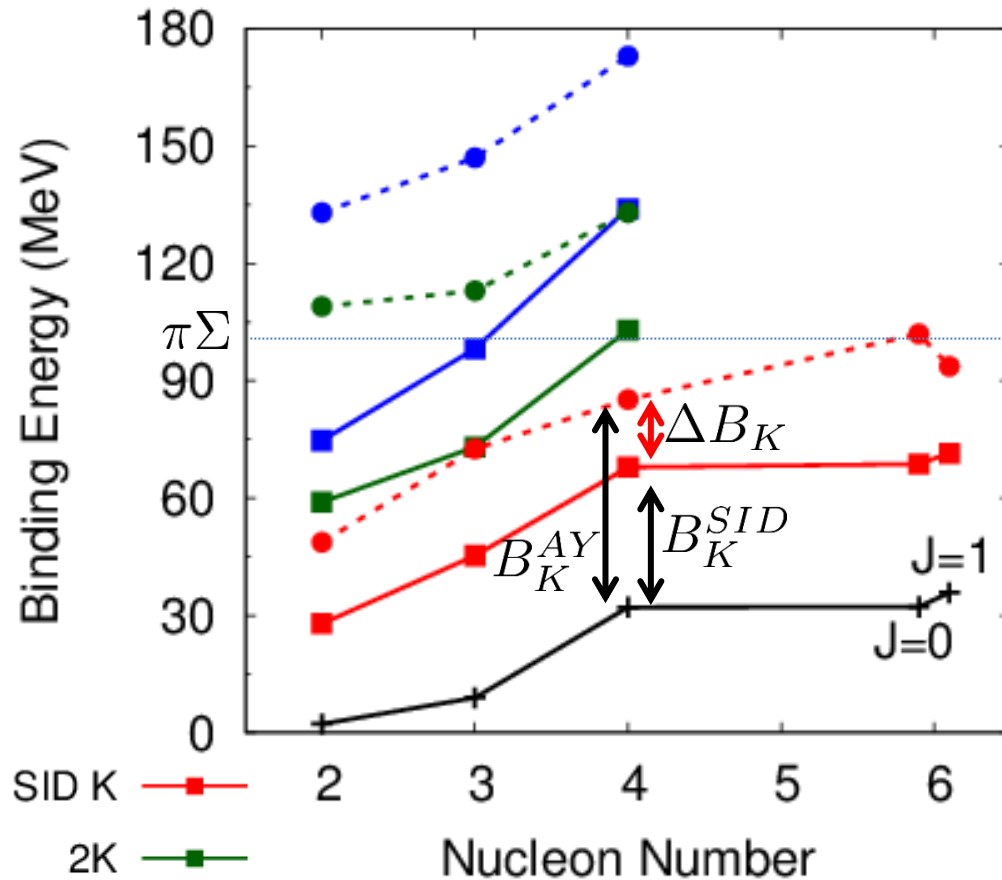
## Summary

- We have investigated the structure of light kaonic nuclei,  $K^{\text{bar}}NN$ ,  $K^{\text{bar}}NNN$ ,  $K^{\text{bar}}NNNN$  and  $K^{\text{bar}}NNNNNN$
- Width largely depends how to deal with two-body energy in N-body systems, and it is around 25-30 MeV for Type I and 60-75 MeV for Type II
- B.E is not sensitive how to deal with two-body energy in N-body systems
- Difference between B.E. for SIDDHARTA and AY is  $\Delta B_K \sim 20$  MeV
- In the seven-body systems,  $J^\pi=1^-$  and  $0^-$  states are degenerate for SIDDHARTA potential, but  $0^-$  state is ground state for AY potential

## Future plan

- Channel-coupling between  $K^{\text{bar}}N$ -  $\pi\Sigma$
- Kaonic atom (T. Hoshino, S.O, W. Horiuchi)

# *N and K number dependence of B.E.*



$$B_K^{SID} \equiv B^{SID} - B_N \sim 34 \text{ MeV}$$

$$B_K^{AY} \equiv B^{AY} - B_N \sim 58 \text{ MeV}$$

$$\Delta B_K \equiv B^{AY} - B^{SID} \sim 24 \text{ MeV}$$

(averaged value)

$$B_{2K}^{SID} \sim 64 \text{ MeV}$$

$$B_{2K}^{AY} \sim 104 \text{ MeV}$$

$$\Delta B_{2K} \sim 40 \text{ MeV}$$

$$B_{3K}^{SID} \sim 88 \text{ MeV}$$

$$B_{3K}^{AY} \sim 137 \text{ MeV}$$

$$\Delta B_{3K} \sim 49 \text{ MeV}$$

Difference of KN interaction  
is enhanced in multi kaonic nuclei

# Structure of double kaonic nuclei

$K^-K^-pp-K^-\bar{K}^0pn-\bar{K}^0\bar{K}^0nn (J^\pi = 0^+)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	59.0	54.8	109
$\Gamma[\text{MeV}]$	64.0	124	143
${}^3\text{Li}K^-K^- - {}^3\text{He}K^-\bar{K}^0 - {}^3\text{H}\bar{K}^0\bar{K}^0 (J^\pi = 1/2^+)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	73.1	72.8	113
$\Gamma[\text{MeV}]$	58.6	135	160
${}^4\text{Li}K^-K^- - {}^4\text{He}K^-\bar{K}^0 - {}^4\text{H}\bar{K}^0\bar{K}^0 (J^\pi = 0^+)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	103	111	133
$\Gamma[\text{MeV}]$	60.1	149	187

- Binding energies are similar values for Type I and II
- Width of Type II is 2 times larger than Type I
- Large decay width



# Structure of triple kaonic nuclei

$K^- K^- K^- pp\text{-}K^- K^- \bar{K}^0 pn\text{-}K^- \bar{K}^0 \bar{K}^0 nn \ (J^\pi = 0^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	74.7	58.1	133
$\Gamma[\text{MeV}]$	112	163	205
${}^3\text{Li}K^- K^- K^- {}^3\text{He}K^- K^- \bar{K}^0 {}^3\text{H}K^- \bar{K}^0 \bar{K}^0 {}^3n\bar{K}^0 \bar{K}^0 \bar{K}^0 \ (J^\pi = 1/2^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	98.2	91.4	147
$\Gamma[\text{MeV}]$	97.9	190	245
${}^4\text{Be}K^- K^- K^- {}^4\text{Li}K^- K^- \bar{K}^0 {}^4\text{He}K^- \bar{K}^0 \bar{K}^0 {}^4\text{H}\bar{K}^0 \bar{K}^0 \bar{K}^0 \ (J^\pi = 0^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	134	140	173
$\Gamma[\text{MeV}]$	96.4	219	294

# $\Lambda(1405)$ in multi-kaonic nuclei

$$|[\bar{K}N]_{I=0}\rangle = \frac{1}{\sqrt{2}}[|K^-p\rangle - |\bar{K}^0n\rangle]$$

$K^-K^-pp\text{-}K^-\bar{K}^0pn\text{-}\bar{K}^0\bar{K}^0nn\ (J^\pi = 0^+)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$P_{K^-K^-}$	0.35	0.35	0.34
$P_{K^-\bar{K}^0}$	0.37	0.36	0.36
$P_{\bar{K}^0\bar{K}^0}$	0.29	0.29	0.30

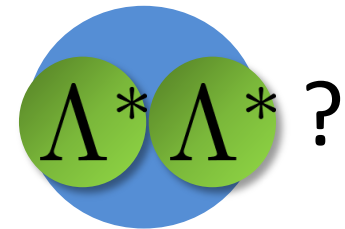
${}^3\text{Li}K^-K^-K^- \text{-} {}^3\text{He}K^-K^-\bar{K}^0 \text{-} {}^3\text{H}K^-\bar{K}^0\bar{K}^0 \text{-} {}^3n\bar{K}^0\bar{K}^0\bar{K}^0\ (J^\pi = 1/2^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$P_{K^-K^-K^-}$	0.02	0.01	0.05
$P_{K^-K^-\bar{K}^0}$	0.50	0.51	0.46
$P_{K^-\bar{K}^0\bar{K}^0}$	0.47	0.47	0.44
$P_{\bar{K}^0\bar{K}^0\bar{K}^0}$	0.01	0.01	0.05

$2\Lambda^*$

0.25

0.50

0.25



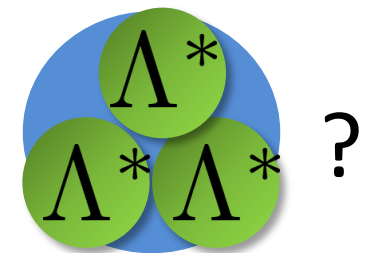
$3\Lambda^*$

0.125

0.375

0.375

0.125



Multi- $\Lambda(1405)$  is not clustered in ground state of multi-KN systems

$K^- K^- pp\text{-}K^- \bar{K}^0 pn\text{-}\bar{K}^0 \bar{K}^0 nn$ ( $J^\pi = 0^+$ )			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	59.0	54.8	109
$\Gamma[\text{MeV}]$	64.0	124	143
$B_{\bar{K}}[\text{MeV}]$	$122 + i51.4$	$119 + i98.7$	$166 + i94.5$
$\delta\sqrt{s}[\text{MeV}]$	$60.8 + i25.7$	$29.6 + i24.7$	
$P_{K^- K^-}$	0.35	0.35	0.34
$P_{K^- \bar{K}^0}$	0.37	0.36	0.36
$P_{\bar{K}^0 \bar{K}^0}$	0.29	0.29	0.30

${}^3\text{Li}K^- K^- {}^3\text{He}K^- \bar{K}^0 {}^3\text{H}\bar{K}^0 \bar{K}^0$ ( $J^\pi = 1/2^+$ )			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	73.1	72.8	113
$\Gamma[\text{MeV}]$	58.6	135	160
$B_{\bar{K}}[\text{MeV}]$	$127 + i49.3$	$128 + i112$	$164 + i104$
$\delta\sqrt{s}[\text{MeV}]$	$63.4 + i24.7$	$21.4 + i18.6$	
$P_{K^- K^-}$	0.03	0.02	0.09
$P_{K^- \bar{K}^0}$	0.38	0.37	0.37
$P_{\bar{K}^0 \bar{K}^0}$	0.60	0.61	0.55

${}^4\text{Li}K^- K^- {}^4\text{He}K^- \bar{K}^0 {}^4\text{H}\bar{K}^0 \bar{K}^0$ ( $J^\pi = 1/2^+$ )			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	103	111	133
$\Gamma[\text{MeV}]$	60.1	149	187
$B_{\bar{K}}[\text{MeV}]$	$130 + i48.0$	$141 + i118$	$159 + i119$
$\delta\sqrt{s}[\text{MeV}]$	$65.1 + i24.0$	$17.6 + i14.7$	
$P_{K^- K^-}$	0.05	0.04	0.11
$P_{K^- \bar{K}^0}$	0.90	0.92	0.78
$P_{\bar{K}^0 \bar{K}^0}$	0.05	0.04	0.11

$K^-K^-K^-pp\text{-}K^-K^-\bar{K}^0pn\text{-}K^-\bar{K}^0\bar{K}^0nn\ (J^\pi = 0^+)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	74.7	58.1	133
$\Gamma[\text{MeV}]$	112	163	205
$B_{\bar{K}}[\text{MeV}]$	$152 + i90.2$	$129 + i132$	$202 + i135$
$\delta\sqrt{s}[\text{MeV}]$	$50.7 + i30.1$	$21.5 + i22.0$	
$P_{K^-K^-}$	0.47	0.48	0.48
$P_{K^-\bar{K}^0}$	0.36	0.35	0.35
$P_{\bar{K}^0\bar{K}^0}$	0.17	0.17	0.17

${}^3\text{Li}K^-K^-K^-{}^3\text{He}K^-K^-\bar{K}^0{}^3\text{H}K^-\bar{K}^0\bar{K}^0{}^3n\bar{K}^0\bar{K}^0\bar{K}^0\ (J^\pi = 1/2^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	98.2	91.4	147
$\Gamma[\text{MeV}]$	97.9	190	245
$B_{\bar{K}}[\text{MeV}]$	$173 + i79.6$	$166 + i155$	$217 + i158$
$\delta\sqrt{s}[\text{MeV}]$	$57.7 + i26.5$	$18.5 + i17.2$	
$P_{K^-K^-K^-}$	0.02	0.01	0.05
$P_{K^-K^-\bar{K}^0}$	0.50	0.51	0.46
$P_{K^-\bar{K}^0\bar{K}^0}$	0.47	0.47	0.44
$P_{\bar{K}^0\bar{K}^0\bar{K}^0}$	0.01	0.01	0.05

${}^4\text{Be}K^-K^-K^-{}^4\text{Li}K^-K^-\bar{K}^0{}^4\text{He}K^-\bar{K}^0\bar{K}^0{}^4\text{H}\bar{K}^0\bar{K}^0\bar{K}^0\ (J^\pi = 0^-)$			
Model	SIDDHARTA		AY
	Type I	Type II	
$B[\text{MeV}]$	134	140	173
$\Gamma[\text{MeV}]$	96.4	219	294
$B_{\bar{K}}[\text{MeV}]$	$185 + i75.3$	$195 + i171$	$219 + i189$
$\delta\sqrt{s}[\text{MeV}]$	$61.6 + i25.1$	$16.3 + i14.2$	
$P_{K^-K^-K^-}$	0.0007	0.0005	0.006
$P_{K^-K^-\bar{K}^0}$	0.06	0.05	0.12
$P_{K^-\bar{K}^0\bar{K}^0}$	0.91	0.92	0.79
$P_{\bar{K}^0\bar{K}^0\bar{K}^0}$	0.03	0.03	0.08

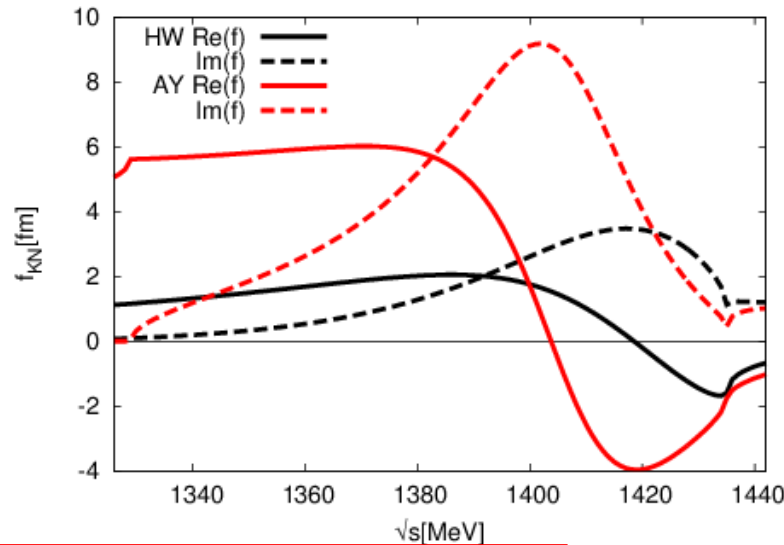
# *NN interaction*

- AV4' potential:  $\{1, \sigma.\sigma, \tau.\tau, \sigma.\sigma\tau.\tau\}$

*R.B.Wiringa, S.C.Pieper, PRL89,182501 (2002).*

	AV4'		Expt.
	$B$ [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]	$B$ [MeV]
$^2\text{H}$	2.24	2.02	2.22
$^3\text{H}$	8.99	1.67	8.48
$^4\text{He}$	32.11	1.39	28.30
$^6\text{He}$	32.22	2.66	29.27
$^6\text{Li}$	35.81	2.43	31.99

# Pole position of $\Lambda(1405)$ and energy dependence of potential



Hyodo, Weise, PRC77, 035204 (2008).

## Phenomenological potential

Akaishi, Yamazaki, PRC65, 04400(2002).  
Shevchenko, PRC85, 034001(2012).

$\Lambda(1405)$ , one pole  
Energy independent

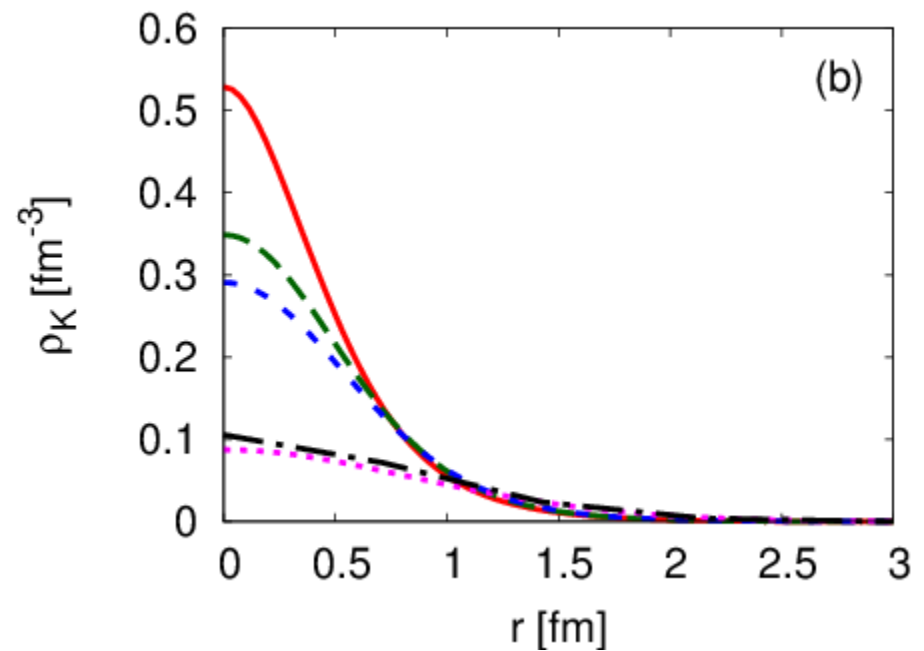
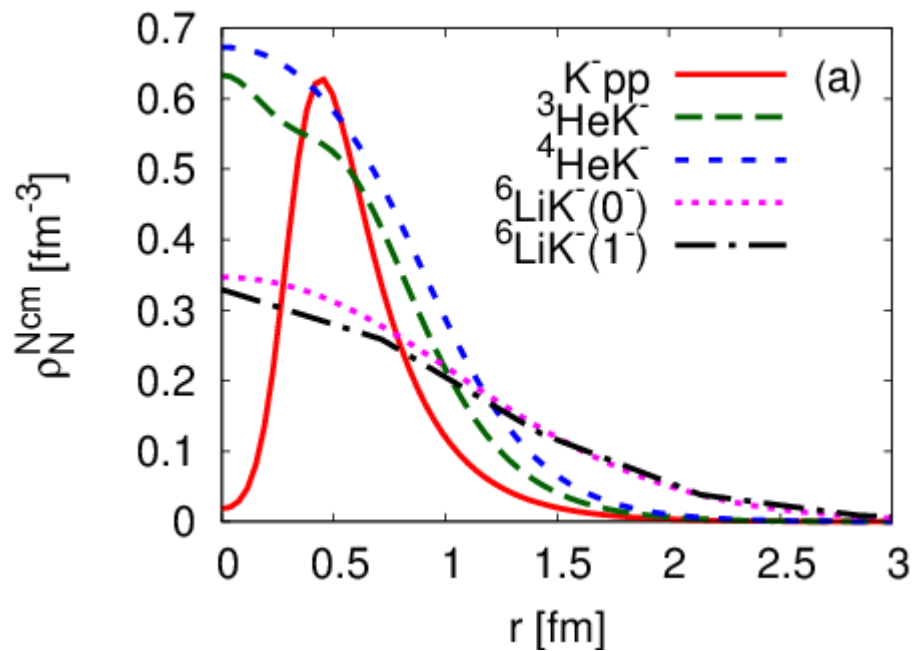
## Chiral SU(3) dynamics

Kaiser, Siegel, Weise, NPA594, 325(1995).  
Oset, Ramos, NPA635, 99(1998).  
Hyodo, Jido, PPNP67, 55(2012).

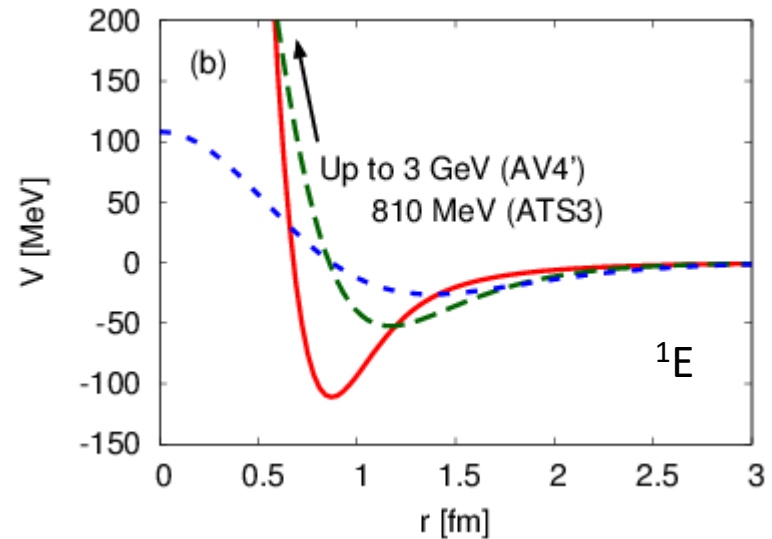
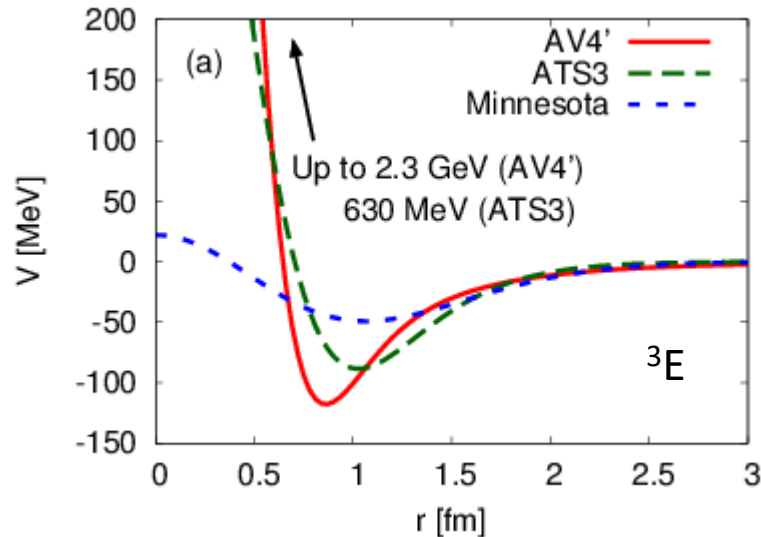
$\Lambda(1420)$ , two pole  
Energy dependent

This difference is enhanced in  
kaon-nucleus quasi-bound states

# *N and K distribution*



# Dependence on $NN$ interaction

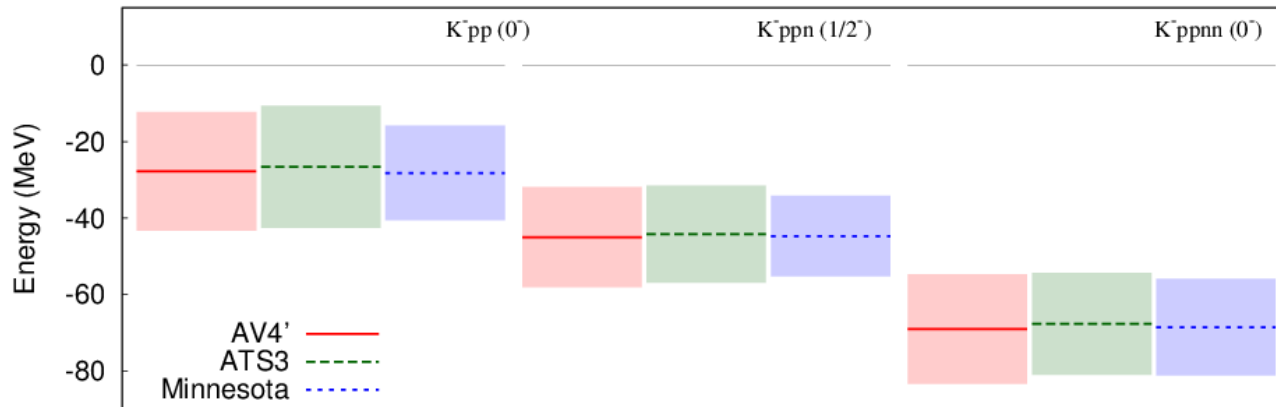


We investigate the  $NN$  interaction dependence by using AV4', ATS3, and Minnesota potential model, which well reproduce the binding energy of s-shell nuclei



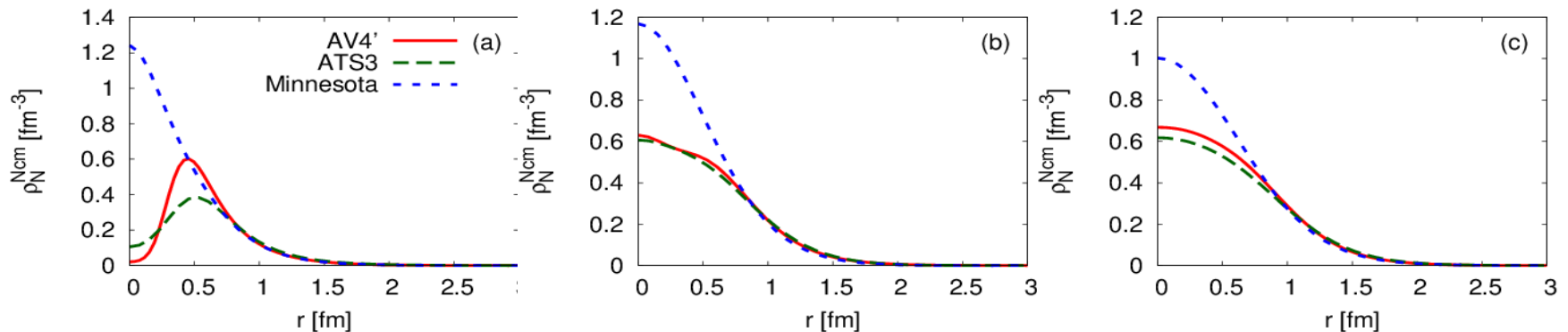
# Dependence on NN interaction

Binding energy and decay width



➤ Binding energy and decay width are not sensitive to NN interaction model

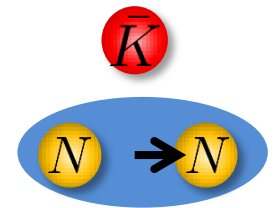
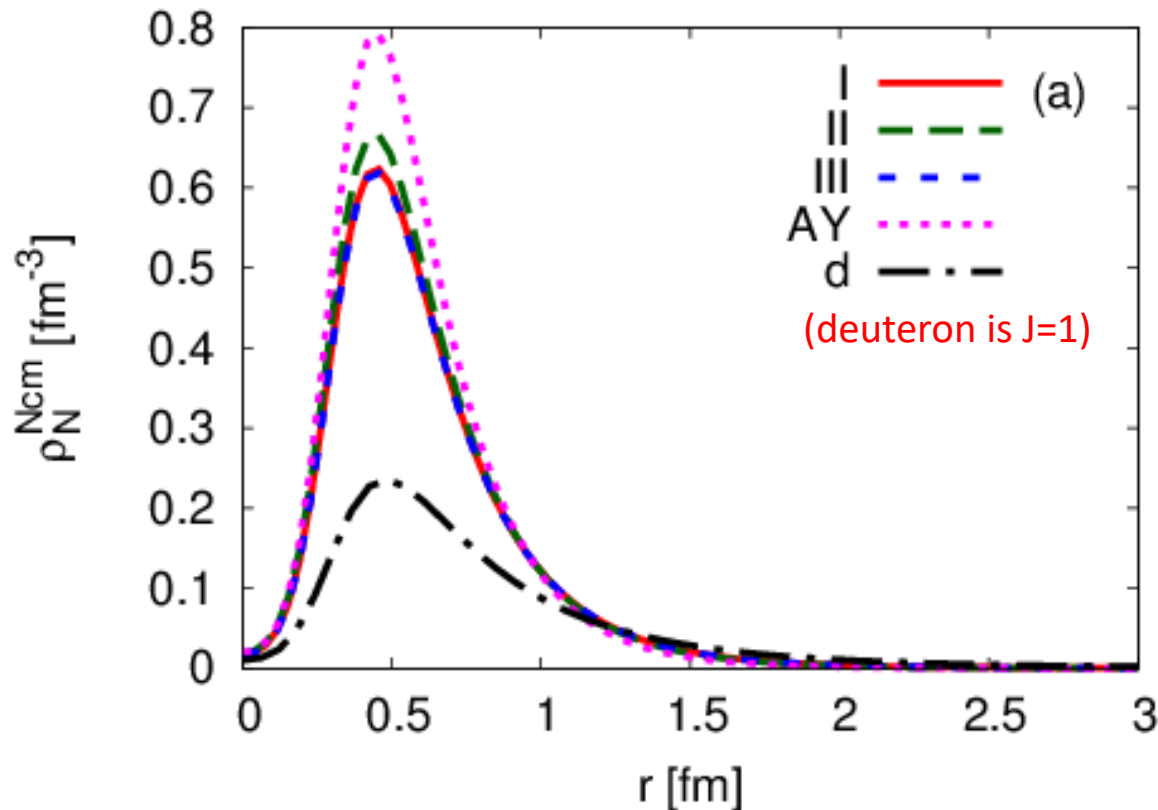
Nucleon distribution



➤ AV4' and ATS3 potential with strong repulsive core produce similar density distribution, but the central density for Minnesota potential with soft core become high.

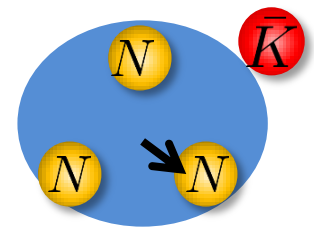
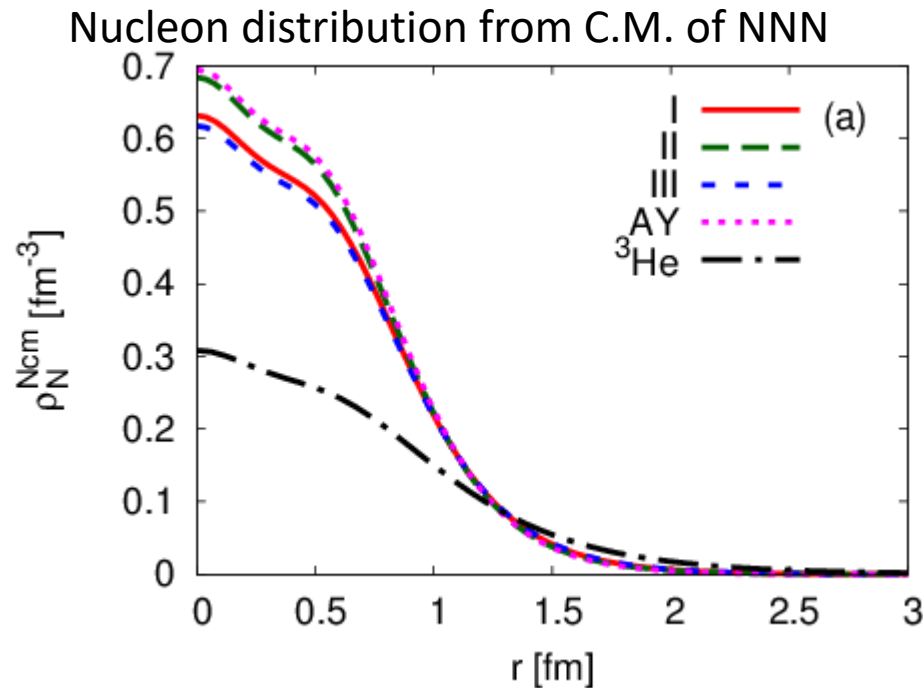
# Density distribution of $K^-pp-K^0\bar{p}n$

Nucleon distribution from C.M. of NN



Central density for SIDDHARTA potential is slightly smaller than density for AY-potential

# Density distribution of $K$ -ppn- $K^{0bar}$ pnn

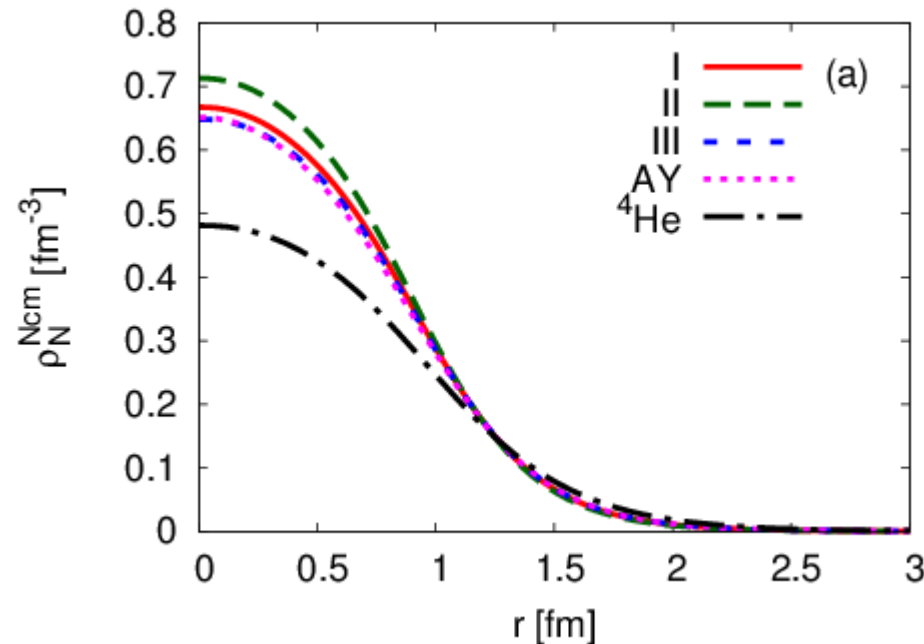


- Central nucleon density  $\rho(0) \sim 0.6 fm^{-3}$  is two times larger than  $^3He$ , but smaller than the density  $\rho(0) = 1.4 fm^{-3}$  predicted by g-matrix effective KN and NN interactions

*Dote, et. al., PLB590, 51(2004).*

# Density distribution of $K^-ppnn-K^{0bar}pnnn$

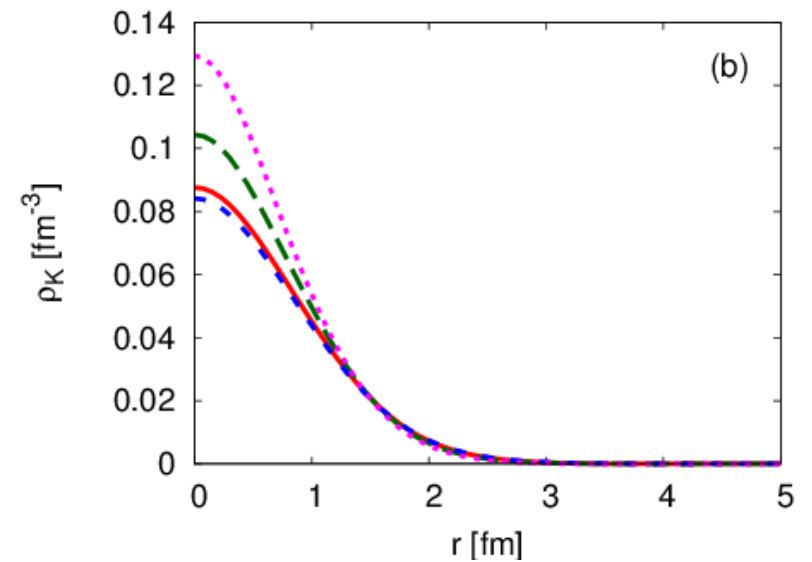
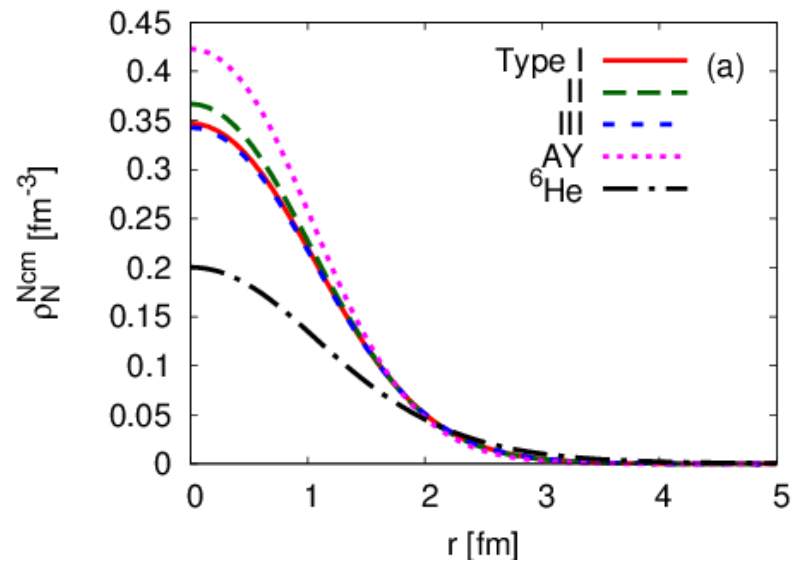
Nucleon distribution from C.M. of NNNN



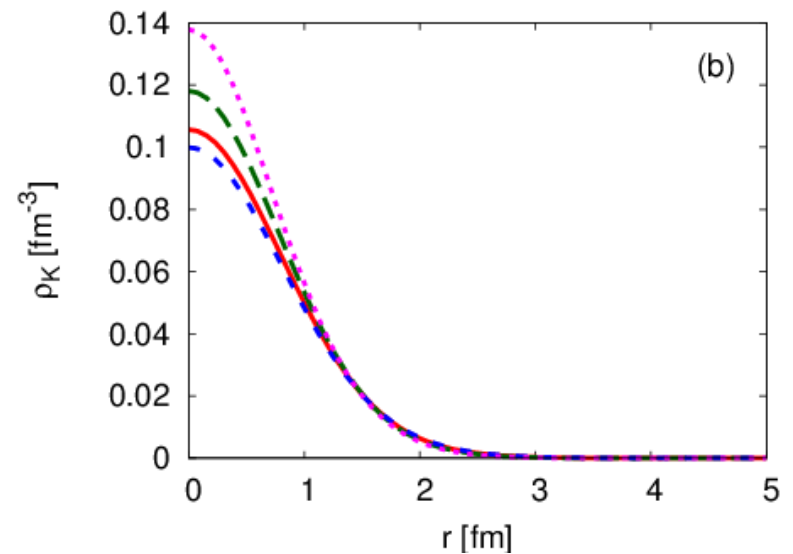
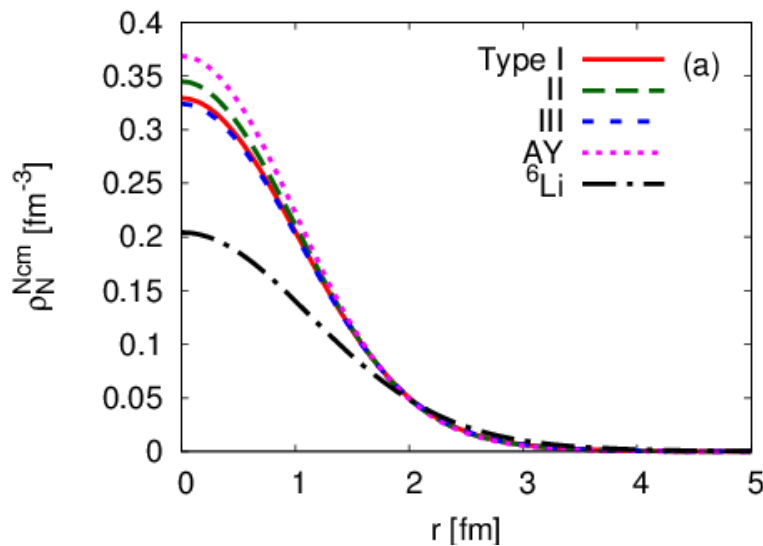
➤ Central nucleon density  $\rho(0) \sim 0.7 \text{ fm}^{-3}$  is 1.5 times larger than  $^4\text{He}$

# Density distribution of $K^-ppppnnnn-K^{0bar}ppppnnnn$

$J^\pi=0^-$

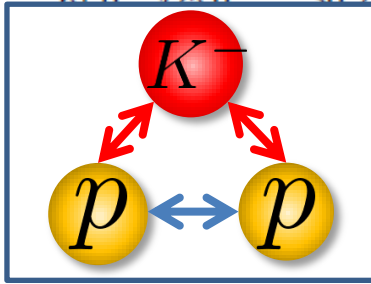


$J^\pi=1^-$



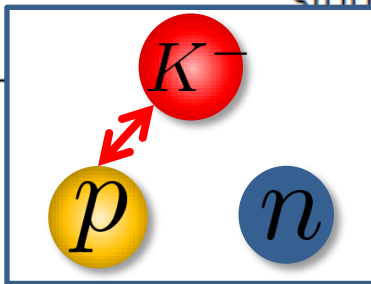
# Structure of $K^{\text{bar}}NN$ with $J^\pi=0^-$

$K^-pp-\bar{K}^0pn (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	27.9	26.1	27.3	48.7
$\Gamma[\text{MeV}]$	30.9	59.3	30.5	61.9
$\delta\sqrt{s}[\text{MeV}]$	61.0 - i25.0	30.2 - i23.7	-61.5 - i24.2	
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$		2.07	2.16	1.84
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$		1.73	1.81	1.55
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$		1.08	1.12	0.958
$\sqrt{\langle r_K^2 \rangle} [\text{fm}]$		1.10	1.15	0.988



- Coulomb splitting is small ( $\sim 0.5\text{MeV}$ )
- Binding energies are almost same between Type I, II, and III, but width of Type II is two times larger than Type I and III

$K^-pn-\bar{K}^0nn (J^\pi = 0^-)$				
Model	SIDDHARTA		AY	
	Type II	Type III		
$B[\text{MeV}]$	5.3	27.0	48.1	
$\Gamma[\text{MeV}]$	9.4	31.0	61.6	
$\delta\sqrt{s}[\text{MeV}]$	-i23.8	-60.8 - i24.7		
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	2.10	2.19	1.85	
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	1.82	1.75	1.56	
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.13	1.09	0.963	
$\sqrt{\langle r_K^2 \rangle} [\text{fm}]$	1.15	1.11	0.993	



- Binding energy for AY-potential is 48 MeV
- The radii for AY-potential become smaller than SIDDHARTA potential

# Structure of $K^{bar}NNNN$ with $J^\pi=0^-$

${}^4\text{Li}K^- - {}^4\text{He}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	67.9	72.7	61.6	85.2
$\Gamma[\text{MeV}]$	28.3	74.1	23.1	86.5
$\delta\sqrt{s}[\text{MeV}]$	$-67.6 - i23.0$	$-18.4 - i15.0$	$-77.0 - i19.2$	
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	1.98	1.91	2.01	2.07
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	1.83	1.72	1.90	1.81
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.22	1.18	1.24	1.27
$\sqrt{\langle r_K^2 \rangle} [\text{fm}]$	1.22	1.12	1.28	1.14
${}^4\text{He}K^- - {}^4\text{H}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	69.6	75.5	63.4	87.4
$\Gamma[\text{MeV}]$	28.0	74.5	23.0	87.2
$\delta\sqrt{s}[\text{MeV}]$	$-68.7 - i22.4$	$-19.1 - i14.9$	$-78.3 - i18.8$	
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	1.96	1.89	1.99	2.04
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	1.82	1.71	1.89	1.79
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.21	1.17	1.23	1.26
$\sqrt{\langle r_K^2 \rangle} [\text{fm}]$	1.21	1.11	1.28	1.13

- Coulomb splitting is large ( $\sim 2$  MeV), since Coulomb effect is repulsive for  ${}^4\text{He}K^0$ , but attractive for  ${}^4\text{He}K^-$
- Binding energy is about 60-75 MeV for SIDDHARTA potential
- width of Type II is three times larger than Type I and III
- Binding energy for AY-potential is about 86 MeV

# Gamow vector

$$\langle \psi | \psi \rangle = \int dr |\psi(r)|^2 = 1,$$

$$_G \langle \psi | \psi \rangle_G = \int dr \psi(r)^2 = 1,$$

$$\langle r^2 \rangle = \int dr r^2 |\psi(r)|^2$$

$$\langle r^2 \rangle_G = \int dr r^2 \psi_G(r)^2$$

$$\langle r^2 \rangle = \langle r^2 \rangle_G = \frac{1}{2\kappa^2} \quad (\text{bound state}).$$

$$\langle r^2 \rangle_G = \frac{1}{2\kappa^2 + 4i\kappa\gamma - 2\gamma^2} \quad (\text{quasibound state}).$$



# Correlated Gaussian basis

Varga, Suzuki, Phys. Rev C52 (1995) 2885.

Varga, Suzuki, Comp. Phys. Com. 106 (1997) 157.

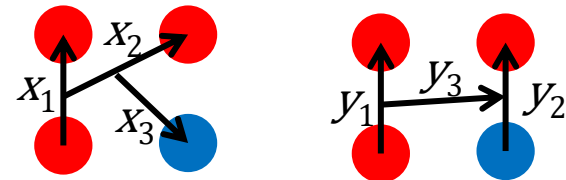
$$\Psi = \sum_{i=1}^K c_i \phi_i, \quad \phi_i = \mathcal{A} \left\{ e^{-\frac{1}{2} \tilde{\mathbf{x}} A_i \mathbf{x}} \chi_{iJM} \eta_{iTM_t} \right\}$$

$A_i$ :  $(N-1) \times (N-1)$  matrix (parameters of coordinates)

$\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}\}$ ,  $\chi_{iJM}$ : spin function,  $\eta_{iTM_t}$ : isospin function

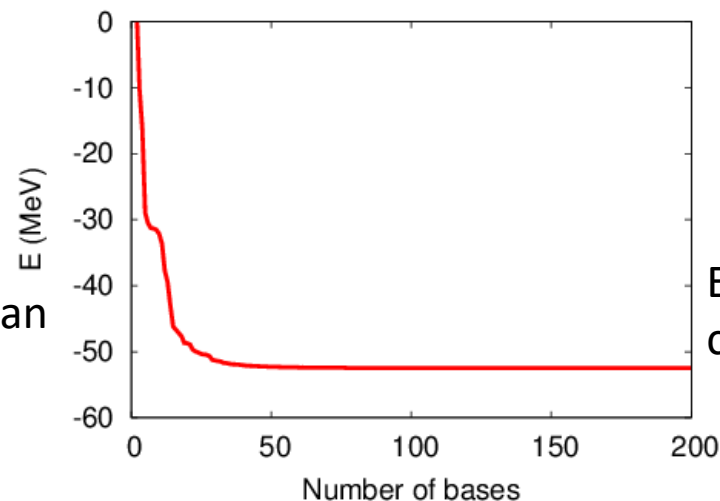
- Correlated Gaussian basis represent the total angular momentum  $L=0$ , but higher partial wave for each  $\mathbf{x}_i$  are included by off-diagonal component of  $A_i$ .
- Matrix elements are analytically calculable for  $N$ -body systems
- Functional form of the correlated Gaussian remains unchanged under the coordinate transformation

$$\mathbf{y} = T\mathbf{x} \Rightarrow \tilde{\mathbf{y}} B \mathbf{y} = \tilde{\mathbf{x}} \tilde{T} B T \mathbf{x}$$



## Stochastic variational method

- To obtain the well variational basis, we increase the basis size one-by-one by searching for the best variational parameter  $A_i$  among many random trials
- Diagonalize full complex Hamiltonian by using basis optimized for the real part of the Hamiltonian



Energy convergence curve for KNN

# Correlated Gaussian basis

Varga, Suzuki, Phys. Rev C52 (1995) 2885.

Varga, Suzuki, Comp. Phys. Com. 106 (1997) 157.

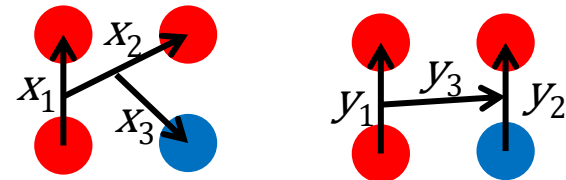
$$\Psi = \sum_{i=1}^K c_i \phi_i, \quad \phi_i = \mathcal{A} \{ e^{-\frac{1}{2} \tilde{\mathbf{x}} A_i \mathbf{x}} \chi_{iJM} \eta_{iTM_t} \}$$

$A_i$ :  $(N-1) \times (N-1)$  matrix (parameters of coordinates)

$\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}\}$ ,  $\chi_{iJM}$ : spin function,  $\eta_{iTM_t}$ : isospin function

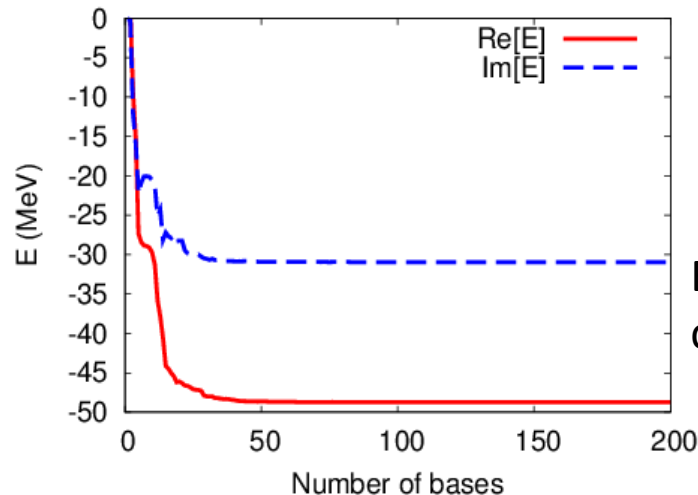
- Correlated Gaussian basis represent the total angular momentum  $L=0$ , but higher partial wave for each  $\mathbf{x}_i$  are included by off-diagonal component of  $A_i$ .
- Matrix elements are analytically calculable for  $N$ -body systems
- Functional form of the correlated Gaussian remains unchanged under the coordinate transformation

$$\mathbf{y} = T\mathbf{x} \Rightarrow \tilde{\mathbf{y}} B \mathbf{y} = \tilde{\mathbf{x}} \tilde{T} B T \mathbf{x}$$



## Stochastic variational method

- To obtain the well variational basis, we increase the basis size one-by-one by searching for the best variational parameter  $A_i$  among many random trials
- Diagonalize full complex Hamiltonian by using basis optimized for the real part of the Hamiltonian



Energy convergence curve for KNN

# $K^{\text{bar}}N$ interactions

## SIDDHARTA potential

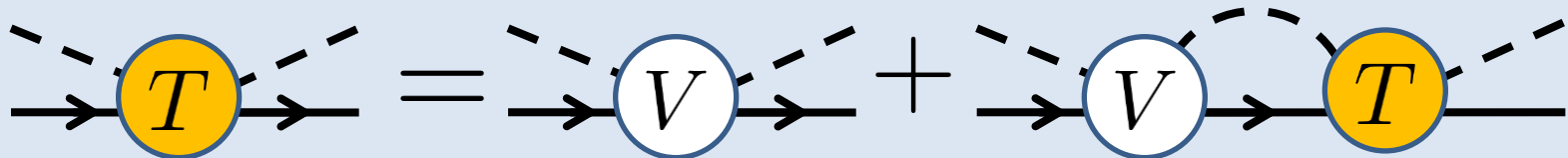
*K.Miyahara, T.Hyodo, PRC 93 (2016) 1, 015201.*

- Reproduce the scattering amplitude by chiral SU(3) dynamics using driving interaction at NLO  
*Y.Ikeda, T.Hyodo, W.Weise, NPA881 (2012) 98.*

### Chiral SU(3) dynamics

#### Description of $S=-1$ , $K^{\text{bar}}N$ $s$ -wave scattering

- ✓ Interaction  $\leftarrow$  chiral symmetry
- ✓ Amplitude  $\leftarrow$  unitarity in coupled channel



Kaiser, Siegel, Weise, NPA594, 325(1995).

Oset, Ramos, NPA635, 99(1998).

Hyodo, Jido, PPNP67, 55(2012).

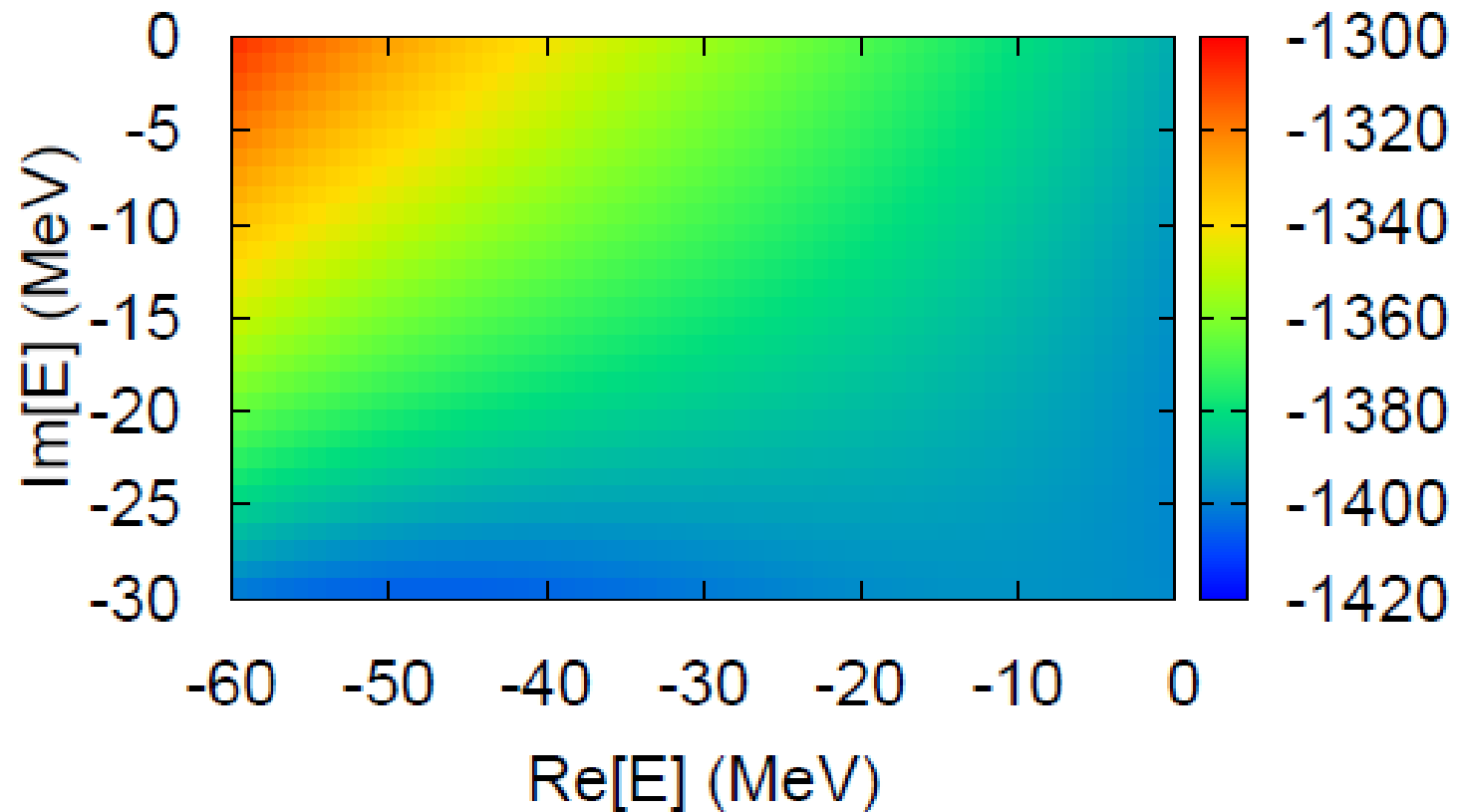


FIG. 1. (Color online) Strength of the  $\bar{K}N$  potential  $V_{\bar{K}N}^{I=0}(r=0, E)$  on the complex energy plane.

$K^-pp\bar{K}^0pn$ ( $J^\pi = 0^-$ )				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	27.9	26.1	27.3	48.7
$\Gamma[\text{MeV}]$	30.9	59.3	30.5	61.9
$B_{\bar{K}}[\text{MeV}]$	$61.0 + i25.0$	$60.4 + i47.4$	$60.1 + i24.7$	$77.7 + i41.8$
$\delta\sqrt{s}[\text{MeV}]$	$-61.0 - i25.0$	$-30.2 - i23.7$	$-61.5 - i24.2$	
$P_{K^-}$	0.65	0.65	0.65	0.64
$P_{\bar{K}^0}$	0.35	0.35	0.35	0.36
$\sqrt{\langle r_{NN}^2 \rangle}$ [fm]	2.16	2.07	2.16	1.84
$\sqrt{\langle r_{KN}^2 \rangle}$ [fm]	1.80	1.73	1.81	1.55
$\sqrt{\langle r_N^2 \rangle}$ [fm]	1.12	1.08	1.12	0.958
$\sqrt{\langle r_{\bar{K}}^2 \rangle}$ [fm]	1.14	1.10	1.15	0.988
$\sqrt{\langle r_N^2 \rangle}_G$ [fm]	$1.10 - i0.119$	$1.02 - i0.182$	$1.10 - i0.121$	$0.918 - i0.153$
$\sqrt{\langle r_{\bar{K}}^2 \rangle}_G$ [fm]	$1.11 - i0.171$	$1.00 - i0.256$	$1.11 - i0.173$	$0.941 - i0.182$
$\langle T \rangle_G^{K^-}$ [MeV]	$117 + i28.8$	$124 + i53.1$	$116 + i28.7$	$102 + i31.4$
$\langle V \rangle_G^{K^-}$ [MeV]	$-113 - i33.7$	$-120 - i63.9$	$-112 - i33.7$	$-102 - i47.0$
$\langle T \rangle_G^{\bar{K}^0}$ [MeV]	$74.3 + i18.4$	$76.3 + i33.1$	$73.7 + i18.2$	$63.1 + i15.5$
$\langle V \rangle_G^{\bar{K}^0}$ [MeV]	$-62.0 - i19.1$	$-64.3 - i35.6$	$-61.3 - i19.0$	$-48.6 - i21.6$
$\langle V \rangle_G^{K^- \bar{K}^0}$ [MeV]	$-44.1 - i9.76$	$-41.9 - i16.4$	$-43.9 - i9.50$	$-64.0 - i9.24$
$\langle V_{\bar{K}N}^{I=0} \rangle$ [MeV]	$-193 - i14.3$	$-201 - i27.2$	$-191 - i14.0$	$-186 - i26.0$
$\langle V_{\bar{K}N}^{I=1} \rangle$ [MeV]	$-8.67 - i1.19$	$-10.8 - i2.43$	$-8.52 - i1.20$	$-8.26 - i4.95$

$K^-pn\text{-}\bar{K}^0nn$ ( $J^\pi = 0^-$ )				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	27.6	25.3	27.0	48.1
$\Gamma[\text{MeV}]$	31.6	59.4	31.0	61.6
$B_{\bar{K}}[\text{MeV}]$	$60.2 + i25.6$	$58.7 + i47.5$	$59.2 + i25.1$	$76.3 + i41.5$
$\delta\sqrt{s}[\text{MeV}]$	$-60.2 - i25.6$	$-29.4 - i23.8$	$-60.8 - i24.7$	
$P_{K^-}$	0.38	0.38	0.38	0.37
$P_{\bar{K}^0}$	0.62	0.62	0.62	0.63
$\sqrt{\langle r_{NN}^2 \rangle}$ [fm]	2.18	2.10	2.19	1.85
$\sqrt{\langle r_{KN}^2 \rangle}$ [fm]	1.82	1.75	1.83	1.56
$\sqrt{\langle r_N^2 \rangle}$ [fm]	1.13	1.09	1.14	0.963
$\sqrt{\langle r_{\bar{K}}^2 \rangle}$ [fm]	1.15	1.11	1.16	0.993
$\sqrt{\langle r_N^2 \rangle}_G$ [fm]	$1.11 - i0.123$	$1.03 - i0.187$	$1.11 - i0.125$	$0.923 - i0.155$
$\sqrt{\langle r_{\bar{K}}^2 \rangle}_G$ [fm]	$1.11 - i0.176$	$1.01 - i0.263$	$1.12 - i0.179$	$0.946 - i0.185$
$\langle T \rangle_G^{K^-}$ [MeV]	$67.6 + i20.6$	$81.4 + i34.9$	$78.5 + i19.4$	$65.3 + i16.1$
$\langle V \rangle_G^{K^-}$ [MeV]	$-44.5 - i10.2$	$-69.9 - i38.0$	$-66.7 - i20.4$	$-51.3 - i22.6$
$\langle T \rangle_G^{\bar{K}^0}$ [MeV]	$112 + i28.7$	$118 + i52.2$	$111 + i28.5$	$99.5 + i30.8$
$\langle V \rangle_G^{\bar{K}^0}$ [MeV]	$-107 - i33.3$	$-112 - i62.2$	$-106 - i33.0$	$-97.3 - i45.8$
$\langle V \rangle_G^{K^- \bar{K}^0}$ [MeV]	$-44.5 - i10.2$	$-42.2 - i16.7$	$-44.2 - i9.93$	$-64.3 - i9.40$
$\langle V_{\bar{K}N}^{I=0} \rangle$ [MeV]	$-194 - i14.7$	$-201 - i27.4$	$-192 - i14.4$	$-186 - i25.9$
$\langle V_{\bar{K}N}^{I=1} \rangle$ [MeV]	$-8.39 - i1.14$	$-10.4 - i2.34$	$-8.22 - i1.15$	$-8.10 - i4.86$

${}^3\text{He}K^- - {}^3\text{H}\bar{K}^0 (J^\pi = 1/2^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	45.3	49.7	42.0	72.6
$\Gamma[\text{MeV}]$	25.5	69.4	21.7	78.6
$B_{\bar{K}}[\text{MeV}]$	$70.4 + i20.7$	$78.4 + i55.9$	$64.8 + i17.5$	$94.3 + i51.9$
$\delta\sqrt{s}[\text{MeV}]$	$-70.4 - i20.7$	$-26.1 - i18.6$	$-75.7 - i18.2$	
$P_{K^-}$	0.53	0.53	0.53	0.51
$P_{\bar{K}^0}$	0.47	0.47	0.47	0.49
$\sqrt{\langle r_{NN}^2 \rangle}$ [fm]	1.99	1.90	2.01	1.87
$\sqrt{\langle r_{KN}^2 \rangle}$ [fm]	1.79	1.68	1.83	1.63
$\sqrt{\langle r_N^2 \rangle}$ [fm]	1.17	1.11	1.18	1.09
$\sqrt{\langle r_{\bar{K}}^2 \rangle}$ [fm]	1.17	1.08	1.21	1.03
$\sqrt{\langle r_N^2 \rangle}_G$ [fm]	$1.16 - i0.0539$	$1.09 - i0.0952$	$1.18 - i0.0575$	$1.07 - i0.124$
$\sqrt{\langle r_{\bar{K}}^2 \rangle}_G$ [fm]	$1.15 - i0.115$	$1.02 - i0.196$	$1.19 - i0.118$	$0.996 - i0.176$
$\langle T \rangle_G^{K^-}$ [MeV]	$114 + i17.4$	$126 + i42.2$	$109 + i15.6$	$107 + i27.6$
$\langle V \rangle_G^{K^-}$ [MeV]	$-118 - i22.4$	$-135 - i54.0$	$-112 - i20.9$	$-114 - i44.6$
$\langle T \rangle_G^{\bar{K}^0}$ [MeV]	$103 + i15.9$	$113 + i39.8$	$98.6 + i13.7$	$101 + i26.1$
$\langle V \rangle_G^{\bar{K}^0}$ [MeV]	$-105 - i20.2$	$-118 - i50.0$	$-99.5 - i18.3$	$-107 - i42.1$
$\langle V \rangle_G^{K^- \bar{K}^0}$ [MeV]	$-39.0 - i3.56$	$-36.0 - i12.7$	$-37.6 - i1.05$	$-59.6 - i6.33$
$\langle V_{\bar{K}N}^{I=0} \rangle$ [MeV]	$-170 - i9.20$	$-188 - i26.6$	$-160 - i7.07$	$-189 - i26.3$
$\langle V_{\bar{K}N}^{I=1} \rangle$ [MeV]	$-22.8 - i3.58$	$-34.0 - i8.07$	$-20.8 - i3.80$	$-21.7 - i13.0$

${}^4\text{Li}K^- - {}^4\text{He}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	67.9	72.7	61.6	85.2
$\Gamma[\text{MeV}]$	28.3	74.1	23.1	86.5
$B_{\bar{K}}[\text{MeV}]$	$67.6 + i23.0$	$73.5 + i59.9$	$57.6 + i17.5$	$85.2 + i55.2$
$\delta\sqrt{s}[\text{MeV}]$	$-67.6 - i23.0$	$-18.4 - i15.0$	$-77.0 - i19.2$	
$P_{K^-}$	0.08	0.06	0.07	0.16
$P_{\bar{K}^0}$	0.92	0.94	0.93	0.84
$\sqrt{\langle r_{NN}^2 \rangle} \text{ [fm]}$	1.98	1.91	2.01	2.07
$\sqrt{\langle r_{KN}^2 \rangle} \text{ [fm]}$	1.83	1.72	1.90	1.81
$\sqrt{\langle r_N^2 \rangle} \text{ [fm]}$	1.22	1.18	1.24	1.27
$\sqrt{\langle r_{\bar{K}}^2 \rangle} \text{ [fm]}$	1.22	1.12	1.28	1.14
$\sqrt{\langle r_N^2 \rangle}_G \text{ [fm]}$	$1.21 - i0.0324$	$1.17 - i0.0627$	$1.24 - i0.0431$	$1.26 - i0.125$
$\sqrt{\langle r_{\bar{K}}^2 \rangle}_G \text{ [fm]}$	$1.19 - i0.123$	$1.05 - i0.201$	$1.26 - i0.139$	$1.09 - i0.210$
$\langle T \rangle_G^{K^-} \text{ [MeV]}$	$32.6 + i6.75$	$26.7 + i16.2$	$30.7 + i1.60$	$50.3 + i7.22$
$\langle V \rangle_G^{K^-} \text{ [MeV]}$	$-25.1 - i6.74$	$-20.0 - i15.9$	$-23.2 - i2.51$	$-42.3 - i12.0$
$\langle T \rangle_G^{\bar{K}^0} \text{ [MeV]}$	$214 + i27.8$	$240 + i66.0$	$200 + i27.6$	$183 + i52.7$
$\langle V \rangle_G^{\bar{K}^0} \text{ [MeV]}$	$-265 - i38.4$	$-300 - i94.3$	$-245 - i38.5$	$-232 - i88.3$
$\langle V \rangle_G^{K^- \bar{K}^0} \text{ [MeV]}$	$-24.8 - i3.66$	$-20.2 - i9.13$	$-23.9 + i0.218$	$-43.9 - i2.82$
$\langle V_{\bar{K}N}^{I=0} \rangle \text{ [MeV]}$	$-132 - i7.68$	$-136 - i20.4$	$-118 - i4.57$	$-158 - i22.1$
$\langle V_{\bar{K}N}^{I=1} \rangle \text{ [MeV]}$	$-46.6 - i6.49$	$-67.2 - i16.7$	$-39.1 - i7.00$	$-35.3 - i21.2$



${}^4\text{He}K^- - {}^4\text{H}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	69.6	75.5	63.4	87.4
$\Gamma[\text{MeV}]$	28.0	74.5	23.0	87.2
$B_{\bar{K}}[\text{MeV}]$	$68.7 + i22.4$	$76.4 + i59.7$	$58.8 + i17.1$	$86.5 + i55.6$
$\delta\sqrt{s}[\text{MeV}]$	$-68.7 - i22.4$	$-19.1 - i14.9$	$-78.3 - i18.8$	
$P_{K^-}$	0.93	0.94	0.93	0.86
$P_{\bar{K}^0}$	0.07	0.06	0.07	0.14
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	1.96	1.89	1.99	2.04
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	1.82	1.71	1.89	1.79
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.21	1.17	1.23	1.26
$\sqrt{\langle r_{\bar{K}}^2 \rangle} [\text{fm}]$	1.21	1.11	1.28	1.13
$\sqrt{\langle r_N^2 \rangle}_G [\text{fm}]$	$1.21 - i0.0338$	$1.16 - i0.0633$	$1.23 - i0.0441$	$1.24 - i0.120$
$\sqrt{\langle r_{\bar{K}}^2 \rangle}_G [\text{fm}]$	$1.19 - i0.120$	$1.04 - i0.196$	$1.26 - i0.136$	$1.13 - i0.208$
$\langle T \rangle_G^{K^-} [\text{MeV}]$	$216 + i28.1$	$244 + i66.6$	$202 + i27.9$	$188 + i53.4$
$\langle V \rangle_G^{K^-} [\text{MeV}]$	$-269 - i39.1$	$-306 - i95.7$	$-250 - i39.3$	$-241 - i90.0$
$\langle T \rangle_G^{\bar{K}^0} [\text{MeV}]$	$30.5 + i5.50$	$25.1 + i14.7$	$28.6 + i0.588$	$47.0 + i6.54$
$\langle V \rangle_G^{\bar{K}^0} [\text{MeV}]$	$-23.0 - i5.54$	$-18.5 - i14.2$	$-21.2 - i1.63$	$-38.7 - i10.9$
$\langle V \rangle_G^{K^- \bar{K}^0} [\text{MeV}]$	$-24.4 - i3.00$	$-19.9 - i8.71$	$-23.2 + i0.893$	$-42.7 + i2.64$
$\langle V_{\bar{K}N}^{I=0} \rangle [\text{MeV}]$	$-130 - i7.27$	$-136 - i20.3$	$-117 - i4.18$	$-157 - i21.9$
$\langle V_{\bar{K}N}^{I=1} \rangle [\text{MeV}]$	$-46.7 - i6.72$	$-68.5 - i17.0$	$-39.5 - i7.35$	$-36.1 - i21.7$

${}^6\text{Be}K^- - {}^6\text{Li}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	68.0	76.9	62.7	101
$\Gamma[\text{MeV}]$	23.9	73.4	19.5	86.4
$B_{\bar{K}}[\text{MeV}]$	$75.0 + i18.3$	$87.6 + i59.5$	$65.8 + i13.4$	$113 + i56.5$
$\delta\sqrt{s}[\text{MeV}]$	$-75.0 - i18.3$	$-14.6 - i9.92$	$-85.0 - i15.0$	
$P_{K^-}$	0.73	0.75	0.73	0.65
$P_{\bar{K}^0}$	0.27	0.25	0.27	0.35
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	2.85	2.83	2.87	2.57
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	2.56	2.47	2.59	2.29
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.85	1.83	1.86	1.67
$\sqrt{\langle r_{\bar{K}}^2 \rangle} [\text{fm}]$	1.54	1.48	1.66	1.44
$\sqrt{\langle r_N^2 \rangle_G} [\text{fm}]$	$1.85 - i0.0179$	$1.82 - i0.0329$	$1.86 - i0.0148$	$1.66 - i0.0728$
$\sqrt{\langle r_{\bar{K}}^2 \rangle_G} [\text{fm}]$	$1.62 - i0.0764$	$1.50 - i0.117$	$1.65 - i0.0886$	$1.42 - i0.167$
$\langle T \rangle_G^{K^-} [\text{MeV}]$	$185 + i18.9$	$210 + i46.1$	$177 + i19.7$	$181 + i30.6$
$\langle V \rangle_G^{K^-} [\text{MeV}]$	$-220 - i28.1$	$-257 - i67.6$	$-210 - i30.6$	$-221 - i57.4$
$\langle T \rangle_G^{\bar{K}^0} [\text{MeV}]$	$80.4 + i4.38$	$76.8 + i24.1$	$75.8 - i2.32$	$107 + i15.3$
$\langle V \rangle_G^{\bar{K}^0} [\text{MeV}]$	$-84.4 - i7.14$	$-82.8 - i29.5$	$-79.6 + i0.994$	$-117 - i29.1$
$\langle V \rangle_G^{K^- \bar{K}^0} [\text{MeV}]$	$-29.3 + i0.0277$	$-24.7 - i9.71$	$-26.5 + i4.52$	$-51.9 - i2.69$
$\langle V_{\bar{K}N}^{I=0} \rangle [\text{MeV}]$	$-137 - i6.30$	$-152 - i23.4$	$-123 - i3.17$	$-178 - i24.8$
$\langle V_{\bar{K}N}^{I=1} \rangle [\text{MeV}]$	$-31.3 - i5.64$	$-51.3 - i13.3$	$-28.5 - i6.59$	$-30.7 - i18.4$

${}^6\text{Li}K^- - {}^6\text{He}\bar{K}^0 \ (J^\pi = 0^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	68.7	77.0	63.2	102
$\Gamma[\text{MeV}]$	24.0	73.2	19.4	86.4
$B_{\bar{K}}[\text{MeV}]$	$74.4 + i18.6$	$86.2 + i59.6$	$65.0 + i13.5$	$111 + i56.5$
$\delta\sqrt{s}[\text{MeV}]$	$-74.4 - i18.6$	$-14.4 - i9.93$	$-84.5 - i15.1$	
$P_{K^-}$	0.36	0.35	0.36	0.39
$P_{\bar{K}^0}$	0.64	0.65	0.64	0.61
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	2.83	2.80	2.85	2.57
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	2.54	2.46	2.58	2.28
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.83	1.81	1.84	1.66
$\sqrt{\langle r_{\bar{K}}^2 \rangle} [\text{fm}]$	1.63	1.53	1.66	1.44
$\sqrt{\langle r_N^2 \rangle_G} [\text{fm}]$	$1.83 - i0.0200$	$1.80 - i0.0346$	$1.84 - i0.0195$	$1.66 - i0.0722$
$\sqrt{\langle r_{\bar{K}}^2 \rangle_G} [\text{fm}]$	$1.62 - i0.0756$	$1.49 - i0.119$	$1.65 - i0.0880$	$1.42 - i0.167$
$\langle T \rangle_G^{K^-} [\text{MeV}]$	$101 + i7.71$	$104 + i26.5$	$96.4 + i3.10$	$117 + i18.1$
$\langle V \rangle_G^{K^-} [\text{MeV}]$	$-111 - i11.7$	$-117 - i34.4$	$-105 - i8.02$	$-131 - i33.8$
$\langle T \rangle_G^{\bar{K}^0} [\text{MeV}]$	$165 + i15.9$	$184 + i43.8$	$158 + i14.5$	$171 + i27.7$
$\langle V \rangle_G^{\bar{K}^0} [\text{MeV}]$	$-194 - i23.6$	$-221 - i62.5$	$-184 - i23.4$	$-206 - i52.4$
$\langle V \rangle_G^{K^- \bar{K}^0} [\text{MeV}]$	$-30.1 - i0.369$	$-25.6 - i9.99$	$-27.4 + i4.14$	$-52.3 - i2.82$
$\langle V_{\bar{K}N}^{I=0} \rangle [\text{MeV}]$	$-139 - i6.56$	$-154 - i23.8$	$-125 - i3.36$	$-178 - i24.9$
$\langle V_{\bar{K}N}^{I=1} \rangle [\text{MeV}]$	$-30.8 - i5.46$	$-49.7 - i12.9$	$-27.8 - i6.37$	$-30.5 - i18.3$

${}^6\text{Be}K^- - {}^6\text{Li}\bar{K}^0 (J^\pi = 1^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	69.5	75.6	63.4	91.0
$\Gamma[\text{MeV}]$	26.7	73.5	21.8	86.2
$B_{\bar{K}}[\text{MeV}]$	$69.1 + i21.8$	$76.8 + i60.7$	$59.1 + i16.1$	$93.8 + i56.1$
$\delta\sqrt{s}[\text{MeV}]$	$-69.1 - i21.8$	$-12.8 - i10.1$	$-79.4 - i18.3$	
$P_{K^-}$	0.07	0.06	0.06	0.14
$P_{\bar{K}^0}$	0.93	0.94	0.94	0.86
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	3.00	2.97	3.01	2.87
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	2.57	2.50	2.61	2.41
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.94	1.92	1.95	1.85
$\sqrt{\langle r_{\bar{K}}^2 \rangle} [\text{fm}]$	1.55	1.48	1.59	1.41
$\sqrt{\langle r_N^2 \rangle_G} [\text{fm}]$	$1.94 - i0.0147$	$1.91 - i0.0163$	$1.95 - i0.0212$	$1.84 - i0.0690$
$\sqrt{\langle r_{\bar{K}}^2 \rangle_G} [\text{fm}]$	$1.54 - i0.0809$	$1.43 - i0.136$	$1.58 - i0.0880$	$1.38 - i0.143$
$\langle T \rangle_G^{K^-} [\text{MeV}]$	$30.7 + i5.41$	$25.0 + i16.0$	$28.6 + i0.0899$	$52.3 + i7.93$
$\langle V \rangle_G^{K^-} [\text{MeV}]$	$-23.7 - i5.54$	$-18.8 - i15.5$	$-21.7 - i1.27$	$-44.7 - i13.1$
$\langle T \rangle_G^{\bar{K}^0} [\text{MeV}]$	$237 + i23.6$	$261 + i58.1$	$225 + i23.0$	$215 + i40.5$
$\langle V \rangle_G^{\bar{K}^0} [\text{MeV}]$	$-290 - i34.1$	$-324 - i86.3$	$-273 - i34.0$	$-272 - i74.9$
$\langle V \rangle_G^{K^- \bar{K}^0} [\text{MeV}]$	$-23.0 - i2.73$	$-18.5 - i8.93$	$-21.8 + i1.28$	$-42.0 - i3.56$
$\langle V_{\bar{K}N}^{I=0} \rangle [\text{MeV}]$	$-122 - i6.85$	$-129 - i20.1$	$-110 - i3.70$	$-155 - i21.7$
$\langle V_{\bar{K}N}^{I=1} \rangle [\text{MeV}]$	$-43.8 - i6.51$	$-64.5 - i16.7$	$-37.3 - i7.21$	$-35.8 - i21.5$

${}^6\text{Li}K^- - {}^6\text{He}\bar{K}^0 \ (J^\pi = 1^-)$				
Model	SIDDHARTA			AY
	Type I	Type II	Type III	
$B[\text{MeV}]$	71.5	78.8	65.5	93.7
$\Gamma[\text{MeV}]$	26.3	74.0	21.7	86.7
$B_{\bar{K}}[\text{MeV}]$	$70.6 + i21.0$	$80.2 + i60.5$	$60.7 + i15.7$	$95.7 + i56.3$
$\delta\sqrt{s}[\text{MeV}]$	$-70.6 - i21.0$	$-13.4 - i10.1$	$-81.0 - i17.8$	
$P_{K^-}$	0.94	0.95	0.94	0.87
$P_{\bar{K}^0}$	0.06	0.05	0.06	0.13
$\sqrt{\langle r_{NN}^2 \rangle} [\text{fm}]$	2.99	2.96	3.00	2.85
$\sqrt{\langle r_{KN}^2 \rangle} [\text{fm}]$	2.56	2.49	2.60	2.40
$\sqrt{\langle r_N^2 \rangle} [\text{fm}]$	1.94	1.91	1.94	1.85
$\sqrt{\langle r_{\bar{K}}^2 \rangle} [\text{fm}]$	1.55	1.47	1.59	1.41
$\sqrt{\langle r_N^2 \rangle_G} [\text{fm}]$	$1.93 - i0.0150$	$1.91 - i0.0155$	$1.94 - i0.0216$	$1.84 - i0.0680$
$\sqrt{\langle r_{\bar{K}}^2 \rangle_G} [\text{fm}]$	$1.53 - i0.0791$	$1.42 - i0.131$	$1.58 - i0.0865$	$1.37 - i0.141$
$\langle T \rangle_G^{K^-} [\text{MeV}]$	$238 + i23.7$	$264 + i58.6$	$227 + i23.2$	$219 + i41.0$
$\langle V \rangle_G^{K^-} [\text{MeV}]$	$-294 - i34.5$	$-331 - i87.5$	$-278 - i34.7$	$-279 - i76.3$
$\langle T \rangle_G^{\bar{K}^0} [\text{MeV}]$	$29.0 + i4.23$	$23.6 + i14.7$	$26.8 - i0.890$	$49.3 + i7.21$
$\langle V \rangle_G^{\bar{K}^0} [\text{MeV}]$	$-21.9 - i4.46$	$-17.6 - i14.2$	$-20.0 - i0.454$	$-41.7 - i12.0$
$\langle V \rangle_G^{K^- \bar{K}^0} [\text{MeV}]$	$-22.6 - i2.04$	$-18.2 - i8.57$	$-21.2 + i1.98$	$-41.1 - i3.36$
$\langle V_{\bar{K}N}^{I=0} \rangle [\text{MeV}]$	$-120 - i6.39$	$-129 - i20.0$	$-108 - i3.27$	$-154 - i21.5$
$\langle V_{\bar{K}N}^{I=1} \rangle [\text{MeV}]$	$-43.7 - i6.75$	$-65.6 - i17.0$	$-37.5 - i7.61$	$-36.4 - i21.9$