JAEA/ASRC Reimei Workshop: New exotic hadron matter at J-PARC 2016
Dates_ 24–26 October 2016 Venue_ Inha University
Organizer_ Department of Physics, Inha University Sponsors_ ASRC, JAEA / MEXT

Effects of Structurization of Dense Matter in Compact Stars

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Matter of neutron stars

Density

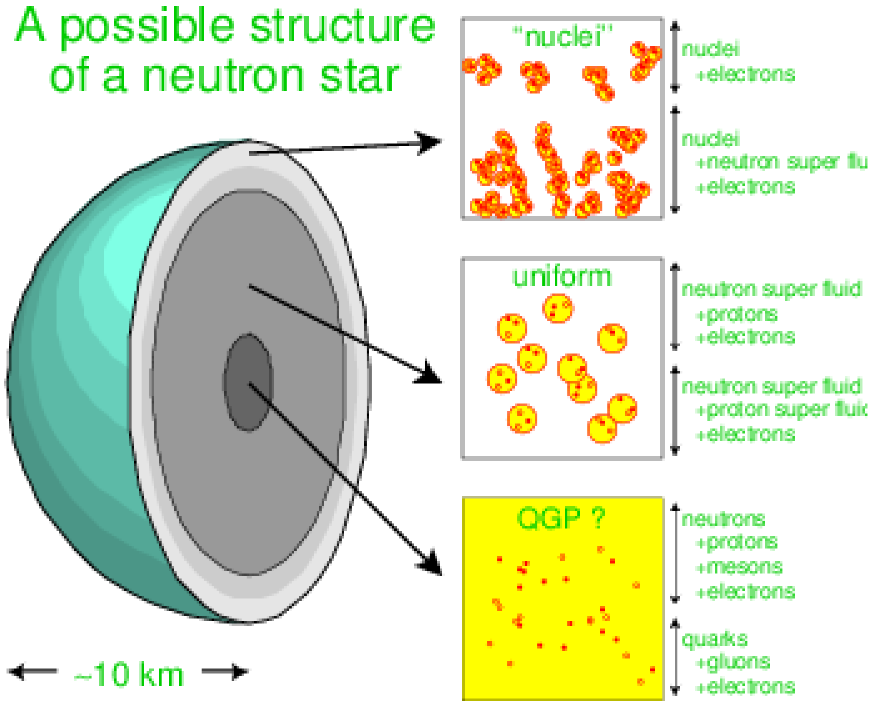
$\approx 0 \cdots \approx 10\rho_0$

Composition

- Nucleons + leptons
- . . . + mesons, hyperons
- quarks + gluons + leptons

Structure & correlation

- uniform
- crystal
- pasta
- amorphous
- pairing
- …..



EOS and structure of neutron stars

Tolman-Oppenheimer-Volkoff (TOV) eq. gives density profile of isotropic material in static gravitational equilibrium.

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} f$$ Balance of pressure and gravity

$$f \equiv \left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1+\frac{P}{\rho c^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}$$ Relativistic correction

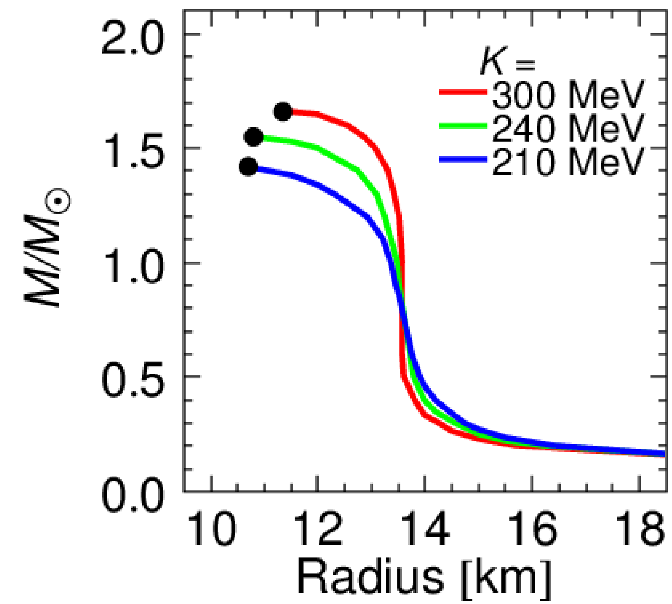
$P = P(\rho)$ Density dependence of pressure (EOS)

$\rho = \rho(r)$ Density at r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ Mass inside r

$M = m(R)$ Overall mass

$R = r(\rho \approx 0)$ Radius



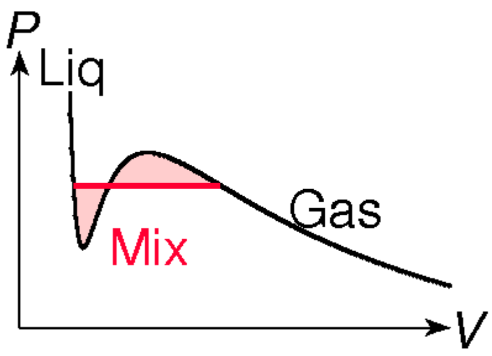
Matter with stiff EOS can sustain heavy neutron stars and soft EOS cannot.

EOS (relation between density ρ and pressure P) determines the neutron star structure.

First-order phase transition and EOS

- Single component ***congruent***
  (e.g. water)

  Maxwell construction **satisfies** the Gibbs cond. $T^{I}=T^{II}$, $P^{I}=P^{II}$, $\mu^{I}=\mu^{II}$.



- Many components ***non-congruent***
  (e.g. water+ethanol)

  Gibbs cond. $T^{I}=T^{II}$, $P_i^{I}=P_i^{II}$, $\mu_i^{I}=\mu_i^{II}$.
  No Maxwell construction *!*

- Many charged components
  (nuclear matter)

  Gibbs cond. $T^{I}=T^{II}$, $\mu_i^{I}=\mu_i^{II}$.
  No Maxwell construction *!*
  No constant *pressure !*

$$\frac{dP_i}{dr} = -\frac{\partial U_i(\rho_i; r)}{\partial r}$$

Explicit dependence on $\boldsymbol{r}$ by Coulomb int.

This is the case for nuclear matter !

Mixed phases in compact stars

- Low-density nuclear matter
  liquid-gas
  neutron drip
- High density matter
  meson condensation
  hyperon mixture (?)
  hadron-quark

Low-density nuclear matter

General behavior of neutron star matter

Slve the following conditions of *n p e* matter in chemical equilibrium

$\rho_p = \rho_e$ (neutrality)

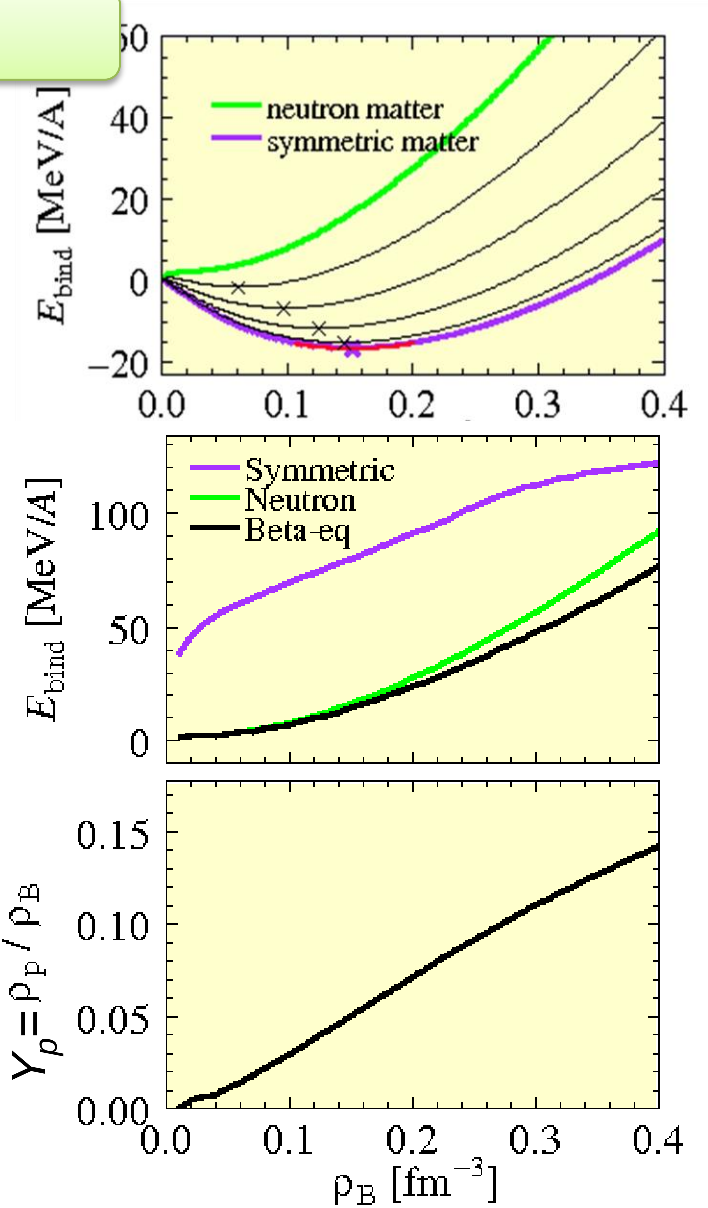
$\mu_n = \mu_p + \mu_e$ (beta equilibrium: $n \leftrightarrow p + e + \nu$)

$\mu_\nu \approx 0$

$$\mu_{n,p} = \sqrt{p_{F(n,p)}^2 + m_N^{\ 2}} + U_{n,p}$$
$$= \sqrt{(3\pi^3 \rho_{n,p})^{2/3} + m_N^{\ 2}} + U_{n,p}$$

$$\mu_e = \left(3\pi^3 \rho_e\right)^{1/3}$$

- Total energy density is monotonically increasing function due to the electron.
- Beta-equilibrium matter is neutron-rich.
- Proton fraction increases with density.



ρ_B = baryon number density

Model-independent explanation

- uniform electron at T=0

$$\rho_e = 2\frac{4\pi\left(p_{Fe}/2\pi\hbar\right)^3}{3} = \frac{p_{Fe}{}^3}{3\pi^2\hbar^3} = \frac{\mu_e{}^3}{3\pi^2\hbar^3}$$

$$\mu_e = p_{Fe} = \left(3\pi^2\rho_e\right)^{1/3}\hbar \quad \text{chemical pot}$$

$$\varepsilon_e = \int_0^{p_F}\frac{d^3p}{(2\pi)^3}p = \frac{p_{Fe}^4}{4\pi^2} = \frac{\left(3\pi^2\rho_e\right)^{4/3}}{4\pi^2} \quad \text{energy density}$$

- uniform nucleon at T=0

$$\rho_N = 2\frac{4\pi\left(p_{FN}/2\pi\hbar\right)^3}{3} = \frac{p_{FN}{}^3}{3\pi^2\hbar^3} \qquad (N = p, n)$$

$$\mu_N = \sqrt{p_{FN}{}^2 + m_N^2} + U_N = \sqrt{\left(3\pi^2\hbar^3\rho_N\right)^{2/3} + m_N^2} + U_N \quad \text{chem pot}$$

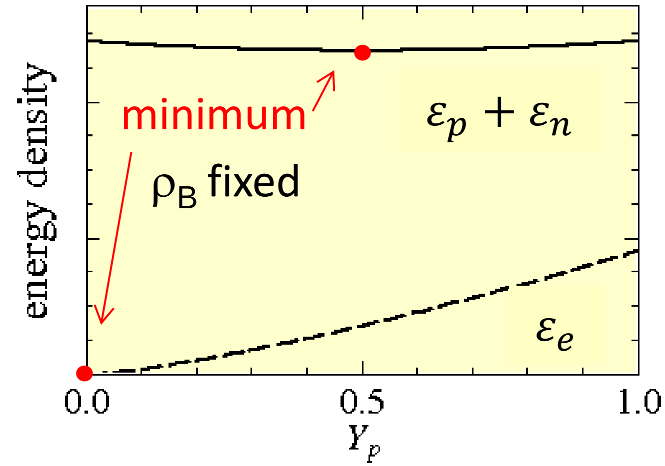
$$\varepsilon_N = 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[\sqrt{p^2 + m_N^2} + U_N\right] \quad \text{energy density}$$

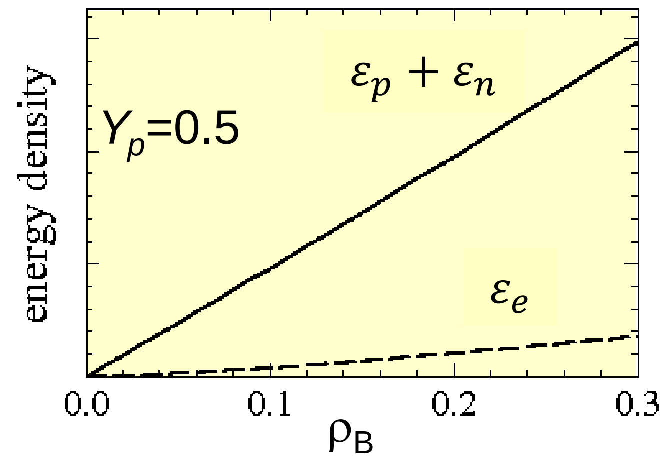
$$\approx 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[m_N + \frac{p^2}{2m_N} + U_N\right] \quad \text{non-relativistic approx}$$

$$= (m_N + U_N)\rho_N + \frac{\left(3\pi^2\rho_N\right)^{5/3}\hbar^2}{10\pi^2 m_N}$$

Even if m_N may be large or small, $d\varepsilon_N/d\rho_N$ is large.

In anyway, N is stiffer than e





- with fixed $\rho_B = \rho_p + \rho_n$, min point of $\varepsilon_p + \varepsilon_n + \varepsilon_e$ exists at a small Y_p → neutron-rich matter.

- energy density of baryon increases more rapidly than electron → proton fraction increases with density. (characteristics of uniform matter)

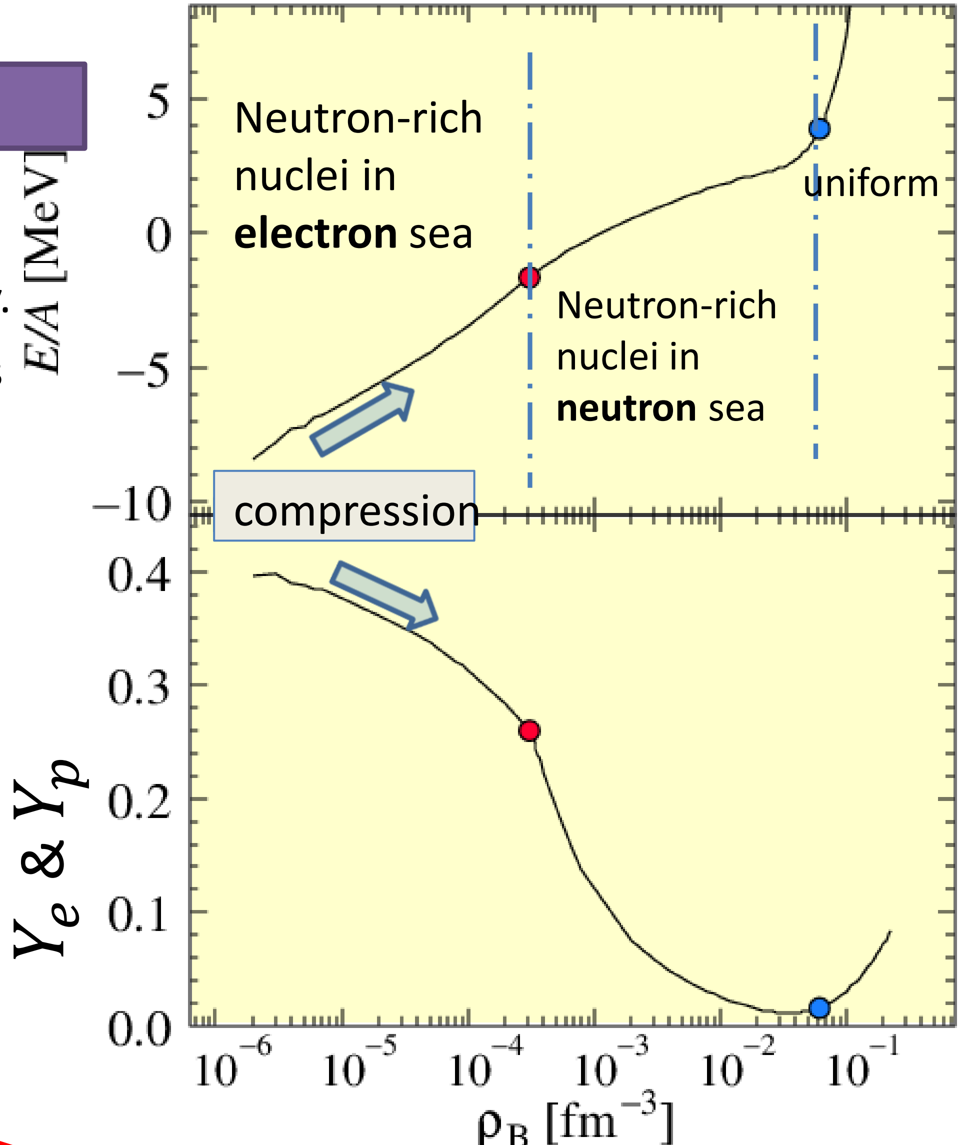
From inhomogeneous to uniform

First, electrons degenerate.
→ electron energy depends on the density.
→ Y_e and Y_p decrease. Baryon energy is not directly dependent on the density.

As Y_p decreases, neutrons begin to drip (●)
→ Neutrons found space to escape.
→ Y_e and Y_p decrease rapidly.

As neutrons degenerate, increase of Y_n (decrease of Y_p) is suppressed.

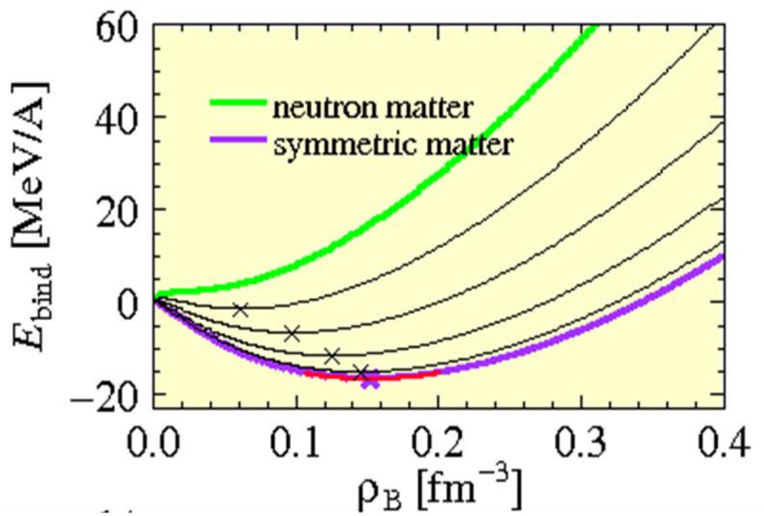
→???
→ at last, protons degenerate. (●)
→ Y_p starts to increase in Uniform matter.



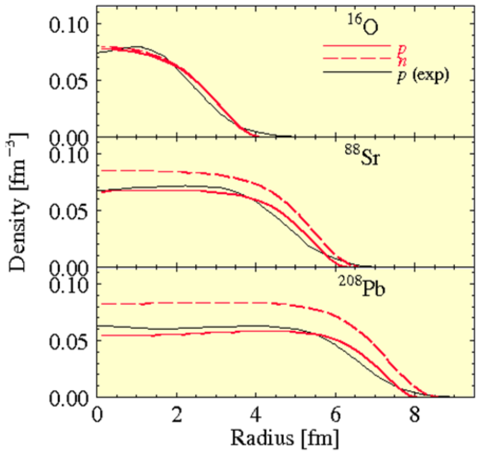
Not clear yet

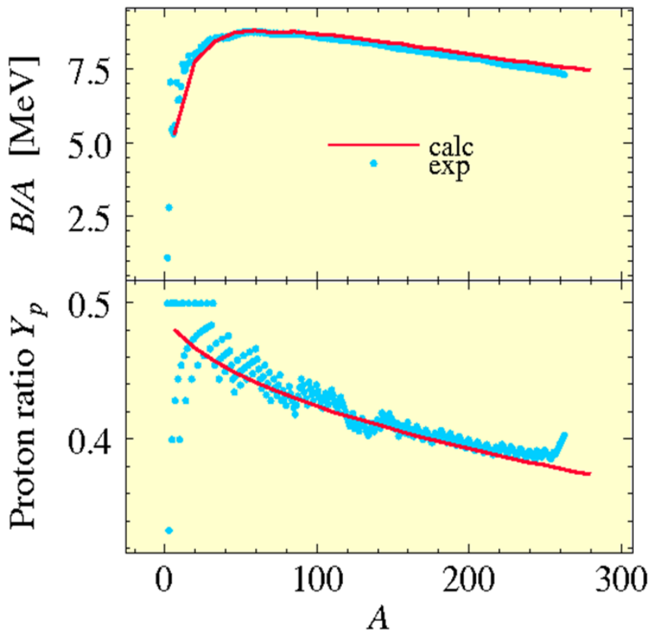
RMF + Thomas-Fermi model

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but realistic enough.



Binding energies ↓, proton fractions ↓, and density profiles → of nuclei are well reproduced.





Saturation property of symmetric nuclear matter : minimum energy $E/A \approx -16$ MeV at $\rho_B \approx 0.16\ \mathrm{fm}^{-3}$.

Inhomogeneous nuclear matter

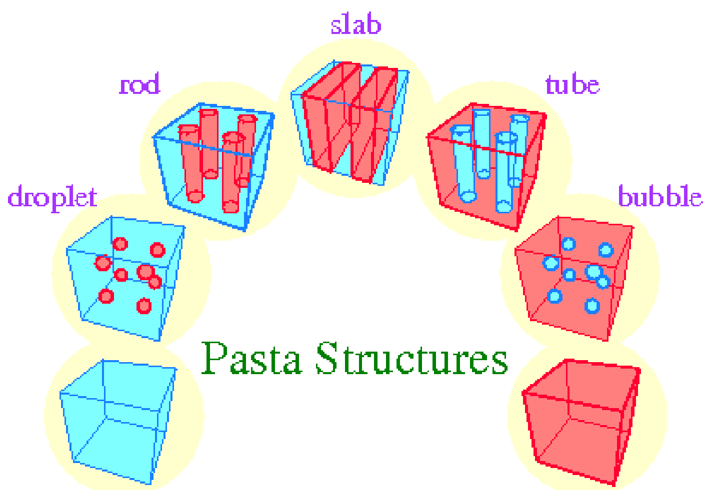
[Ravenhall et al, PRL50(1983)2066]

Another aspect: “structured mixed phase” in the first-order phase transition

Regular structure by the balance between Coulomb repulsion and surface tension.
(cf. water: neutral → no regular structure)

$$\frac{E_C}{A} \propto \frac{Z^2/R}{A} \propto R^2 \quad \Rightarrow \frac{E_C}{A} = aR^2$$

$$\frac{E_S}{A} \propto \frac{R^2}{R^3} \propto R^{-1} \quad \Rightarrow \frac{E_S}{A} = bR^{-1}$$

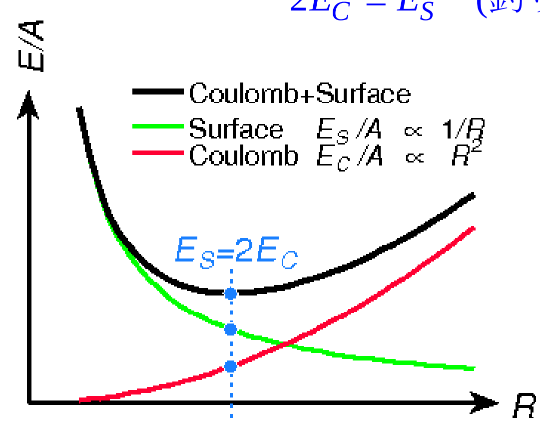


$$\frac{d((E_C + E_S)/A)}{dR} = 0 \quad \text{(エネルギー最少)}$$

$$\frac{d(aR^2 + bR^{-1})}{dR} = 2aR - bR^{-2} = 0$$

$$2aR^2 = bR^{-1}$$

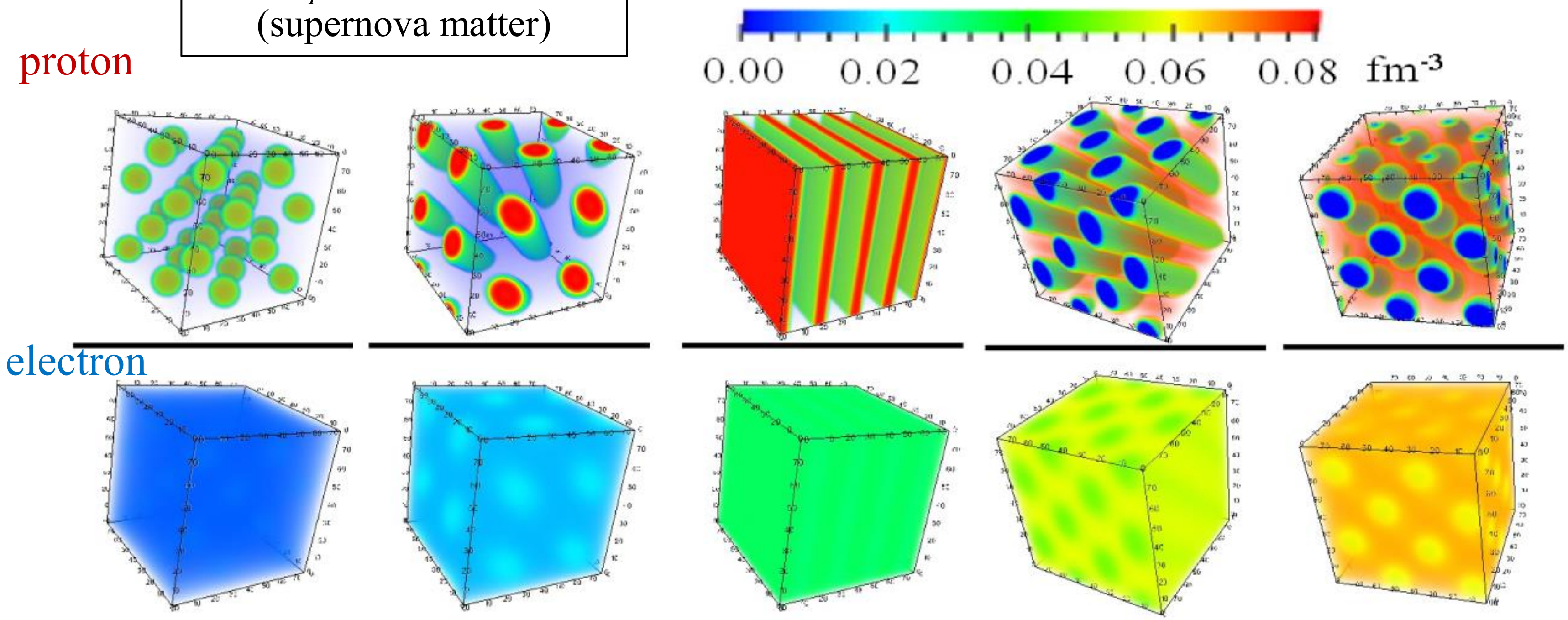
$$2E_C = E_S \quad \text{(釣り合い条件)}$$



Result of fully 3D calculation by RMF+Thomas-Fermi

[Phys.Lett. B713 (2012) 284]

Symmetric nuclear matter
$Y_p = Z/A = 0.5$
(supernova matter)



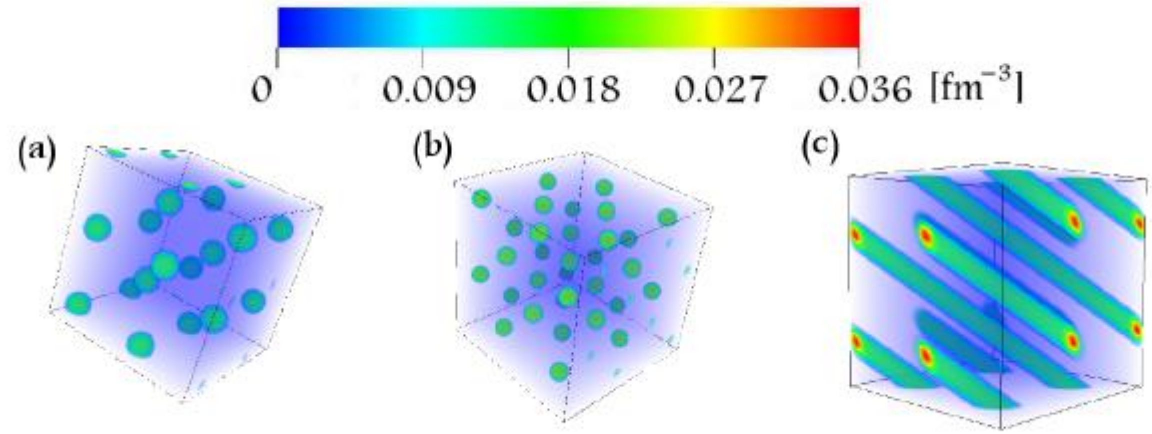
"droplet"	"rod"	"slab"	"tube"	"bubble"
$\boxed{\text{[fcc]}}$	[honeycomb]		[honeycomb]	[fcc]
ρ_B = 0.012 fm^{-3}	0.024 fm^{-3}	0.05 fm^{-3}	0.08 fm^{-3}	0.094 fm^{-3}

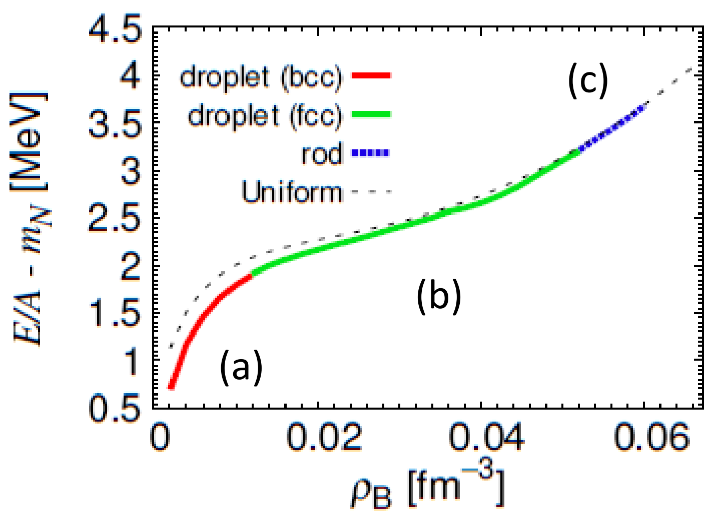
Confirmed the appearance of pasta structures.

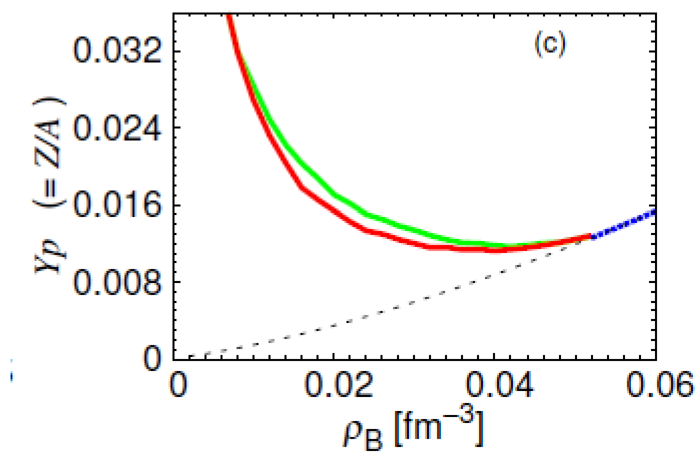
Beta equilibrium case
(neutron star crust)

[Phys. Rev. C **88**, 025801]

Crystalline structures bcc & fcc.
Rod phase appears.

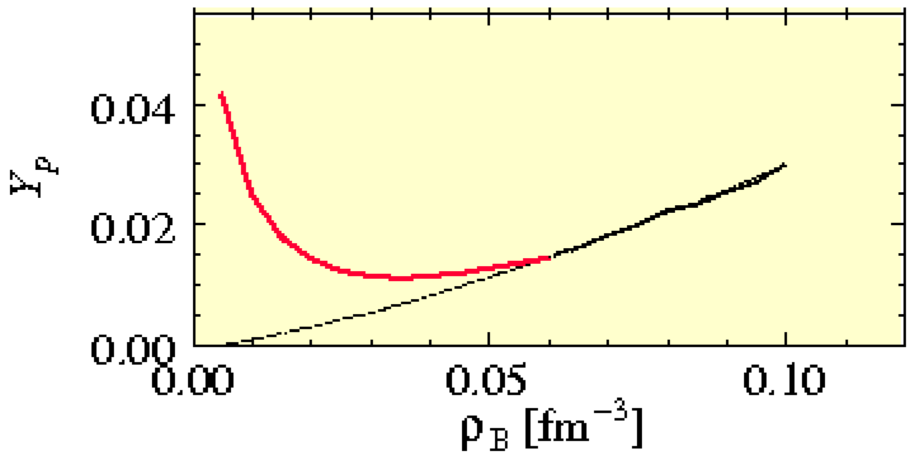






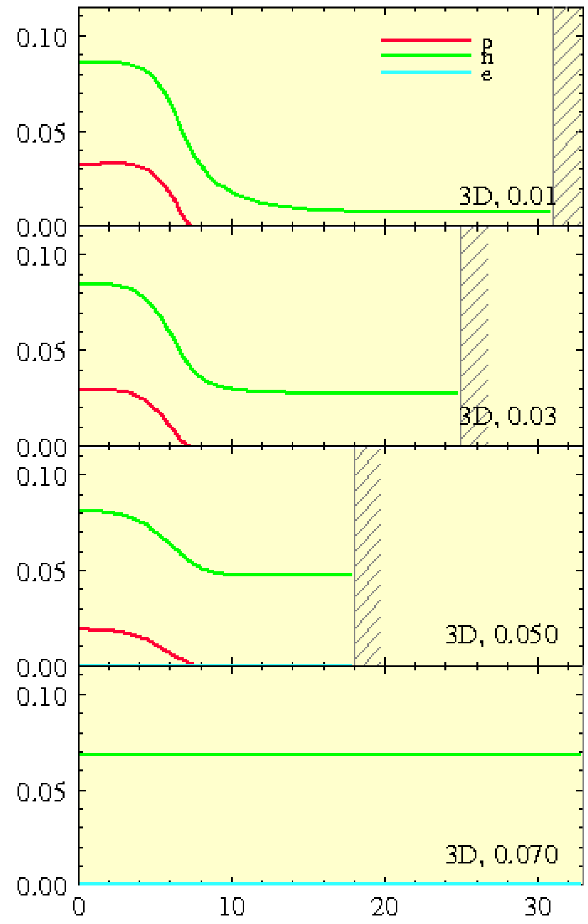
Beta-equilibrium case

by Wigner-Seitz approx



Proton fractions of uniform matter and inhomogeneous matter are different.

Uniform: locally neutral
Inhomog: can be charged



Low density → weaker Coulomb
→ Larger inter-nuclear distance
→ Local charge neutrality disappears

(2) Kaon condensation and htperons

[Phys. Rev. C **73**, 035802]

K single particle energy (model-independent form)

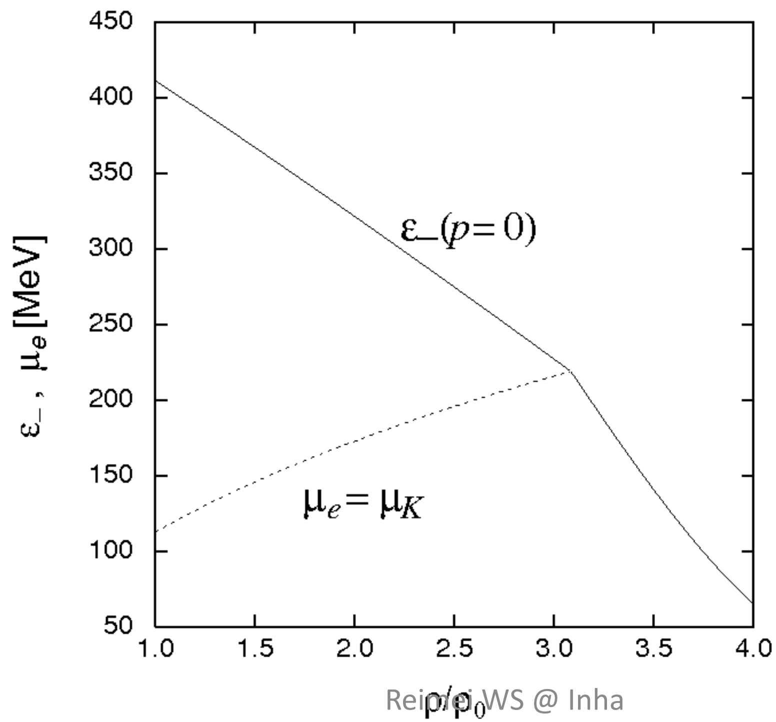
$$\varepsilon_{\pm}(\mathbf{p}) = \sqrt{p^2 + m_K^{*\,2} + \left(\left(\rho_n + 2\rho_p\right)/4f^2\right)^2} \pm \left(\rho_n + 2\rho_p\right)/4f^2,$$

From a Lagrangian with chiral symmetry

$$m_K^{*\,2} = m_K^{\,2} - \Sigma_{KN}\left(\rho_n + 2\rho_p\right)/4f^2,$$

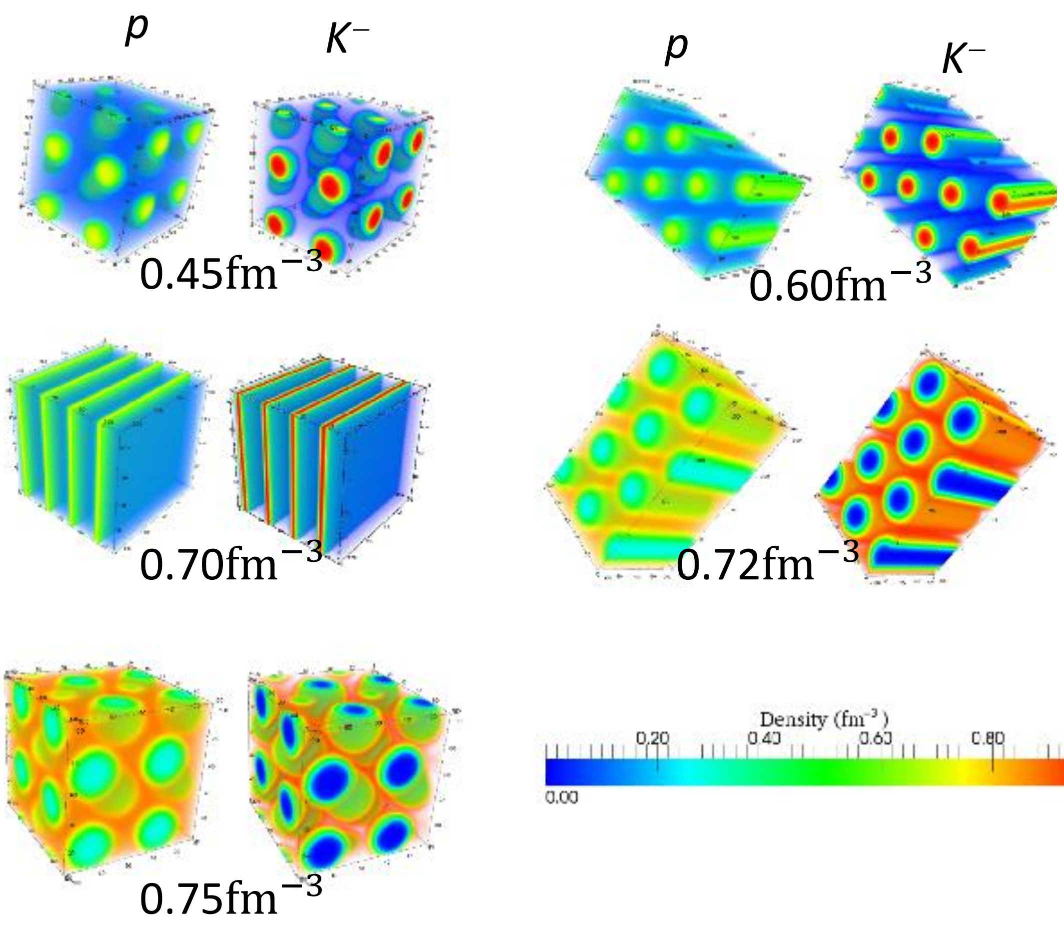
$$\mu_K = \varepsilon_{-}(p=0) = \mu_n - \mu_p = \mu_e$$

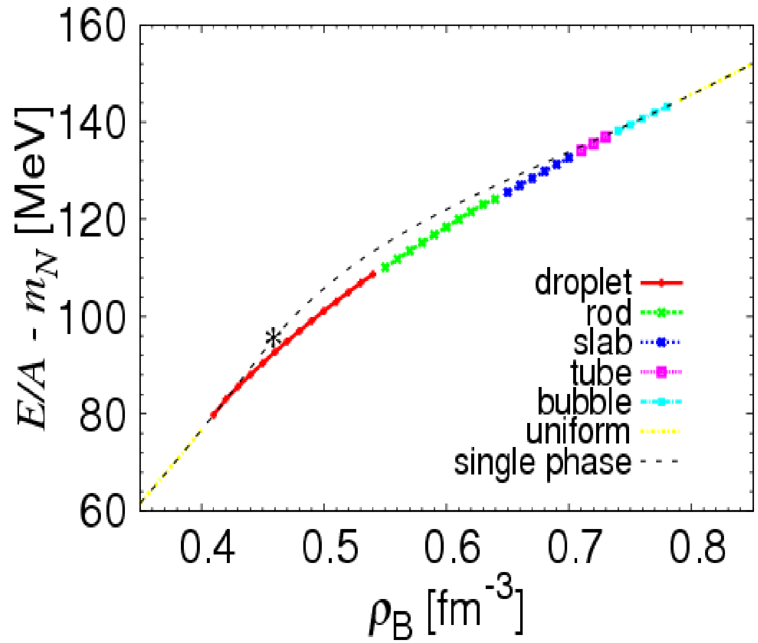
Threshold condition of condensation



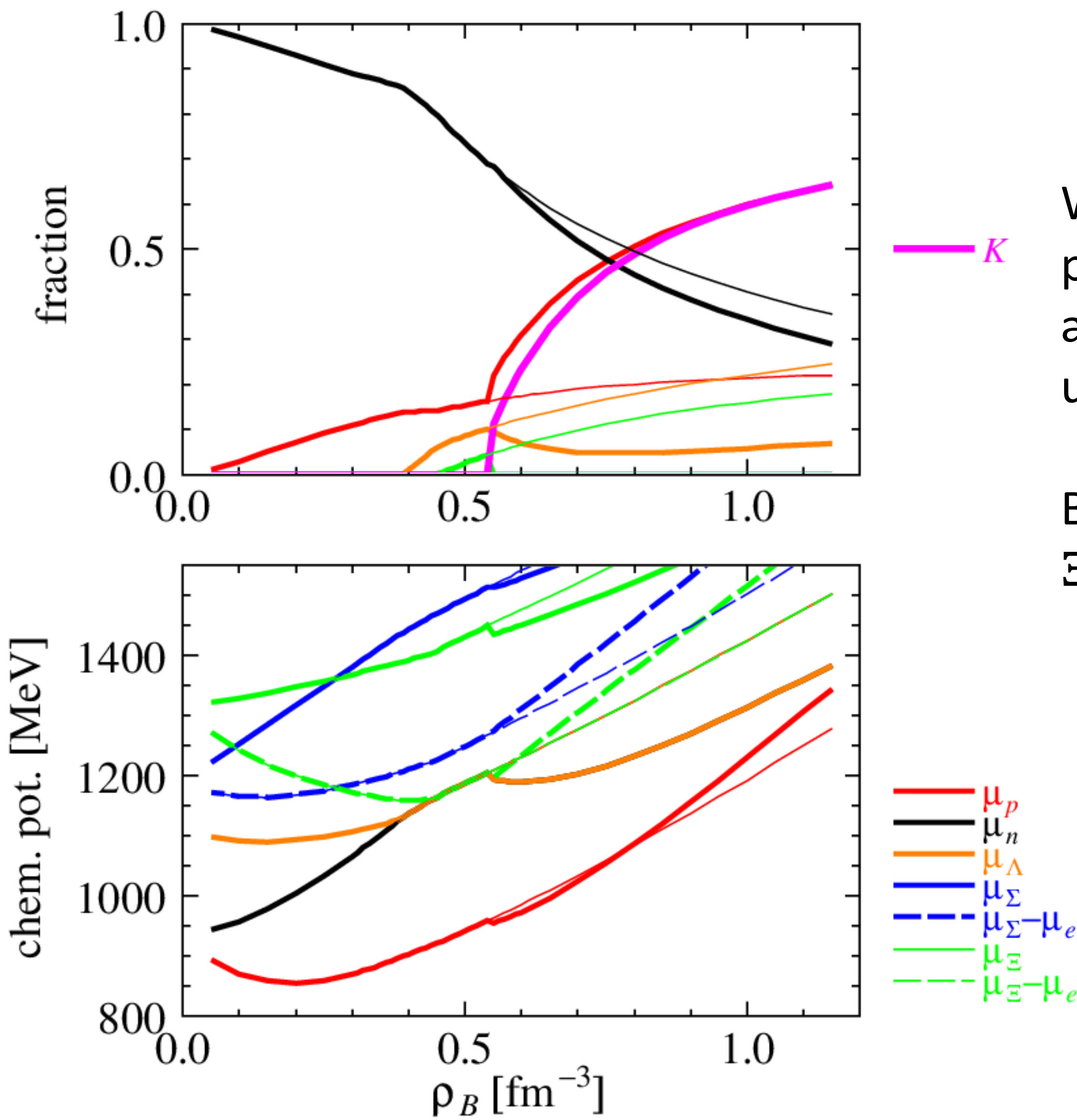
Kaonic pasta phases
--- Fully 3D calculation

[unpublished yet]





Hyperon versus kaon in uniform matter



Without kaon, in the present parameter set, firstly appears Λ and then Ξ^-in the case of uniform.

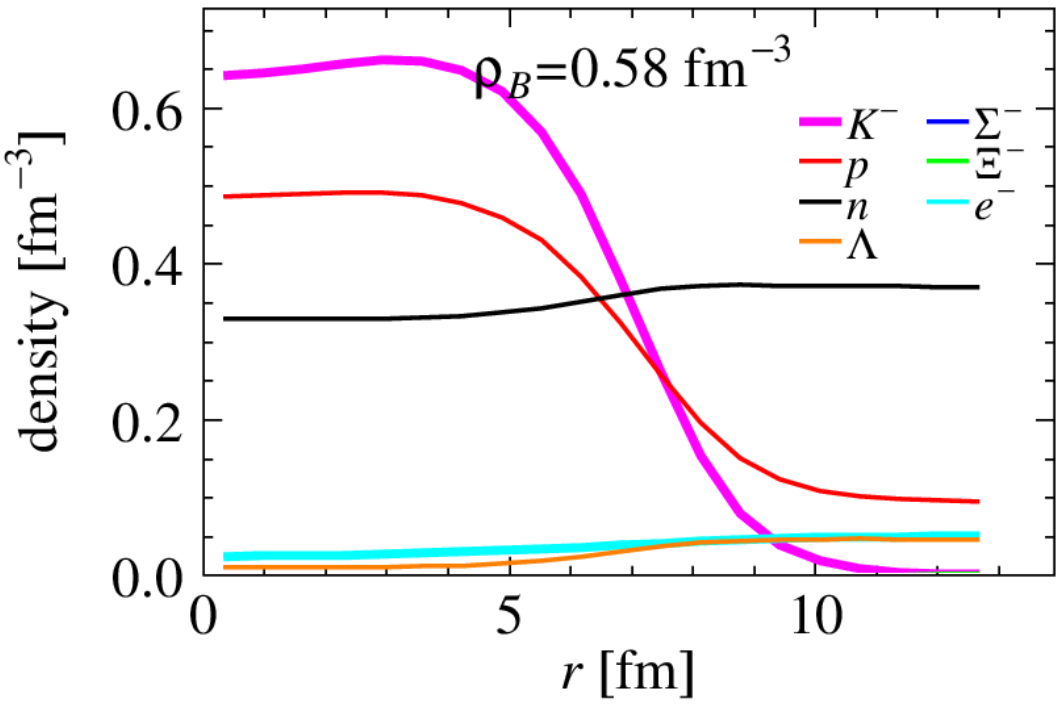
By the appearance of Kaon, Ξ^- disappears and Λ decreases.

Inhomogeneous structure is considered

Density profile in a spherical WS cell.

Λ(orange line near the bottom) and Kaon(purple) avoid with each other.

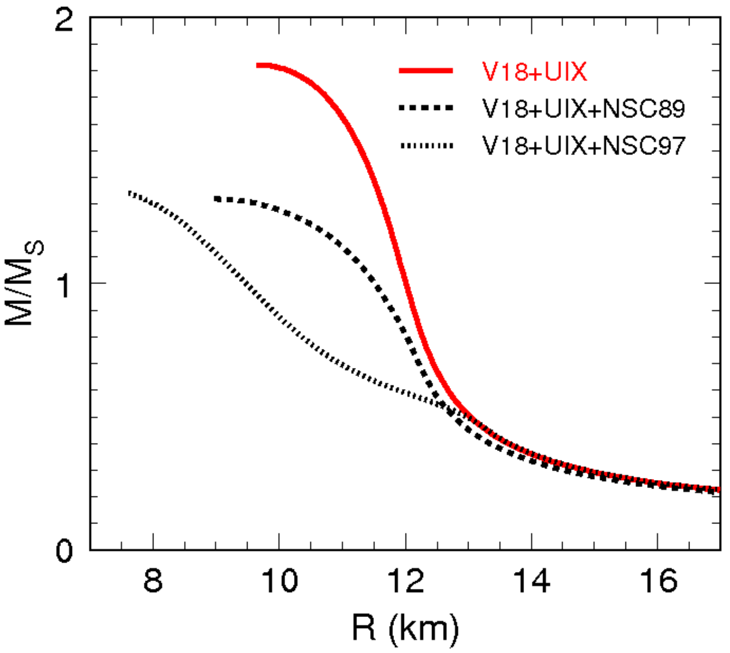
→ Inhomogeneous structure may enhance (moderate the suppression of) the appearance of hyperons in kaon condensation (?)



(3) Hadron-quark phase transition

[Phys. Rev. D **76**, 123015]

At 2--3ρ_0, hyperons are expected to appear.
→ Softening of EOS
→ Maximum mass of neutron star becomes less than 1.4 solar mass and far from 2 .0 solar mass.
→ Contradicts the obs $> 1.5 M_\odot$



Schulze et al, PRC73 (2006) 058801

Quark-hadron mixed phase

to get density profile, energy, pressure, etc of the system

$$\mu_u + \mu_e = \mu_d = \mu_s, \quad \mu_n = \mu_u + 2\mu_d, \quad \mu_p + \mu_e = \mu_n = \mu_\Lambda = \mu_\Sigma - \mu_e$$

$$\mu_i = \frac{\partial \varepsilon(\mathbf{r})}{\partial \rho_i(\mathbf{r})} \quad (i = u, d, s, p, n, \Lambda, \Sigma^-, e)$$

$$\varepsilon(\mathbf{r}) \equiv \varepsilon_B(\mathbf{r}) + \varepsilon_e(\mathbf{r}) + \left(\nabla V_C(\mathbf{r})\right)^2 / 8\pi e^2$$

$$\varepsilon_B(\mathbf{r}) = \begin{cases} \varepsilon_H(\mathbf{r}) & (\text{hadron phase: BHF}) \\ \varepsilon_Q(\mathbf{r}) & (\text{quark phase: MIT bag}) \end{cases}$$

$$\varepsilon_e(\mathbf{r}) = \left(3\pi^2 \rho_e(\mathbf{r})\right)^{4/3} / 4\pi^2$$

$$E/A = \frac{1}{\rho_B V}\left[\int_V d^3 r\, \varepsilon(\mathbf{r}) + \tau S\right] \quad \begin{pmatrix} \rho_B = \text{average baryon density} \\ S = \text{Q-H boundary area} \\ V = \text{cell volume} \end{pmatrix}$$

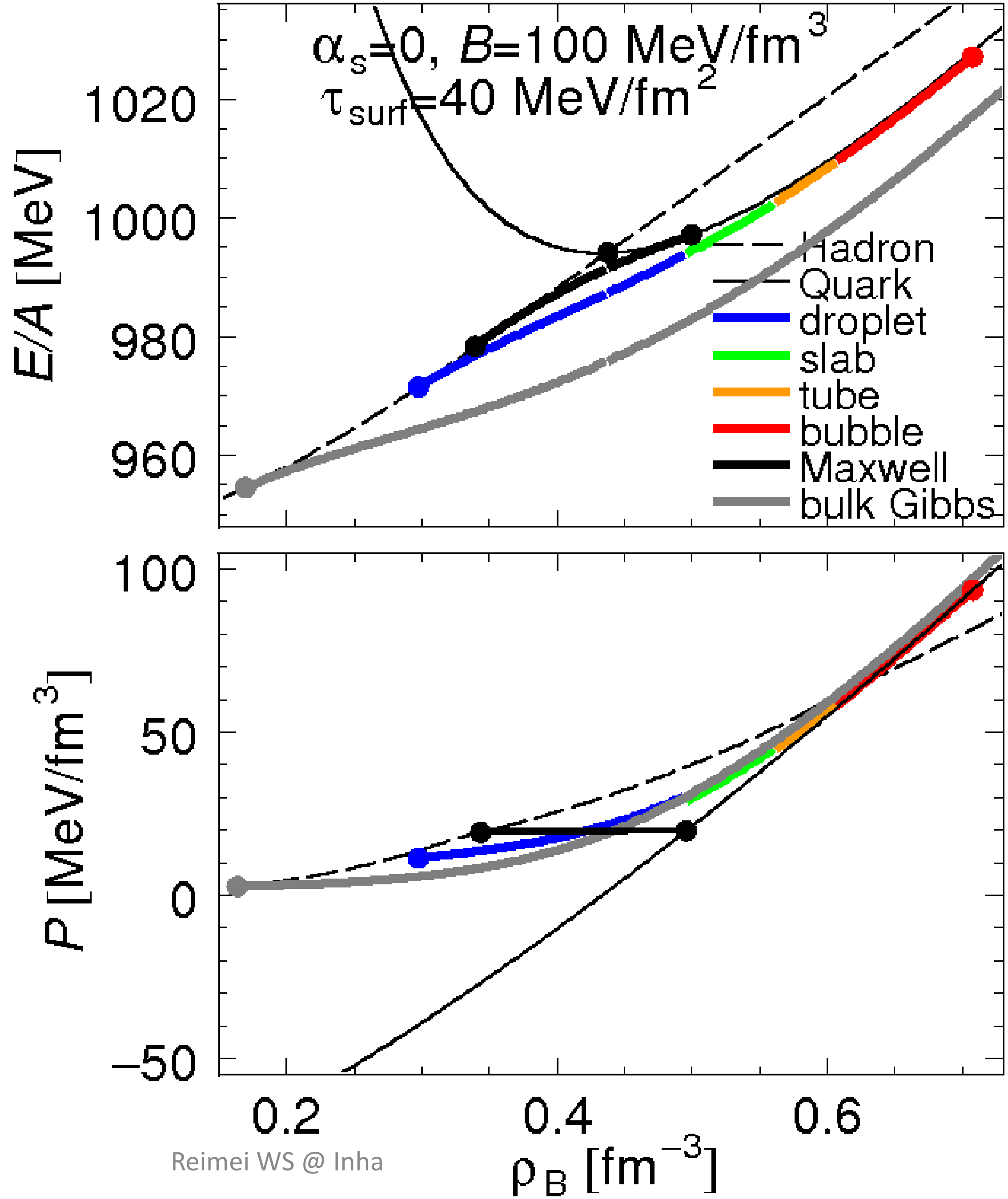
$$\int_V d^3 r \left[\rho_p(\mathbf{r}) - \rho_\Sigma(\mathbf{r}) + \frac{2}{3}\rho_u(\mathbf{r}) - \frac{1}{3}\rho_d(\mathbf{r}) - \frac{1}{3}\rho_s(\mathbf{r}) - \rho_e(\mathbf{r})\right] = 0 \quad (\text{total charge})$$

$$\frac{1}{V}\int_V d^3 r \left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r}) + \rho_\Lambda(\mathbf{r}) + \rho_\Sigma(\mathbf{r}) + \frac{1}{3}\rho_u(\mathbf{r}) + \frac{1}{3}\rho_d(\mathbf{r}) + \frac{1}{3}\rho_s(\mathbf{r})\right] = \rho_B \quad (\text{given})$$

EOS of matter

Full calculation is between the Maxwell construction (local charge neutral) and the bulk Gibbs calculation (neglects the surface and Coulomb).

Closer to the Maxwell.



Structure of compact stars

TOV equation
$$\frac{dP}{dr} = -\rho\frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

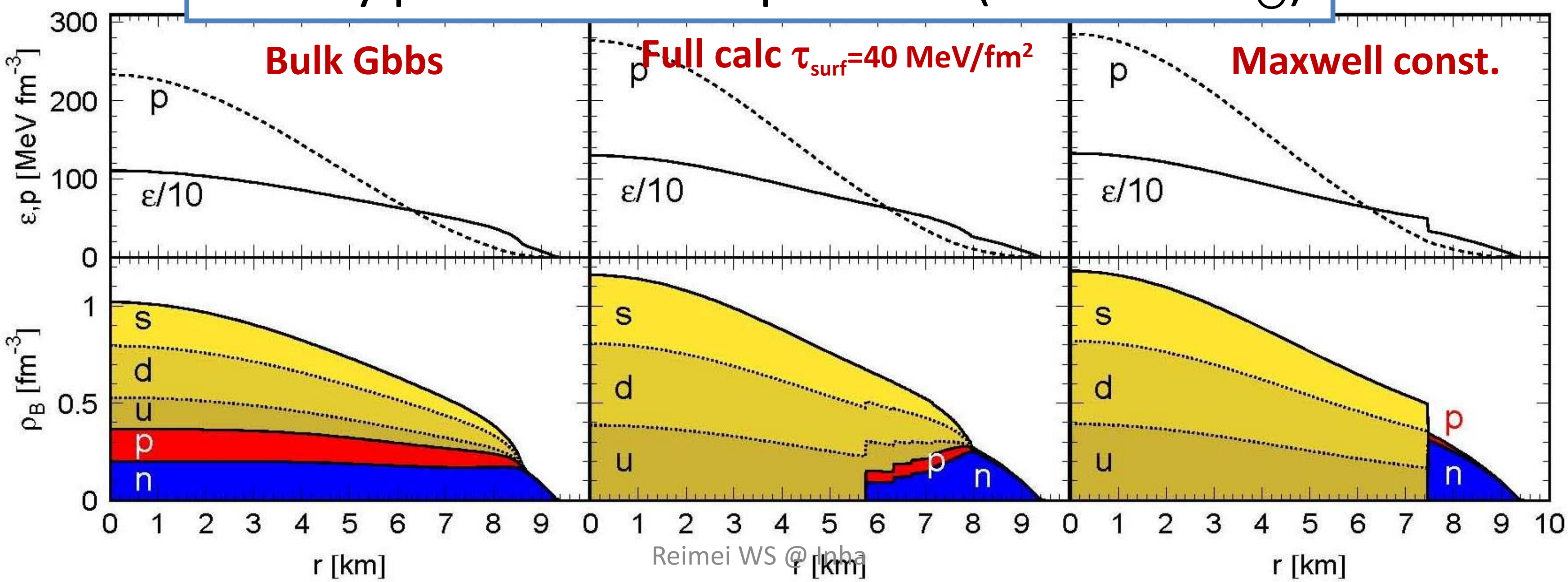
$P = P(\rho)$ Pressure (input of TOV eq.)

$\rho = \rho(r)$ Density at position r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s)ds$ mass inside the position r

$M = m(R), \quad R = R(\rho \approx 0)$ total mass and radius.

Density profile of a compact star ($M = 1.4\ M_\odot$)

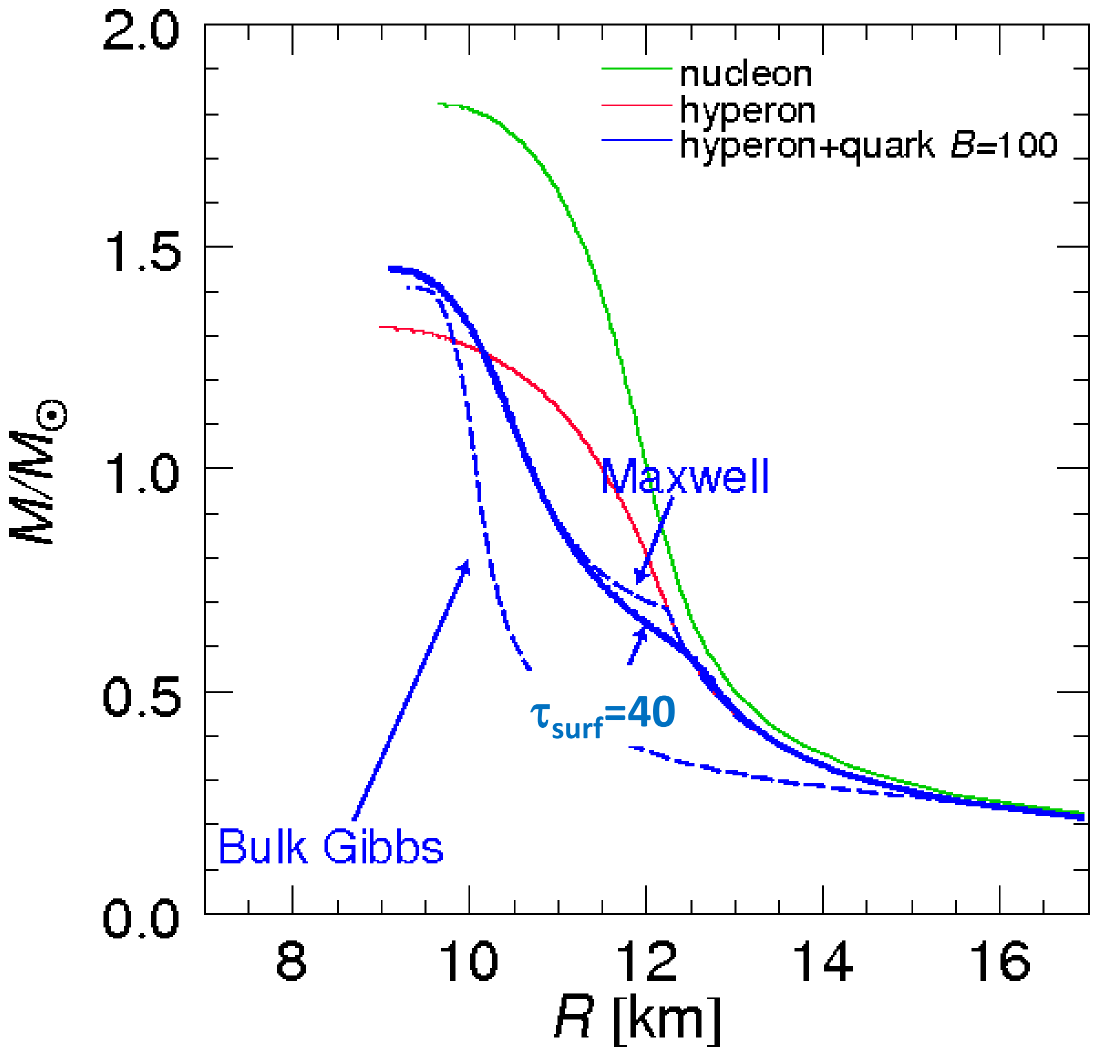


Mass-radius relation of a cold neutron star

Full calculation with pasta structures yields similar result to the Maxwell construction.

Maximum masses are almost the same for 3 cases.

We need to improve largely the quark EOS or hadron EOS to get $\sim 2M_{\odot}$



Summary

First-order phase transition of nuclear matter
→ mixed phase of multi-components with charge
→ Structured mixed phase (pasta).
important for EOS.

It also affects the chemical composition.

Important subjects on NS

- Maximum mass of NS: $M_{\max} \sim 2M_{\odot}$
  EOS too soft if Y mixed.
- Cooling of NS by neutrino emission :
  Too fast if hyperons are mixed.
- Magnetar:
  Origin of strong magnetic field