## HIDDEN-CHARM PENTAQUARK STATES IN THE QCD SUM RULE

## HUA-XING CHEN

BEIHANG UNIVERSITY

## ONTENTS

- Experimental status of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$
- Experimental status of other multiquark states
- The history of multiquark states
- Identifying exotic hidden-charm pentaquarks

Rui Chen, Xiang Liu, Xue-Qian Li, Shi-Lin Zhu
Method: one pion exchange (OPE) model

- Towards exotic hidden-charm pentaquarks in QCD

Hua-Xing Chen, Wei Chen, Xiang Liu, T. G. Steele, Shi-Lin Zhu
Method: QCD sum rule

## Experimental status of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$

|일 Selected for a Viewpoint in Physics

## $0^{5}$

Observation of $J / \psi p$ Resonances Consistent with Pentaquark States in $\Lambda_{b}^{0} \rightarrow \boldsymbol{J} / \boldsymbol{\psi} \boldsymbol{K}^{-} \boldsymbol{p}$ Decays

R. Aaij et al. ${ }^{*}$
(LHCb Collaboration)



FIG. 1 (color online). Feynman diagrams for (a) $\Lambda_{b}^{0} \rightarrow J / \psi \Lambda^{*}$ and (b) $\Lambda_{b}^{0} \rightarrow P_{c}^{+} K^{-}$decay.

## The measured invariant mass spectra




FIG. 2 (color online). Invariant mass of (a) $K^{-} p$ and (b) $J / \psi p$ combinations from $\Lambda_{b}^{0} \rightarrow J / \psi K^{-} p$ decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.


FIG. 4 (color online). Invariant mass spectrum of $J / \psi K^{-} p$ combinations, with the total fit, signal, and background components shown as solid (blue), solid (red), and dashed lines, respectively.

## LHCb performed the analysis of the above experimental data



About CERN
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- The LHCb experiment at CERN's Large Hadron Collider has reported the discovery of a class of particles known as pentaquarks.

Posted by Corinne Pralavorio on 14 Jul 2015. Last updated 14 Jul 2015, 10.19. Voir en français


Possible layout of the quarks in a pentaquark particle. The five quarks might be tightly bound (left). They might also be assembled into a meson (one quark and one antiquark) and a baryon (three quarks), weakly bound together (Image: Daniel Dominguez)

## A summary of the observed XYZ states

| ${ }^{\text {Enx }}$ (3872 ${ }^{\text {a }}$ | $Y(4260)$ | $X(3940)$ | $X(3915)$ | $Z_{b}(10610)$ |
| :---: | :---: | :---: | :---: | :---: |
| $Y(3940)$ | $Y(4008)$ | $X(4160)$ | $X(4350)$ | $Z_{b}(10650)$ |
| $Z^{+}(4430)$ | $Y(4360)$ | - | Z(3930) | $Z_{c}(3900)$ |
| $Z^{+}(4051)$ | $Y(4660)$ | - | - | $Z_{c}(4025)$ |
| $Z^{+}$(4248) | $Y(4630)$ | - | - | $Z_{c}(4020)$ |
| $Y(4140)$ | - | - | - | $Z_{c}(3885)$ |
| $Y(4274)$ | - | - | - | - |

In past decade, more and more XYZ states have been reported by experiments

BaBar, Belle, CDF, D0, CLEOc, LHCb, CMS, BESIII


BESIII

## -Theoretical studies

- Some earlier studies
- Studies at the hadron level
one-boson exchange model
- Studies at the quark-gluon level

QCD sum rule

## The history of multiquark states



Volume 8, number 3

Phys.Lett. 8 (1964) 214-215

A SCHEMATIC MODEL OF BARYONS AND MESONS *

## M. GELL-MANN

California Institute of Technology, Pasadena, California

## Received 4 January 1964

: -
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon $b$ if we assign to the triplet $t$ the following properties: spin $\frac{1}{2}, z=-\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}, d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) $q$ and the members of the anti-triplet as_anti-quarks $\bar{q}$. Baryons can now be constructed fron quarks by using the combinations (qqq), (qqqqī) etc., while mesons are made out of ( $\mathrm{q} \overline{\mathrm{q}})$, ( $\mathrm{q} q \overline{\mathrm{q}} \overline{\mathrm{q}})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations 1,8 , and 10 that have been observed, while

8419/TH. 412
21. February 1964

AN $\mathrm{SU}_{3}$ MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING II *)
G. Zẇeig

CERN.-.Geneva
*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.
6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A} A A A A, \overline{A A A A A A A}$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formied from $\overline{A A}, \overline{\mathrm{AA} A}$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A} A$ and $A A A$, that is, "deuces and treys".

## The muliquark states were predicted at the birth of Quark Model

## Quark Model



# Multiquark hadrons．I．Phenomenology of $Q^{2} \bar{Q}^{2}$ mesons＊ 

## R．J．Jaffe ${ }^{\dagger}$

Stanford Linear Accelerator Center，Stanford University，Stanford，California 94305 and Laboratory for Nuclear Science and Department of Physics，Massachusetts Institute of Technology，Cambridge，Massachusetts 02139 （Received 15 July 1976）

The spectra and dominant decay couplings of $Q^{2} \bar{Q}^{2}$ mesons are presented as calculated in the quark－bag model．Certain known $0^{+}$mesons $\left[\epsilon(700), S^{*}, \delta, \kappa\right]$ are assigned to the lightest cryptoexotic $Q^{2} \bar{Q}^{2}$ nonet．The usual quark－model $0^{+}$nonet（ $Q \bar{Q} L=1$ ）must lie higher in mass．All other $Q^{2} \bar{Q}^{2}$ mesons are predicted to be broad，heavy，and usually inelastic in formation processes．Other $Q^{2} \bar{Q}^{2}$ states which may be experimentally prominent are discussed．

## The hadron with four quarks plus one antiquark was developed by Strottman in 1979

# Multiquark baryons and the MIT bag model 

## D．Strottman

Theoretical Division，Los Alamos Scientific Laboratory，University of California，Los Alamos，New Mexico 87545
（Received 4 December 1978）
The calculation of masses of $q^{4} \bar{q}$ and $q^{5} q^{2}$ baryons is carried out within the framework of Jaffe＇s approximation to the MIT bag model．A general method for calculating the necessary $\mathrm{SU}(6) \supset \mathrm{SU}(3) \otimes \mathrm{SU}(2)$ coupling coefficients is outlined and tables of the coefficients necessary for $q^{4} \bar{q}$ and $q^{5} \bar{q}^{2}$ calculations are given．An expression giving the decay amplitude of an arbitrary multiquark state to arbitrary two－body final states is given in terms of $\operatorname{SU}(3)$ Racah and $9-\lambda \mu$ recoupling coefficients．The decay probabilities for low－ lying $1 / 2^{-} q^{4} \bar{q}$ baryons are given and compared with experiment．All low－lying $1 / 2^{-}$baryons are found to belong to the same $\operatorname{SU}(6)$ representation and all known $1 / 2^{-}$resonances below 1900 MeV may be accounted for without the necessity of introducing $P$－wave states．The masses of many exotic states are predicted including a $1 / 2^{-} Z_{0}^{*}$ at 1650 MeV and $1 / 2^{-}$hypercharge -2 and +3 states at 2.25 and 2.80 GeV ， respectively．The agreement with experiment for the $3 / 2^{-}$and $5 / 2^{-}$baryons is less good．The lowest $q^{5} \bar{q}^{2}$ state is predicted to be a $1 / 2^{+} \Lambda^{*}$ at 1900 MeV ．

## The name pentaquark was first proposed by Lipkin in 1987



New Possibilities for Exotic Hadrons - Anticharmed Strange Baryons*
Harry J. Lipkin
Department of Nuclear Physics Weizmann Institute of Science 76100 Rehovot, Israel Submitted to Physics Letters

May 20, 1987

## ABSTRACT

$$
Y=2 \text { STATES IN SU(6) THEORY* }
$$

Freeman J. Dyson $\dagger$ and Nguyen-Huu Xuong Department of Physics, University of California, San Diego, La Jolla, California (Received 30 November 1964)

Two-baryon states. - The SU(6) theory of strongly interacting particles ${ }^{\mathbf{1}, 2}$ predicts a classification of two-baryon states into multiplets according to the scheme

$$
\begin{equation*}
\underline{56} \otimes \underline{56}=\underline{462} \oplus \underline{1050} \oplus \underline{1134} \oplus \underline{490} . \tag{1}
\end{equation*}
$$

We now propose the hypothesis that all lowlying resonant states of the two-baryon system belong to the 490 multiplet. ${ }^{3}$ This means that six zero-strangeness states shown in Table I should be observed. In all these states odd $T$ goes with even $J$ and vice versa.

## Prediction of narrow $N^{*}$ and $\Lambda^{*}$ resonances with hidden charm above 4 GeV

Jia-Jun Wu ${ }^{1,2}$, R. Molina ${ }^{2,3}$,<br>E. Oset ${ }^{2,3}$ and B. S. Zou ${ }^{1,3}$<br>1. Institute of High Energy Physics, CAS, Beijing 100049, China<br>2. Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC,<br>Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain<br>3. Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China

(Dated: June 25, 2010) arXiv:1007.0573


$$
\begin{aligned}
& \mathcal{L}_{V V V}=i g\left\langle V^{\mu}\left[V^{\nu}, \partial_{\mu} V_{\nu}\right]\right\rangle \\
& \mathcal{L}_{P P V}=-i g\left\langle V^{\mu}\left[P, \partial_{\mu} P\right]\right\rangle \\
& \mathcal{L}_{B B V}=g\left(\left\langle\bar{B} \gamma_{\mu}\left[V^{\mu}, B\right]\right\rangle+\left\langle\bar{B} \gamma_{\mu} B\right\rangle\left\langle V^{\mu}\right\rangle\right) \\
& \text { (0, -1) } \\
& 4213 \\
& D_{s} \Lambda_{c}^{+} \\
& 1.37 \\
& 3.25 \\
& 0 \\
& D \Xi_{c}^{\prime} \\
& 4403 \\
& 0 \\
& 0 \\
& 2.64
\end{aligned}
$$

TABLE III: Pole positions $z_{R}$ and coupling constants $g_{a}$ for the states from $P B \rightarrow P B$.

| $(I, S)$ | $z_{R}(\mathrm{MeV})$ |  | $g_{a}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $(1 / 2,0)$ |  | $\bar{D}^{*} \Sigma_{c}$ | $\bar{D}^{*} \Lambda_{c}^{+}$ |  |
|  | 4418 | 2.75 | 0 |  |
| $(0,-1)$ |  | $\bar{D}_{s}^{*} \Lambda_{c}^{+}$ | $\bar{D}^{*} \Xi_{c}$ | $\bar{D}^{*} \Xi_{c}^{\prime}$ |
|  | 4370 | 1.23 | 3.14 | 0 |
|  | 4550 | 0 | 0 | 2.53 |

TABLE IV: Pole position and coupling constants for the bound states from $V B \rightarrow V B$.

# $\mathcal{L}_{\mathcal{L}_{3} i} \quad$ Possible hidden－charm molecular baryons composed $\mathcal{L}_{\mathcal{B}_{3} i}$ of an anti－charmed meson and a charmed baryon <br> $\mathcal{L}_{\mathcal{B}_{6}}$ <br> YANG Zhong－Cheng（杨忠诚）${ }^{1}$ SUN Zhi－Feng（孙志峰）${ }^{2,4}$ HE Jun（何军 $)^{1,3 ; 1)}$ 

$\mathcal{L}_{\mathcal{B}_{6} i} \quad$ LIU Xiang $\left(\right.$ 刘翔 ${ }^{2,4,42)}$ ZHU Shi－Lin（朱世琳）${ }^{1 ; 3)}$

$$
\begin{aligned}
& V \lambda_{S} g_{V}\left\langle\overline{\mathcal{B}}_{c} \gamma^{\mu} \quad\right. \text { In this work, we have employed the OBE model to }
\end{aligned}
$$ study whether there exist the loosely bound hidden－ charm molecular states composed of an S－wave anti－

$\mathcal{L}_{\mathcal{B}_{6} \mathcal{B}_{6} \sigma}=-\ell_{S}\left\langle\overline{\mathcal{B}}_{6} \sigma \mathcal{B}_{6}\right\rangle$. charmed meson and an S－wave charmed baryon．Our numerical results indicate that there do not exist $\Lambda_{c} \bar{D}$ and $\Lambda_{c} \bar{D}^{*}$ molecular states due to the absence of bound state solution，which is an interesting observa－ tion in this work．Additionally，we notice the bound state solutions only for five hidden－charm states，i．e．， $\Sigma_{c} \bar{D}^{*}$ states with $I\left(J^{P}\right)=\frac{1}{2}\left(\frac{1}{2}^{-}\right), \frac{1}{2}\left(\frac{3}{2}^{-}\right) \frac{3}{2}\left(\frac{1}{2}^{-}\right), \frac{3}{2}\left(\frac{3}{2}^{-}\right)$ and $\Sigma_{c} \bar{D}$ state with $\frac{3}{2}\left(\frac{1}{2}^{-}\right)$．We also extend the same

- There hidden-charm pentaquarks are studied in the chiral unitary appraoch:
J. J.Wu, R.Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010)
T. Uchino, W. H. Liang and E. Oset, arXiv: 1504.05726
- Especially, the hidden-charm molecular baryons of $I\left(J^{P}\right)=\frac{1}{2}\left(\frac{3^{-}}{2}\right)$ were first investigated and predicted to exist within the one boson exchange model in
Z. C. Yang, Z. F. Sun, J. He, X. Liu and S. L. Zhu, Chin. Phys. C 36, 6 (2012)
- More references:


## chiral quark model

## hyperfine interaction

photoproduction
kaon-induced reaction
isospin-exchange attraction

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M. Karliner and J. L. Rosner, arXiv:1506.06386

## Identify exotic hidden-charm pentaquarks

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| PHYSICAL REVIEW LETTERS <br> moving physics forward <br> Highlights <br> Recent <br> Accepted <br> Authors <br> Referees <br> Sear |  |  |
|  | EDITORS' SUGGESTION <br> Identifying Exotic HiddenCharm Pentaquarks <br> The pentaquarks discovered by the LHCb Collaboration could be molecular bound states of a charmed baryon and a meson. Observing the predicted isospin partners would allow for this interpretation to be verified. <br> Rui Chen, Xiang Liu, Xue-Qian Li, and Shi-Lin Zhu <br> Phys. Rev. Lett. 115, 132002 (2015) |  |

## The peculiarity of two Pc states:

- The masses of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$ are close to the $\boldsymbol{\Sigma}_{\mathrm{c}} \mathbf{( 2 4 5 5 ) \mathrm { D } ^ { * }}$ and $\boldsymbol{\Sigma c}^{\mathrm{c}}{ }^{*}(\mathbf{2 5 2 0}) \mathrm{D}^{*}$ thresholds, respectively.
- According to their final state $\mathrm{J} / \psi+\mathrm{p}$, we conclude that the two observed Pc must not be an isosinglet state, and the two $\mathrm{P}_{\mathrm{c}}$ states contain hidden-charm quantum numbers.
- The discovery of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$ inspires us interest in revealing their underlying structures under molecular state assignment

The corresponding flavor wave functions
| | 3

$$
\frac{\left|\frac{1}{2}, \frac{1}{2}\right\rangle=\sqrt{\frac{2}{3}}\left|\Sigma_{c}^{(*)++} D^{*-}\right\rangle-\frac{1}{\sqrt{3}}\left|\Sigma_{c}^{(*)+} \bar{D}^{* 0}\right\rangle}{\left.\frac{1}{2},-\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}\left|\Sigma_{c}^{(*)+} D^{*-}\right\rangle-\sqrt{\frac{2}{3}}\left|\Sigma_{c}^{(*) 0} \bar{D}^{* 0}\right\rangle,}
$$

## These favor wave

 functions with l=1/2 match the discussed $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$We need to perform a dynamical calculation of the structures of
$\mathrm{Ec}(2455) \mathrm{D}^{*}$ and $\mathrm{Ec}^{*}(2520) \mathrm{D}^{*}$

## One pion exchange (OPE) model

## Deuteron: loosely bound state of proton and neutron Nucleon force: short-range, mid-range, long-range

$\varrho$ and $\omega$ exchanges

Scalar $\sigma$ with mass around 600 MeV

The coupling of $\pi$ with nucleons reads

$$
\mathcal{L}=g_{N N \pi} \bar{\psi} i \gamma_{5} \tau \psi \cdot \pi,
$$

the non-relativistic nucleon-nucleon potential via $\pi$ meson exchange can be obtained as

$$
V_{\pi}=\frac{g_{N N \pi}^{2}}{4 \pi} \frac{m_{\pi}^{2}}{12 m_{N}^{2}}\left(\boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2}\right)\left\{\sigma_{1} \cdot \boldsymbol{\sigma}_{2}+\left[\frac{3\left(\boldsymbol{\sigma}_{\mathbf{1}} \cdot \boldsymbol{r}\right)\left(\boldsymbol{\sigma}_{2} \cdot \boldsymbol{r}\right)}{r^{2}}-\sigma_{\mathbf{1}} \cdot \boldsymbol{\sigma}_{\mathbf{2}}\right]\left[1+\frac{3}{m_{\pi} r}+\frac{3}{m_{\pi}^{2} r^{2}}\right]\right\} \frac{e^{-m_{\pi} r}}{r}
$$

## In the past decade, one boson exchange was extensively applied to the studies of newly observed hadron states

## Long list:

```
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```


## One conclusion:

Pion exchange play crucial role to form heavy flavor molecular states

## It is the reason why we adopt one pion exchange model to study two Pc states

## The effective Lagrangian relevant to the deduction of OPE potential:

$$
\begin{gathered}
\mathcal{L}_{\bar{D}^{*} \bar{D}^{*} \mathbb{P}}=i \frac{2 g}{f_{\pi}} v^{\alpha} \varepsilon_{\alpha \mu \nu \lambda} \bar{D}_{a}^{* \mu \dagger} \bar{D}_{b}^{* \lambda} \partial^{\nu} \mathbb{P}_{a b}, \\
\mathcal{L}_{\mathcal{B}_{6} \mathcal{B}_{6} \mathbb{P}}=i \frac{g_{1}}{2 f_{\pi}} \varepsilon^{\mu \nu \lambda \kappa} v_{\kappa} \operatorname{Tr}\left[\overline{\mathcal{B}}_{6} \gamma_{\mu} \gamma_{\lambda} \partial_{\nu} \mathbb{P} \mathcal{B}_{6}\right], \\
\mathcal{L}_{\mathcal{B}_{6}^{*} \mathcal{B}_{6}^{*} \mathbb{P}}=-i \frac{3 g_{1}}{2 f_{\pi}} \varepsilon^{\mu \nu \lambda \kappa} v_{\kappa} \operatorname{Tr}\left[\overline{\mathcal{B}}_{6 \mu}^{*} \partial_{\nu} \mathbb{P} \mathcal{B}_{6 \nu}^{*}\right],
\end{gathered}
$$

where $g=0.59 \pm 0.07 \pm 0.01$ is extracted from the width of $D^{*}$ [25] as is done in Ref. [26], and $g_{1}=0.94$ was fixed in Refs. [12,24].
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## Scattering amplitude

 momentum spaceThe effective potential in

## $\sqrt{2 E_{A}} \sqrt{2 E_{B}} \sqrt{2 E_{C}} \sqrt{2 E_{D}}$. <br> Fourier transformation <br> $V(\mathbf{r}) \rightarrow$ The effective potential in coordinate space

## The effective potentials of $\Sigma_{c}(2455) D^{*}$ and $\Sigma_{c}{ }^{*}(2520) D^{*}$ systems

$$
\begin{aligned}
& V_{\Sigma_{c} \bar{D}^{*}}(r)=\frac{1}{3} \frac{g g_{1}}{f_{\pi}^{2}} \nabla^{2} Y\left(\Lambda, m_{\pi}, r\right) \mathcal{J}_{0} \mathcal{G}_{0}, \\
& V_{\Sigma_{c}^{*} \bar{D}^{*}}(r)=\frac{1}{2} \frac{g g_{1}}{f_{\pi}^{2}} \nabla^{2} Y\left(\Lambda, m_{\pi}, r\right) \mathcal{J}_{1} \mathcal{G}_{1},
\end{aligned}
$$

$$
Y(\Lambda, m, r)=\frac{1}{4 \pi r}\left(e^{-m r}-e^{-\Lambda r}\right)-\frac{\Lambda^{2}-m^{2}}{8 \pi \Lambda} e^{-\Lambda r} .
$$

TABLE I. The values of the $\mathcal{J}_{i}$ and $\mathcal{G}_{i}$ coefficients. Here, $S, L$, and $J$ denote the spin, orbital, and total angular quantum numbers, respectively. $\mathbb{S}$ denotes $L=1$ since we are interested in the $S$-wave interaction of the $\Sigma_{c}(2455) \bar{D}^{*}$ and $\Sigma_{c}^{*}(2520) \bar{D}^{*}$ systems.

| $I$ | $\mathcal{G}_{0}$ | $\mathcal{G}_{1}$ | $\left.\left.\right\|^{2 S+1} L_{J}\right\rangle$ | $\mathcal{J}_{0}$ | $\mathcal{J}_{1}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $1 / 2$ | 1 | -1 | $\left.\left.\right\|^{2} \mathbb{S}_{(1 / 2)}\right\rangle$ | -2 | $5 / 3$ |
| $3 / 2$ | $-1 / 2$ | $1 / 2$ | $\left.\left.\right\|^{4} S_{(3 / 2)}\right\rangle$ | 1 | $2 / 3$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\left\|{ }^{6} \mathbb{S}_{(5 / 2)}\right\rangle$ | $\cdots$ | -1 |

## By solving Shrouding equation, we try to find bound state solutions

## Numerical results



FIG. 1 (color online). The variations of the obtained OPE effective potentials for the $\Sigma_{c}^{(*)} \bar{D}^{*}$ systems to $r$, and obtained bound state solutions. Here, the masses of $P_{c}(4380)$ and $P_{c}(4450)$ can be reproduced well under the $\Sigma_{c} \bar{D}^{*}$ with $(I=1 / 2$, $J=3 / 2)$ and $\Sigma_{c}^{*} \bar{D}^{*}$ with $(I=1 / 2, J=5 / 2)$ molecular assignments, respectively. $\Lambda=2.35 \mathrm{GeV}$ and $\Lambda=1.77 \mathrm{GeV}$ are taken for the $\Sigma_{c} \bar{D}^{*}$ and $\Sigma_{c}^{*} \bar{D}^{*}$ systems, respectively. The blue curves are the effective potentials, and the red line stands for the corresponding energy levers. Additionally, the obtained spatial wave functions are given here.

## Conclusions

- The masses of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$ can be reproduced
- $\operatorname{Pc}(4380)$ and $\operatorname{Pc}(4450)$ as $\Sigma_{c}(2455) D^{*}$ and $\Sigma_{c}{ }^{*}(2520) D^{*}$ molecular states with ( $\mathrm{I}=1 / 2, \mathrm{~J}=1 / 2$ ) and ( $\mathrm{l}=1 / 2, \mathrm{~J}=3 / 2$ ), respectively.
- Qualitatively explain why the width of $\mathrm{Pc}(4450)$ is much narrower than that of $\operatorname{Pc}(4380)$

$$
\begin{aligned}
& \text { S-wave: } \operatorname{Pc}(4380) \rightarrow>J / \Psi+p \\
& \text { D-wave: } \operatorname{Pc}(4450) \rightarrow>J / \Psi+p
\end{aligned}
$$

- Predict two isospin partners of $\mathrm{Pc}(4380)$ and $\mathrm{Pc}(4450)$ $\Sigma_{C}(2455) D^{*}$ with ( $I=3 / 2, \mathrm{~J}=1 / 2$ ) and binding energy -80 MeV $\Sigma^{*}{ }^{\star}(2520) \mathrm{D}^{*}$ with (I=3/2,J=1/2) and binding energy -28 MeV Decay modes: $\Delta(1232) J / \psi$ and $\Delta(1232) \eta_{c}$


## Our Study through QCD Sum Rule

## Towards exotic hidden-charm pentaquarks in QCD

Hua-Xing Chen ${ }^{1}$, Wei Chen ${ }^{2},{ }^{*}$ Xiang Liu ${ }^{3,4},{ }^{\dagger}$ T. G. Steele ${ }^{2},{ }^{\ddagger}$ and Shi-Lin Zhu ${ }^{5,6,78}$<br>${ }^{1}$ School of Physics and Nuclear Energy Engineering and International Research Center for Nuclei and Particles in the Cosmos, Beihang University, Beijing 100191, China<br>${ }^{2}$ Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, SK, S7N 5E2, Canada<br>${ }^{3}$ School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China<br>${ }^{4}$ Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China<br>${ }^{5}$ School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China<br>${ }^{6}$ Collaborative Innovation Center of Quantum Matter, Beijing 100871, China<br>${ }^{7}$ Center of High Energy Physics, Peking University, Beijing 100871, China

Inspired by the $P_{c}(4380)$ and $P_{c}(4450)$ recently observed by LHCb, a QCD sum rule investigation is performed, by which $P_{c}(4380)$ and $P_{c}(4450)$ can be identified as exotic hidden-charm pentaquarks composed of an anti-charmed meson and a charmed baryon. Our results suggest that the $P_{c}(4380)$ and $P_{c}(4450)$ states have quantum numbers $J^{P}=3 / 2^{-}$and $5 / 2^{+}$, respectively. As an important extension, the mass predictions of hiddenbottom pentaquarks are given. Searches for these partners of $P_{c}(4380)$ and $P_{c}(4450)$ is especially accessible at future experiments like LHCb .

## - $\mathbf{A}[J / \psi p]$ current

$$
\left[\bar{c}_{d}(x) \gamma_{\mu} c_{d}(x)\right]\left[\varepsilon^{a b c}\left(u_{a}^{T}(x) C d_{b}(x)\right) \gamma_{5} u_{c}(x)\right]
$$

flavor contents
color indices

Lorentz indices

## Two Configurations:

$$
\left[\bar{c}_{d} c_{d}\right]\left[\epsilon^{a b c} q_{a} q_{b} q_{c}\right] \text { and }\left[\bar{c}_{d} q_{d}\right]\left[\epsilon^{a b c} c_{a} q_{b} q_{c}\right]
$$

These two configurations, as if they are local, can be related to each other through

- The Fierz transformation

$$
\begin{aligned}
\left(\bar{s}_{a} u_{b}\right)\left(\bar{s}_{b} d_{a}\right)= & -\frac{1}{4}\left\{\left(\bar{s}_{a} u_{a}\right)\left(\bar{s}_{b} d_{b}\right)+\left(\bar{s}_{a} \gamma_{\mu} u_{a}\right)\left(\bar{s}_{b} \gamma^{\mu} d_{b}\right)+\frac{1}{2}\left(\bar{s}_{a} \sigma_{\mu \nu} u_{a}\right)\left(\bar{s}_{b} \sigma^{\mu \nu} d_{b}\right)\right. \\
& \left.-\left(\bar{s}_{a} \gamma_{\mu} \gamma_{5} u_{a}\right)\left(\bar{s}_{b} \gamma^{\mu} \gamma_{5} d_{b}\right)+\left(\bar{s}_{a} \gamma_{5} u_{a}\right)\left(\bar{s}_{b} \gamma_{5} d_{b}\right)\right\}
\end{aligned}
$$

- The color rearrangement

$$
\delta^{d e} \epsilon^{a b c}=\delta^{d a} \epsilon^{e b c}+\delta^{d b} \epsilon^{a e c}+\delta^{d c} \epsilon^{a b e}
$$

## Configuration $\left[\bar{c}_{d} c_{d}\right]\left[\epsilon^{a b c} q_{a} q_{b} q_{c}\right]$

- There are three independent local light baryon fields of flavor-octet and having a positive parity:
H. X. Chen, V. Dmitrasinovic, A. Hosaka, K. Nagata and S. L. Zhu, Phys. Rev. D 78, 054021 (2008)

$$
\begin{aligned}
N_{1}^{N} & =\epsilon_{a b c} \epsilon^{A B D} \lambda_{D C}^{N}\left(q_{A}^{a T} C q_{B}^{b}\right) \gamma_{5} q_{C}^{c}, \\
N_{2}^{N} & =\epsilon_{a b c} \epsilon^{A B D} \lambda_{D C}^{N}\left(q_{A}^{a T} C \gamma_{5} q_{B}^{b}\right) q_{C}^{c}, \\
N_{3 \mu}^{N} & =\epsilon_{a b c} \epsilon^{A B D} \lambda_{D C}^{N}\left(q_{A}^{a T} C \gamma_{\mu} \gamma_{5} q_{B}^{b}\right) \gamma_{5} q_{C}^{c},
\end{aligned}
$$

- Together with light baryon fields having negative parity and the charmonium fields:

$$
\begin{gathered}
\bar{c}_{d} c_{d}\left[0^{+}\right], \bar{c}_{d} \gamma_{5} c_{d}\left[0^{-}\right], \\
\bar{c}_{d} \gamma_{\mu} c_{d}\left[1^{-}\right], \bar{c}_{d} \gamma_{\mu} \gamma_{5} c_{d}\left[1^{+}\right], \bar{c}_{d} \sigma_{\mu v} c_{d}\left[1^{ \pm}\right],
\end{gathered}
$$

- We can construct the currents of the configuration $\left[\bar{c}_{d} c_{d}\right]\left[\epsilon^{a b c} q_{a} q_{b} q_{c}\right]$.
- Those containing $J=3 / 2$ components are $\left[\bar{c}_{d} c_{d}\right]\left[N_{3 \mu}^{N}\right],\left[\bar{c}_{d} \gamma_{5} c_{d}\right]\left[N_{3 \mu}^{N}\right],\left[\bar{c}_{d} \gamma_{\mu} c_{d}\right]\left[N_{1,2}^{N}\right]$, $\left[\bar{c}_{d} \gamma_{\mu} \gamma_{5} c_{d}\right]\left[N_{1,2}^{N}\right],\left[\bar{c}_{d} \gamma_{\mu} c_{d}\right]\left[N_{3 v}^{N}\right],\left[\bar{c}_{d} \gamma_{\mu} \gamma_{5} c_{d}\right]\left[N_{3 v}^{N}\right]$, $\left[\bar{c}_{d} \sigma_{\mu \nu} c_{d}\right]\left[N_{1,2}^{N}\right],\left[\bar{c}_{d} \sigma_{\mu \nu} c_{d}\right]\left[N_{3 \rho}^{N}\right]$,
- Three of them of $J=3 / 2 \& 5 / 2$ couple well to the combination of $J / \psi$ and proton

$$
\begin{aligned}
& \eta_{1 \mu}^{c \bar{c} u d d}=\left[\bar{c}_{d} \gamma_{\mu} c_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C d_{b}\right) \gamma_{5} u_{c}\right], \\
& \eta_{2 \mu}^{c \overline{c u u d}}=\left[\bar{c}_{d} \gamma_{\mu} c_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{5} d_{b}\right) u_{c}\right], \\
& \eta_{3 \mu \nu l}^{c \overline{c i u d}}=\left[\bar{c}_{d} \gamma_{\mu} c_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{v} \gamma_{5} d_{b}\right) u_{c}\right]+\{\mu \leftrightarrow \nu\} .
\end{aligned}
$$

## Configuration $\left[\bar{c}_{d} q_{d}\right]\left[\epsilon^{a b c} c_{a} q_{b} q_{c}\right.$ ]

2. Currents of $\left[\tau_{d} d_{d}\right]\left[e^{2} u_{s} d_{0} d_{c}\right]$

In this subsection, we construct the currents of the color

$\xi_{1 \mu}=\left[e^{\left.\left.a k\left(u_{a}^{T} C d_{b}\right) \gamma_{\mu} \gamma_{s} c_{c}\right] \bar{c}_{d} u_{d}\right]}\right.$
$\zeta_{z_{\mu}}=\mid \epsilon^{\left.a k k_{( }\left(u_{a}^{T} C d_{b}\right)_{\mu} \mu_{\mu} c_{c} \mid \bar{c}_{d} d y_{s} u_{d}\right]}$

$\xi_{4}=\left[e^{\left.\text {akx }\left(u_{a}^{T} c_{\gamma_{s}} d_{b}\right) \gamma_{\mu} \gamma_{s} c_{c} \| \bar{c}_{d} \gamma_{s} u_{d}\right] .}\right.$
$\xi_{*}=\left[e^{a k( }\left(u_{a}^{T} C d_{6}\right) \gamma s c_{c}\left[\| c_{d} d \gamma_{p} u_{1}\right]\right.$.
$\zeta_{\epsilon_{\psi}}=\left[e^{e k\left(u_{a}^{T} C d_{b}\right) c_{c}\left[c_{d} \gamma_{p} \gamma_{p} \gamma_{s} u_{d}\right]}\right.$
$\zeta_{\mu}=\left[\epsilon^{e l k}\left(u_{a}^{T} c_{\gamma s} d d_{b}\right) c_{c}\left[\tilde{c}_{\mu} c_{\gamma_{p}} \psi_{d}\right]\right.$,

$\xi_{s}=\left[e^{a k( }\left(u_{a}^{T} C d_{b}\right) \sigma_{\mu} \gamma \gamma_{s} c_{\|} \| c_{c} d \gamma u_{d}\right]$,
$\xi_{19 \psi}=\left[e^{e k\left(u_{a}^{T}\right.}\left(u_{d} C d_{b}\right) \sigma_{\mu} c_{c}\left[\bar{c}_{c} \gamma_{2} \gamma_{s} u_{d}\right]\right.$.


$\xi_{13}=\left[e^{e l k}\left(u_{a}^{T} C d_{b}\right) \gamma \gamma_{y} r_{c} c_{[ }\left[\bar{c}_{d} \sigma_{p} u_{d}\right]\right.$,
$\xi_{14}=\left[e^{e k x}\left(u_{d}^{T} C d_{b}\right) \gamma_{v} c_{c} \| c_{d} \sigma_{p} \gamma_{s} u_{d}\right]$,
$\xi_{1 L_{p}}=\left\lfloor\ell^{a l k}\left(u_{a}^{T} c \gamma_{y} d_{b}\right) \gamma_{1} c_{c} \| c_{d} \sigma_{p} u_{d}\right]$.



$\xi_{19 \mu}=\left[e^{a k\left(u_{a}\right.} u_{a}^{T} \gamma_{\mu} \gamma_{y} \gamma_{d} d_{b}\right) c_{c}\left[\bar{c}_{d} u_{d}\right]$.

$\xi_{2 l \mu}=\left[e^{e k( }\left(u_{a}^{T} C_{\gamma} d_{b}\right) \sigma_{\mu} \gamma_{s} \gamma_{c}\left[c_{d} u_{d}\right]\right.$,

$\zeta_{23}=\left[\epsilon^{\left.\text {akc }\left(u_{d}^{T} C_{\gamma} \gamma_{\gamma} \gamma_{s} d_{b}\right) \sigma_{\mu} c_{c} \| c_{d} u_{d}\right] .}\right.$




$\epsilon_{2 s_{4}}=\left[\epsilon^{\left.\text {elk }\left(u_{d}^{T} C \gamma_{\mu} \gamma_{y} d_{b}\right)_{\gamma} \gamma_{\gamma} \gamma_{s} c_{c} \| \bar{c}_{d} \gamma_{\gamma} \gamma_{s} u_{d}\right]}\right.$.


$\epsilon_{31 \mu}=\left[e^{a t k}\left(u_{u}^{T} C_{\gamma} C_{\gamma} d_{d} d_{b} \gamma_{\mu} c_{c}\right]\left[\bar{c}_{d \gamma}, u d\right]\right.$.
$\epsilon_{3 z_{4}}=\left[\epsilon^{\left.e t k\left(u_{d}^{T} C \gamma_{y} \gamma_{\gamma} d_{b}\right) \gamma_{\mu} \gamma_{s} c_{c}\right]\left[\tilde{c}_{d} \gamma_{y} \gamma_{s} u_{d}\right] .}\right.$




$\xi_{ग 刃 \mu}=\left[\epsilon^{a k( }\left(u_{d}^{T} C \gamma_{,} d_{b}\right)_{s} c_{c} \|\left[\bar{c}_{d} \sigma_{\mu} u_{d} u_{d}\right]\right.$.

$\xi_{3 \mu}=\left[\epsilon^{\left.\epsilon^{k}\left(w_{a}^{T} C \gamma_{\gamma} \gamma_{s} d_{b}\right) c_{c l \mid} \bar{c}_{d} \sigma_{\mu} \sigma_{d}\right]}\right.$.














 $\xi_{s s \psi}=\left[\epsilon^{d \tau}\left(u_{a}^{T} C \sigma_{\mu \mu} \gamma_{s} d_{b}\right)_{y} \gamma_{v} \gamma_{s} c_{c} \| \bar{c}_{d} \gamma_{s} u_{d}\right]$.



 $\xi_{G 1 \mu}=\left[\epsilon^{d \tau}\left(u_{a}^{T} C \sigma_{p} d_{b} d_{b}\right) \sigma_{v p} \gamma_{3} c_{c} \mid \bar{c}_{d} \gamma_{f} \mu_{d}\right]$.













 $\zeta_{T \sigma_{\psi}}=\left[e^{e c_{c}\left(u_{d}^{T} u_{d} \sigma_{p} \sigma_{s} \gamma_{d} d_{b} \gamma_{p} \gamma_{s} c_{c} \| \tau_{d} \sigma_{p p} \gamma_{s} u_{d}\right]}\right.$





- The currents of this type are more complicated.
- The physical states are probably their mixings.
- Some currents may well couple to the physical states, but the problem is that we do not know this at the beginning.
- Hence, we select some of them to perform the QCD sum rule to see whether we can obtain reliable/stable sum rule results.
- In the present work we selected altogether 30 currents.


## Configuration $\left[\bar{c}_{d} q_{d}\right]\left[\epsilon^{a b c} c_{a} q_{b} q_{c}\right.$ ]

- The currents of this type can not be systematically constructed so easily, so we just transform the previous currents to this configuration, and select those related to $D / D^{*}$ and $\Lambda_{c} / \Sigma_{c} / \Sigma_{c}^{*}$.
- We shall investigate the following currents of $J=3 / 2$

$$
\begin{aligned}
J_{\mu}^{\bar{D}^{*} \Sigma_{c}} & =\left[\bar{c}_{d} \gamma_{\mu} d_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{v} u_{b}\right) \gamma^{v} \gamma_{5} c_{c}\right], \\
J_{\mu}^{\bar{D} \Sigma_{c}^{*}} & =\left[\bar{c}_{d} \gamma_{5} d_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{\mu} u_{b}\right) c_{c}\right],
\end{aligned}
$$

- We shall investigate the following currents of $J=5 / 2$

$$
\begin{aligned}
J_{\{\mu \nu\rangle}^{\bar{D}^{*} \Sigma_{c}^{*}} & =\left[\bar{c}_{d} \gamma_{\mu} d_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{\nu} u_{b}\right) \gamma_{5} c_{c}\right]+\{\mu \leftrightarrow v\}, \\
J_{\{\mu v\}}^{\bar{D} \Sigma_{c}^{*}} & =\left[\bar{c}_{d} \gamma_{\mu} \gamma_{5} d_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{v} u_{b}\right) c_{c}\right]+\{\mu \leftrightarrow v\}, \\
J_{\{\mu \nu\}}^{\bar{D}^{*} \Lambda_{c}} & =\left[\bar{c}_{d} \gamma_{\mu} u_{d}\right]\left[\epsilon_{a b c}\left(u_{a}^{T} C \gamma_{v} \gamma_{5} d_{b}\right) c_{c}\right]+\{\mu \leftrightarrow v\},
\end{aligned}
$$

## QCD SUM RULE

- In sum rule analyses, we consider two-point correlation functions:

$$
\begin{aligned}
\Pi\left(q^{2}\right) & \stackrel{\text { def }}{=} i \int d^{4} x e^{i q x}\langle 0| \mathrm{T} \eta(x) \eta^{+}(0)|0\rangle \\
& \approx \sum_{\mathrm{n}}\langle 0| \eta|\mathrm{n}\rangle\langle\mathrm{n}| \eta^{+}|0\rangle
\end{aligned}
$$

where $\eta$ is the current which can couple to hadronic states.

- By using the dispersion relation, we can obtain the spectral density

$$
\Pi\left(q^{2}\right)=\int_{s_{<}}^{\infty} \frac{\rho(s)}{s-q^{2}-i \varepsilon} d s
$$

- In QCD sum rule, we can calculate these matrix elements from QCD (OPE) and relate them to observables by using dispersion relation.



## Quark and Gluon Level

(Convergence of OPE)

$$
\Pi_{\mathrm{OPE}}\left(\mathrm{q}^{2}\right) \frac{\text { dispersion relation }}{s=-q^{2}}
$$

## Hadron Level

$$
\Pi_{p h y s}\left(q^{2}\right)=f_{P}^{2} \frac{d+M}{q^{2}-M^{2}} \longleftrightarrow \rho_{\text {phys }}(s)=\lambda_{x}^{2} \delta\left(s-M_{x}^{2}\right)+\cdots
$$

(for baryon case)

## Quark-Hadron Duality

(Sufficient amount of Pole contribution)


## -Parity of Pentaquark

- Assuming $J$ is a pentaquark current, $\gamma_{5} J$ is its partner having the opposite parity.
- They can couple to the same physical state through

$$
<0|J| P(q)>=f_{P} u(q), \quad<0\left|\gamma_{5} J\right| P(q)>=f_{P} \gamma_{5} u(q)
$$

- The same pentaq ark current $J$ can couple to states $f$ both positive and negative parities through

$$
<0|J| P(q)>=f_{P} u(q)
$$

$$
<0|J| P^{\prime}(q)>=f_{F} \gamma_{5} u^{\prime}(q)
$$

where $\mid P(q)>$ has the same parity as $J$, while $\left|P^{\prime}(q)>\right|$ as the opposite parity.

$$
f_{P}^{2} \frac{q+M}{q^{2}-M^{2}} \quad f_{P}^{2} \frac{-q q+M}{q^{2}-M^{2}}
$$

## QCD Sum Rule

- Borel transformation to suppress the higher order terms:

$$
\Pi\left(M_{B}^{2}\right) \equiv f^{2} e^{-M^{2} / M_{B}^{2}}=\int_{s_{<}}^{s_{0}} e^{-s / M_{B}^{2}} \rho(s) d s
$$

- Two parameters

$$
M_{B}, \quad s_{0}
$$

We need to choose certain region of $\left(M_{B}, s_{0}\right)$.

- Criteria

1. Stability
2. Convergence of OPE
3. Positivity of spectral density
4. Sufficient amount of pole contribution

The sum rule results obtained using $J_{\mu}^{\bar{D}^{*} \Sigma_{c}}(J=3 / 2)$ are

$$
M_{\left[\bar{D}^{*} \Sigma_{c}\right], 3 / 2^{-}}=4.37_{-0.12}^{+0.18} \mathrm{GeV}
$$



FIG. 1: The variation of $M_{\left[\bar{D}^{*} \Sigma_{c}\right], 3 / 2^{-}}$with respect to the threshold value $s_{0}$ (left) and the Borel mass $M_{B}$ (right). In the left figure, the

The sum rule results obtained using $J_{\{\mu \nu\}}^{\bar{D} \Sigma_{c}^{*}}$ and $J_{\{\mu \nu\}}^{\bar{D}^{*} \Lambda_{c}}$ are not useful. However, their mixing gives a reliable mass sum rule ( $J=5 / 2$ )
$J_{\{\mu \nu\}}^{\bar{D} \Sigma_{c}^{*} \& \bar{D}^{*} \Lambda_{c}}=\sin \theta \times J_{\{\mu \nu\}}^{\bar{D} \Sigma_{c}^{*}}+\cos \theta \times J_{\{\mu \nu\}}^{\bar{D}^{*} \Lambda_{c}}$
$\tan \theta=-1.25$

$$
M_{\left[\bar{D} \Sigma_{c}^{*} \& \bar{D}^{*} \Lambda_{c}\right], 5 / 2^{+}}=4.47_{-0.12}^{+0.19} \mathrm{GeV}
$$



FIG. 2: The variation of $M_{\left[\bar{D} \Sigma_{c}^{*} \& \bar{D}^{*} \Lambda_{c}\right], 5 / 2^{+}}$with respect to the threshold value $s_{0}$ (left) and the Borel mass $M_{B}$ (right).

## More: hidden-charm baryonium states



FIG. 5: Spectrum of hidden-charm baryonium states obtained using the method of QCD sum rules. The blue lines are obtained using the currents of Type D, and the red lines are obtained using the currents of Type F.

## Summary

We still need more theoretical and experimental joint efforts

## Thank you for your attention

