Analysis of nucleon form factors in Lattice QCD

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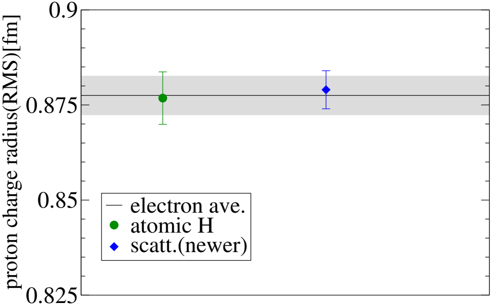
Oct 24, 2016

Proton Radius Problem

- We can measure the size of proton through e-p scattering and hydrogen spectroscopy.



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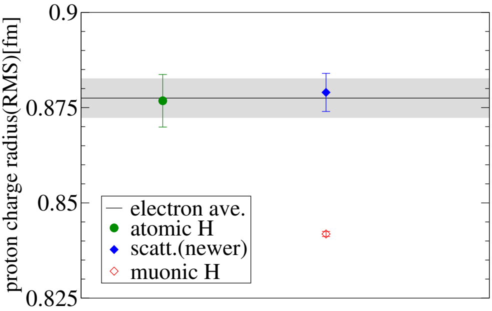

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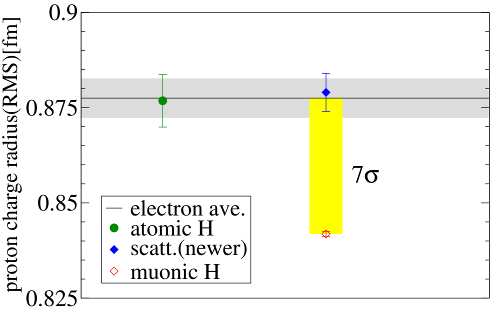
Proton Radius Problem

- We can measure the size of proton through e-p scattering and hydrogen spectroscopy.
- The muonic hydrogen atom offers more precise measurement of the proton charge radius.
- The unexplained discrepancy challenges our understanding of the proton.



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It is difficult to account for this discrepancy.
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- First of all, we would like to know whether results obtained from e-p scattering experiments have no problem.

Determination of the proton radius from e-p scattering

- We first extract a form factor $G(t)$, which is a function of momentum transfer $t = -q^2$
- The form factor is measured in the space-like region $(t > 0)$. The electric form factor $G_E(t)$ corresponds to the Fourier transform of the charge distribution.The charge radius is determined from the slope of the form factor at zero momentum transfer.
- The magnetic moment is given by the magnetic form factor $G_M(t)$ at zero momentum transfer.
- Both electric and magnetic form factors are accessible by lattice QCD simulations.

We can obtain $G_{E,M}$ from matrix elements of vector current V^μ

$$
\begin{aligned}
\langle p'|V^\mu|p\rangle &= \langle p'|\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)|p\rangle \\
&= \langle p'|\frac{(p'+p)^\mu}{2m}\frac{G_E(q^2) - \frac{q^2}{4m^2}G_M(q^2)}{1-q^2/4m^2} + i\frac{\sigma^{\mu\nu}q_\nu}{2m}G_M(q^2)|p\rangle
\end{aligned}
$$

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the goal in this talk

I will show new results of nucleon form factors near the physical point in 2+1 flavor QCD, using model-independent analysis based on the Z-expansion method.

Model-independent analysis of the form factor shape

To access t-dependence of form factors, we need to interpolate the values of the form factors measured at discrete momentum points.
For momentum interpolation, we usually adopt

- dipole form : $G(t) = a/(1 + bt)^2$
- Taylor expansion : $G(t) = \sum_{k=0}^{k_{max}} a_k t^k$

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But,

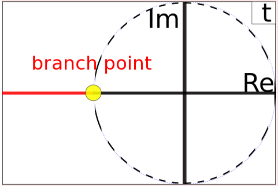
- dipole form is only a model.
- how can we make the right choice of k_{max}?

These points become more severe for a process of extrapolation rather than interpolation.
Mathematically, the latter approach faces a convergence problem due to the presence of cuts due to physical thresholds in the time-like region.

convergence radius

In the time-like region $(t < 0)$ of nucleon form factors, there is a branch cut starting at $t = -(2m_\pi)^2$, which is associated with pion pair creation, running along the negative real axis.

convergence problem

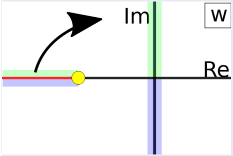


In a complex plain, the expansion $G(t) = \sum_k a_k t^k$ does not converge if $\|t\|^2 > 4{m_\pi}^2$

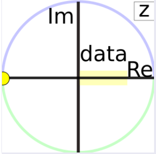
Once the data points are analyzed outside the radius of convergence of a power series, the Taylor expansion becomes non-convergent series.

Except for branch cuts, form factors are analytic functions of t. We thus perform conformal mapping thanks to the analyticity of the form factor.

conformal mapping



By the first transformation $w = \sqrt{t_{cut} + t}(t_{cut} = 4{m_\pi}^2)$, analytic domain is mapped into the right half of the complex plain.



By the final transformation $z = \frac{w - \sqrt{t_{cut}}}{w + \sqrt{t_{cut}}}$, analytic domain is mapped into the open unit disk($\|z\| < 1$).
The region where the data exist is assured to be inside a circular region of analyticity.

We perform conformal mapping in above procedure.

C. Glenn Boyd *et al.*, Physics Letters B 353 (1995) 306-312
Richard J. Hill and Gil Paz,Phys. Rev. D 82, 113005(2010)

After conformal mapping

From the analyticity of G(z) in the region $\|z\| < 1$, where there is no singlularity, the form factor can be expanded in terms of z as $G(z) = \sum_k c_k z^k$ then the coefficients c_k are supposed to have the following relations

$$c_k = \frac{1}{2\pi i} \oint \frac{G(z)}{z^k} \frac{dz}{z}$$

$$\sum_k {c_k}^2 = \frac{1}{2\pi i} \oint G(z) \cdot G(z)^* \frac{dz}{z}$$

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After conformal mapping

Performing a contour integral with $z(t) = e^{i\theta}$,we get

$$c_k = \begin{cases} G(0) & k = 0 \\ -\frac{2}{\pi}\int_{t_{cut}}^{\infty} \frac{dt}{t}\sqrt{\frac{t_{cut}}{t-t_{cut}}}\mathrm{Im}G(t)sin[k\theta(t)] & k \geq 1 \end{cases}$$

$$\sum_k {c_k}^2 = \frac{1}{\pi}\int_{t_{cut}}^{\infty}\frac{dt}{t}\sqrt{\frac{t_{cut}}{t-t_{cut}}}\|G(t)\|^2$$

A good convergence is expected for the coefficients c_k

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Z-Expansion method

$G(t) = \sum_k c_k z^k$, $z = \frac{\sqrt{t_{cut}+t}-\sqrt{t_{cut}}}{\sqrt{t_{cut}+t}+\sqrt{t_{cut}}}$

Excellent properties of z-expansion method:

- Taylor expansion inside a circular region of analyticity.
- Since $\sum_k \|c_k\|^2 < \infty$, expansion coefficients c_k have good convergence behavior.

C. Glenn Boyd et al., Physics Letters B 353 (1995) 306-312
Richard J. Hill and Gil Paz,Phys. Rev. D 82, 113005(2010)

Simulation details for 2+1 flavor QCD

Near Physical Point $2+1$ flavor Lattice QCD Simulation using K computer (PACS Collaboration)*

HPCI Strategic Program Field 5
"The origin of matter and the universe"

(Nos. hp120281,hp130023,hp140209, hp140155,hp150135,hp160125)

- $L^3 \times T = 96^3 \times 96 (La \sim 8\mathrm{fm})$
- cutoff $a^{-1} \sim 2.3\mathrm{GeV}$
- pion mass $m_\pi \sim 0.145\mathrm{GeV}$
- 146 configurations

Using isovector form factors in 9 different non-zero momenta.†

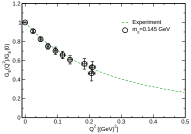

Figure: electric form factor

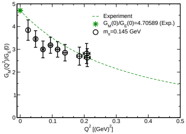

Figure: magnetic form factor

*K.-I.Ishikawa et al. for PACS collaboration arXiv:1511.09222
†T. Yamazaki for PACS collaboration arXiv:1511.09179

Numerial results (electric form factor)

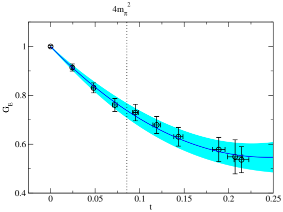

Figure: Taylor expansion(2nd power)

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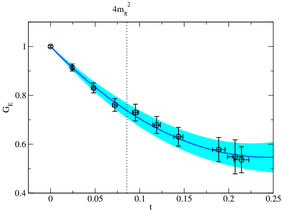

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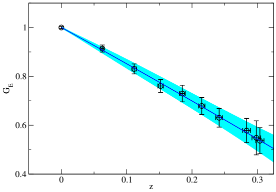

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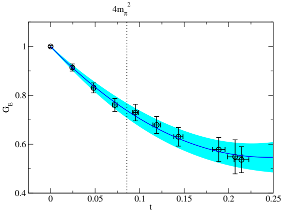

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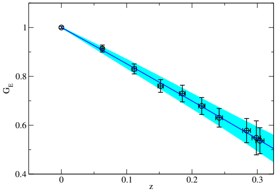

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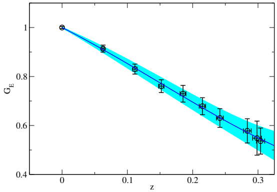

Figure: $z$expansion (4th power)

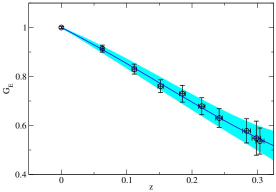

Figure: $z$expansion(8th power)

Numerical result (charge radius)

Root Mean Square (RMS) of charge radius of the isovector form factor is given by the following relation:

$$\langle r_w^2 \rangle = \langle r_p^2 \rangle - \langle r_n^2 \rangle$$

$$\langle r^2 \rangle = -6 \left. \frac{\partial G_E}{\partial t}(t) \right|_{t=0}$$

or

$$\langle r^2 \rangle = -\frac{3}{8{m_\pi}^2} \left. \frac{\partial G_E}{\partial z}(z) \right|_{z=0}$$

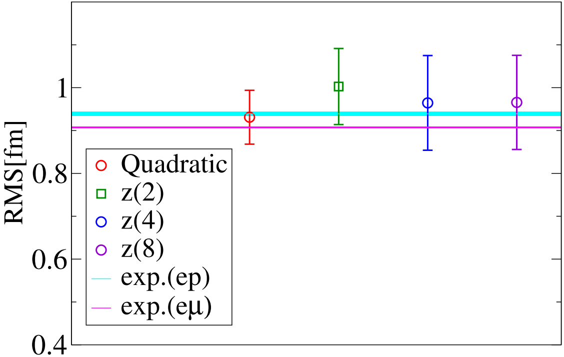

Figure: RMS ,experiment values form PDG

Numerical result (charge radius)

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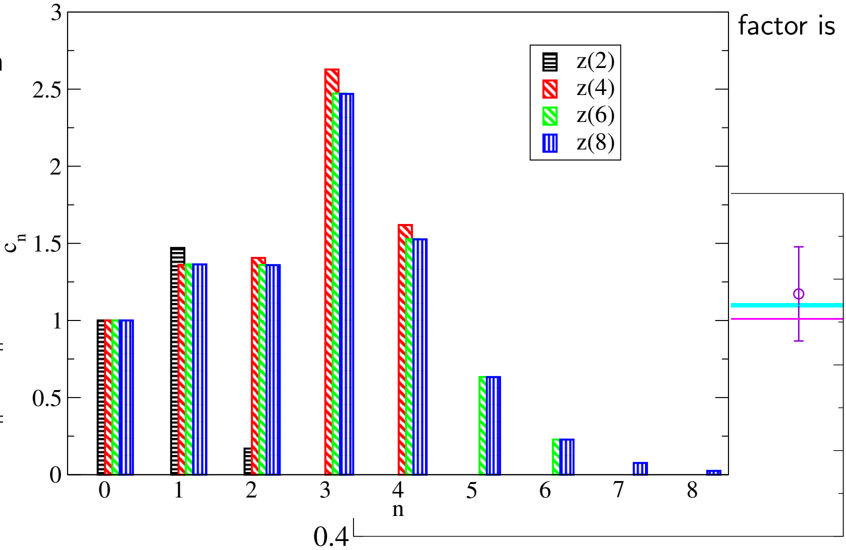

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or

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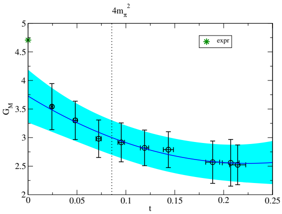
Figure: RMS ,experiment values form PDG

Numerical results (magnetic form factor)



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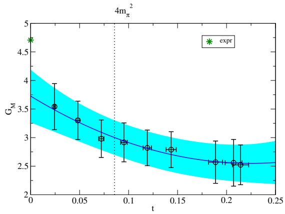

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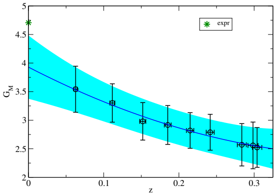

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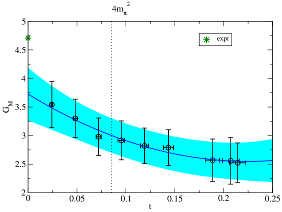

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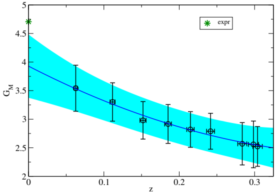

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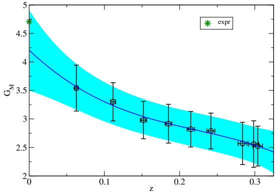

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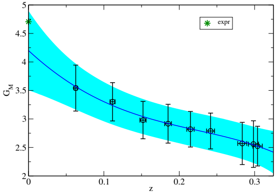
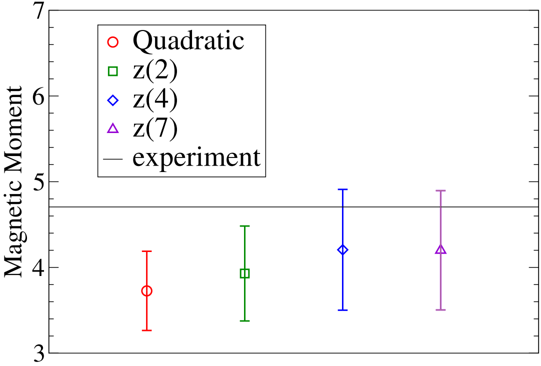


Figure: z expansion (7th power)

Numerical results(magnetic moment)

We can determine the isovector magnetic moment from the relation $G_M(0) = \mu_p - \mu_n$, which should be evaluated by extrapolation of the data given at finite t toward $t = 0$.
Z-Expansion method tends to give more consistent results with the experimental value.



Summary

We analyzed the data of nucleon form factors calculated in 2+1 flavor QCD near the physical point by the PACS collaboration.
We study the form factor shape with a model-independent analysis based on the z-Expansion method.

- confirm that coefficients of Z-Expansion show good convergence behavior.
- reduce systematic uncertainties during fits using Z-Expansion in both cases of interpolation and extrapolation of the form factor data as a function of z.

As a result, we get

- RMS of charge radius from isovector electric form factor
- magnetic moment from isovector magnetic form factor

  which are consistent with experimental values.