

Charm exotic hadrons in coupled-channel scattering from lattice QCD

Yoichi IKEDA (RCNP, Osaka Univ.)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

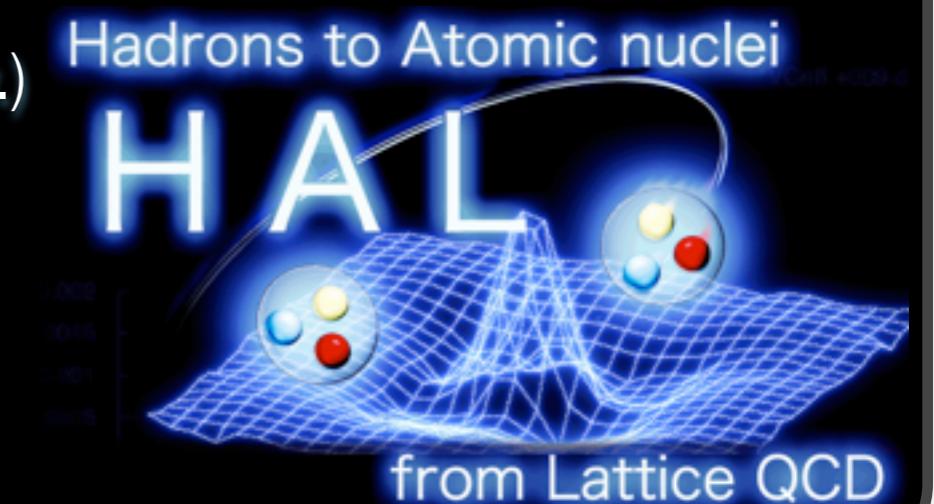
T. Doi, T. Hatsuda, T. Iritani (RIKEN)

S. Gongyo (Univ. Tours)

T. Inoue (Nihon Univ.)

Y. Ikeda, N. Ishii, K. Murano (RCNP, Osaka Univ.)

H. Nemura (Univ. Tsukuba)

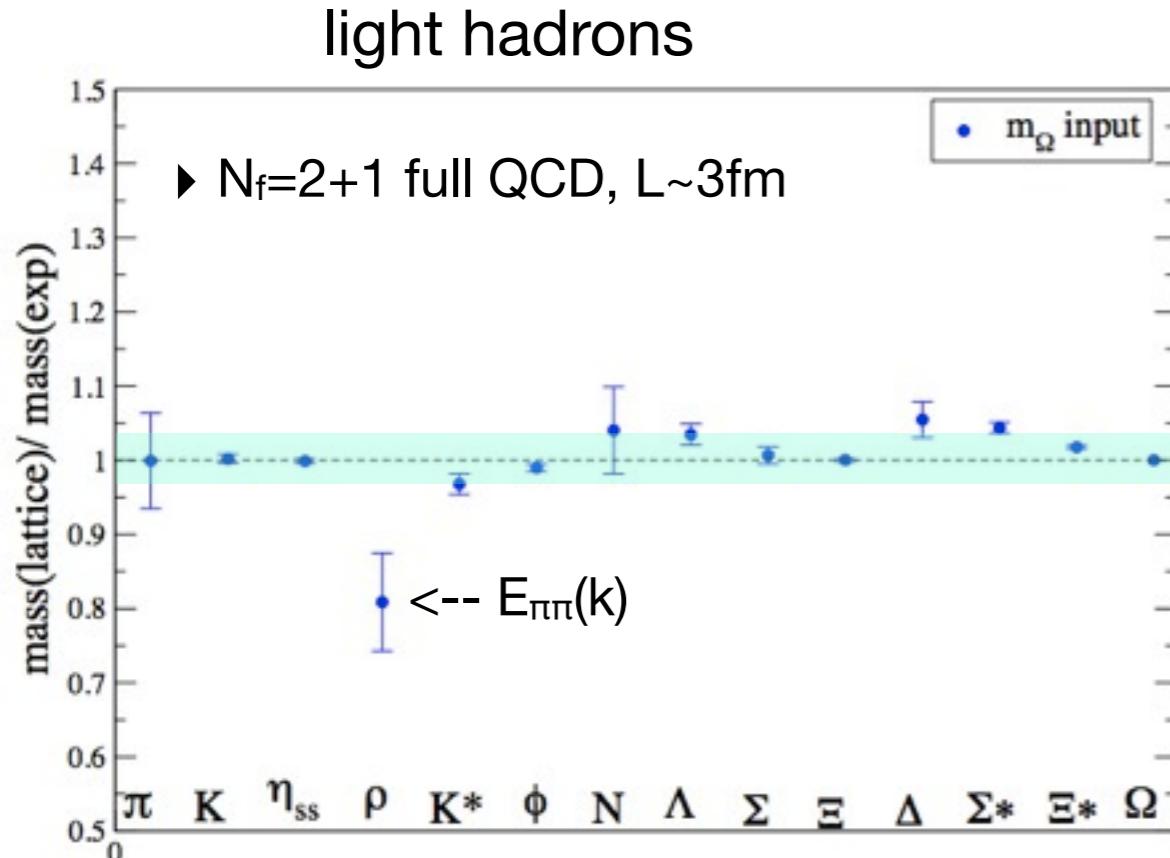


2016 JAEA/ASRC Reimei Workshop: New exotic hadron matter at J-PARC

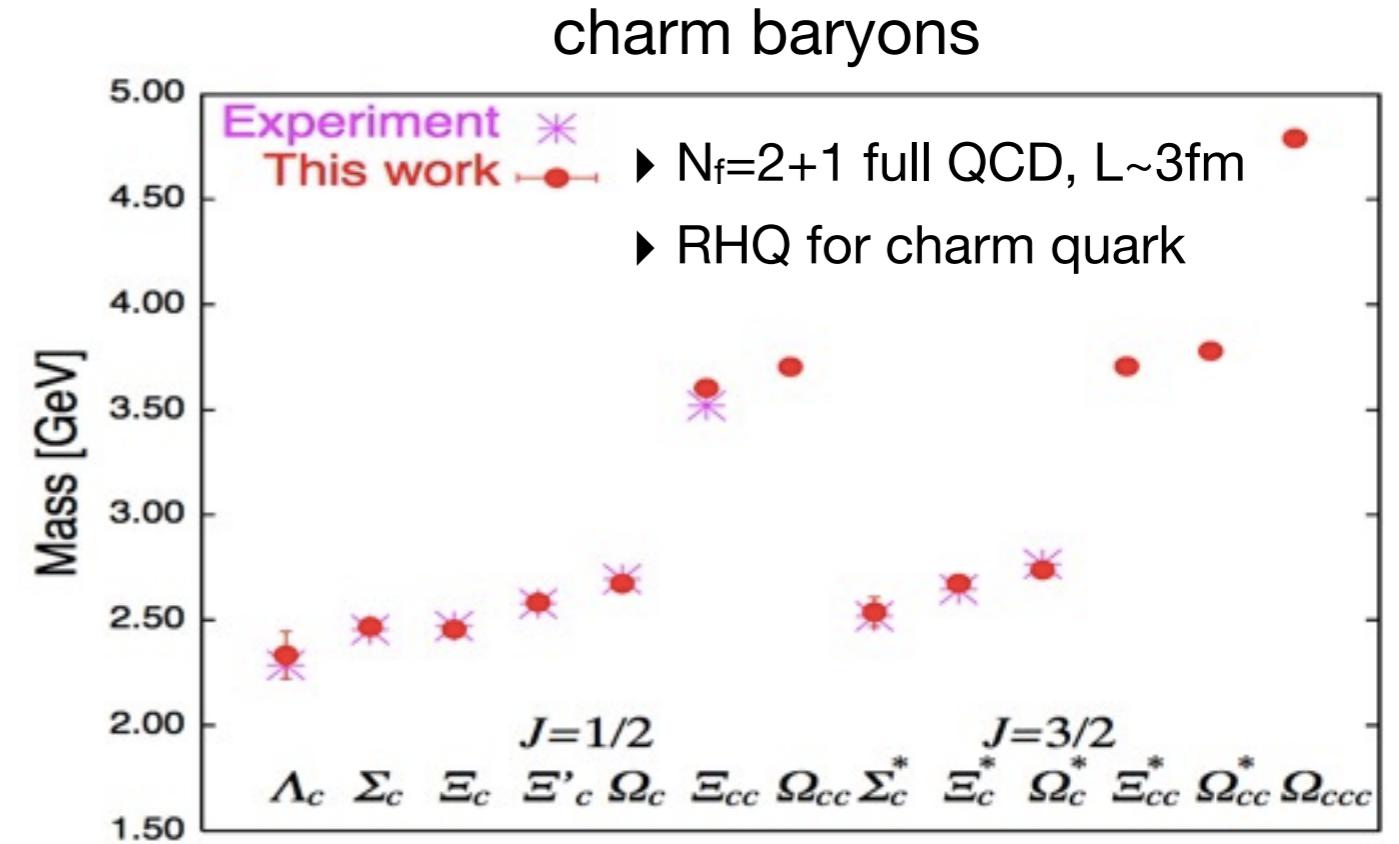
@Inha Univ., Korea (Oct. 24 -- 26, 2016.)

Single hadron spectroscopy from LQCD

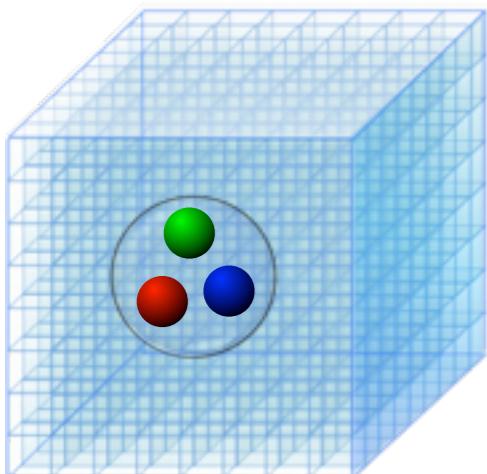
★ Low-lying (stable) hadrons on physical point (physical m_q)



Aoki et al. (PACS-CS), PRD81 (2010).



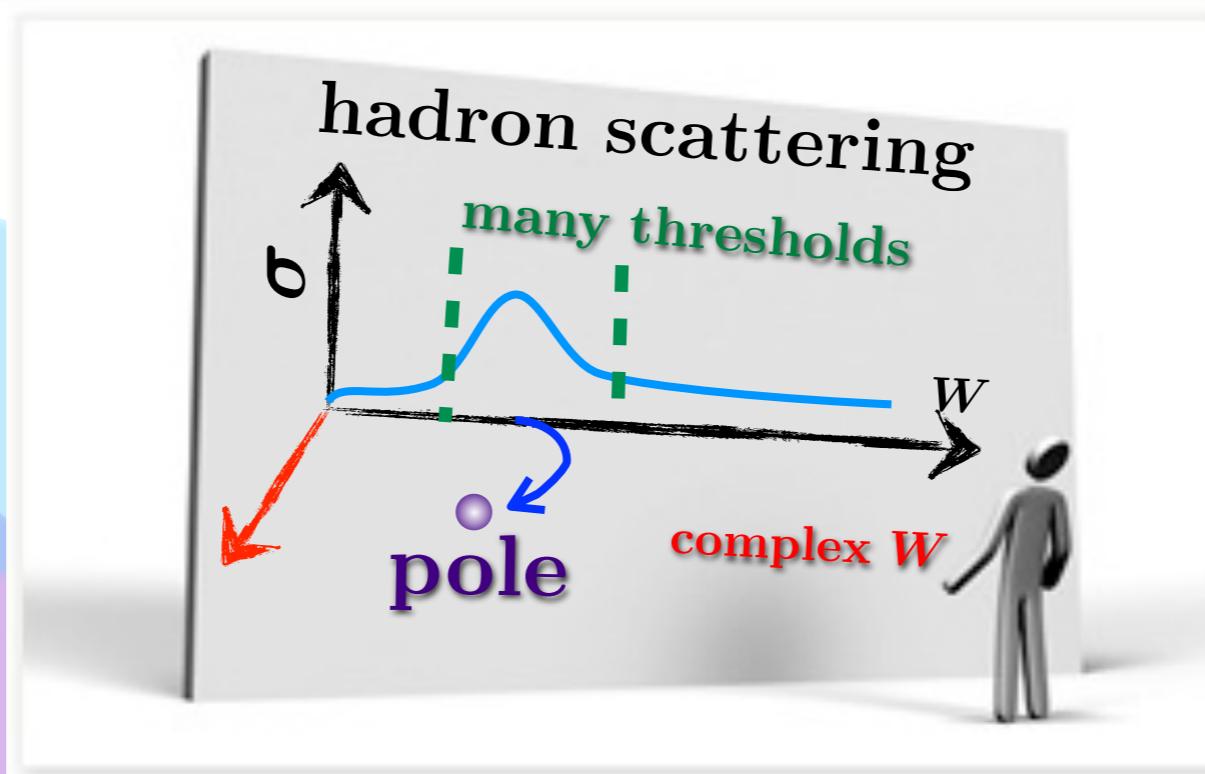
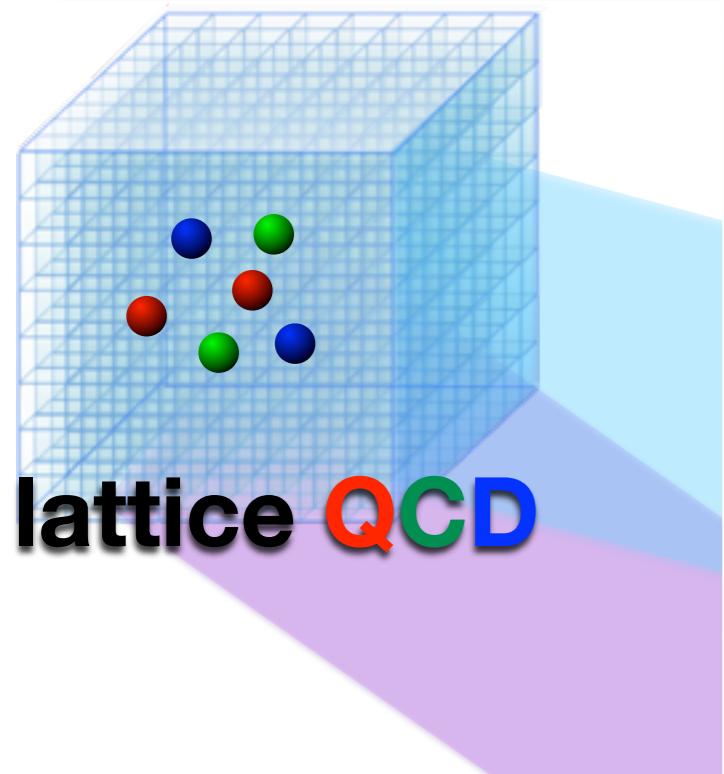
Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



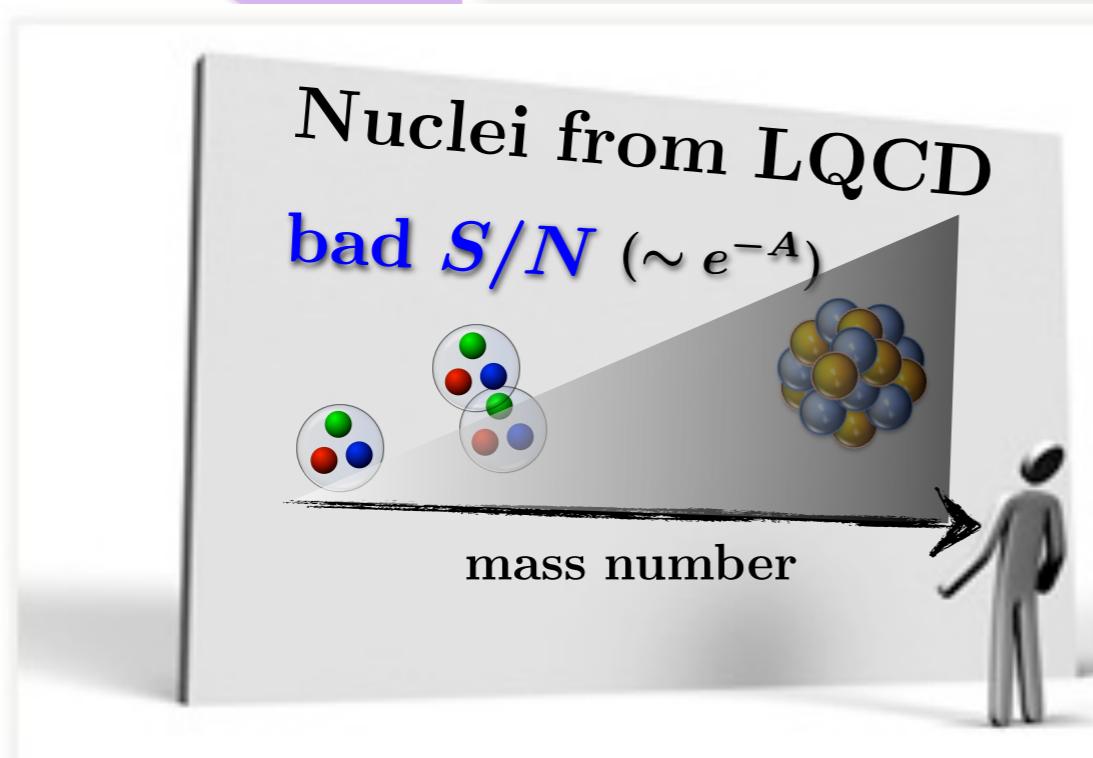
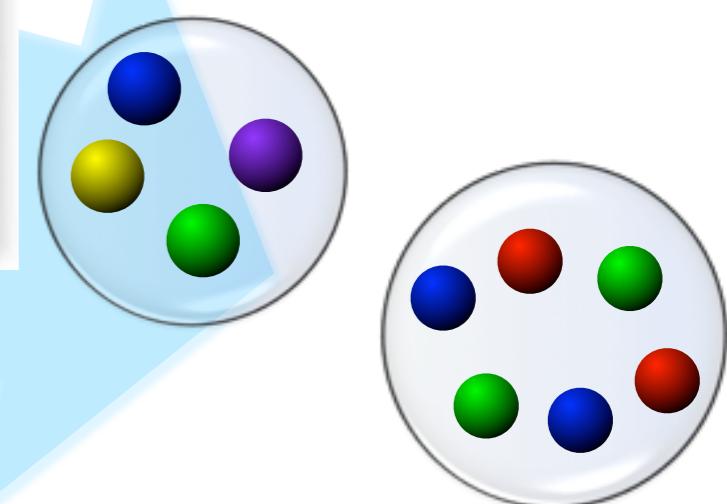
- a few % accuracy already achieved for single hadrons
- LQCD now can predict undiscovered charm hadrons (Ξ_{cc} , Ξ_{cc}^* , Ω_{ccc} , ...)

➡ Next challenge : multi-hadrons (resonances & nuclei)

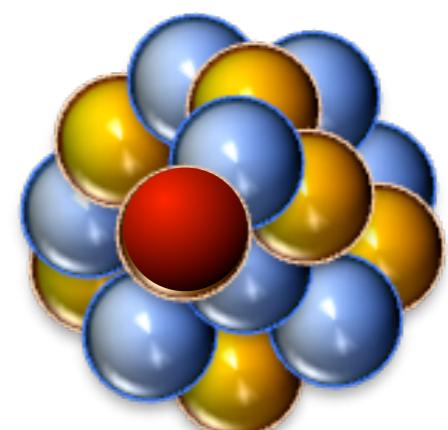
Resonances & nuclei from Lattice QCD



Exotic hadrons

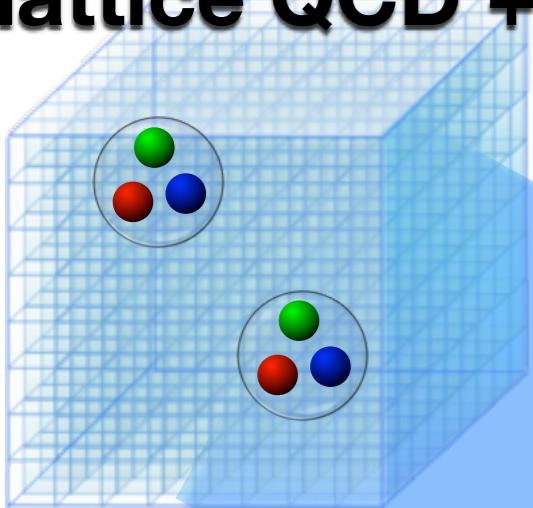


Nuclei



HAL QCD strategy (interactions faithful to S-matrix)

lattice QCD + scattering theory

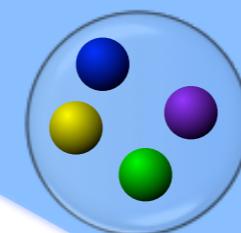


K-computer

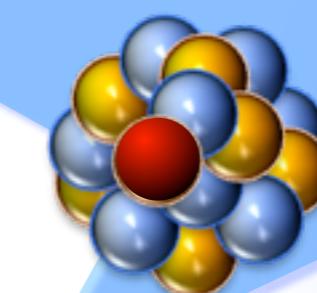
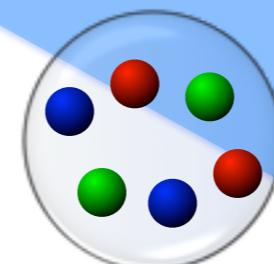
hadron interactions

- ▶ resonance
- ▶ ab initio many-body calc.
- ▶ hadronic EoS

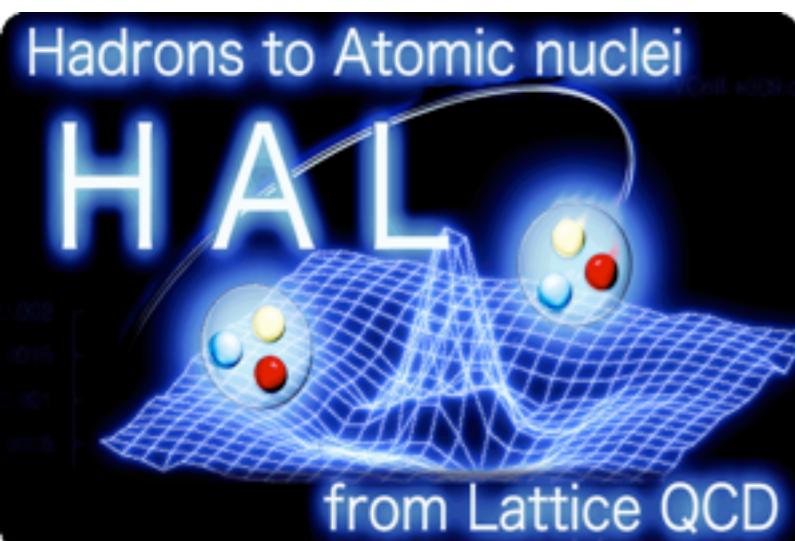
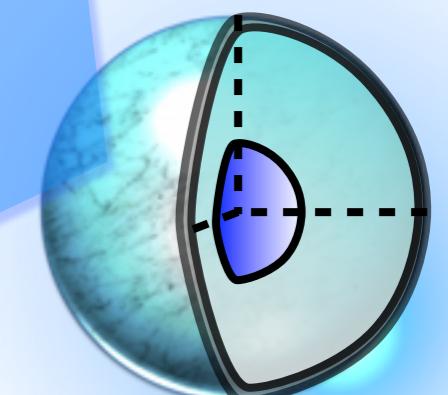
hadron resonances



nuclei



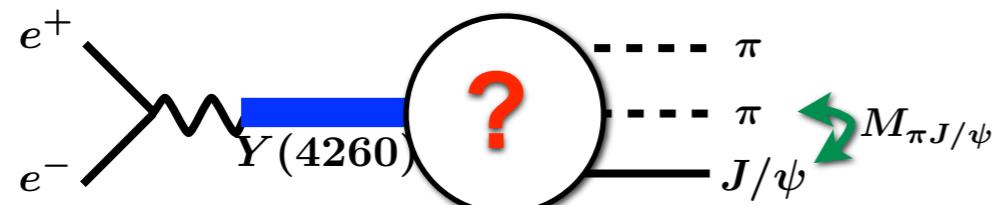
neutron stars



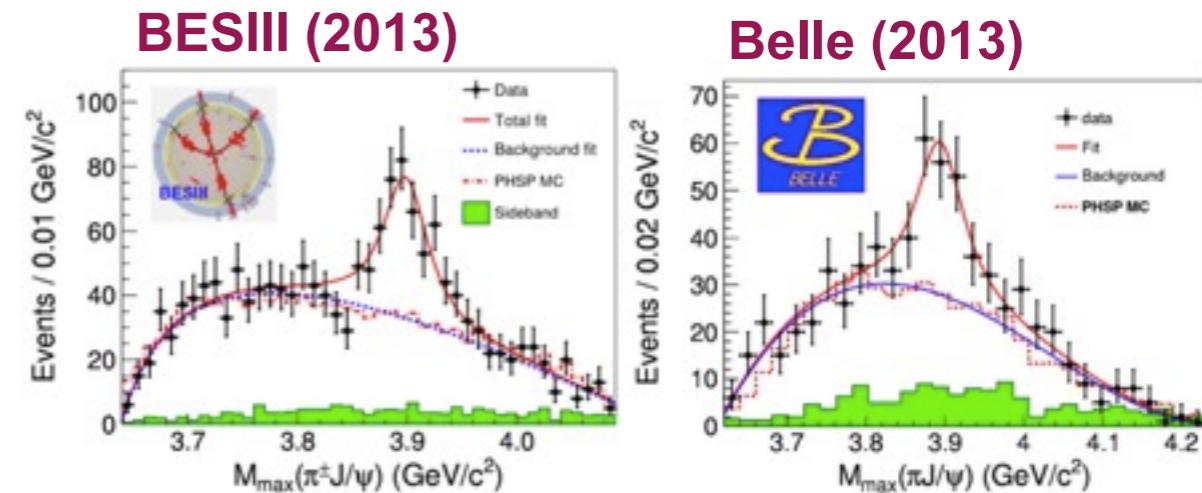
**N. Ishii, Oct. 24 (Mon.)
K. Sasaki, Oct. 24 (Mon.)**

Topics: Charm exotic hadrons & nuclei

✓ Tetraquark candidate $Z_c(3900)$

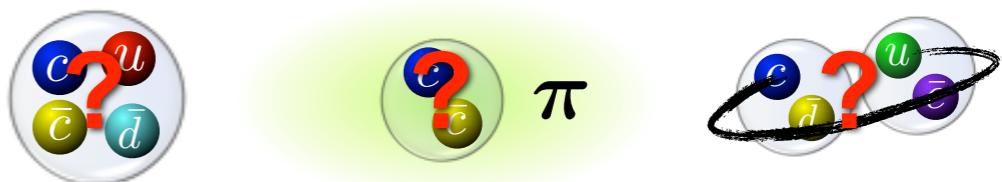


- $Z_c(3900)$ reported in $\pi^{+/-}J/\psi$ ($cc^{\bar{b}ar}ud^{\bar{b}ar}$)
- just above $D^{\bar{b}ar}D^*$ threshold, $J^P=1^+$



structure of $Z_c(3900)$

- tetraquark?, hadro-charmonium?, $D^{\bar{b}ar}D^*$ molecule?
- threshold kinematical effect?

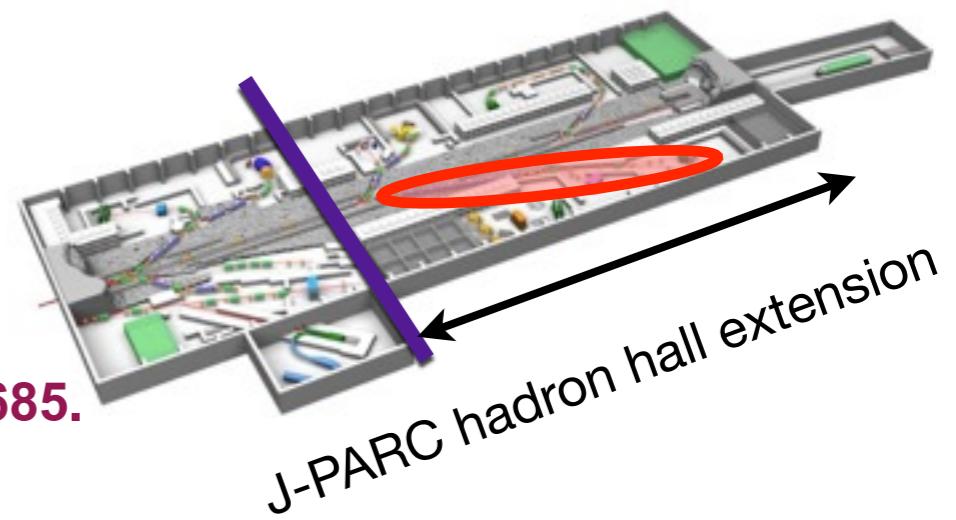


⌚ s-wave coupled-channel $\pi J/\psi$ -- $D^{\bar{b}ar}D^*$ dynamics

✓ $D^{\bar{b}ar}$ & Λ_c nuclei

- Charm nuclei productions @K10, J-PARC
 - $D^{\bar{b}ar}$ & Λ_c -nucleon interactions: no expt. data
 - attractive? repulsive?

see, Hosaka et al., arXiv:1606.08685.



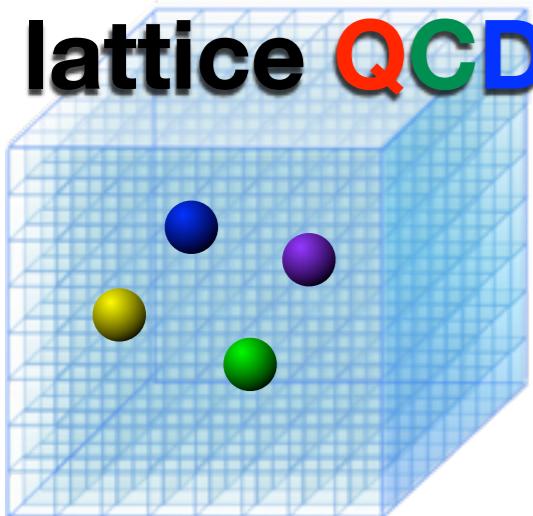
⌚ s-wave $D^{\bar{b}ar}N$ & $\Lambda_c N$ interactions

Contents

- ✓ Introduction
- ✓ Problem in coupled-channel scatterings from conventional LQCD approach
- ✓ HAL QCD approach to extract hadron interactions
- ✓ Results on charm hadron interactions
- ✓ Summary

How to extract hadron interactions?

lattice QCD

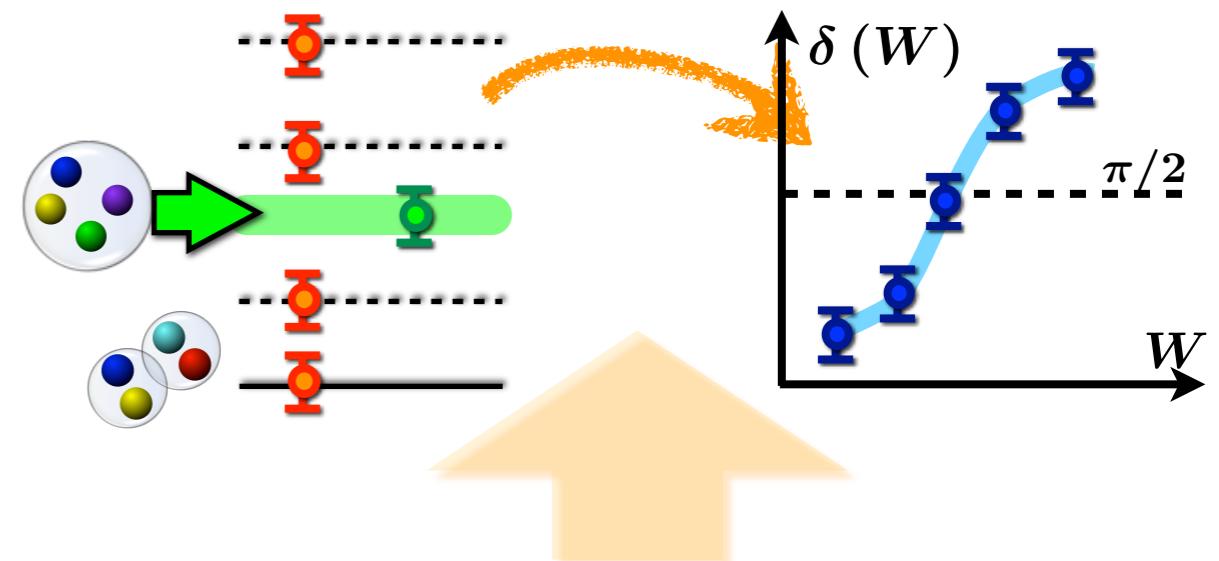
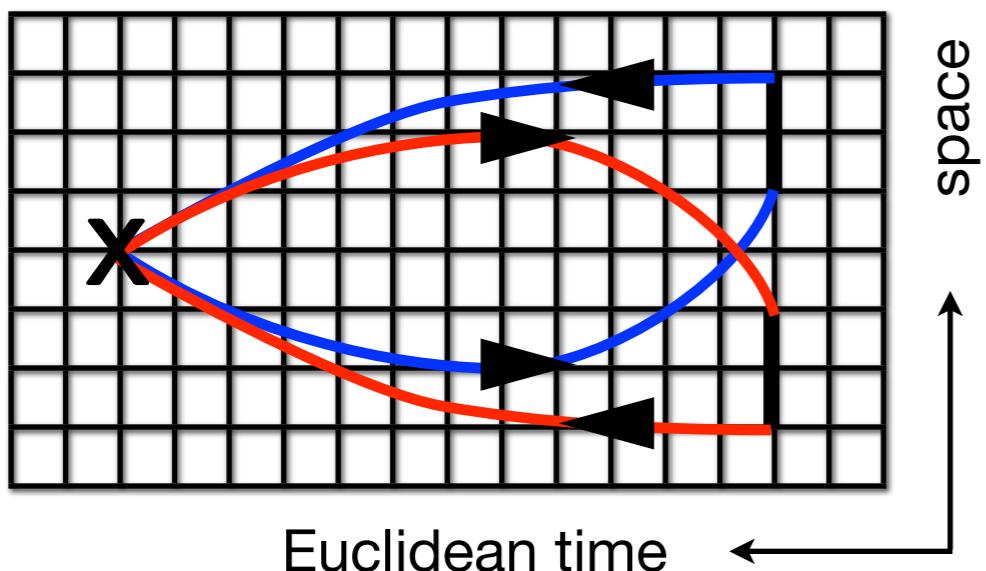


hadron interaction
faithful to S-matrix

✓ exotics, molecules

★ single channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$



❖ Lüscher's formula

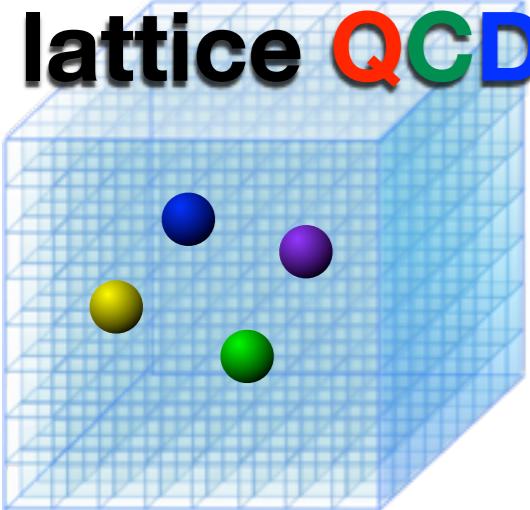
Lüscher, NPB354 (1991).

► finite V spectrum --> phase shift $\delta(W_n)$

$$k_n \cot \delta(k_n) = \frac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} \frac{1}{\vec{p}_m^2 - k_n^2}$$

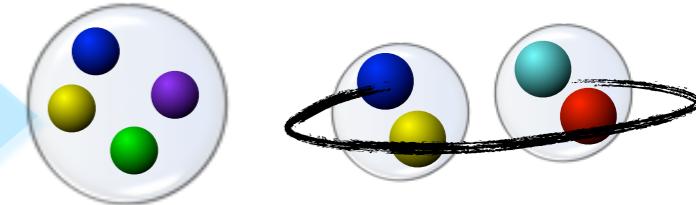
Problem in coupled-channel scattering

lattice QCD

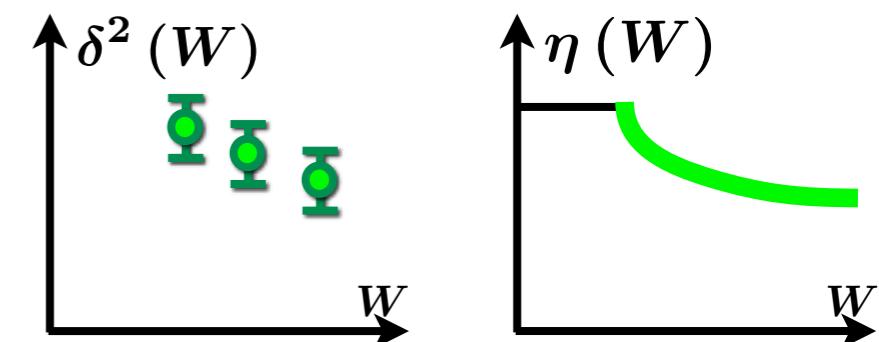
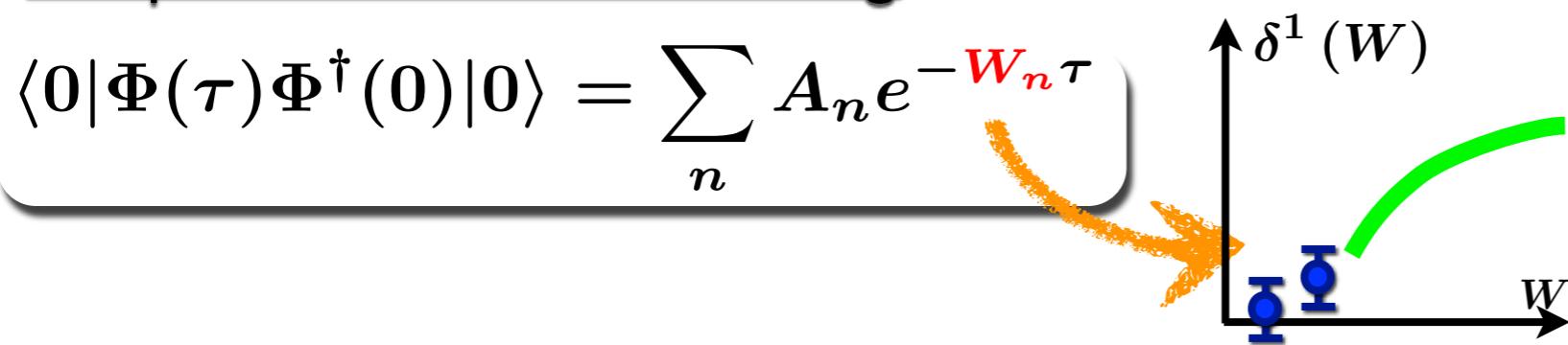


hadron interaction
faithful to S-matrix

✓ exotics, molecules



★ coupled-channel scattering



→ coupled-channel Lüscher's formula

elastic region: $W \rightarrow \delta(W)$

inelastic region: $W \rightarrow \delta^1(W), \delta^2(W), \eta(W) \rightarrow \text{find } W(L_1)=W(L_2)=W(L_3)$

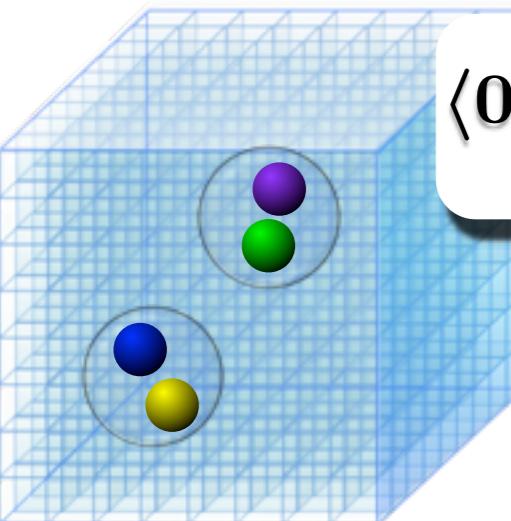
→ **assumptions** about interactions or K-matrices necessary...

★ indicate **more information mandatory** to solve coupled-channel scatterings

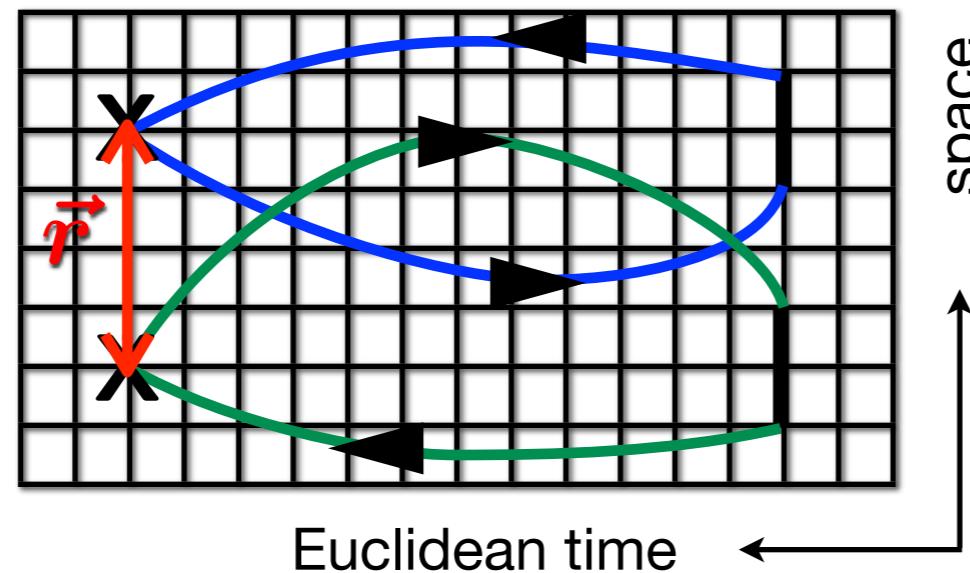
→ What can we measure in addition to temporal correlations?

HAL QCD approach “potential” as representation of S-matrix

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation



$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
 Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
 Ishii et al,(HAL QCD), PLB712, 437(2012).

- ★ Nambu-Bethe-Salpeter wave functions: $\psi_n(\vec{r})$

- ▶ NBS wave functions outside interactions --> **Helmholtz equation**

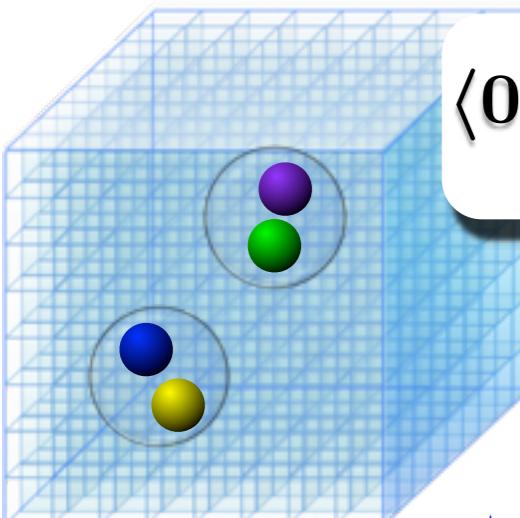
$$(\nabla^2 + \vec{k}_n^2) \psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

→ **S-matrix**

HAL QCD approach “potential” as representation of S-matrix

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation

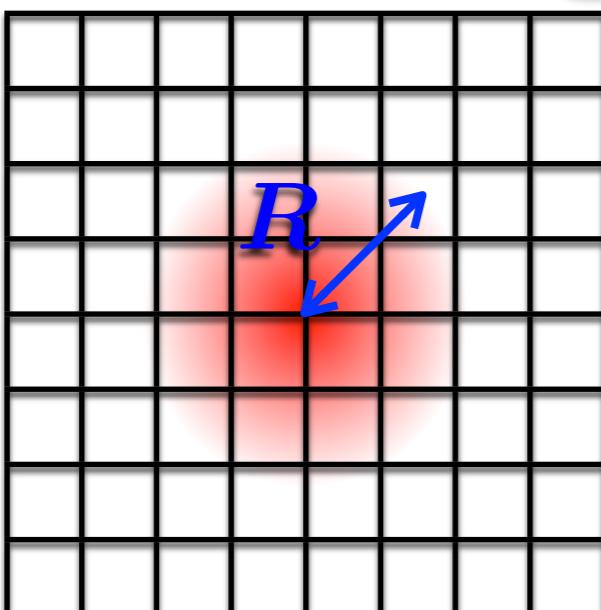
$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-\mathbf{W}_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
 Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
 Ishii et al,(HAL QCD), PLB712, 437(2012).

- ★ NBS wave functions inside interactions: **half-offshell T-matrix**

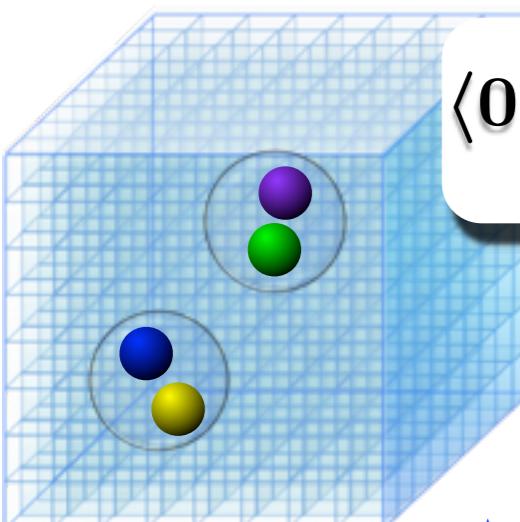
$$(\nabla^2 + \vec{k}_n^2) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$



- $U(r, r')$ is faithful to S-matrix in elastic region
- $U(r, r')$ is energy-independent (until new threshold opens)
- $U(r, r')$ contains all 2PI contributions
- $U(r, r')$ is not an observable (applied to ab initio calc.)

Coupled-channel HAL QCD approach

- ◆ HAL QCD approach: extract **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation


$$\langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1^a Z_2^a} \sum_n A_n \psi_n^a(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

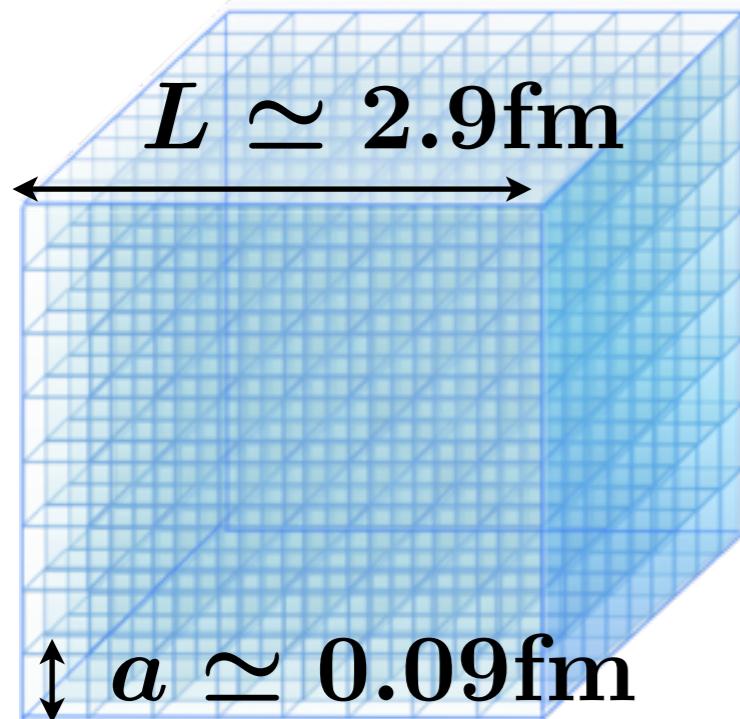
- ★ channel wave functions defined in asymptotic region: $\psi_n^a(\vec{r})$

$$(\nabla^2 + (\vec{k}_n^a)^2) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')$$

- ★ **coupled-channel potential $U^{ab}(r, r')$:**

- $U^{ab}(r, r')$ is faithful to S-matrix in both elastic and inelastic regions
- $U^{ab}(r, r')$ is energy-independent (until new threshold opens)
- $U^{ab}(r, r')$ contains all 2PI contributions

Lattice QCD setup



★ N_f=2+1 full QCD

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- $a=0.0907(13)$ fm $\rightarrow L \sim 2.9$ fm ($32^3 \times 64$)

★ Relativistic Heavy Quark action for charm

S. Aoki et al., PTP109, 383 (2003).

Y. Namekawa et al., PRD84, 074505 (2011).

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{QCD} a)$, ...

⇒ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

- three sets of full QCD gauge configs. used ($m_\pi \sim 410\text{-}700\text{MeV}$)

light hadron mass (MeV)

$m_\pi = 411, 572, 701$

$m_K = 635, 714, 787$

$m_\rho = 896, 1000, 1097$

$m_N = 1215, 1411, 1583$

Charmed meson mass (MeV)

$m_{\eta_c} = 2988, 3005, 3024$

$m_{J/\psi} = 3097, 3118, 3143$

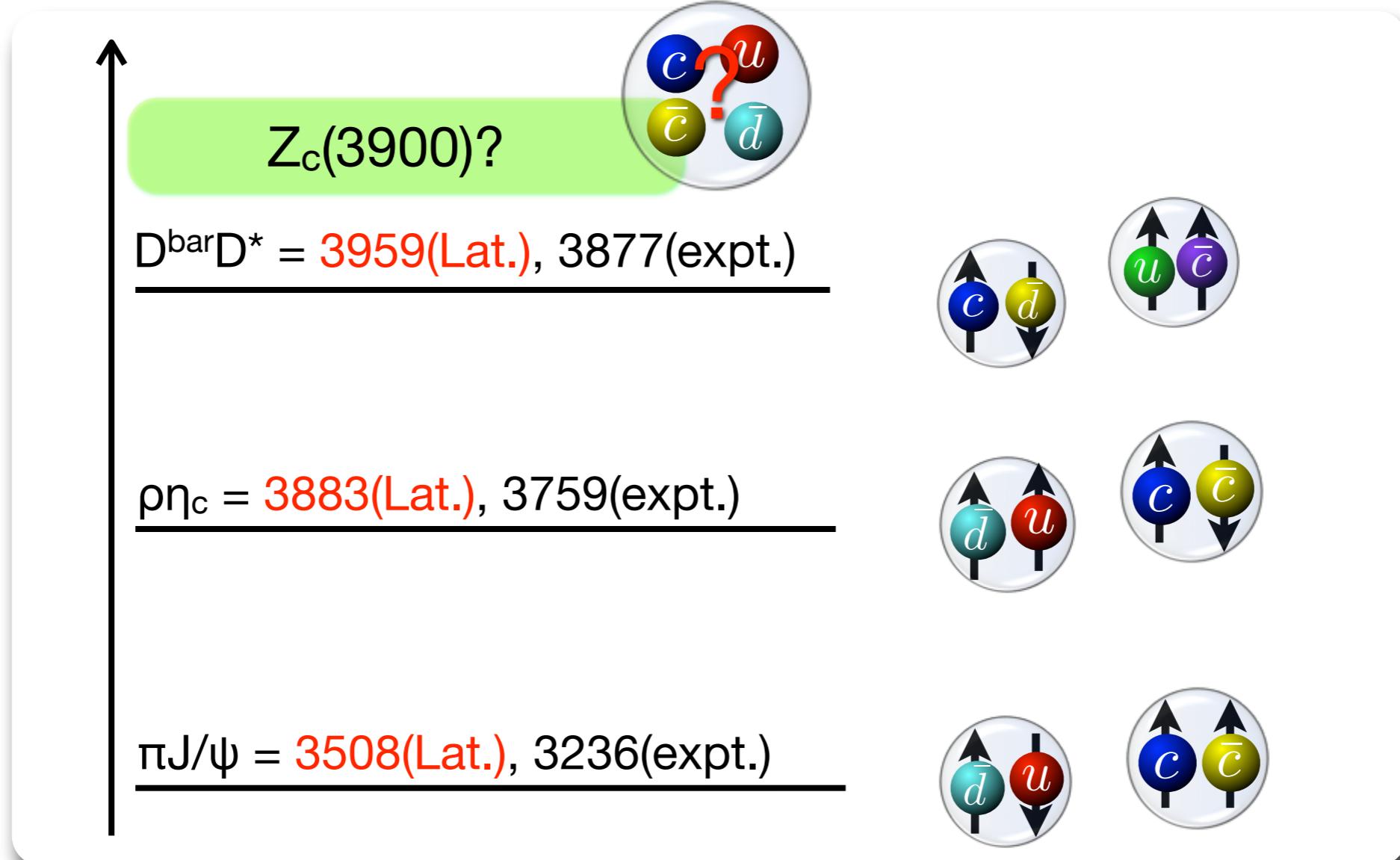
$m_D = 1903, 1947, 2000$

$m_{D^*} = 2056, 2101, 2159$

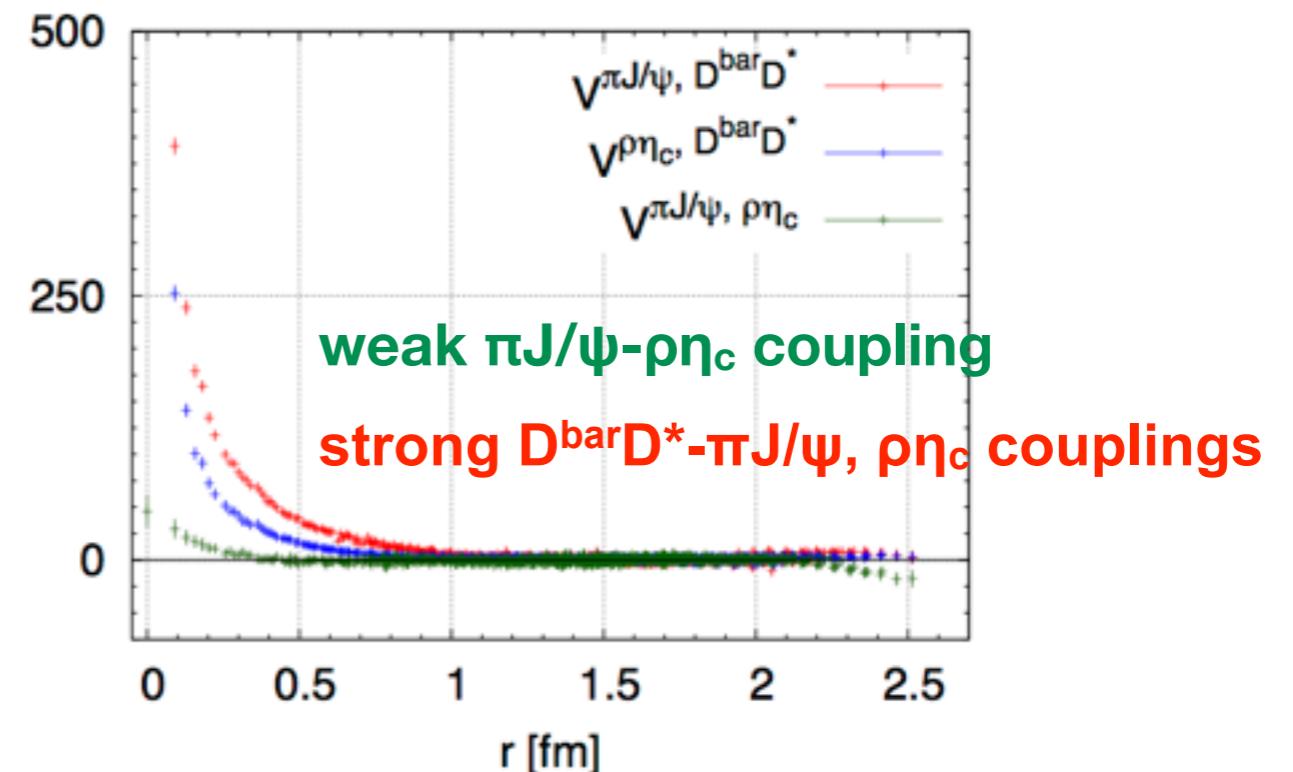
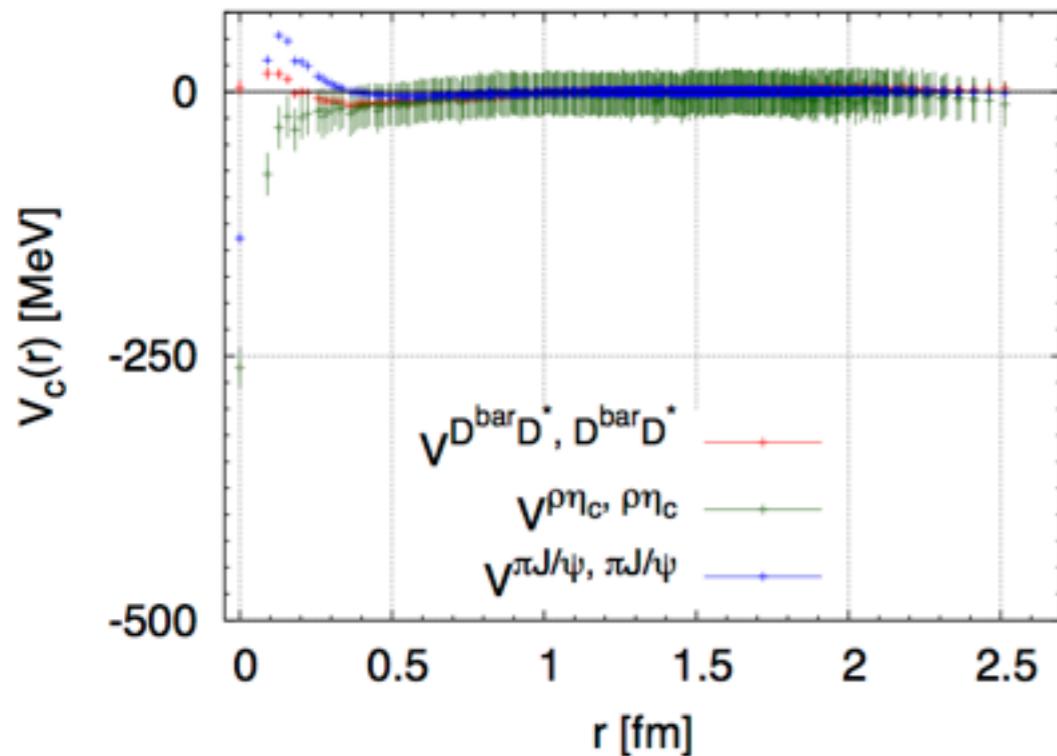
$m_{\Lambda_c} = 2434, 2584, 2710$

Structure of $Z_c(3900)$

-- $\pi J/\psi - \rho \eta_c - D^{\bar{b}a}D^*$ in $|G(J^{PC})=1^+(1^{+-})$ --

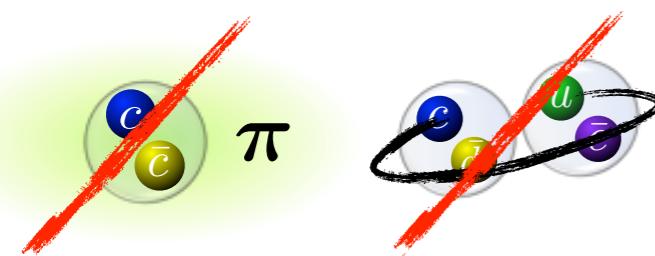


S-wave $\pi J/\psi - \rho \eta_c - D^{\bar{b}ar}D^*$ potential @ $m_\pi=410\text{MeV}$

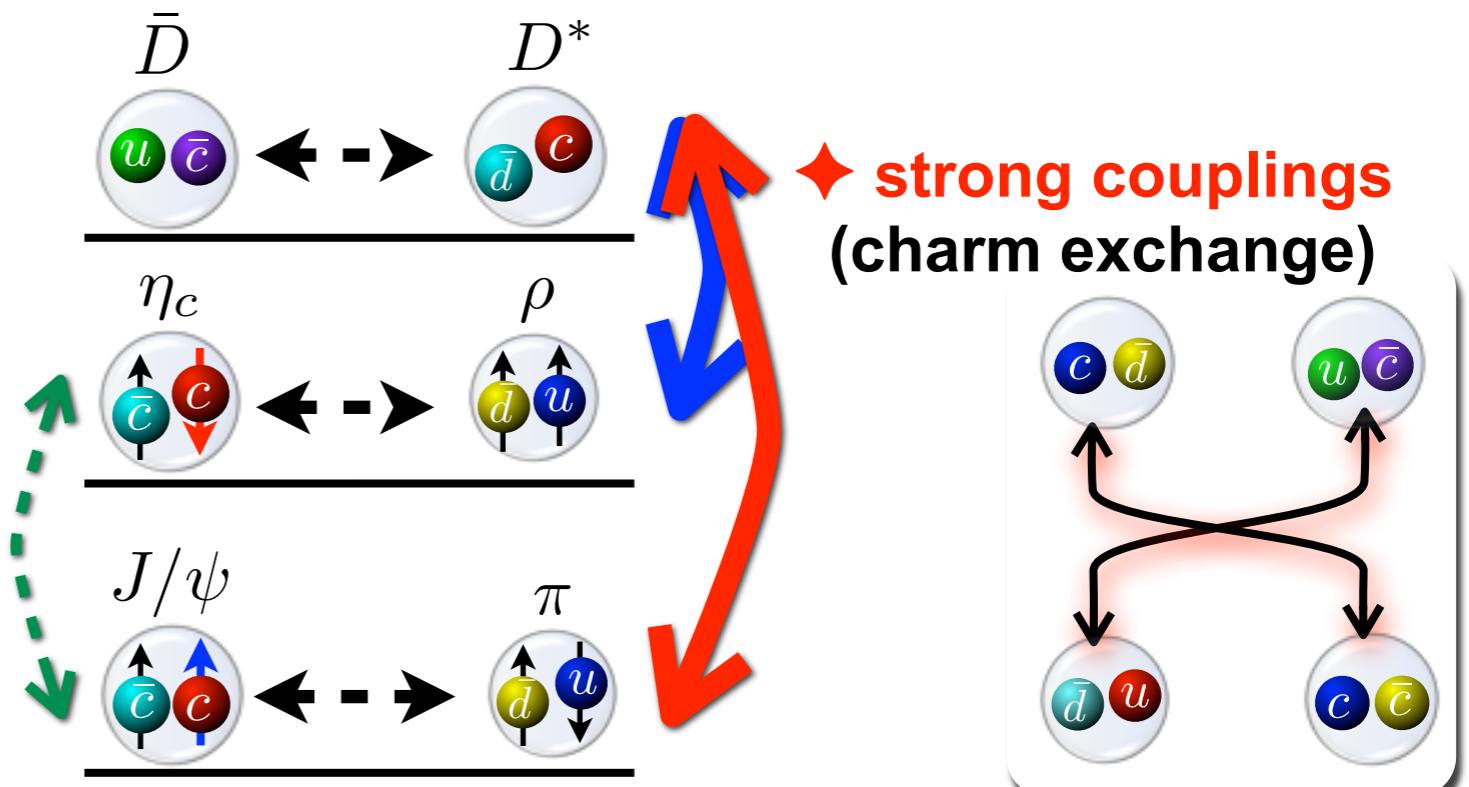


[Y. Ikeda \(HAL QCD\), arXiv.1602.03465\(hep-lat\).](#)

★ **weak diagonal potentials:**
 $Z_c(3900)$ is NOT simple $\pi J/\psi$ & $D^{\bar{b}ar}D^*$



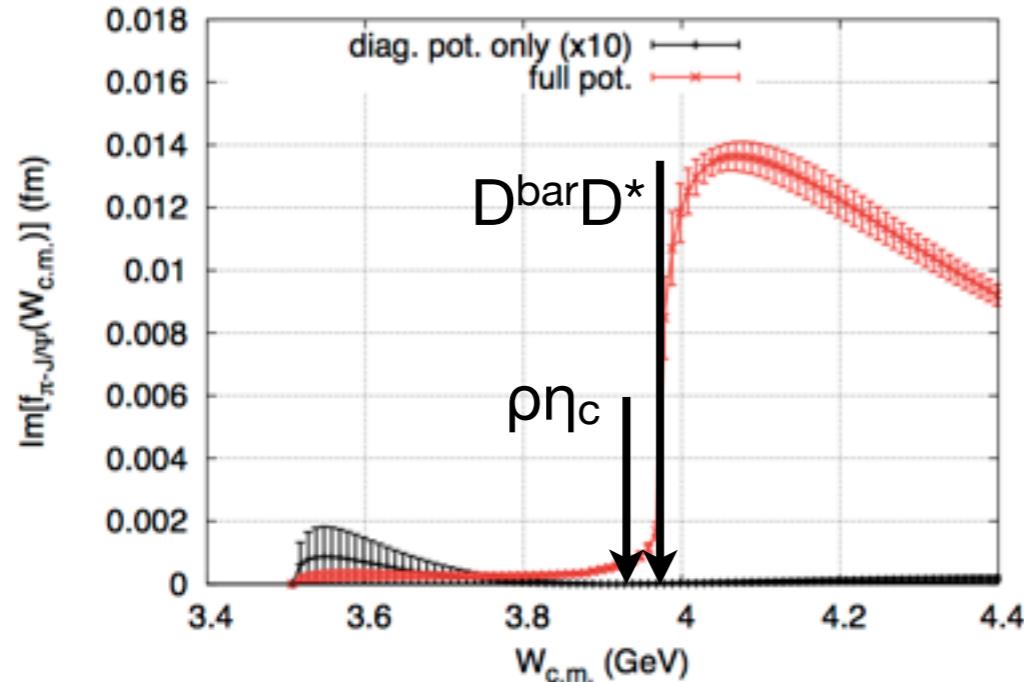
◆ **heavy quark spin symmetry**



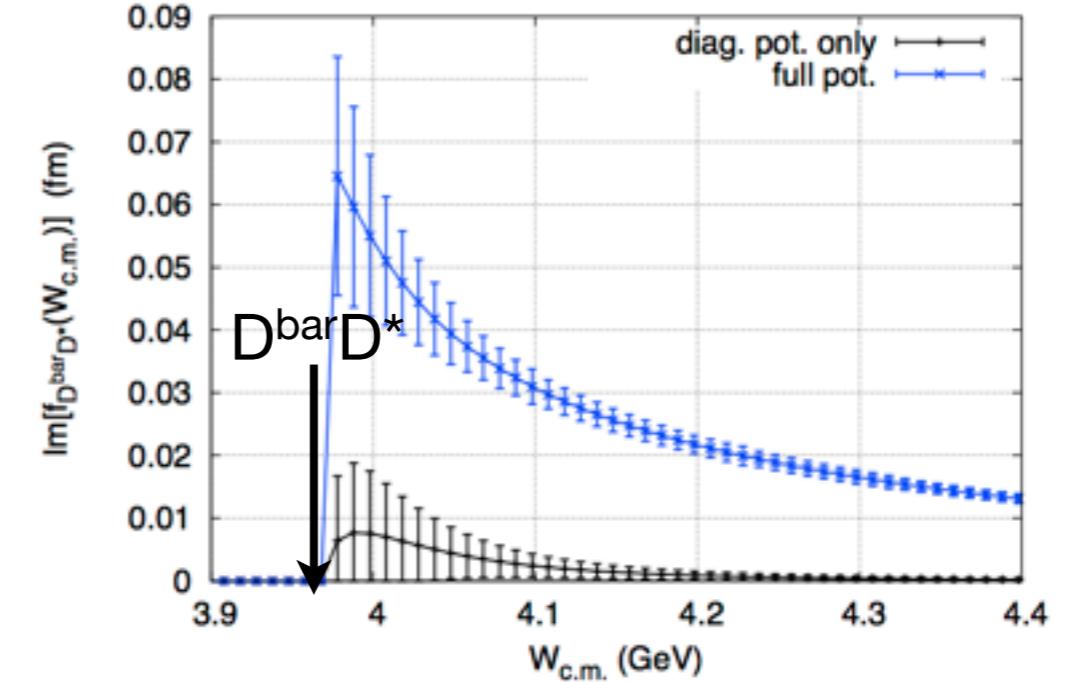
Invariant mass spectra of $\pi J/\psi$ & $D^{\bar{b}ar}D^*$

★ 2-body scattering (ideal setting to understand $Z_c(3900)$ structure)

- $\pi J/\psi$ invariant mass



- $D^{\bar{b}ar}D^*$ invariant mass

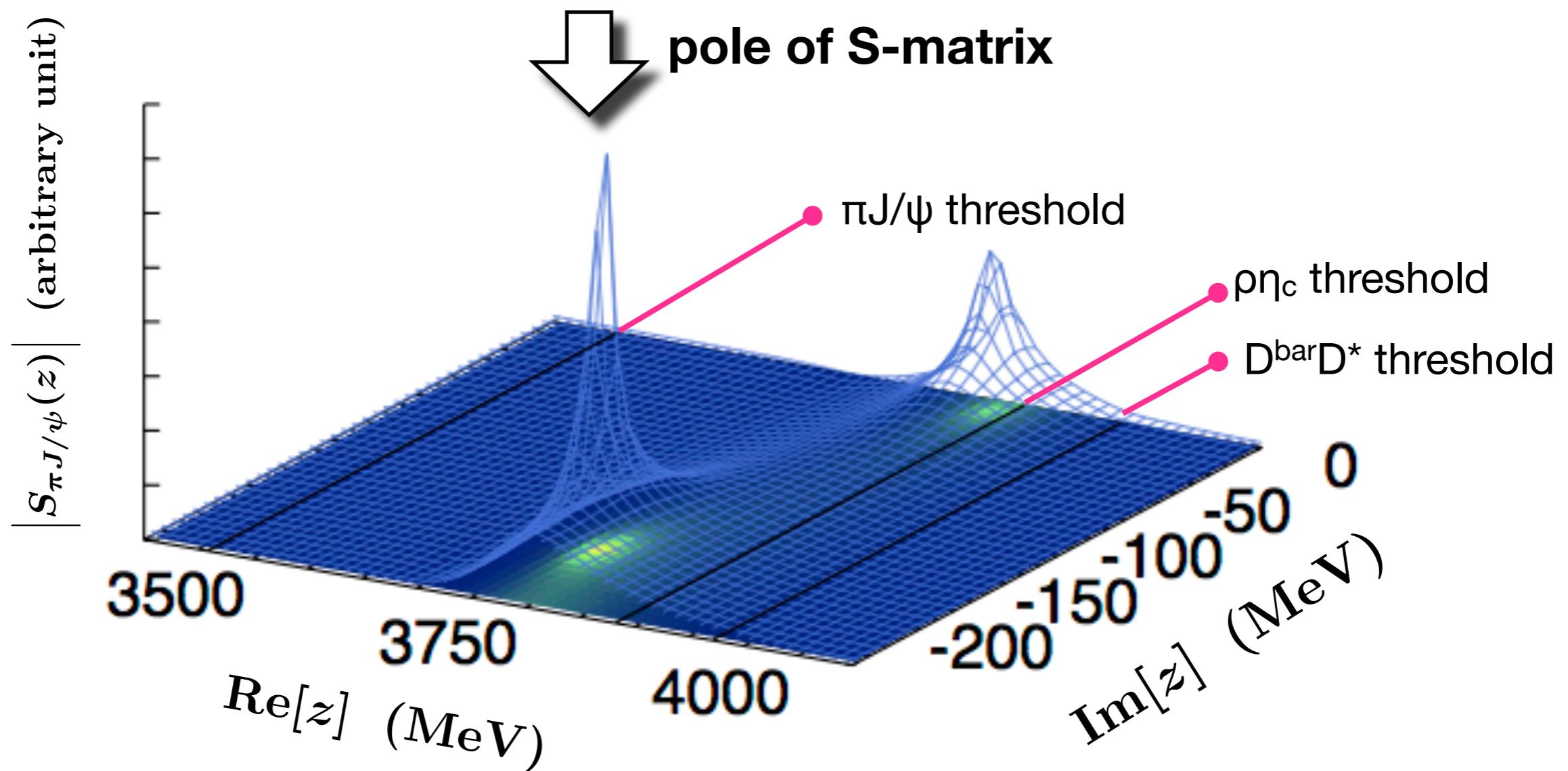


✓ Enhancement near $D^{\bar{b}ar}D^*$ threshold due to strong $V_{\pi J/\psi, D^{\bar{b}ar}D^*}$

- Peak in $\pi J/\psi$ (not Breit-Wigner line shape)
- Threshold enhancement in $D^{\bar{b}ar}D^*$

✓ Is $Z_c(3900)$ a conventional resonance? --> pole position

Complex pole position ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\bar{b}ar}D^*$:2nd)



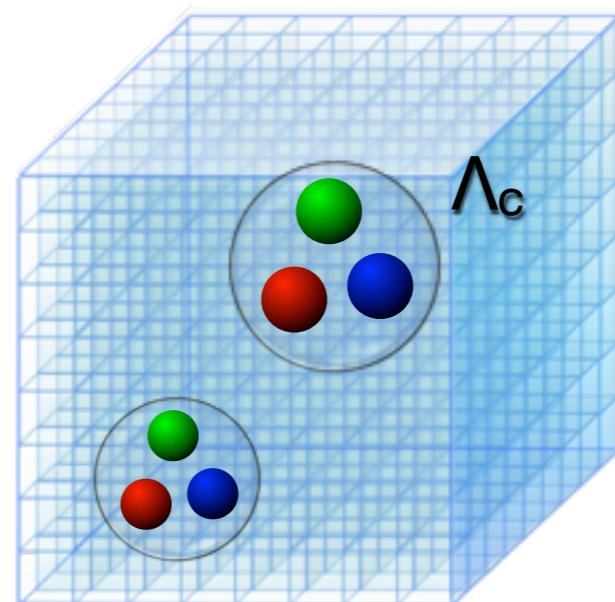
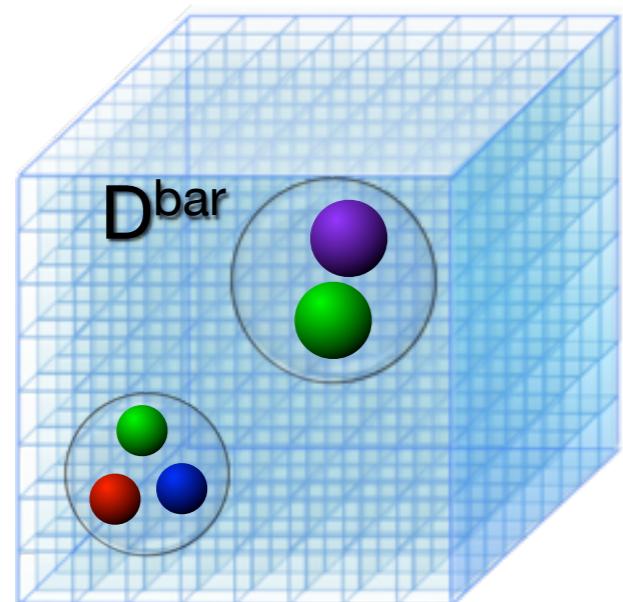
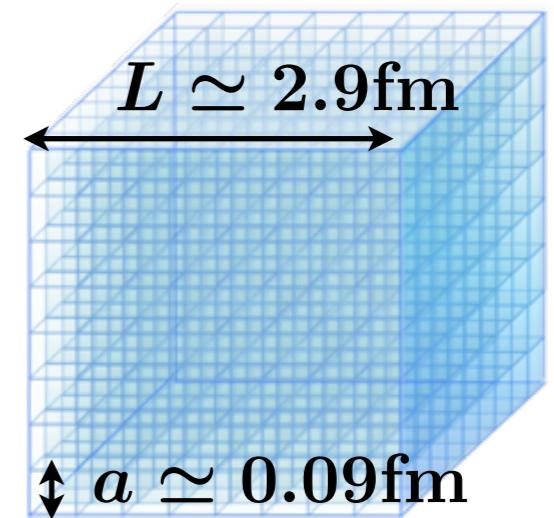
- “Virtual” pole on [2nd, 2nd, 2nd] sheet is found (far below $D^{\bar{b}ar}D^*$ threshold)
- No pole on other relevant sheets to $Z_c(3900)$
- $Z_c(3900)$ is not a conventional resonance
- $Z_c(3900)$ is cusp induced by off-diagonal $V^{\pi\psi}, D^{\bar{b}ar}D^*$

S-wave $D^{\bar{b}ar}N$ & $\Lambda_c N$ interactions

-- implications to $D^{\bar{b}ar}$ & Λ_c nuclei --

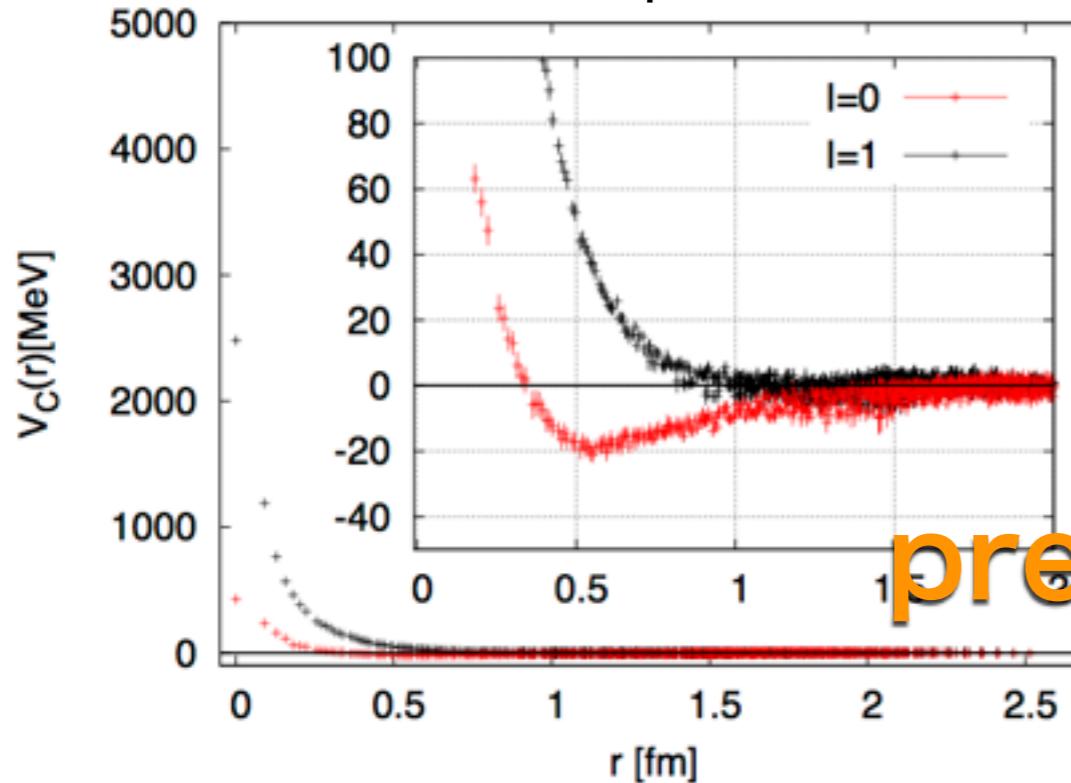
- ▶ $N_f=2+1$ full QCD configs. (PACS-CS collaboration)
- ▶ RHQ for charm

$m_\pi =$	411, 572, 701
$m_N =$	1215, 1411, 1583
$m_D =$	1903, 1947, 2000
$m_{\Lambda_c} =$	2434, 2584, 2710



S-wave $D^{\bar{b}ar}N$ interactions in $|l|=0, 1$ @ $m_\pi=410\text{MeV}$

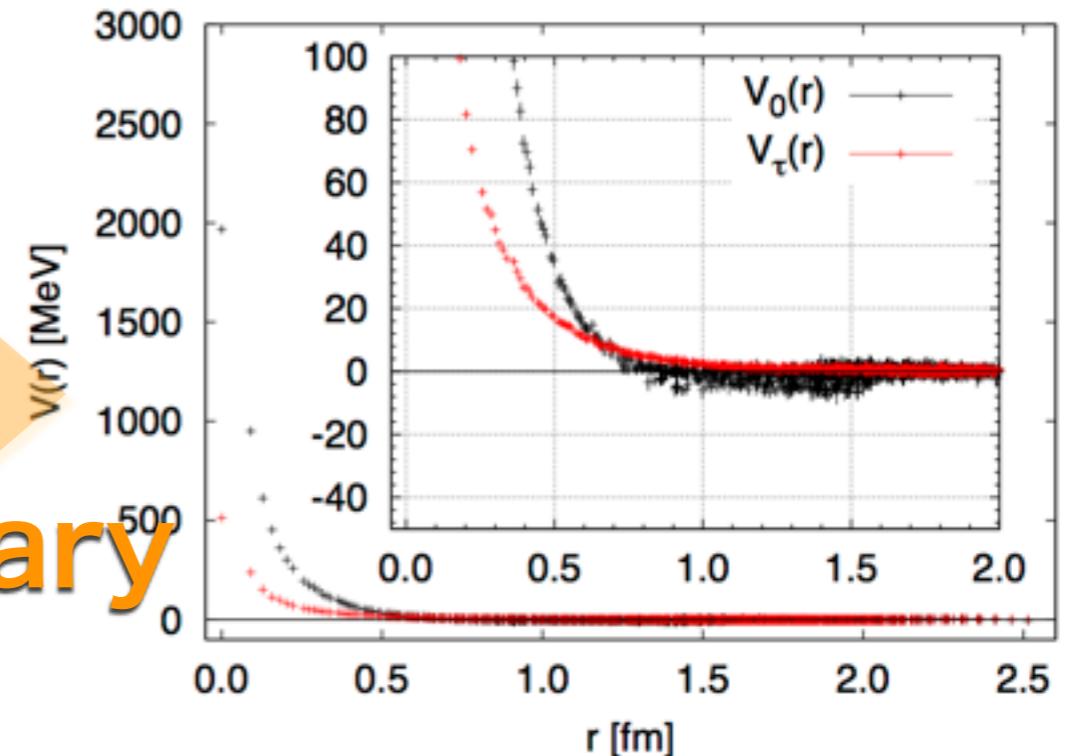
$|l|=0$ & $|l|=1$ potentials



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I = 0) \\ -0.20(1) & (I = 1) \end{cases}$$

preliminary

$$V_{\bar{D}N}(\vec{r}) = V_0(\vec{r}) + V_\tau(\vec{r})(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)$$



► $V_0(r)$ is **repulsive**, $V_\tau(r)$ is **positive**

$D^{\bar{b}ar}N$ interaction

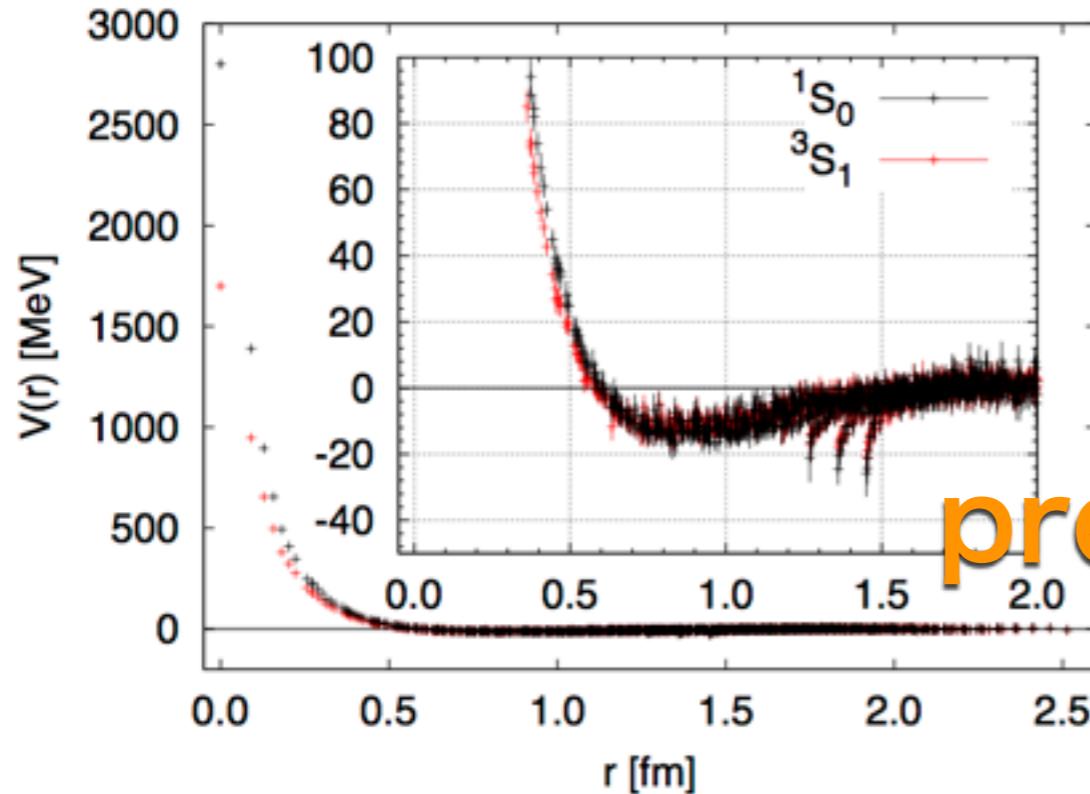
- (1) attractive in $|l|=0$ & repulsive in $|l|=1$
- (2) $V_0(r)$ is repulsive, $V_\tau(r)$ is positive
- (3) $D^{\bar{b}ar}$ would be bound in neutron/proton rich nuclei

[Y. Ikeda \[HAL QCD\], in preparation](#)

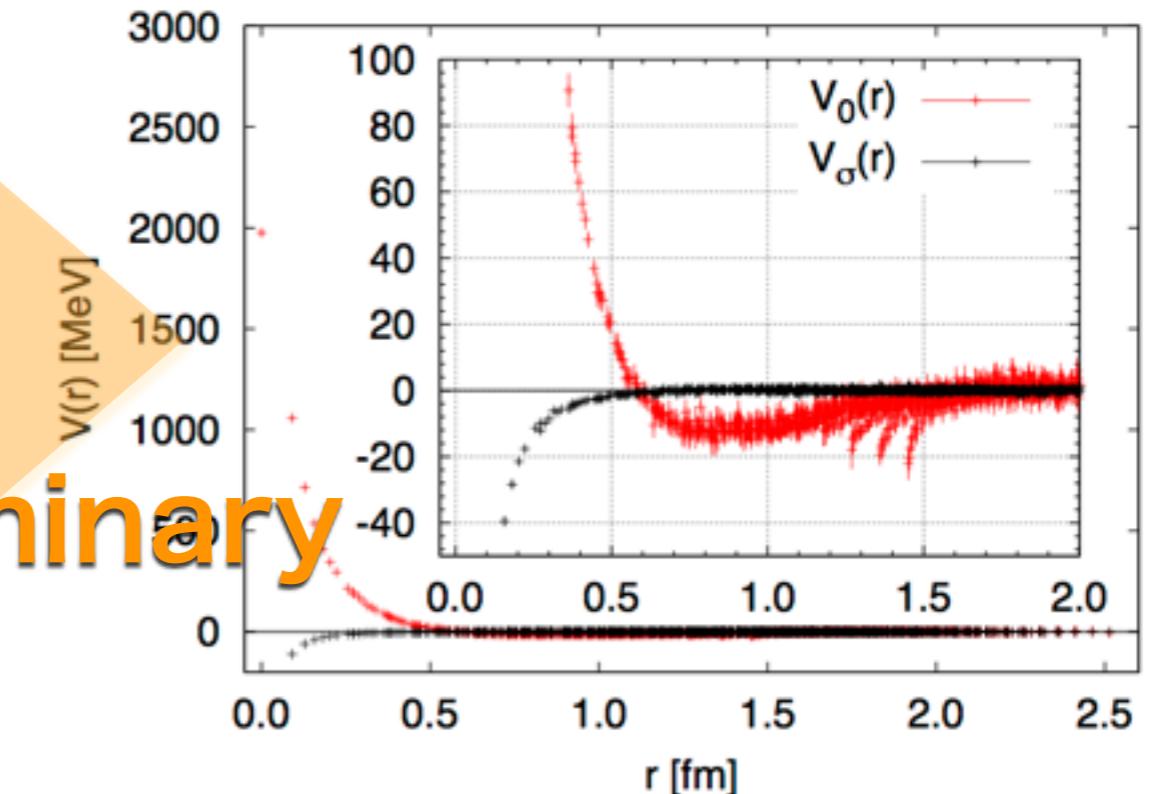
[AMD calc.: M. Isaka, YI, E. Hiyama, in progress](#)

S-wave Λ_c N interactions @ $m_\pi=410\text{MeV}$

1S_0 & 3S_1 channels



$$V_{\Lambda_c N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})(\vec{\sigma}_{\Lambda_c} \cdot \vec{\sigma}_N)$$



$$a_{\Lambda_c N} = \begin{cases} 0.29(16) & (^1S_0) \\ 0.28(13) & (^3S_1) \end{cases}$$

► $V_0(r)$ is **attractive**, $V_\sigma(r)$ is **very small**

Λ_c N potentials in 1S_0 & 3S_1 channels

- (1) weak net attractions in both 1S_0 and 3S_1 channels
- (2) **weak spin dependence --> HQ spin symmetry at work**
- (3) Λ_c will be bound in heavy nuclei

Summary

◆ HAL QCD approach for coupled-channel hadron interactions

- ▶ energy-independent “potentials” from equal-time NBS wave functions
- ▶ a solution of coupled-channel problems

◆ Charmed tetraquark candidate $Z_c(3900)$ in $J^P=1^+$ channel

- ▶ $Z_c(3900)$ is threshold cusp induced by strong $V^{D\bar{D}^* - \pi J/\psi}$

◆ $D\bar{D}$ nuclei

- ▶ Isospin dependent interaction plays important role (neutron/proton rich favored)

◆ Λ_c nuclei

- ▶ Spin independent interaction is weakly attractive (heavy nuclei favored)

◆ Physical point simulation is next step

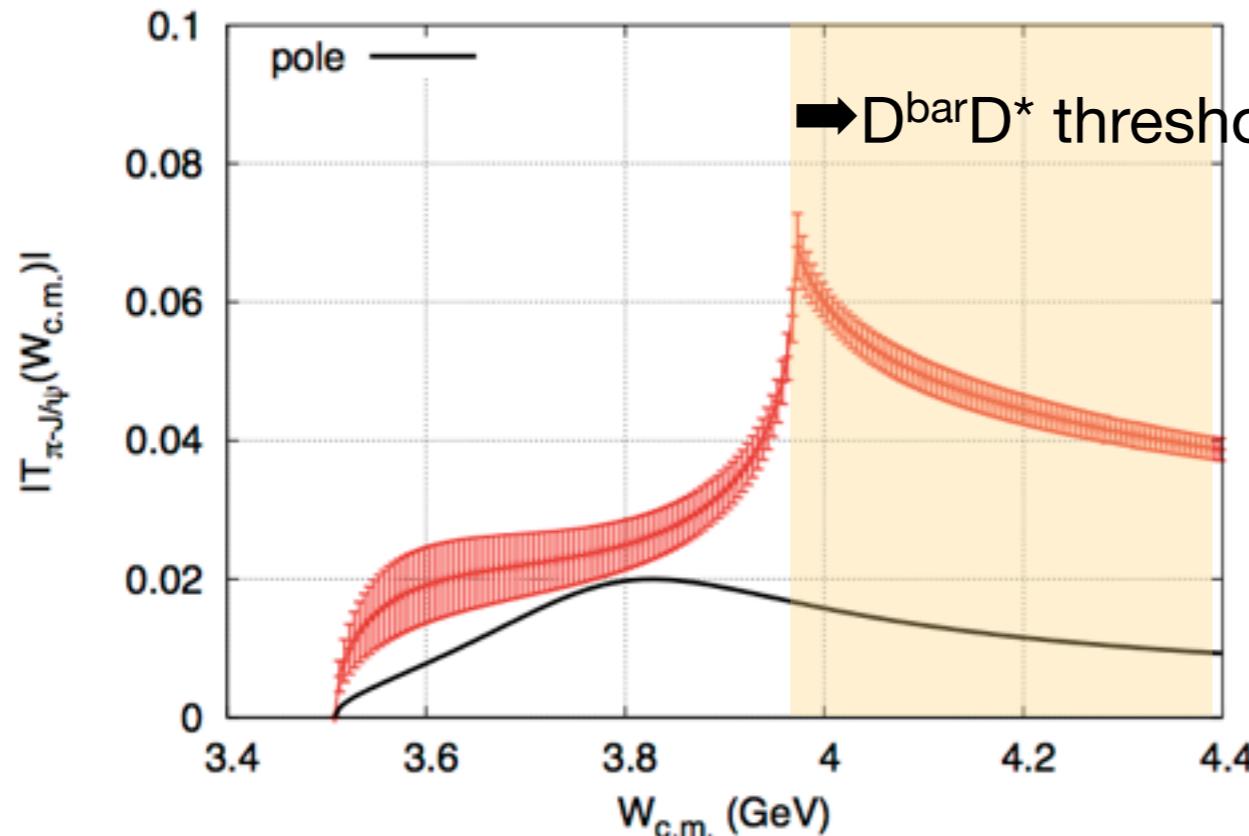
Backup : Z_c(3900)

T-matrix of $\pi J/\psi$ & $D^{\bar{b}ar}D^*$

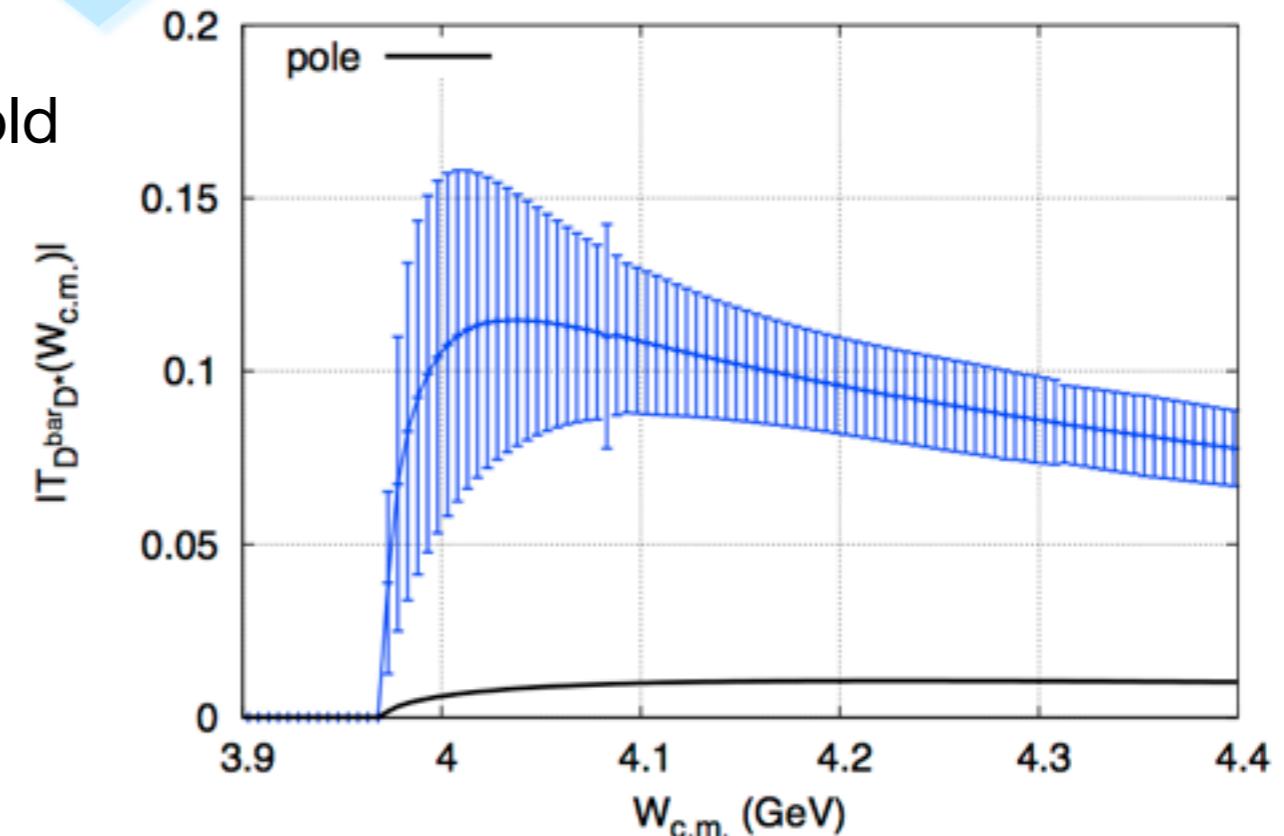
- calculate residues of T-matrices in $\pi J/\psi$ & $D^{\bar{b}ar}D^*$ channels

$$S(k) = 1 + 2iT(k)$$

- $\pi J/\psi - \pi J/\psi$ T-matrix



- $D^{\bar{b}ar}D^* - D^{\bar{b}ar}D^*$ T-matrix



- contribution from virtual pole to T-matrix is small
- $Z_c(3900)$ is cusp at $D^{\bar{b}ar}D^*$ threshold induced by off-diagonal $V^{\pi\psi}, D^{\bar{b}ar}D^*$

Complex energy plane ($\pi J/\psi$ -- $\rho \eta_c$ -- $D^{\bar{D}} D^*$ coupled-channel)

- ✓ $Z_c(3900)$ is **genuine resonance**: pole position in $\text{Re}[z] > m_D + m_{D^*}$ on **[bbb]** sheet
- ✓ $Z_c(3900)$ is **quasi-bound state**: pole position in $m_\rho + m_{\eta_c} < \text{Re}[z] < m_{\bar{D}} + m_{D^*}$ on **[bbt]** sheet

