

Finite-Volume Hamiltonian Method for Baryon Spectrum

Jia-Jun Wu

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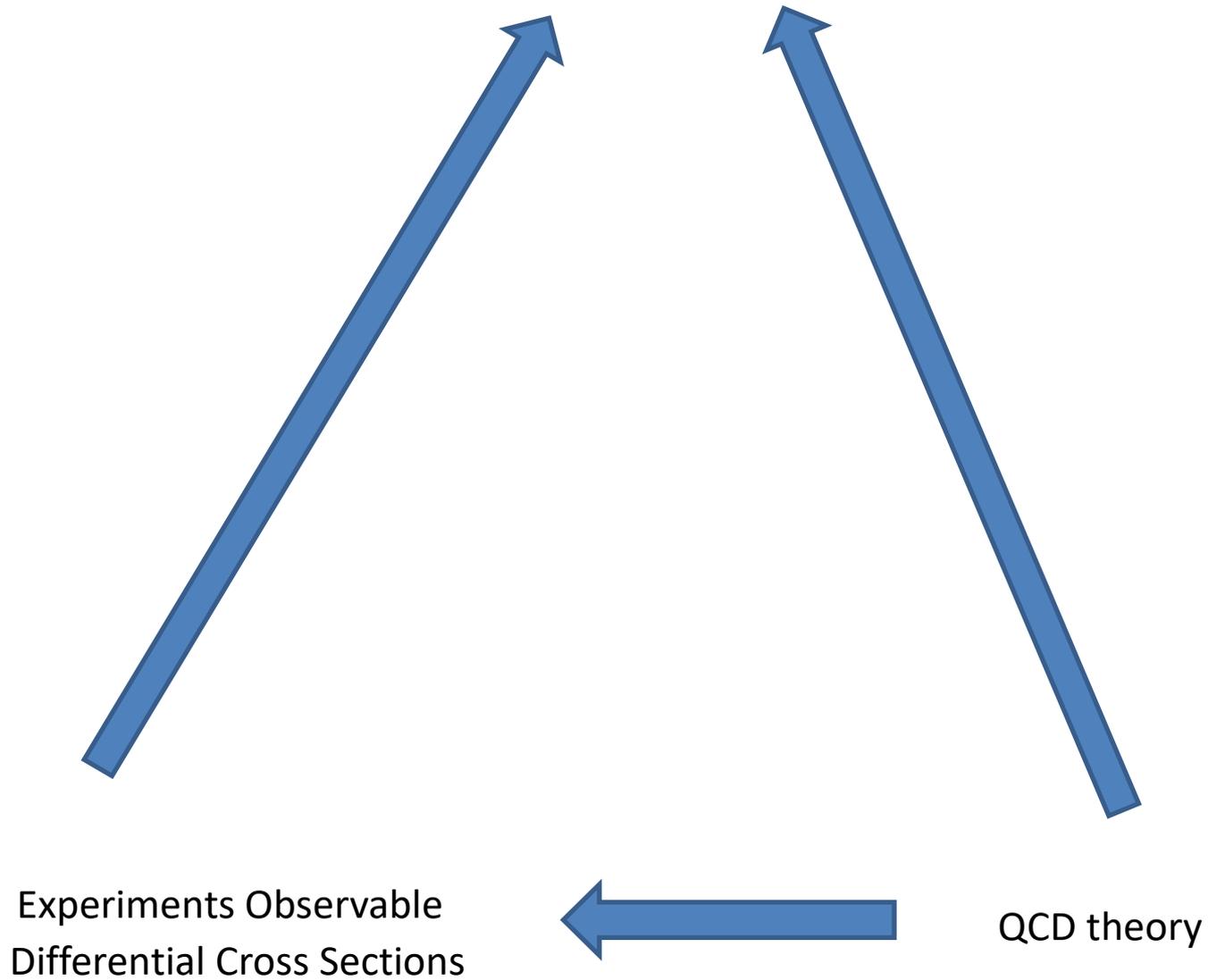
Centre for the Subatomic Structure of Matter (CSSM),
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2016 JAEA/ASRC Reimei(黎明) workshop

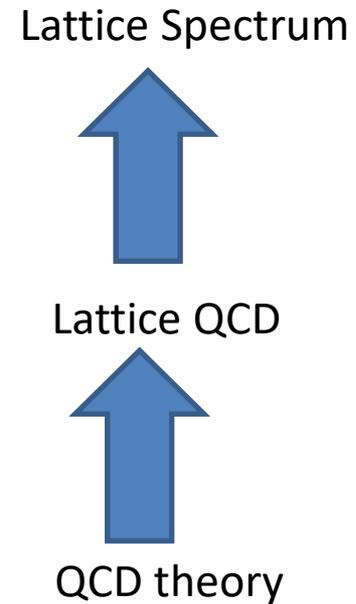
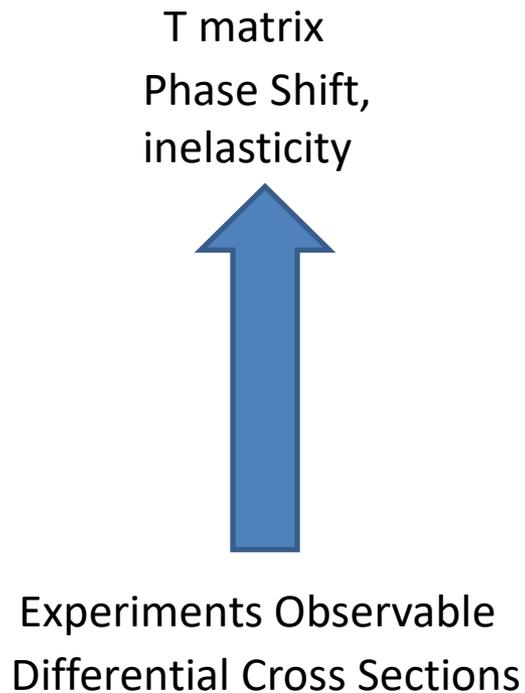
Outline

- Motivation
- Finite-Volume Hamiltonian Method
- Study of $N^*(1535)$
- Study of $\Lambda^*(1405)$
- Study of $N^*(1440)$

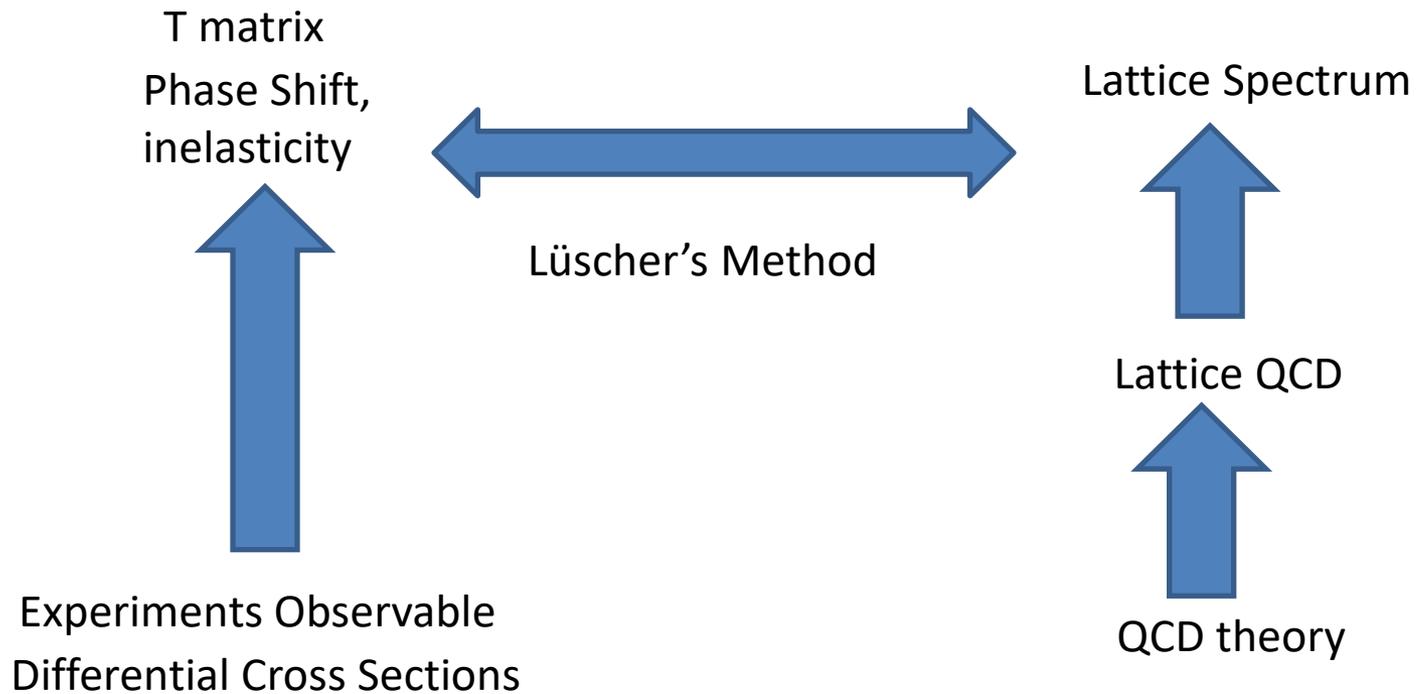
Resonance Properties: Mass, Width,
Pole position, Coupling, structure



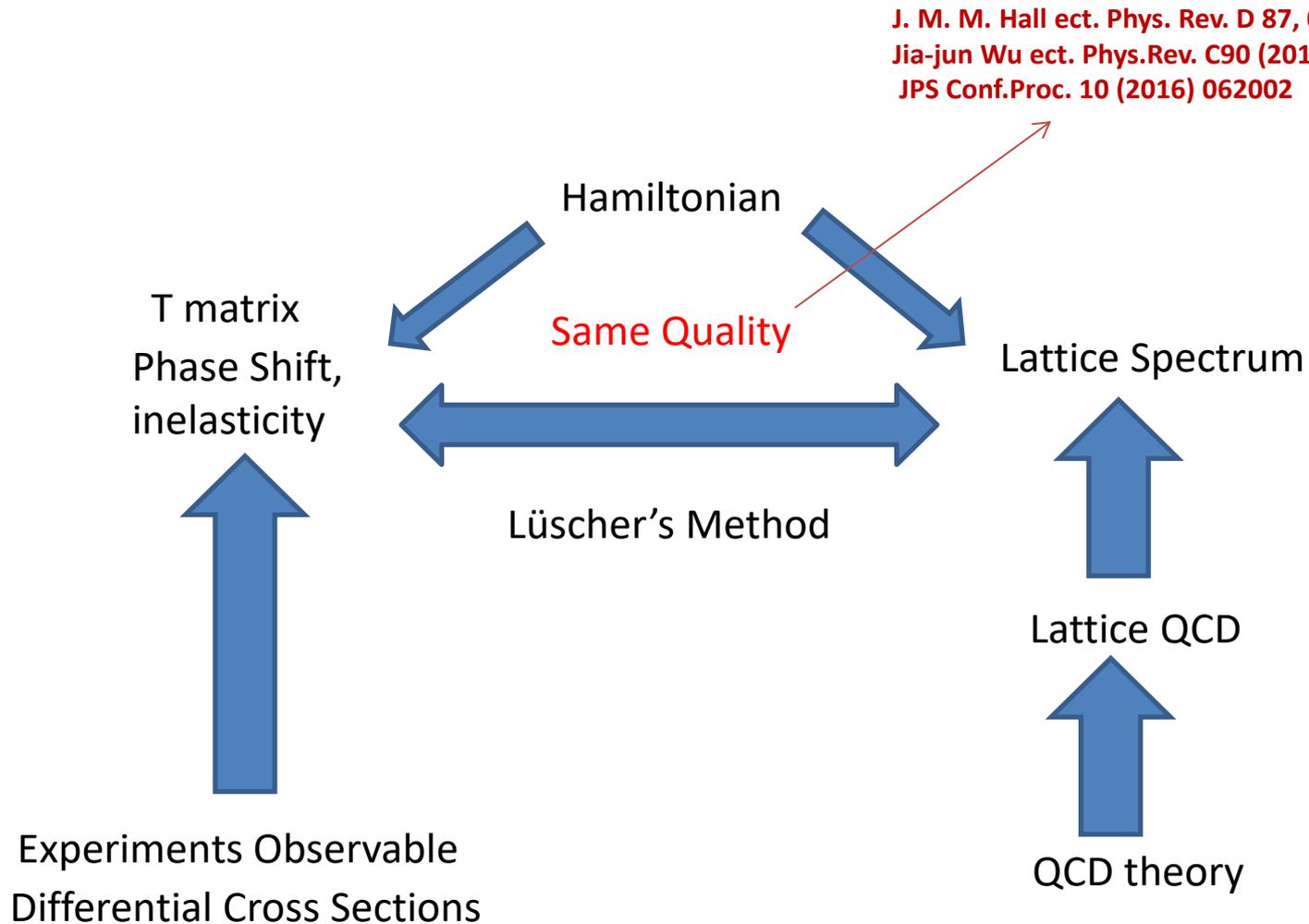
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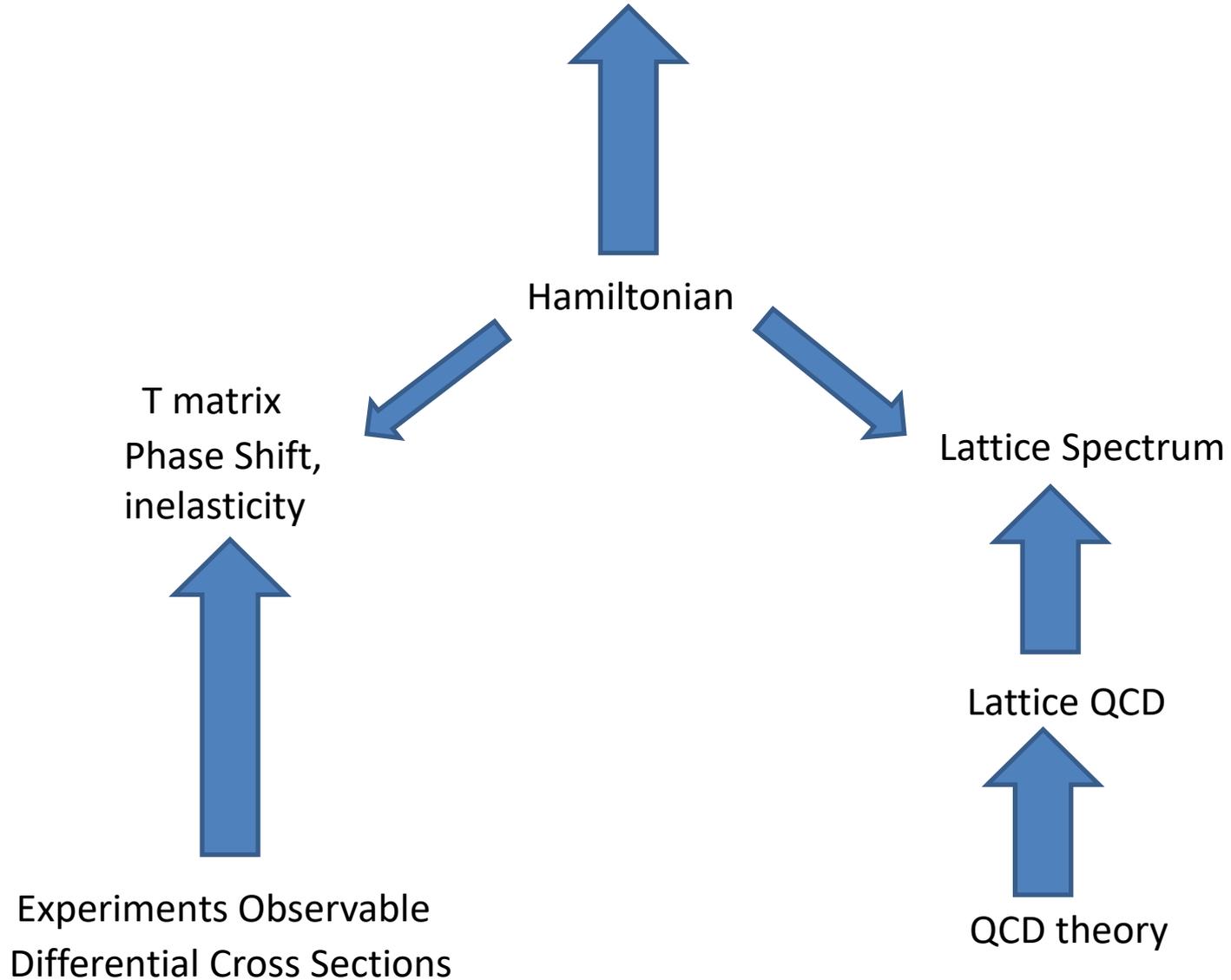
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Resonance Properties: Mass, Width,
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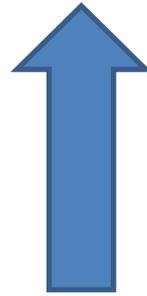


Resonance Properties: Mass, Width,
Pole position, Coupling, structure



Finite-Volume Hamiltonian Method

Resonance Properties: Mass, Width,
Pole position, Coupling, structure



Hamiltonian



T matrix
Phase Shift,
inelasticity



Lattice Spectrum

Hamiltonian

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_{\alpha})\rangle \left[\sqrt{m_{\alpha 1}^2 + k_{\alpha}^2} + \sqrt{m_{\alpha 2}^2 + k_{\alpha}^2} \right] \langle \alpha(k_{\alpha})|$$

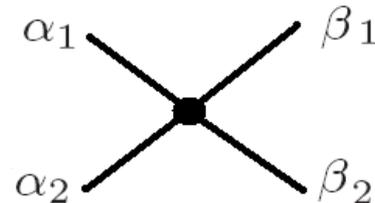
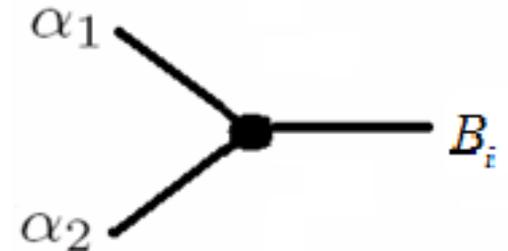
$|B_i\rangle$ bare state with mass m_i

$|\alpha(k_{\alpha})\rangle$ the channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_{\alpha})\rangle g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_{\alpha})| \right]$$

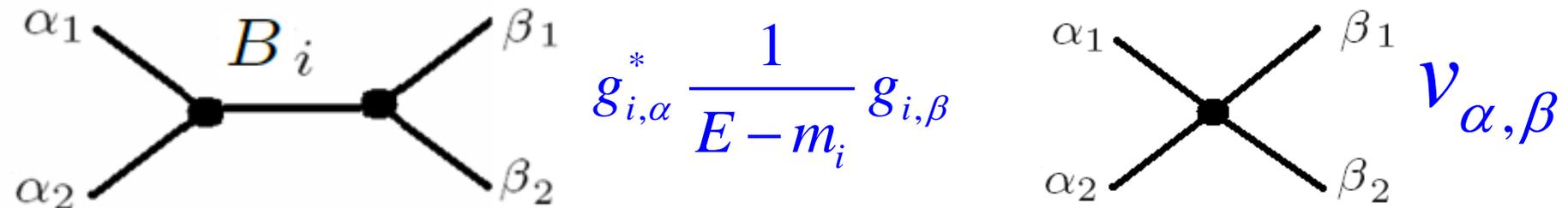
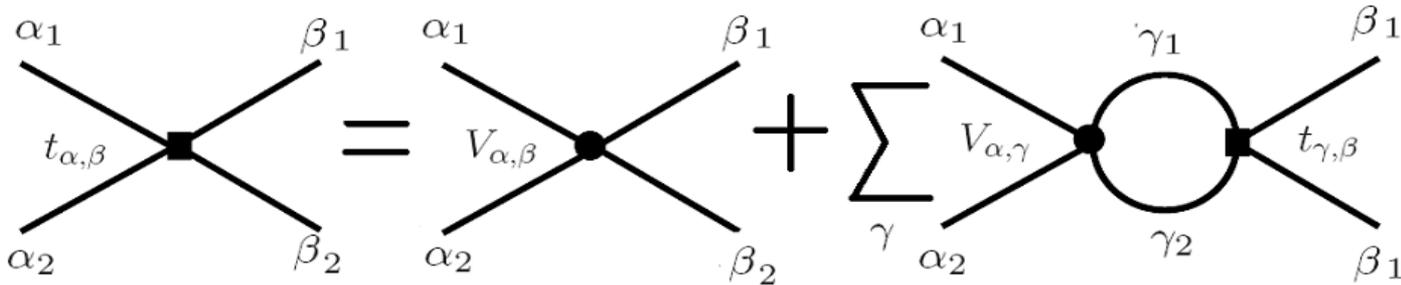
$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_{\alpha})\rangle v_{\alpha,\beta} \langle \beta(k_{\beta})|$$



Hamiltonian In the infinite volume

- T Matrix:

$$t_{\alpha,\beta}(k_\alpha, k_\beta, E) = V_{\alpha,\beta}(k_\alpha, k_\beta) + \sum_\gamma \int k_\gamma^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha, k_\gamma) t_{\gamma,\beta}(k_\gamma, k_\beta, E)}{E - \sqrt{m_{\gamma 1}^2 + k_\gamma^2} - \sqrt{m_{\gamma 2}^2 + k_\gamma^2} + i\varepsilon}$$



Hamiltonian in the infinite volume

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$$t_{\alpha,\beta}(k_\alpha, k_\beta, E) = V_{\alpha,\beta}(k_\alpha, k_\beta) + \sum_\gamma \int k_\gamma^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha, k_\gamma) t_{\gamma,\beta}(k_\gamma, k_\beta, E)}{E - \sqrt{m_{\gamma 1}^2 + k_\gamma^2} - \sqrt{m_{\gamma 2}^2 + k_\gamma^2} + i\varepsilon}$$

$$S_{\alpha,\beta} = 1 - i2\sqrt{\rho_\alpha} t_{\alpha,\beta}(k_{0\alpha}, k_{0\beta}, E) \sqrt{\rho_\beta}$$

$$\rho_\alpha = \frac{\pi k_{0\alpha} \sqrt{m_{\alpha 1}^2 + k_{0\alpha}^2} \sqrt{m_{\alpha 1}^2 + k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha} = S_{\alpha,\alpha}$$

Hamiltonian in the finite volume

- Hamiltonian Matrix for discrete momentum

Continuous	$\int d\vec{k}$	and	$ \alpha(\vec{k}_\alpha)\rangle$	and	$\langle \beta(\vec{k}_\beta) \alpha(\vec{k}_\alpha) \rangle = \delta_{\alpha\beta} \delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓		↓		↓
Discrete	$\sum_j \left(2\pi/L\right)^3$	and	$\left(2\pi/L\right)^{-3/2} \vec{k}_i, -\vec{k}_i\rangle_\alpha$	and	$\langle \vec{k}_j, -\vec{k}_j \vec{k}_i, -\vec{k}_i \rangle_\alpha = \delta_{\alpha\beta} \delta_{ij}$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha,i} |\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2} \right]_\alpha \langle \vec{k}_i, -\vec{k}_i|$$

$$H_I = \sum_j \left(2\pi/L\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[|\vec{k}_j, -\vec{k}_j\rangle_\alpha g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \vec{k}_j, -\vec{k}_j| \right]$$

$$+ \sum_{i,j} \left(2\pi/L\right)^3 \sum_{\alpha,\beta} |\vec{k}_i, -\vec{k}_i\rangle_\alpha v_{\alpha,\beta} \langle \vec{k}_j, -\vec{k}_j|$$

Finite-Volume Hamiltonian Method

- Hamiltonian Matrix for discrete momentum

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (H_0 + H_I) |\Psi\rangle = E |\Psi\rangle$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0, k_0) & v_{1,2}^V(k_0, k_0) & \cdots & v_{1,n_c}^V(k_0, k_0) & v_{1,1}^V(k_0, k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0, k_0) & v_{2,2}^V(k_0, k_0) & \cdots & v_{2,n_c}^V(k_0, k_0) & v_{2,1}^V(k_0, k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0, k_0) & v_{n_c,2}^V(k_0, k_0) & \cdots & v_{n_c,n_c}^V(k_0, k_0) & v_{n_c,1}^V(k_0, k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1, k_0) & v_{1,2}^V(k_1, k_0) & \cdots & v_{1,n_c}^V(k_1, k_0) & v_{1,1}^V(k_1, k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Eigen-Value of
Hamiltonian of two
body system

Eigen-Vector

Lattice Spectrum

Finite-Volume Hamiltonian Method

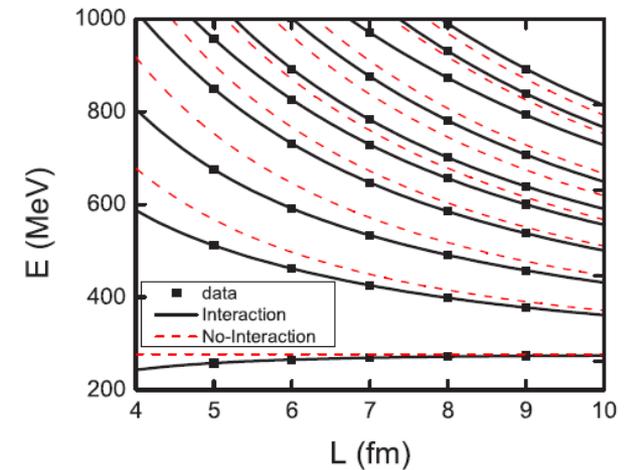
- Comparing with Lüscher method
- For example $\pi\pi$ scattering in S wave

$$H_0 = \begin{pmatrix} m_1 & 0 & 0 & \dots \\ 0 & 2\sqrt{k_0^2 + m_\pi^2} & 0 & \dots \\ 0 & 0 & 2\sqrt{k_1^2 + m_\pi^2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$H_I = \begin{pmatrix} m_0 & g_{\pi\pi}^{fin}(k_0) & g_{\pi\pi}^{fin}(k_1) & \dots \\ g_{\pi\pi}^{fin}(k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_1) & \dots \\ g_{\pi\pi}^{fin}(k_1) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_1) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n) = \left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi\pi}(k_n)$$

$$v_{\pi\pi,\pi\pi}^{fin}(k_n, k_m) = \left(\frac{2\pi}{L}\right)^3 v_{\pi\pi,\pi\pi}(k_n, k_m)$$



Finite-Volume Hamiltonian Method

- Comparing with Lüscher method
- For example $\pi\pi$ scattering in S wave

$$H_0 = \begin{pmatrix} m_1 & 0 & 0 & \dots \\ 0 & 2\sqrt{k_0^2 + m_\pi^2} & 0 & \dots \\ 0 & 0 & 2\sqrt{k_1^2 + m_\pi^2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$H_I = \begin{pmatrix} m_0 & g_{\pi\pi}^{fin}(k_0) & g_{\pi\pi}^{fin}(k_1) & \dots \\ g_{\pi\pi}^{fin}(k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_1) & \dots \\ g_{\pi\pi}^{fin}(k_1) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_1) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n) = \left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi\pi}(k_n)$$

$$v_{\pi\pi,\pi\pi}^{fin}(k_n, k_m) = \left(\frac{2\pi}{L}\right)^3 v_{\pi\pi,\pi\pi}(k_n, k_m)$$

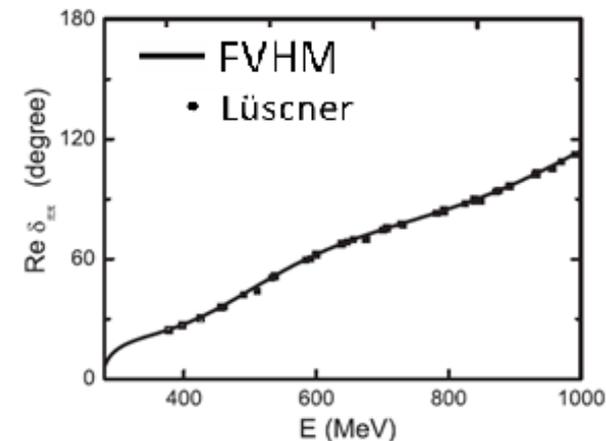
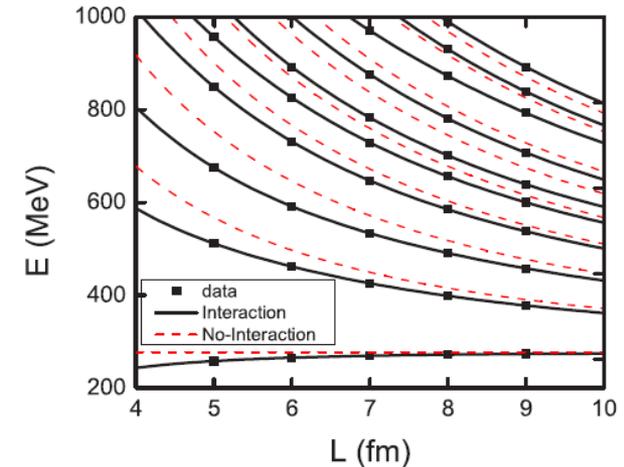
**Lüscher
Method**

$$\delta(k) = -\phi(q) \bmod \pi$$

$$-\phi(q) = \tan^{-1} \left(\frac{q\pi^{3/2}}{Z_{00}(1; q^2)} \right)$$

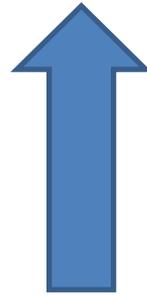
$$q = \frac{kL}{2\pi} = \frac{2\sqrt{E^2/4 - m_\pi^2}L}{2\pi}$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}$$



Finite-Volume Hamiltonian Method

Resonance Properties: Mass , Width, Pole position, Coupling



Hamiltonian



T matrix
Phase Shift,
inelasticity



Lattice Spectrum

The structure of Baryon Resonance

- 3 quark
- 5 quark
- Meson Cloud
- Meson-Baryon Molecule

... ..

Question: How to distinguish them ???

Fitting from the limited data from Experiments and Lattice.

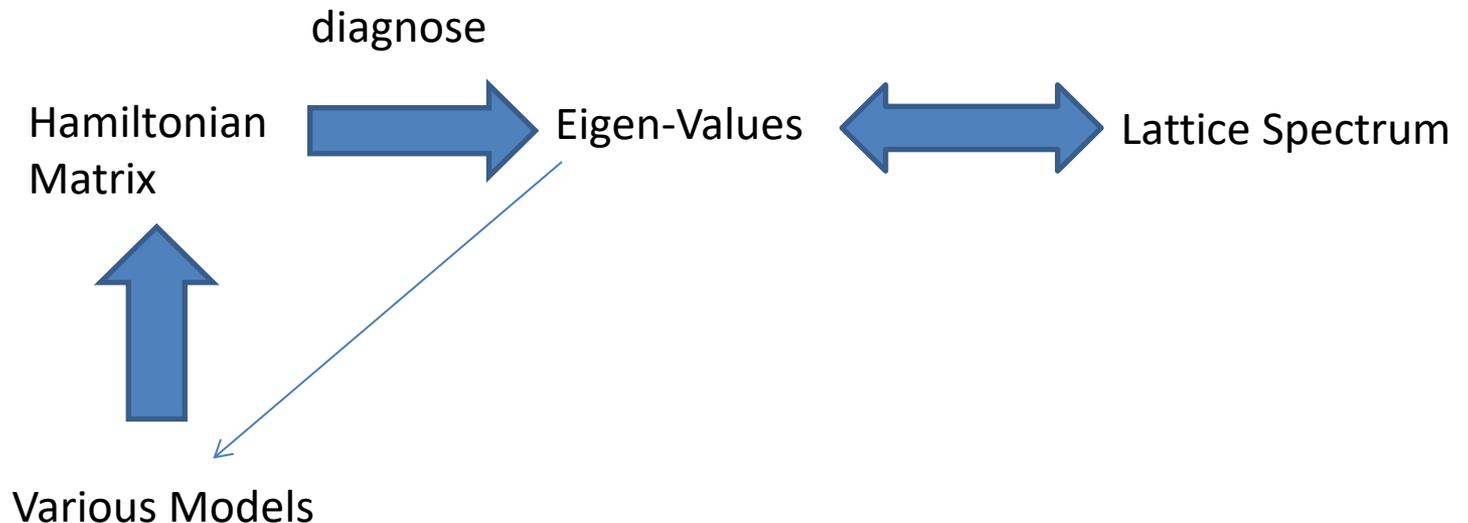
Model with various parameters are always powerful !!!

The structure of Baryon Resonance

- From the Lüscher equation,

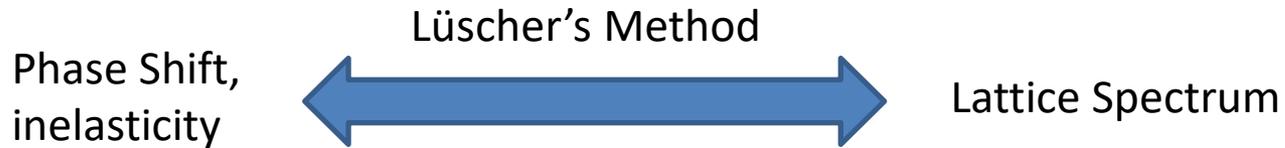


- From our Finite-Volume Hamiltonian matrix:

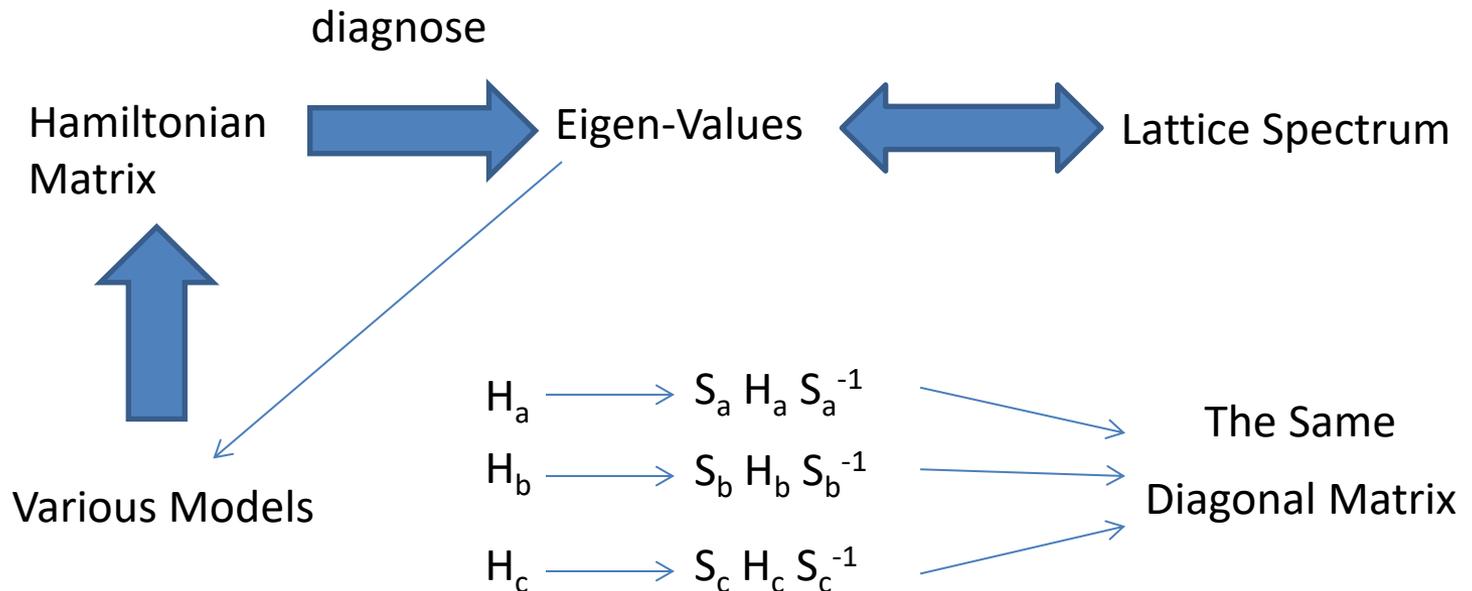


The structure of Baryon Resonance

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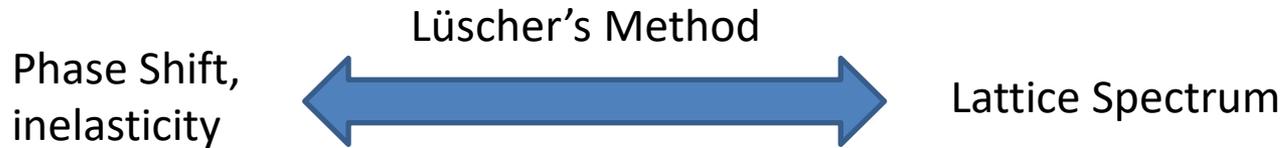


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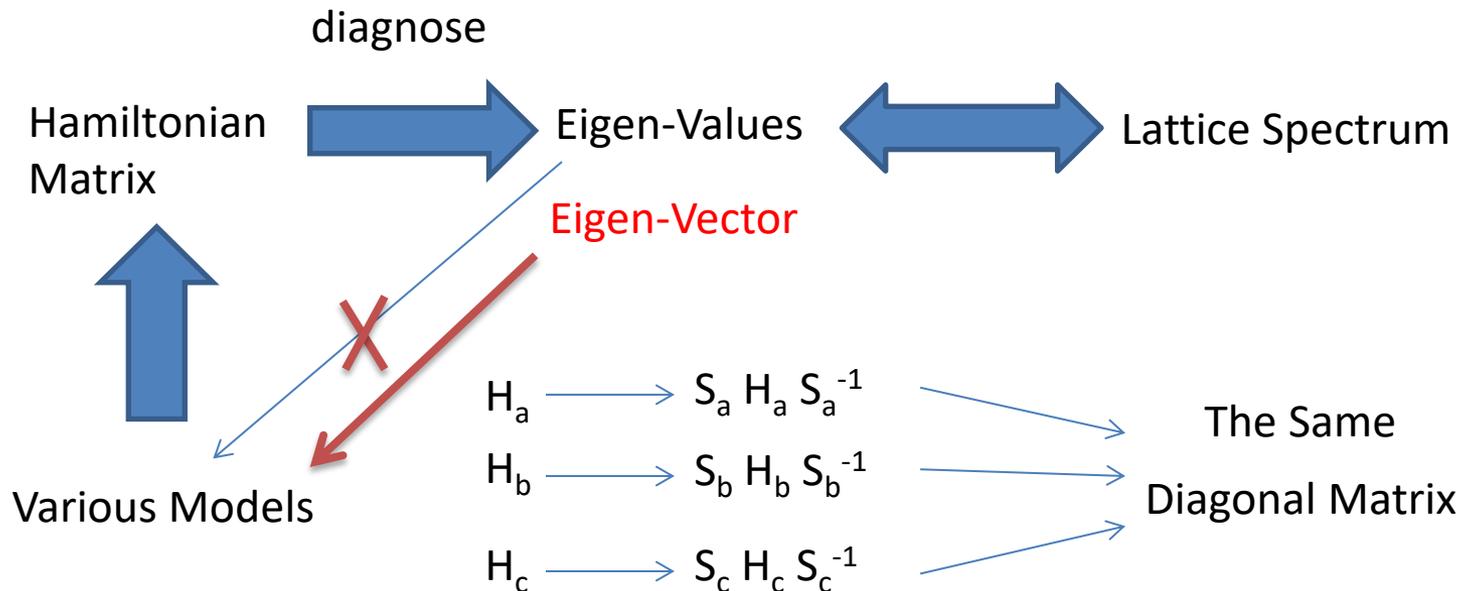


The structure of Baryon Resonance

- From the Lüscher equation,

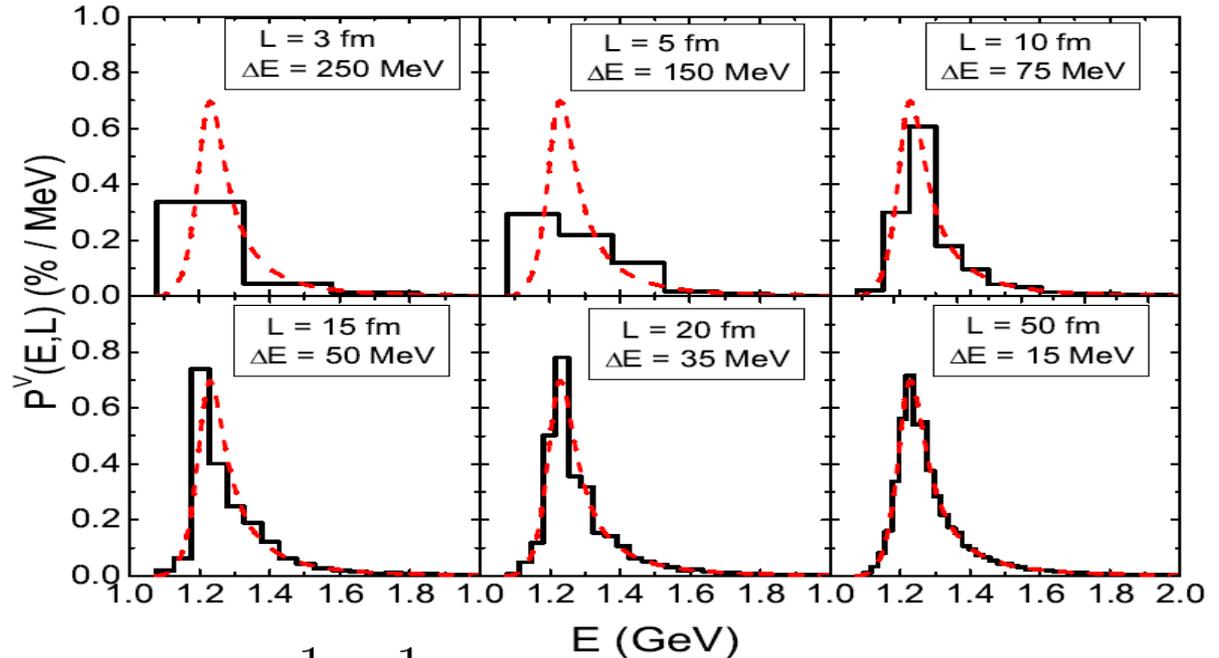


- From our Finite-Volume Hamiltonian matrix:



Eigen-Vector and Resonance

Sato-Lee Model for
 $\Delta(1232)$ region



Finite-Volume
Black Solid line

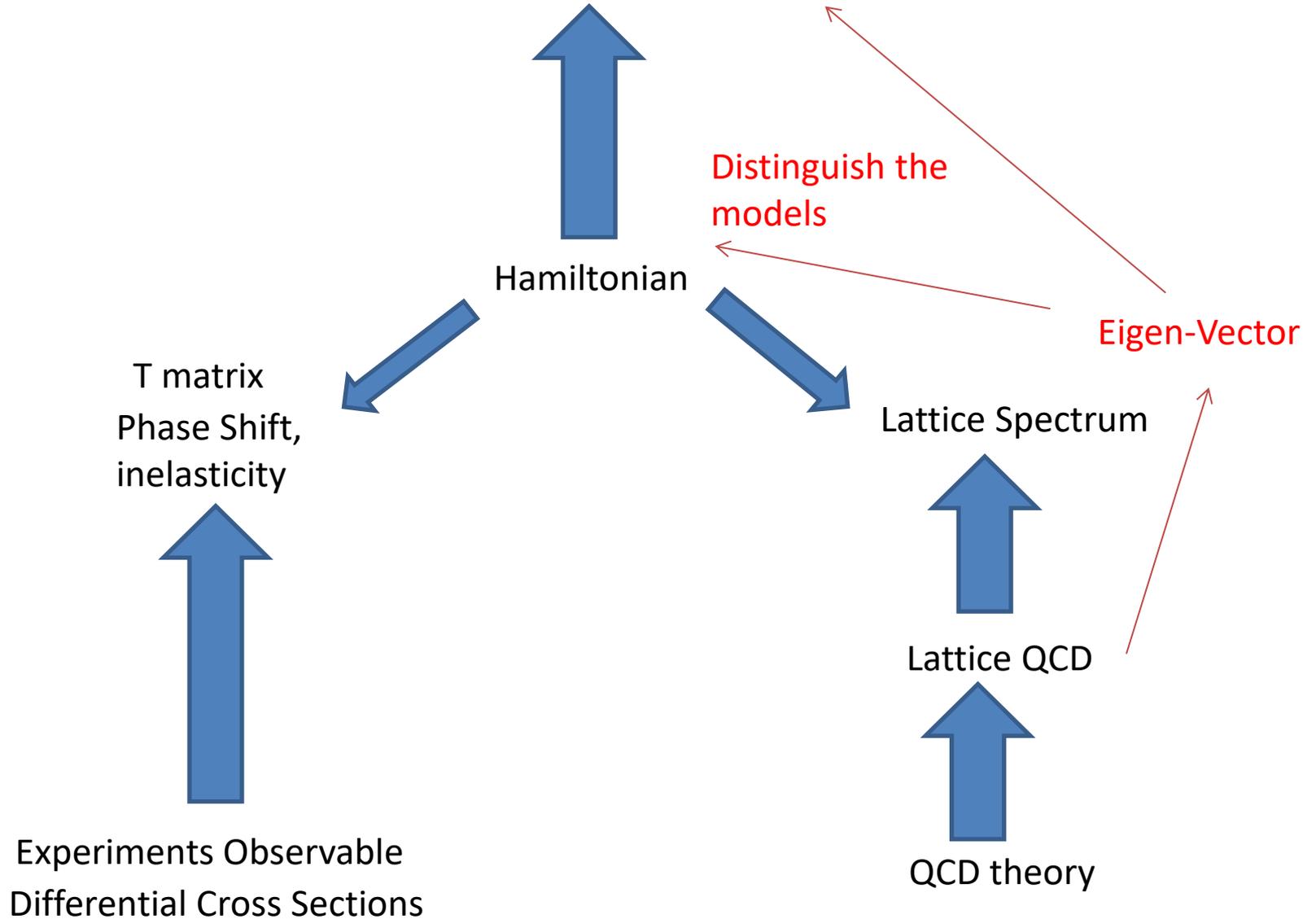
$$P^V(E_k^{ave}, L) = \frac{1}{Z^V} \frac{1}{\Delta E} \sum_{E_k^{ave} - \frac{\Delta E}{2} \leq E_\alpha \leq E_k^{ave} + \frac{\Delta E}{2}} |\langle N^* | \Psi_{E_\alpha}^V \rangle|^2$$

Infinite-Volume
Red dashed line

$$P(E) = \frac{1}{Z} \sum_{i=1, n_c} \pi k_i E_{M_i}(k_i) E_{B_i}(k_i) |\langle N^* | \Psi_{E,i}^{(+)} \rangle|^2$$

$$|\langle N^* | \Psi_{E,i}^{(+)} \rangle|^2 = \frac{\bar{\Gamma}_{\pi N}(k_{\pi N}; E)}{E - m_0 - \Sigma(E)}$$

Resonance Properties: Mass, Width,
Pole position, Coupling, structure



Unfortunately, now the technical of Lattice can not measure all Eigen-states and Eigen-vectors for Baryon Resonances.

Currently, for the Baryon spectrum, pure 3 quark local operator is mostly used, while rare papers use Meson-Baryon non-local operator.

C. B. Lang, ect. arXiv: 1610.01422

So here focus on the 3 quark operator

- In the Lattice, we use 3 quark local operator can extract several Eigen-states.
- 3 quark operator for the vacuum is corresponding to bare state.
- Thus, these lattice Eigen-states should have large bare state components.

Finite-Volume Hamiltonian Method

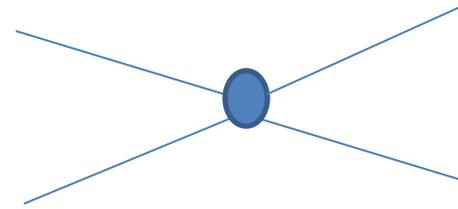
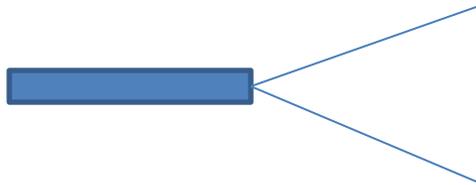
1. Fitting Experimental Data to fix the parameters.
2. Using these fitted parameters to generate the Finite-Volume Hamiltonian matrix. And for high pion mass, other parameters for the mass slope is needed.
3. Calculate the eigenvalue and eigenvector of Hamiltonian matrix to compare with Lattice data.

Actually, if the lattice data is enough, we can directly extract coupling and other parameters from lattice data.

Study of $N^*(1535)$

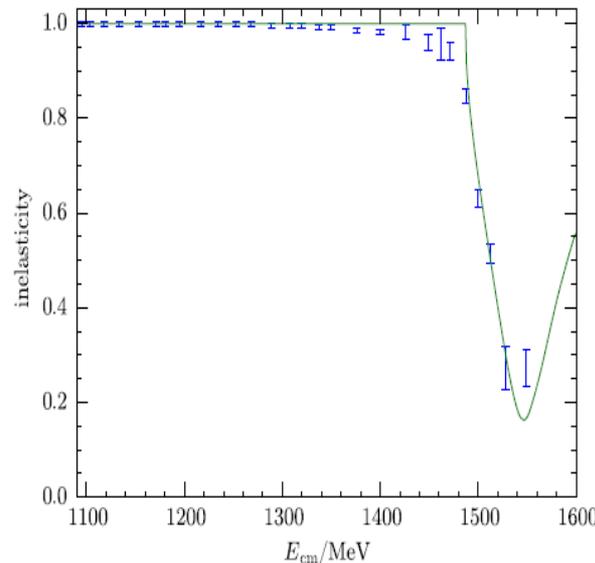
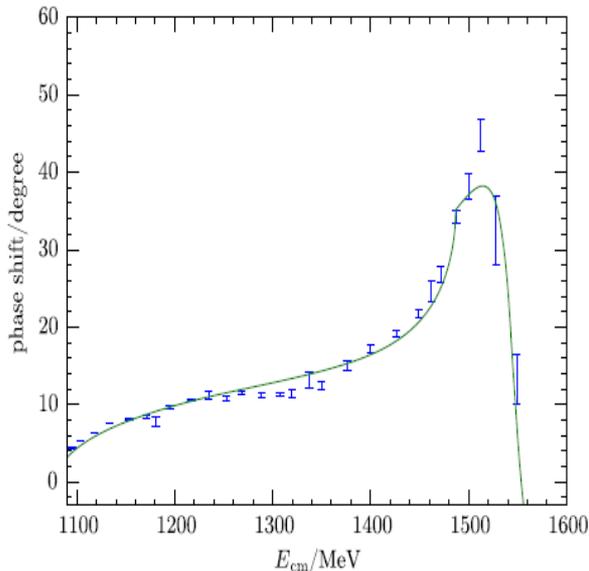
Zhan-wei Liu et al. Phys.Rev.Lett. 116 (2016) no.8, 082004

- 2 Channels: πN and ηN



$$G_{iN}^2(k) = \left(3g_{N_0^*iN}^2 / 4\pi^2 f^2 \right) \omega_i(k) u^2(k)$$

$$\frac{3g_{\pi N}^S \tilde{u}(k) \tilde{u}(k')}{4\pi^2 f^2}$$



$$g_{\pi N}^S = -0.0608 \pm 0.0004$$

$$m_0 = 1601 \pm 14 \text{ MeV}$$

$$g_{N_0^* \pi N} = 0.186 \pm 0.006$$

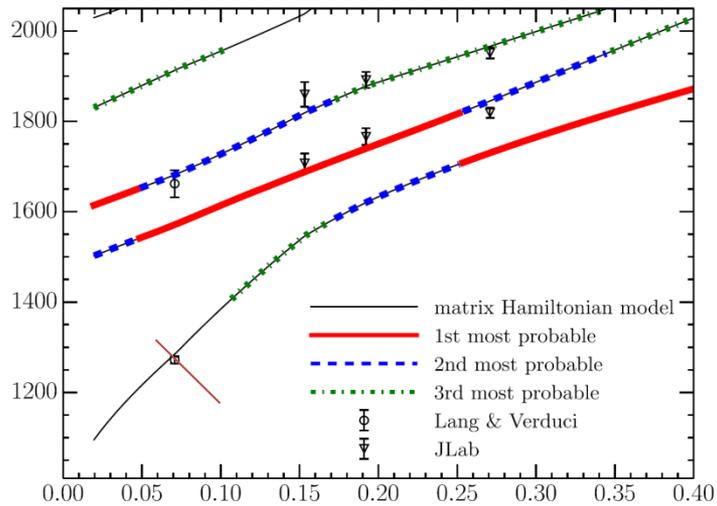
$$g_{N_0^* \eta N} = 0.185 \pm 0.017,$$

$$\chi_{\text{DOF}}^2 = 6.8$$

$$1531 \pm 29 - i88 \pm 2 \text{ MeV}$$

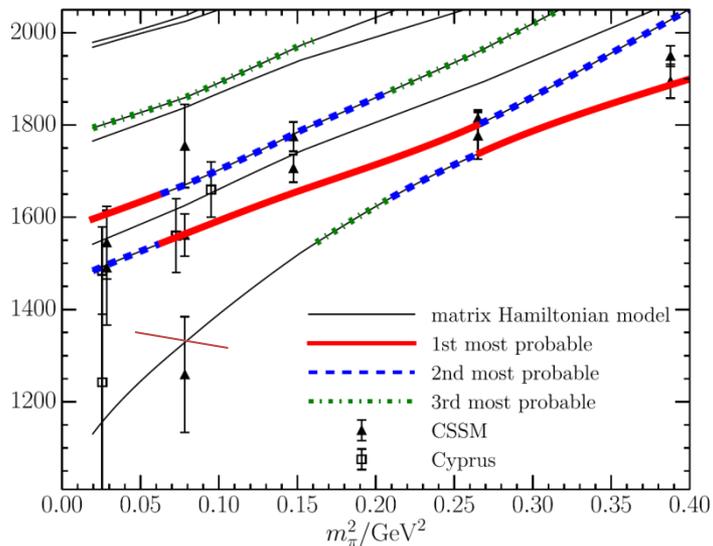
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Zhan-wei Liu et. Phys.Rev.Lett. 116 (2016) no.8, 082004



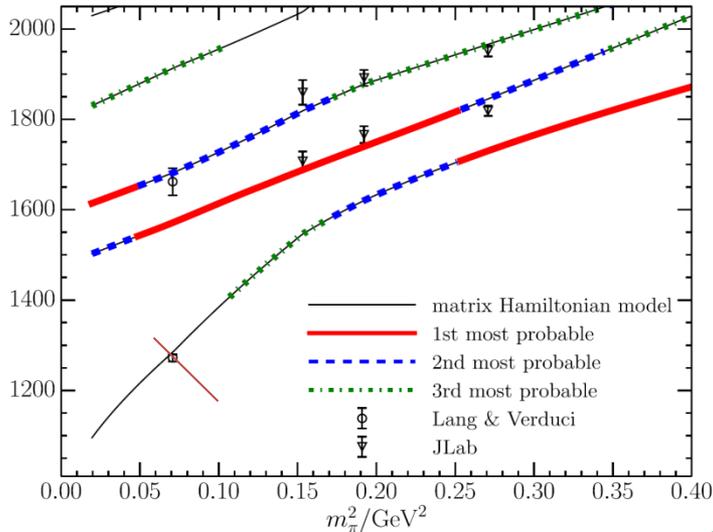
$$m_\eta^2(m_\pi^2) = m_\eta^2|_{phys} + \frac{1}{3}(m_\pi^2 - m_\pi^2|_{phys})$$

$$m_0(m_\pi^2) = m_0|_{phys} + \alpha_0(m_\pi^2 - m_\pi^2|_{phys})$$



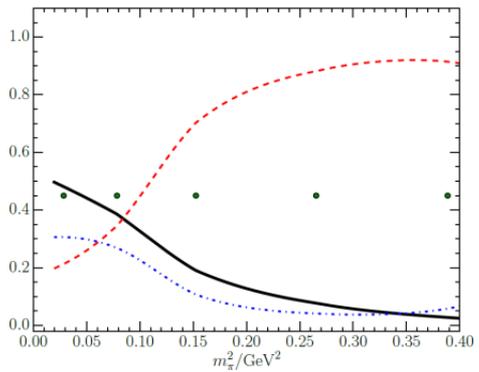
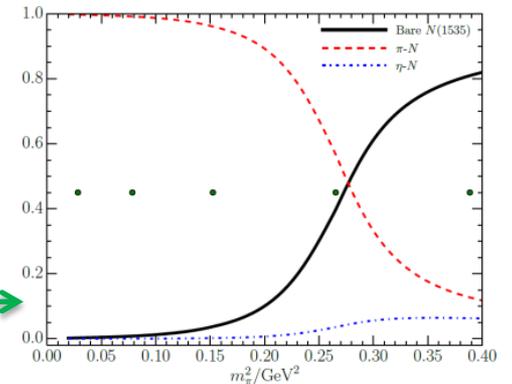
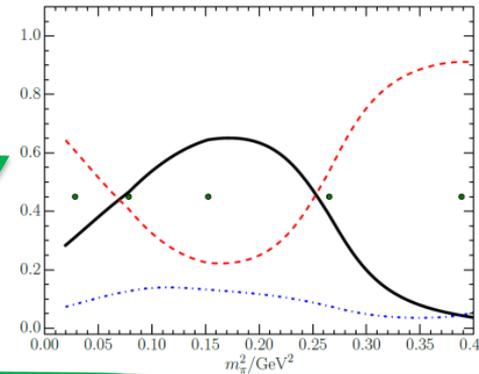
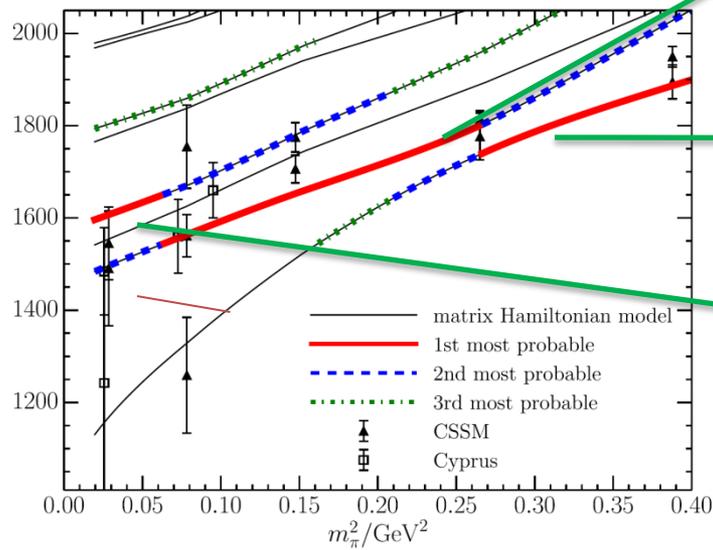
Study of $N^*(1535)$

Zhan-wei Liu et al. Phys.Rev.Lett. 116 (2016) no.8, 082004



$$m_\eta^2(m_\pi^2) = m_\eta^2|_{phys} + \frac{1}{3}(m_\pi^2 - m_\pi^2|_{phys})$$

$$m_0(m_\pi^2) = m_0|_{phys} + \alpha_0(m_\pi^2 - m_\pi^2|_{phys})$$



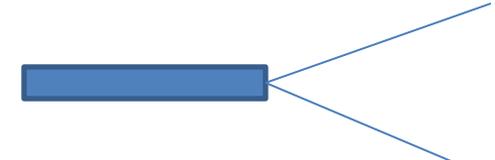
The main components (at least 50%) of $N^*(1535)$ is from the 3 quark core.

Study of $\Lambda^*(1405)$

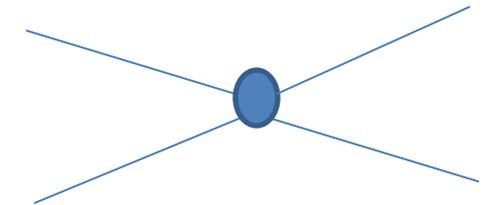
Zhan-wei Liu et. arXiv: 1607.05856

- $I=0$, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$ and $\bar{K}\Xi$
- $I=1$, $\pi\Sigma$, $\bar{K}N$, $\pi\Lambda$

Coupling	No $ B_0\rangle$	With $ B_0\rangle$	Coupling	No $ B_0\rangle$	With $ B_0\rangle$
$g_{\pi\Sigma,\pi\Sigma}^0$	-1.77	-1.59	$g_{\pi\Sigma,\pi\Sigma}^1$	-0.14	<u>-0.14</u>
$g_{\bar{K}N,\bar{K}N}^0$	-2.14	-1.78	$g_{\bar{K}N,\bar{K}N}^1$	-0.06	<u>-0.06</u>
$g_{\bar{K}N,\pi\Sigma}^0$	0.78	0.89	$g_{\bar{K}N,\pi\Sigma}^1$	1.36	<u>1.36</u>
$g_{\bar{K}N,\eta\Lambda}^0$	-0.42	-0.97	$g_{\bar{K}N,\pi\Lambda}^1$	0.96	<u>0.96</u>
$g_{\pi\Sigma,K\Xi}^0$	-0.24	-0.56			
$g_{\eta\Lambda,K\Xi}^0$	0.42	0.97			
$g_{\bar{K}\Xi,K\Xi}^0$	-0.60	-1.37			
$g_{\pi\Sigma,B_0}^0$	-	0.11			
$g_{\bar{K}N,B_0}^0$	-	0.15			
$g_{\eta\Lambda}^0$	-	-0.17	pole 1 (MeV)	1428 - 23 i	1430 - 22 i
$g_{\bar{K}\Xi}^0$	-	-0.08	pole 2 (MeV)	1333 - 85 i	1338 - 89 i
$m_{B_0}^0/\text{MeV}$	-	1713.76			



$$\frac{\sqrt{3}g_{\alpha,B_0}^I}{2\pi f} \sqrt{\omega_\pi(k)} u(k)$$



$$g_{\alpha,\beta}^I \frac{[\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')] u(k)u(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)} \sqrt{2\omega_{\beta_M}(k')}}$$

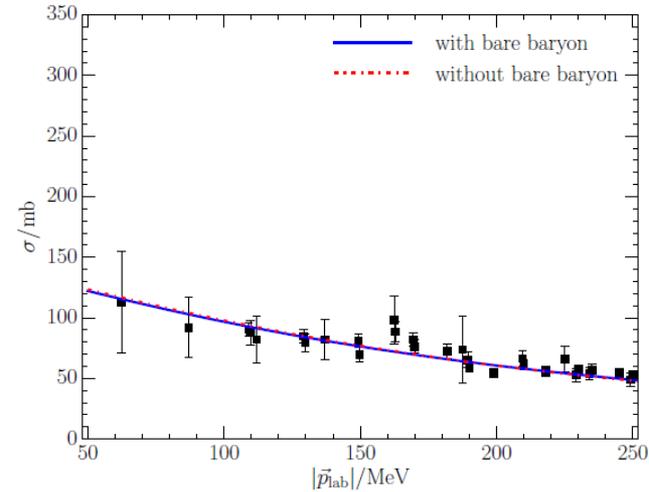
Weinberg – Tomozawa Term

$$g_{\bar{K}N,\eta\Lambda}^0 = -3/\sqrt{2}g_0, \quad g_{\pi\Sigma,K\Xi}^0 = -\sqrt{3/2}g_0$$

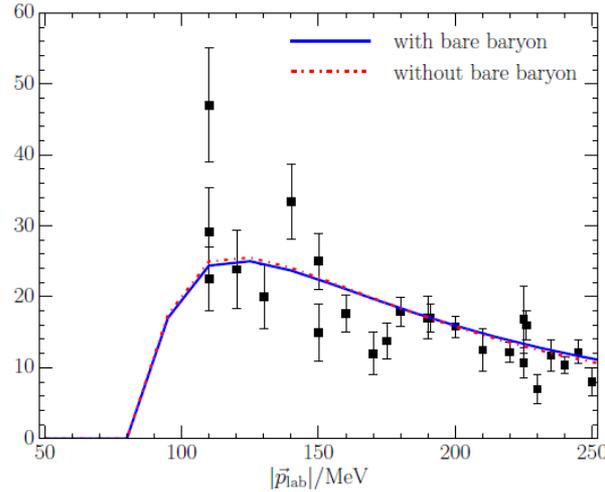
$$g_{\eta\Lambda,K\Xi}^0 = 3/\sqrt{2}g_0, \quad g_{\bar{K}\Xi,K\Xi}^0 = -3g_0.$$

Study of $\Lambda^*(1405)$

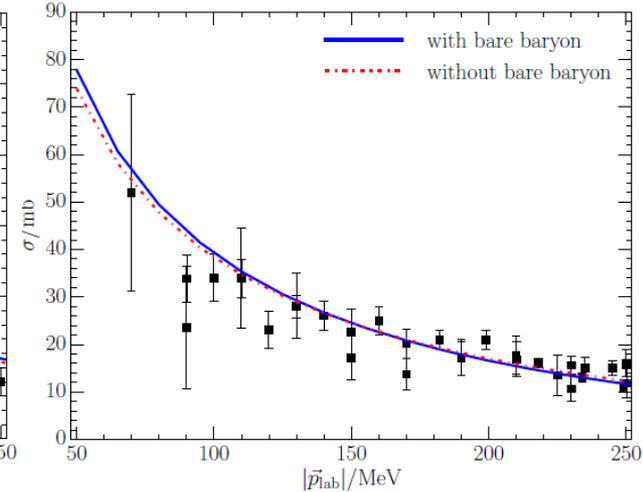
Zhan-wei Liu et. arXiv: 1607.05856



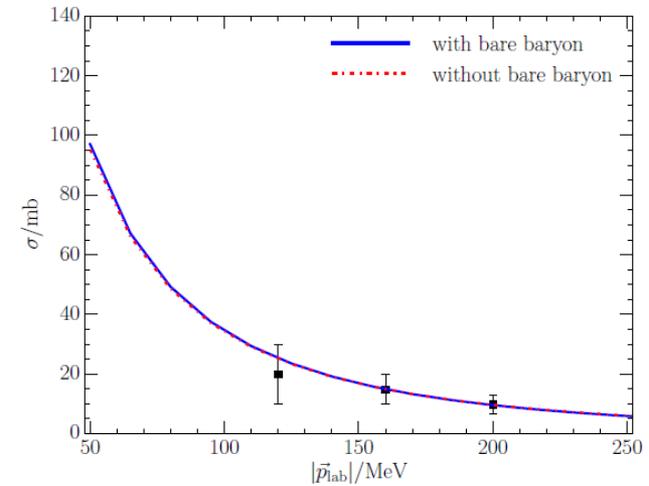
(a) $K^- p \rightarrow K^- p$



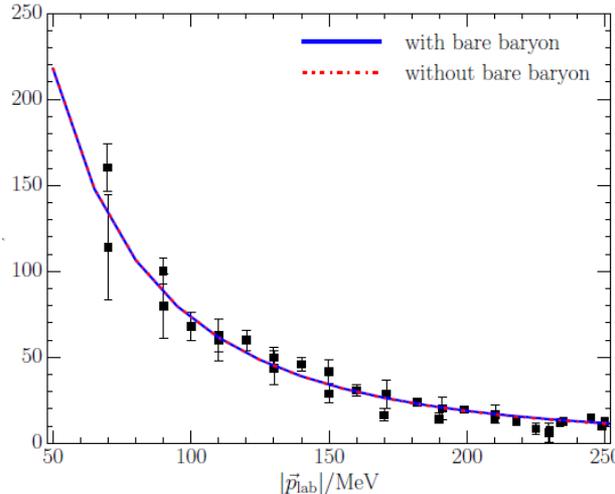
(b) $K^- p \rightarrow \bar{K}^0 n$



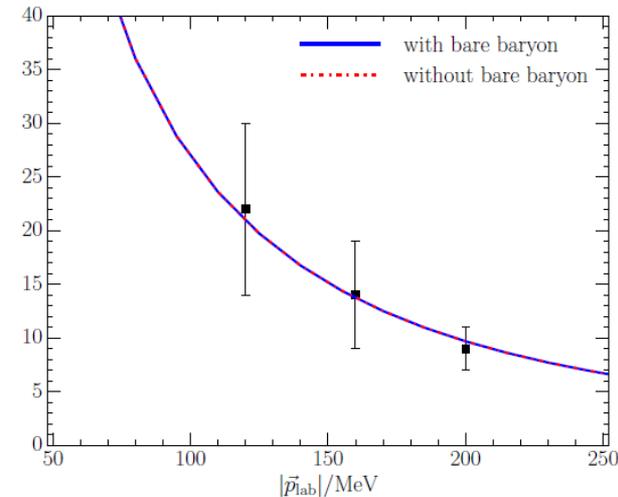
(c) $K^- p \rightarrow \pi^- \Sigma^+$



(d) $K^- p \rightarrow \pi^0 \Sigma^0$



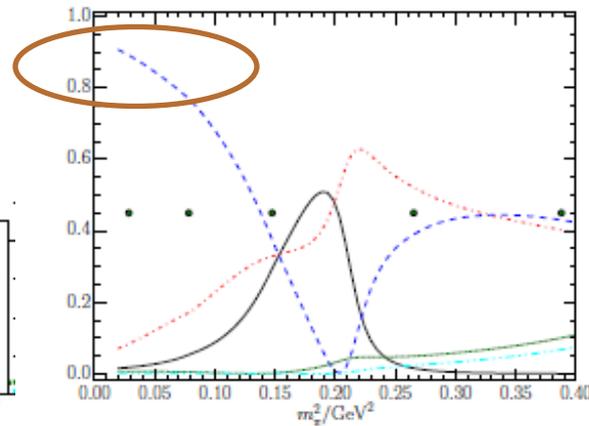
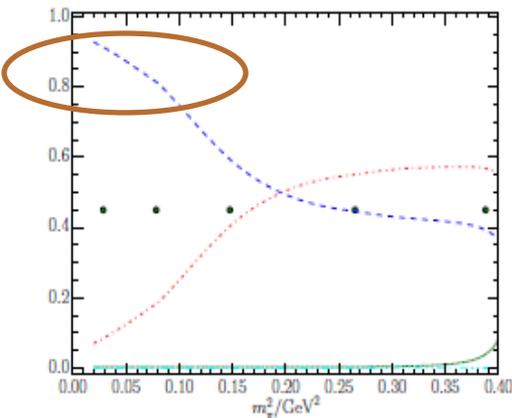
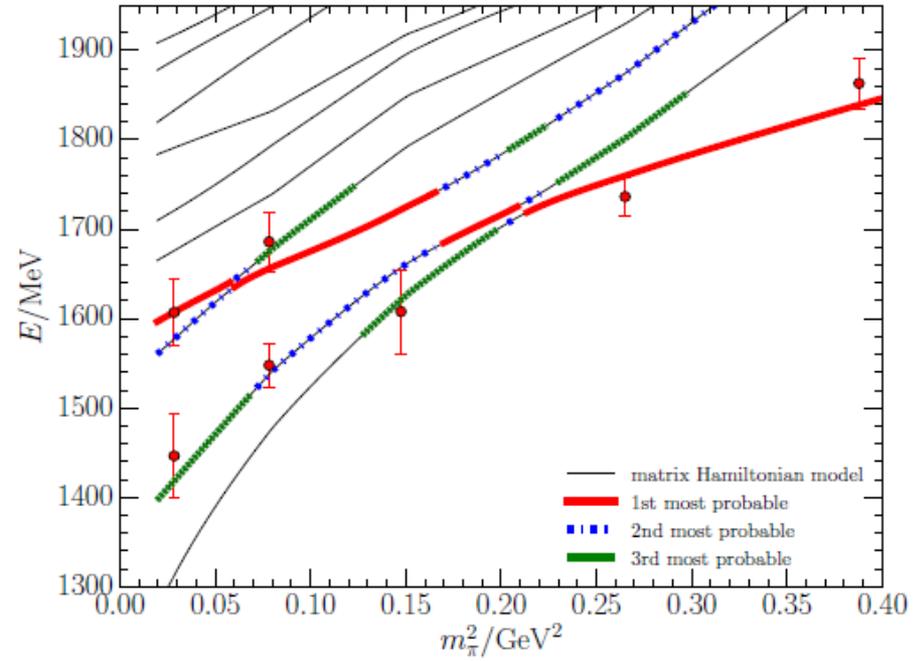
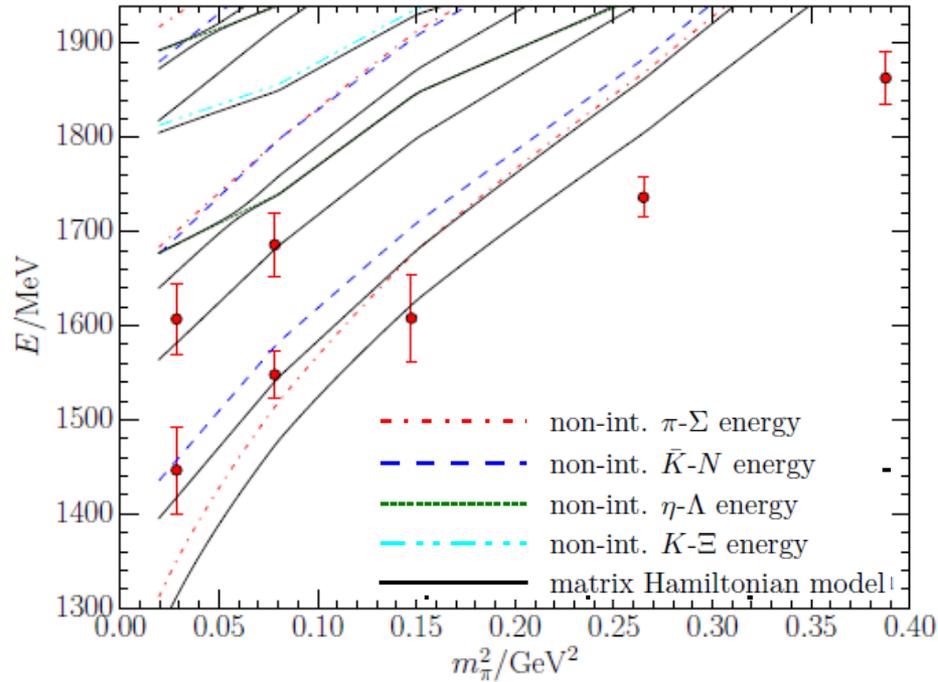
(e) $K^- p \rightarrow \pi^+ \Sigma^-$



(f) $K^- p \rightarrow \pi^0 \Lambda$

Study of $\Lambda^*(1405)$

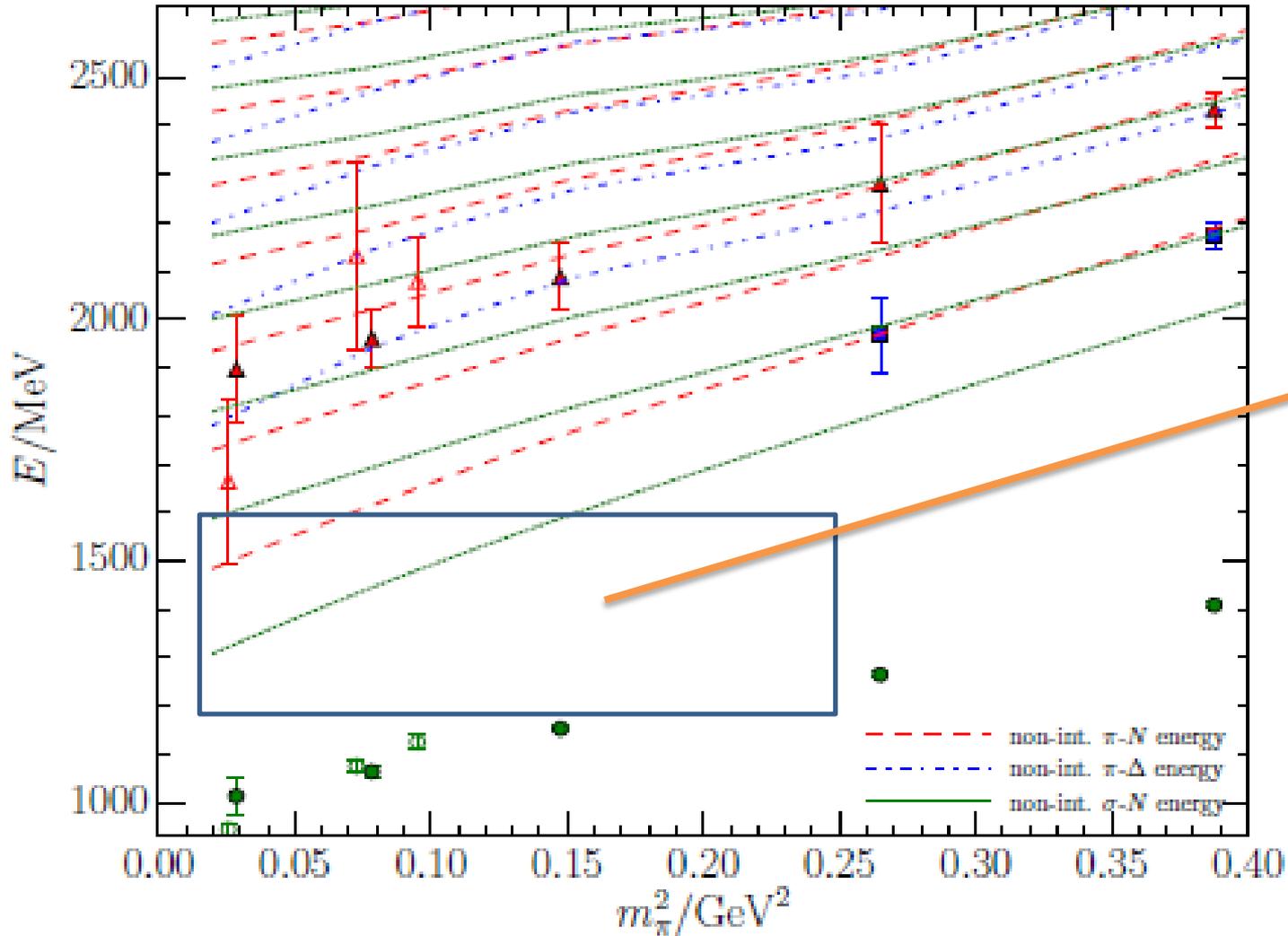
Zhan-wei Liu et. arXiv: 1607.05856



the $\Lambda^*(1405)$ is
predominantly a
molecular $\bar{K} N$ bound
state

Study of $N^*(1440)$

Zhan-wei Liu et. arXiv: 1607.04536



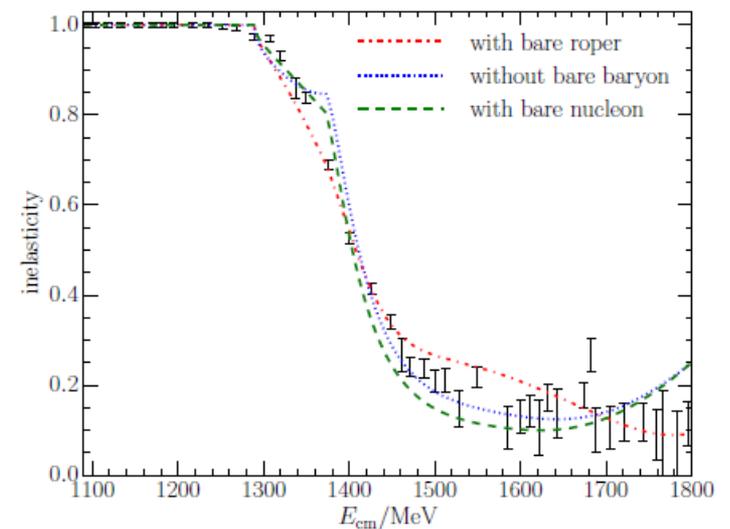
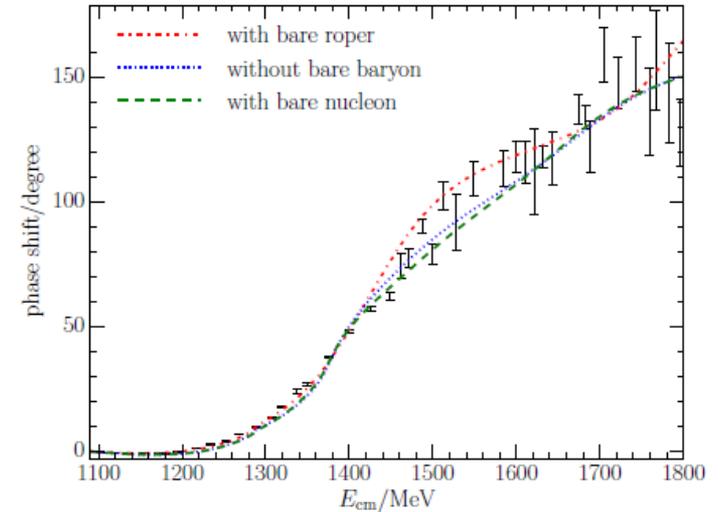
No State
extracted from
3 quark
operator

Study of $N^*(1440)$

Zhan-wei Liu et. arXiv: 1607.04536

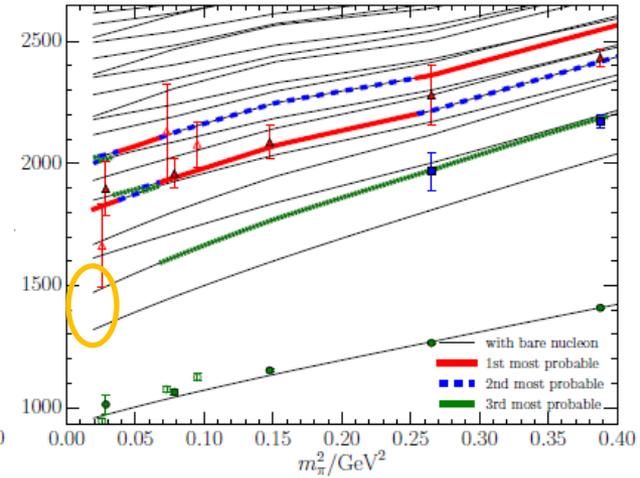
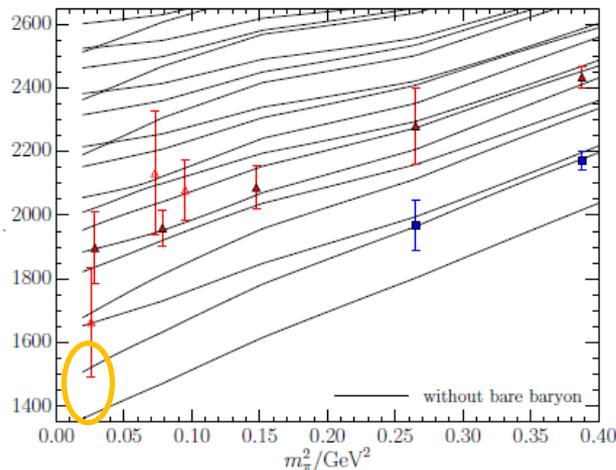
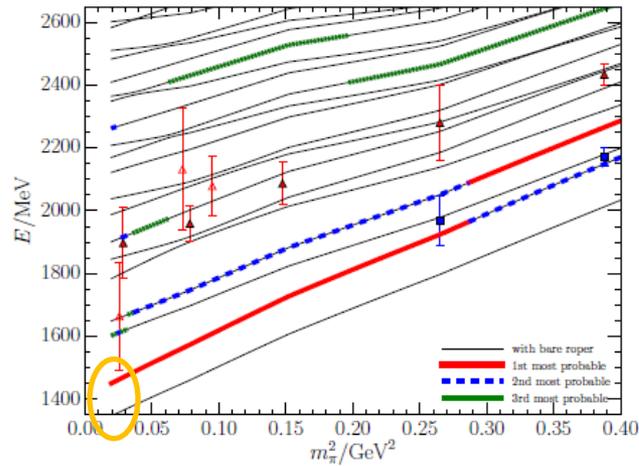
Include 3 channels: πN , $\pi\Delta$ and σN

Parameter	I	II	III
$g_{\pi N}^S$	0.161	0.489	0.213
$g_{\pi\Delta}^S$	-0.046	-1.183	-1.633
$g_{\pi N, \pi\Delta}^S$	0.006	-1.008	-0.640
$g_{\pi N, \sigma N}^S$	<u>0</u>	2.176	2.401
$g_{\sigma N}^S$	<u>0</u>	9.898	9.343
$g_{B_0 \pi N}$	0.640	<u>0</u>	-0.586
$g_{B_0 \pi\Delta}$	1.044	<u>0</u>	1.012
$g_{B_0 \sigma N}$	2.172	<u>0</u>	2.739
m_B^0/GeV	2.033	<u>∞</u>	1.170
$\Lambda_{\pi N}/\text{GeV}$	<u>0.700</u>	0.562	<u>0.562</u>
$\Lambda_{\pi\Delta}/\text{GeV}$	<u>0.700</u>	0.654	<u>0.654</u>
$\Lambda_{\sigma N}/\text{GeV}$	<u>0.700</u>	1.353	<u>1.353</u>
Pole (MeV)	1380 - 87 <i>i</i>	1361 - 39 <i>i</i>	1357 - 36 <i>i</i>



Study of $N^*(1440)$

Zhan-wei Liu *ect.* arXiv: 1607.04536

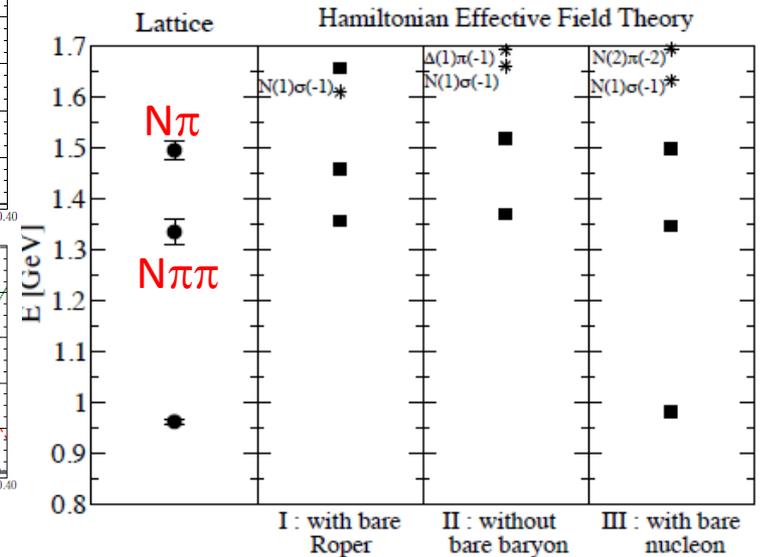
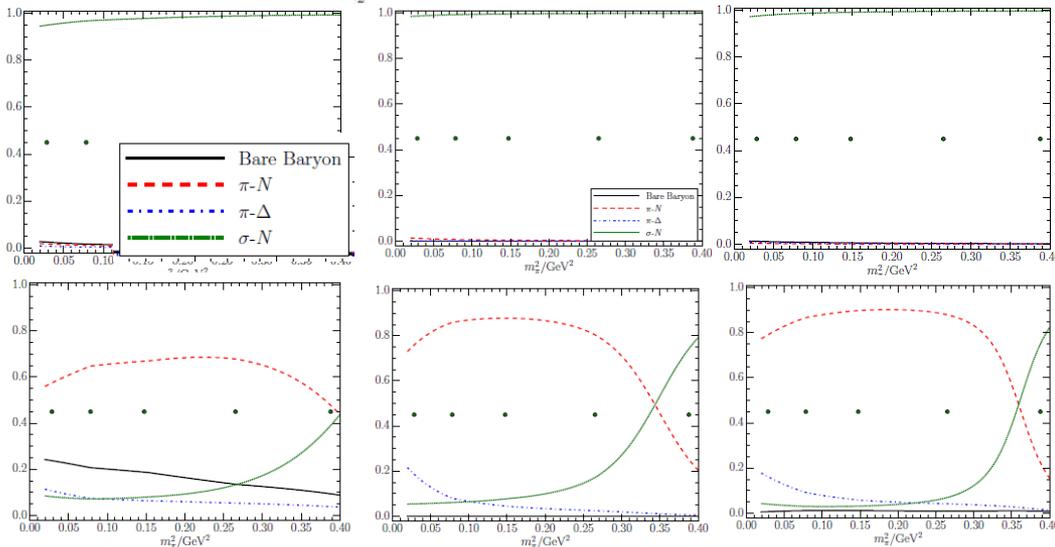
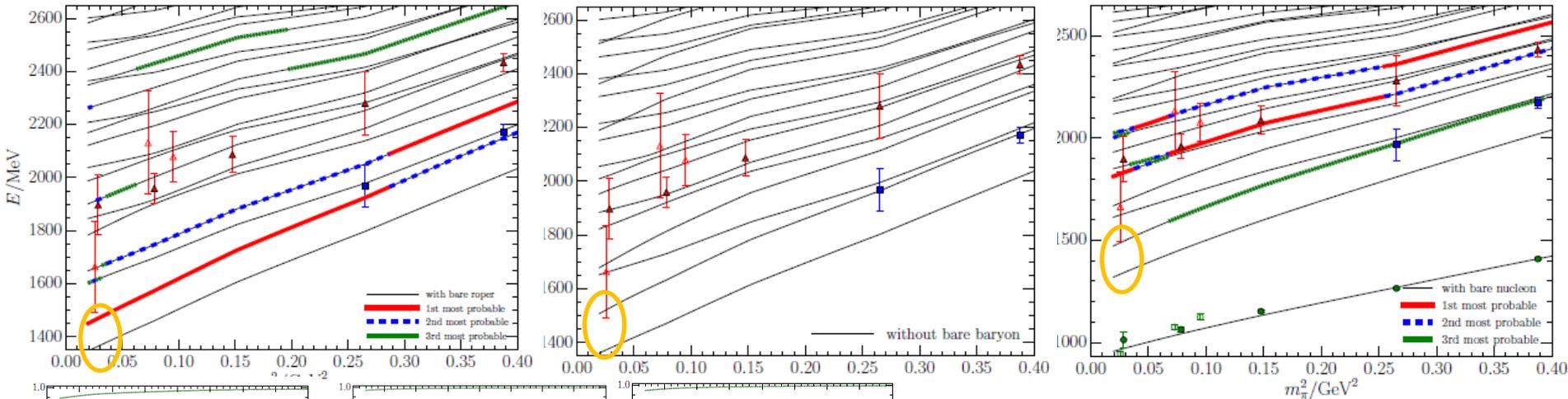


The first scenario with a bare state for $N^*(1440)$ although fit the spectrum well, the largest possibility for bare state does not touch the lattice point. Thus, it fails to explain Lattice data.

The second and third scenarios without a bare state for $N^*(1440)$ also can fit Lattice data well, it indicates that $N^*(1440)$ seems a molecule state.

Study of $N^*(1440)$

Zhan-wei Liu et. arXiv: 1607.04536

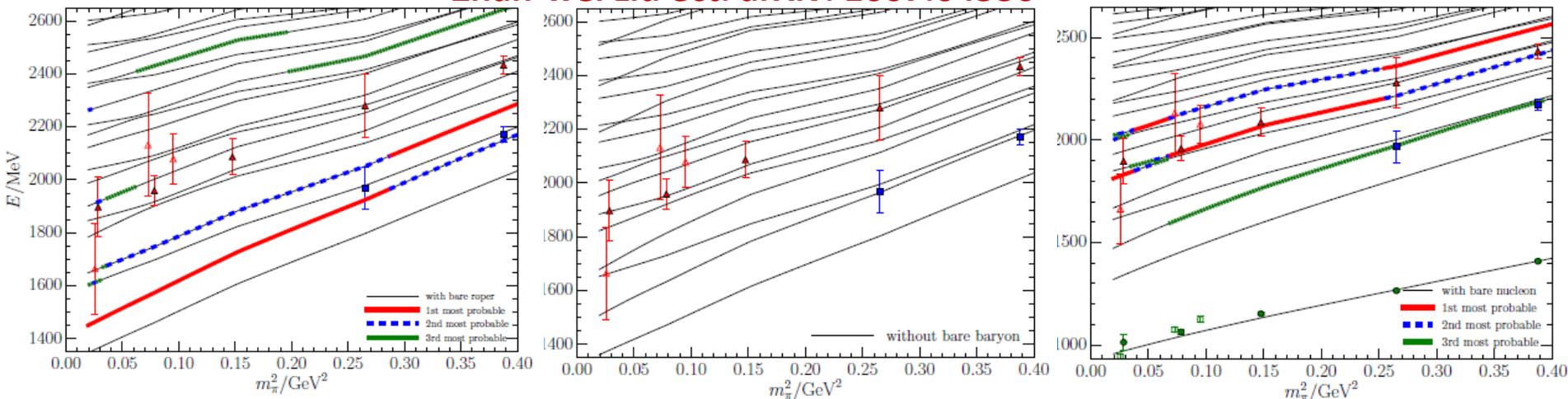


We need more data and detailed study, for the contribution from $N\pi\pi$ three body.

C. B. Lang, et. arXiv: 1610.01422

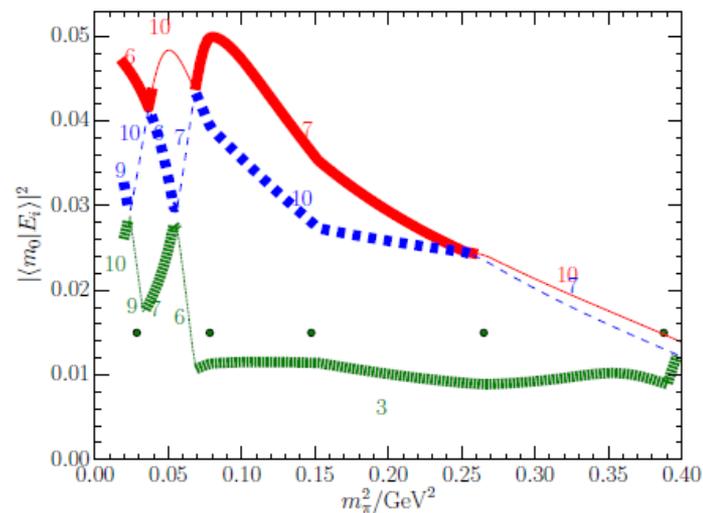
Study of $N^*(1440)$

Zhan-wei Liu *et. al.* arXiv: 1607.04536



Roper does not include so many 3 quark core (at least below 10%) and it seems a molecule state.

For P11 state, where is the first radial excitation state of nucleon. From these results of lattice, it seems around 1900 MeV. We need more detailed calculation for it.



Summary

- Introduce the Finite-Volume Hamiltonian Method
- FVHM connects the experimental data, Lattice data and the properties of resonances.
- The Eigen-Vector in the finite volume will be useful to distinguish the different models and reflect the insight of resonances.
- The $N^*(1535)$, $\Lambda^*(1405)$ and $N^*(1440)$ are studied based on FVHM combine with experimental and Lattice data.

Outlook

- For the Eigen-vector, we expect the new result from Lattice calculation directly.
- Hamiltonian model is need to be improved:
 - $N^*(1535)$: How about $N^*(1650)$
 - $\Lambda^*(1405)$: How about $\Lambda^*(1670)$
 - $N^*(1440)$: Where is the first radial excitation state of nucleon ? Around 1900 MeV ?? What is the contribution of $N\pi\pi$?

Thanks very much