# Mass shift of $\Lambda_C$ baryon in nuclear matter from QCD sum rule

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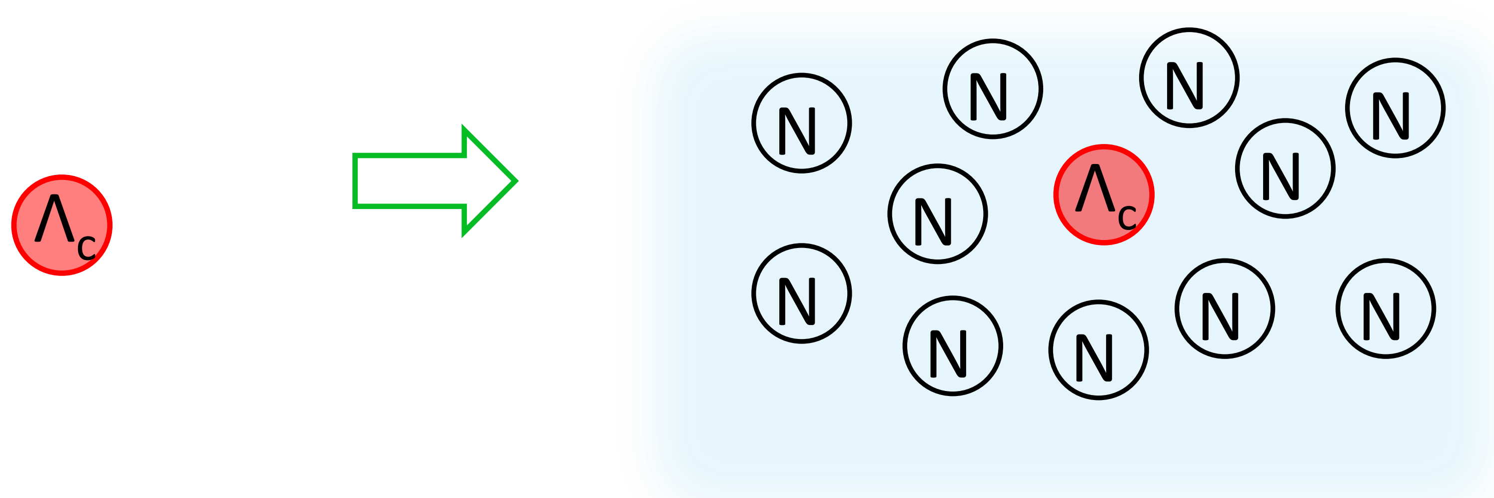
# Outline

- Introduction
- QCD sum rules
- $\Lambda_c$ QCD sum rules
- Results
- Summary

# Introduction

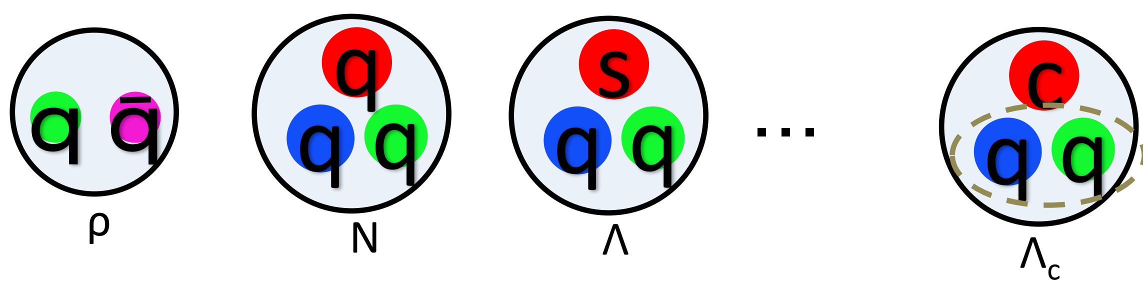
$\Lambda_c$ baryon in nuclear matter:




In vacuum

In nucleus

- Interaction between $\Lambda_C$ and nucleon
- The relation between $\Lambda_C$ mass and the partial restoration of chiral symmetry
- Information on diquark




We investigate the mass modification of $\Lambda_C$ baryon in nuclear matter.

# Introduction

Previous works (QCD sum rules)

E. V. Shuryak, Nucl. Phys. **B198**, 83 (1982)

E. Bagan et al., Phys. Lett. **B287**, 176 (1992)

:

Z.-G. Wang, Eur. Phys. J. **C71**, 1816 (2011)

K. Azizi, N. Er and H. Sundu, arXiv:1605.05535 [hep-ph].

In vacuum

In nuclear matter

# Introduction

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In vacuum

In nuclear matter

| | $\lambda_{\Lambda_c}$ [GeV$^3$] | $\lambda^*_{\Lambda_c}$ [GeV$^3$] | $m_{\Lambda_c}$ [GeV] | $m^*_{\Lambda_c}$ [GeV] | $\Sigma^{\nu}_{\Lambda_c}$ [MeV] | $\Sigma^{S}_{\Lambda_c}$ [MeV] |
|---|---|---|---|---|---|---|
| K. Azizi et al., | $0.044 \pm 0.012$ | $0.023 \pm 0.007$ | $2.235 \pm 0.244$ | $1.434 \pm 0.203$ | $327 \pm 98$ | -801 |
| Z. G. Wang | $0.022 \pm 0.002$ | $0.021 \pm 0.001$ | $2.284^{+0.049}_{-0.078}$ | $2.335^{+0.045}_{-0.072}$ | $34 \pm 1$ | 51 |

Results in Vacuum

Results in nuclear matter

- There are large discrepancies in the results.
- The equations of OPE do not consist with each other.

Our analyses

- Recalculation of OPE
- $\alpha_s$ corrections (NLO)
- Up to dimension 8 condensate (higher order contribution)
- Parity projection

S. Groote, et al., Eur. Phys. J. C58, 355 (2008)

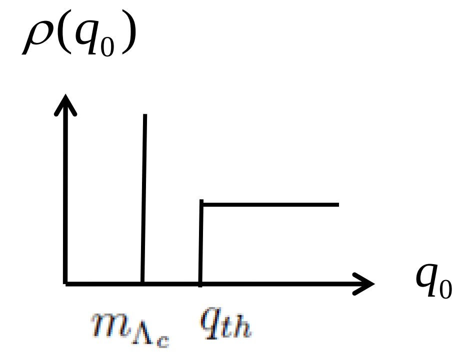
# QCD sum rules

Correlation function: $\Pi(q) = i\int e^{iqx}\langle 0|T[J_B(x)\overline{J}_B(0)]|0\rangle d^4x$

Parity projected
QCD sum rule

Gaussian sum rule: $G_{OPE}(\tau) = \int_0^{\infty} \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - s)^2}{4\tau}\right) \rho(q_0) dq_0$

Hadronic spectral function




$\rho(q_0) = |\lambda|^2\delta(q_0 - m_{\Lambda_c}) + \mathrm{Continuum}(\propto \theta(q_0 - q_{th}))$

# QCD sum rules

Correlation function: $\Pi(q) = i\int e^{iqx}\langle 0|T[J_B(x)\overline{J}_B(0)]|0\rangle d^4x$

$\Downarrow$ Parity projected QCD sum rule

Gaussian sum rule: $\underline{G_{OPE}(\tau)} = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}}\exp\left(-\frac{(q_0^2-s)^2}{4\tau}\right)\underline{\rho(q_0)}dq_0$

Calculated by operator product expansion(OPE)

Non-perturbative contributions are expressed by condensates.

$\langle\overline{q}q\rangle \quad \langle\frac{\alpha_s}{\pi}G^2\rangle \quad \langle\overline{q}q\overline{q}q\rangle \quad \cdots$

(In vacuum)

# QCD sum rules

Correlation function: $\Pi(q) = i \int e^{iqx} \langle 0 | T[J_B(x) \overline{J}_B(0)] | 0 \rangle d^4x$

Parity projected
QCD sum rule

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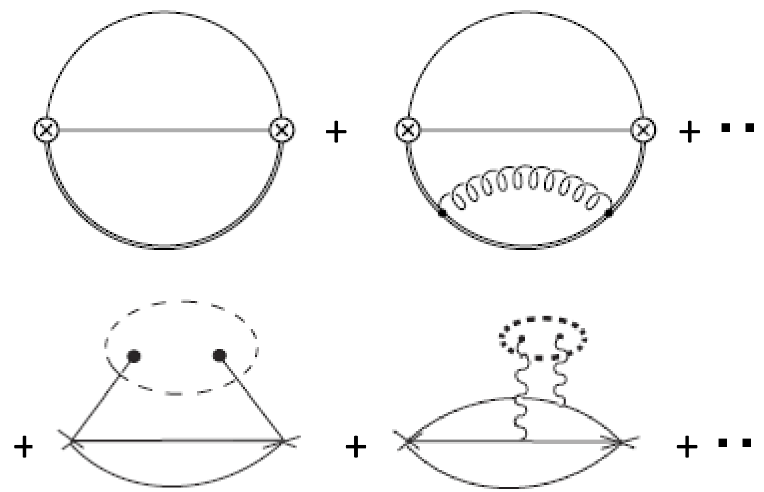
$\langle \overline{q}q \rangle \quad \langle \frac{\alpha_s}{\pi} G^2 \rangle \quad \langle \overline{q}q\overline{q}q \rangle \quad \cdots$

(In vacuum)

$G_{OPE}(\tau)$ =




$\langle \overline{q}q \rangle \qquad \langle \frac{\alpha_s}{\pi} G^2 \rangle$

# QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - s)^2}{4\tau}\right) \rho(q_0) dq_0$$

Application to the analyses in nuclear matter

$$\langle 0|\mathcal{O}_i|0\rangle \Rightarrow \langle \Psi_0|\mathcal{O}_i|\Psi_0\rangle = \langle \mathcal{O}_i\rangle_m$$

New condensates: $\langle 0|\mathcal{O}_i|0\rangle = 0 \Rightarrow \langle \mathcal{O}_i\rangle_m \neq 0$

$$\langle \overline{q}q\rangle_m = \langle \overline{q}q\rangle_0 + \rho\frac{\sigma_N}{2m_q} \qquad \langle \frac{\alpha_s}{\pi}G^2\rangle_m = \langle \frac{\alpha_s}{\pi}G^2\rangle_0 - \rho(0.65\mathrm{GeV}^2)$$

$$\langle \overline{q}g\sigma\cdot Gq\rangle_m = (0.8\mathrm{GeV}^2)\langle \overline{q}q\rangle_m$$

$$\langle q^\dagger q\rangle_m = \rho\frac{3}{2} \qquad \langle q^\dagger iD_0 q\rangle_m = \rho\frac{3}{8}M_N A_2^q \qquad \langle q^\dagger g\sigma\cdot Gq\rangle_m = -\rho(0.33\mathrm{GeV}^2)$$

$$\langle q^\dagger iD_0 iD_0 q\rangle_m + \frac{1}{12}\langle q^\dagger g\sigma\cdot Gq\rangle_m = \rho\frac{1}{4}M_N^2 A_3^q$$

(Linear density approximation)

Condensates have the density dependence.

$\Rightarrow$ In-medium effects can be expressed by the in-medium modifications of the condensates.

# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right)\rho(q_0)dq_0$$

We consider the case of the $\Lambda_c$ baryon.

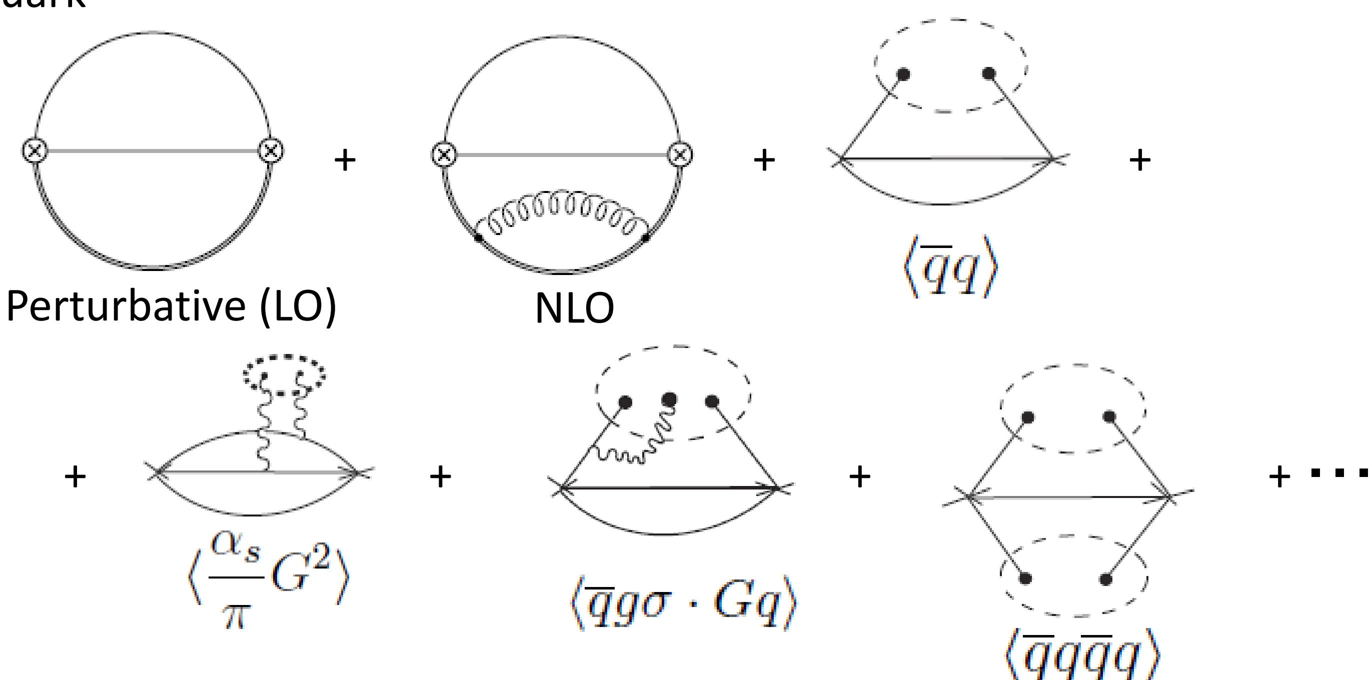
Correlation function : $\Pi(q) = i\int e^{iqx}\langle 0|T[J_{\Lambda_c}(x)\overline{J}_{\Lambda_c}(0)]|0\rangle d^4x$

$$J_{\Lambda_Q} = \epsilon^{abc}(u^{Ta}C\gamma_5 d^b)Q^c$$

## Features of the correlation function:

1. Information of the diquark

$G_{OPE}(\tau)$ =



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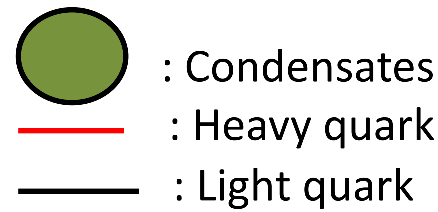


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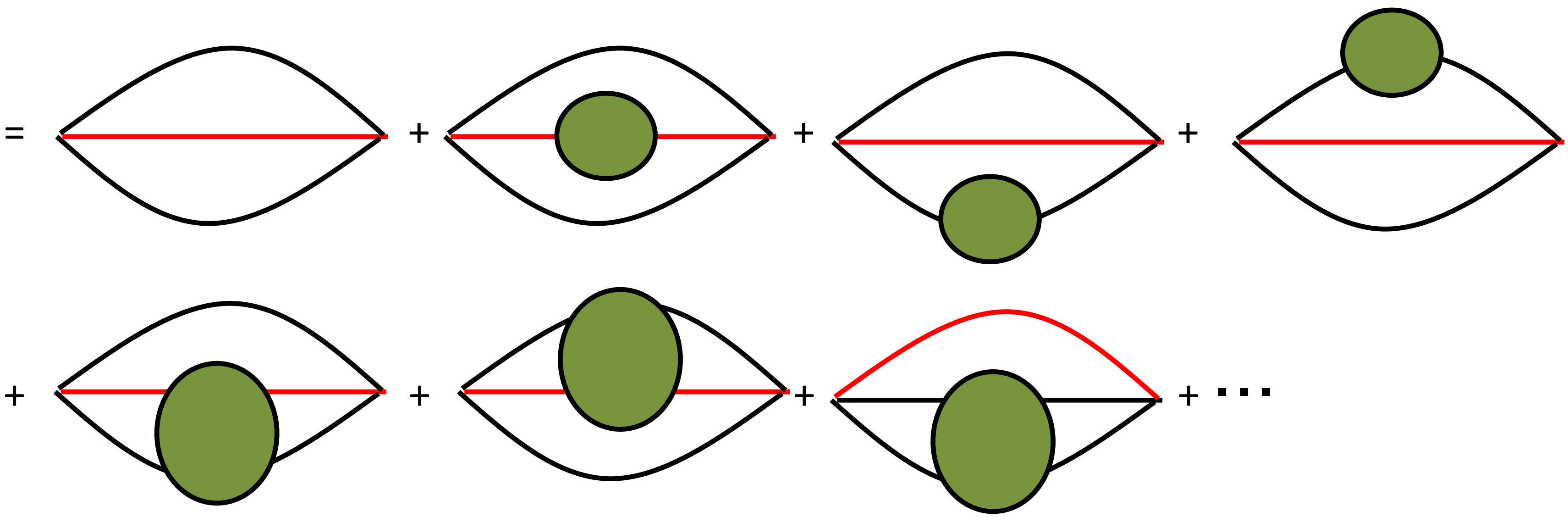

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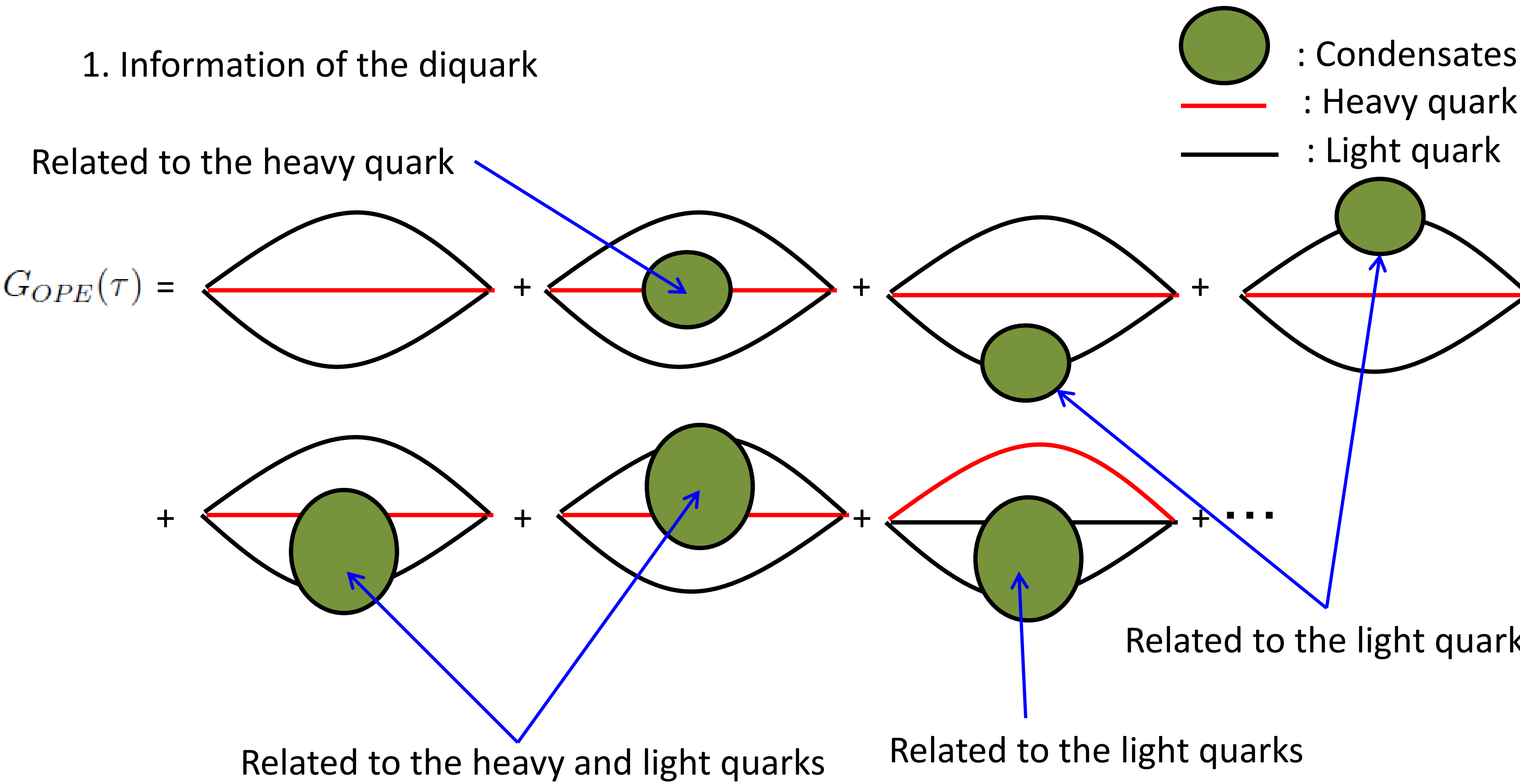

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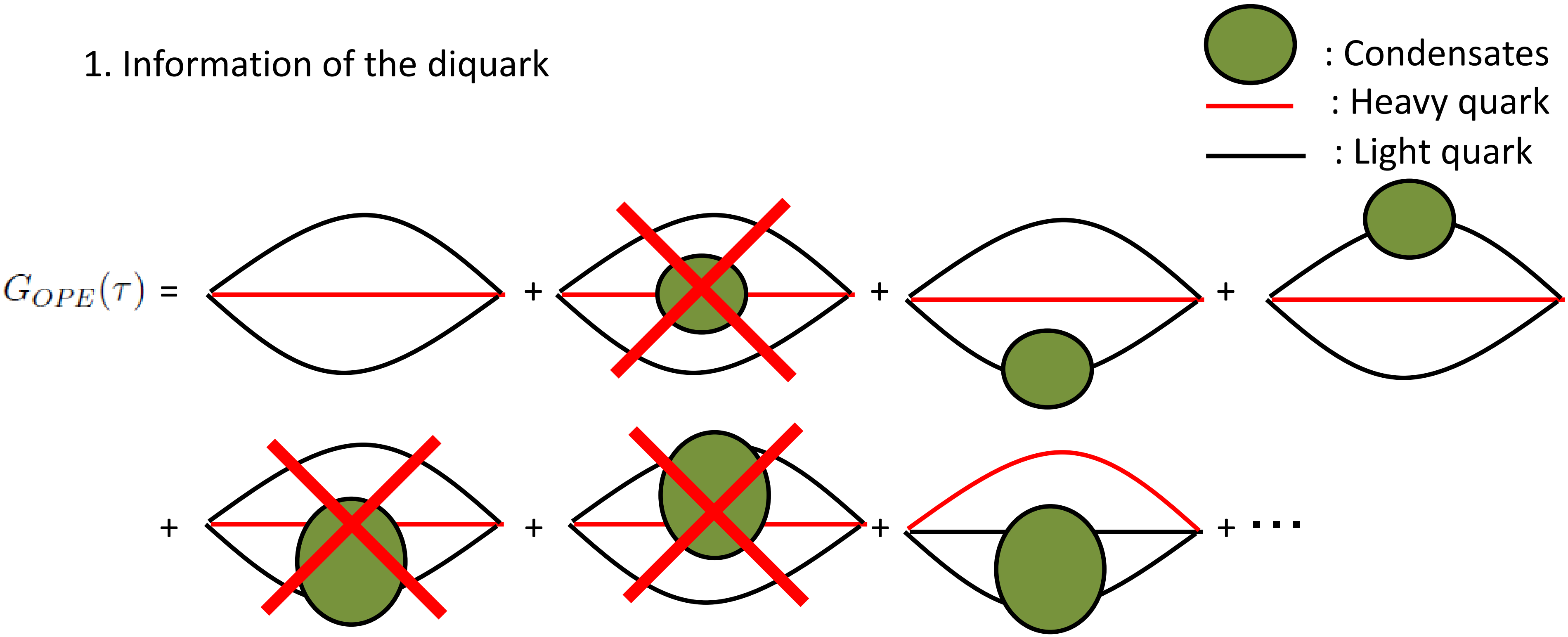




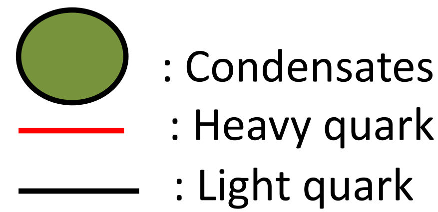
: These contributions are numerically small.

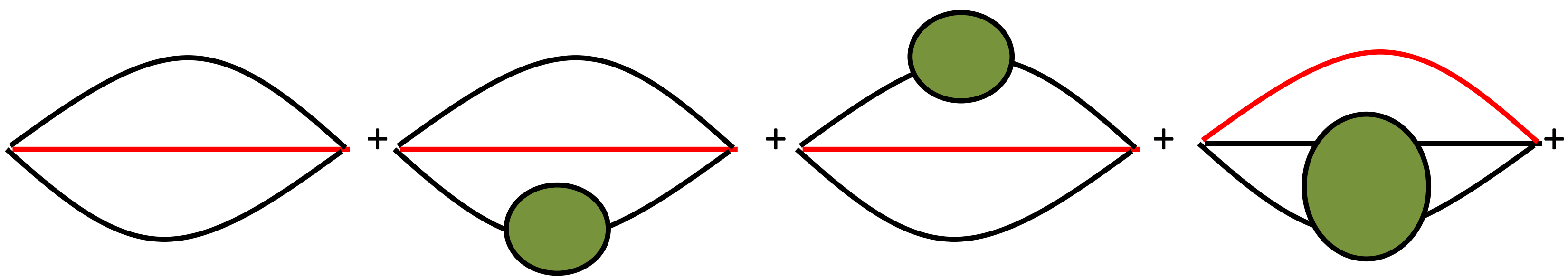
# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

## Feature of the correlation function:

1. Information of the diquark




$G_{OPE}(\tau) \approx$ 


The heavy quark does not affect the condensates and thus the in-medium modifications are expressed by the contributions from the light quarks.

What kind of light quark pair?

$\Lambda_c$ interpolating operator is $J_{\Lambda_Q} = \underline{\epsilon^{abc}(u^{Ta}C\gamma_5 d^b})Q^c$

Good diquark

⇨ The in-modifications of $\Lambda_c$ can be related to the modifications of the good diquark.

# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

We consider the case of the $\Lambda_c$ baryon.

Correlation function : $\Pi(q) = i \int e^{iqx} \langle 0 | T[J_{\Lambda_c}(x) \overline{J}_{\Lambda_c}(0)] | 0 \rangle d^4x$

$$J_{\Lambda_Q} = \epsilon^{abc}(u^{Ta} C\gamma_5 d^b) Q^c$$

## Feature of the correlation function:

2. The partial restoration of the chiral symmetry

The effect from the chiral condensate is small.

$$J_{\Lambda_Q} = \epsilon^{abc}(u^{Ta} C\gamma_5 d^b) Q^c = \epsilon^{abc}(-u_L^T C\gamma_5 d_L + u_R^T C\gamma_5 d_R) Q^c$$

The property of $J_{\Lambda_Q}$

The right handed spinor of u quark is paired with left handed one. $\langle \overline{u} u \rangle$

The right handed spinor of d quark is also paired with left handed one. $m_d$

The contributions appear as $m_q \langle \overline{q} q \rangle$ and are numerically small.

# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

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More explicitly, the contributions of $\langle \overline{q} q \rangle$ are expressed as the following form.

$$\propto \mathrm{Tr}[(\not{q} + m_q) \langle \overline{q} q \rangle] \propto m_q \langle \overline{q} q \rangle$$

⇨ The contributions appear as $m_q \langle \overline{q} q \rangle$ and are numerically small.

# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

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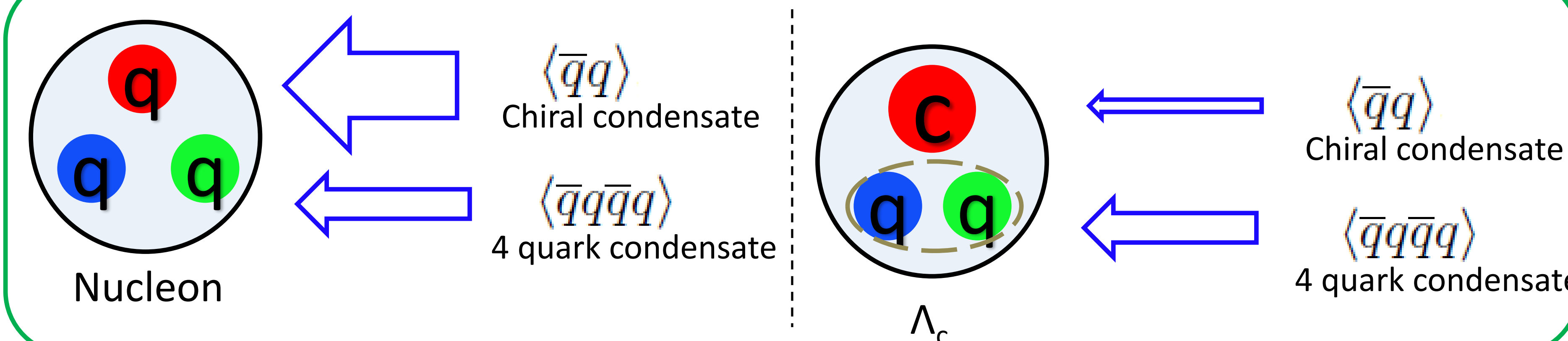
The effect from the partial restoration of the chiral symmetry




$\Lambda_c$ baryon knows the partial restoration of the chiral symmetry breaking through four quark condensates.

# $\Lambda_c$ QCD sum rules

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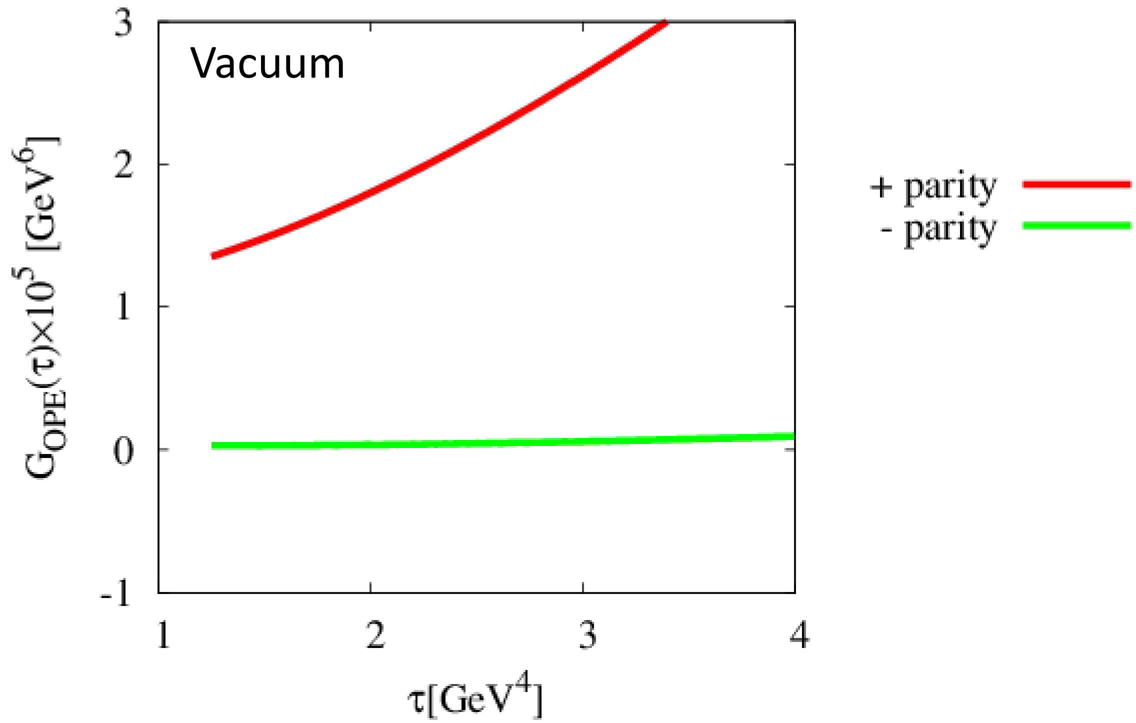
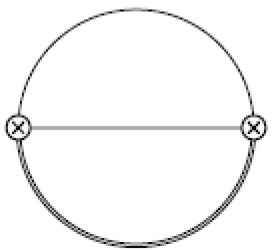



The positive parity states strongly couple to the interpolating operator $J_{\Lambda_Q}$ .

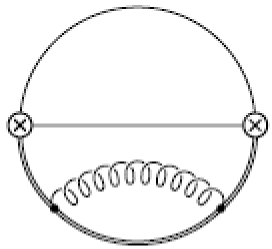
# $\Lambda_c$ QCD sum rules

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

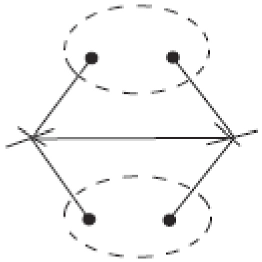
Operator product expansion (OPE) Non-perturbative contributions are expressed by condensates.



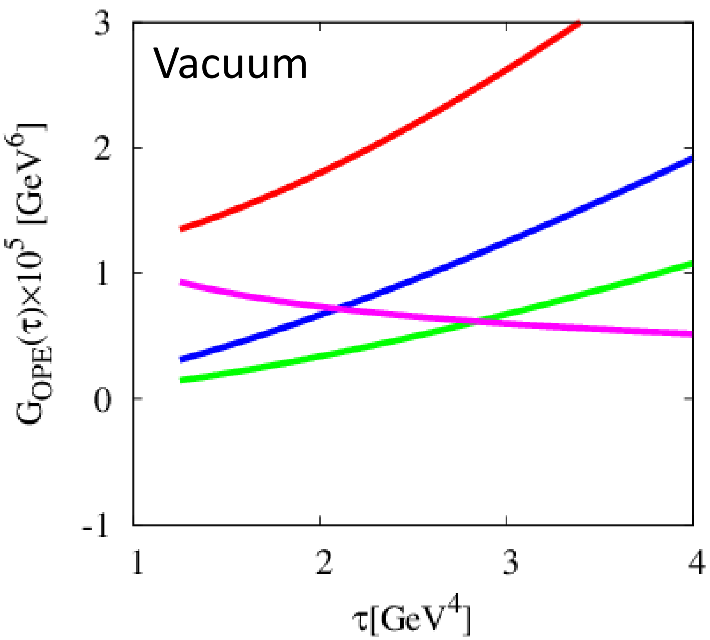
Perturbative (LO)



NLO



$\langle \bar{q}q\bar{q}q \rangle$



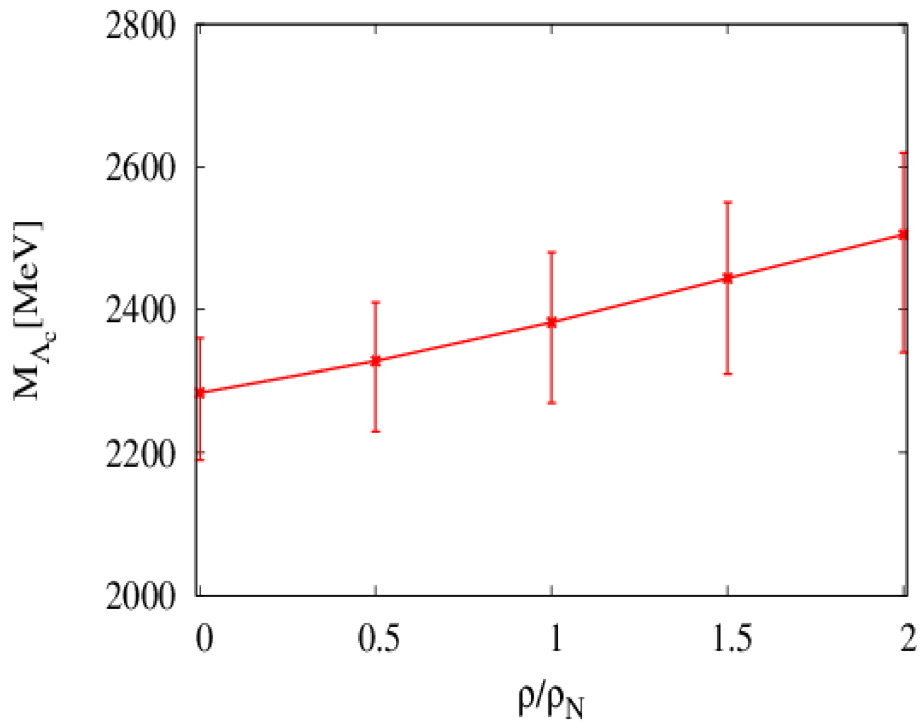

NLO contributions to its leading order are more than 100%.

The contribution of four quark condensate is large.

# Results

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

$$\rho(q_0) = |\lambda|^2 \delta(q_0 - m_{\Lambda_c}) + \mathrm{Continuum}(\propto \theta(q_0 - q_{th}))$$




At $\rho = 1.0\rho_N$, the sfift $\Delta M_{\Lambda_c} \approx 100\mathrm{MeV}$

The density dependence of $M_{\Lambda_c}$

# Summary

- We calculate the parity projected $\Lambda_c$ QCD sum rule.
- The $\Lambda_c$ QCD sum rule has features which are related to the information on diquark and the partial restoration of chiral symmetry.
- We analyze the $\Lambda_c$ spectral function in vacuum and nuclear matter by using QCD sum rules.
- We investigate the density dependence of the mass modification.
- As the density increases, the mass of $\Lambda_c$ increases.

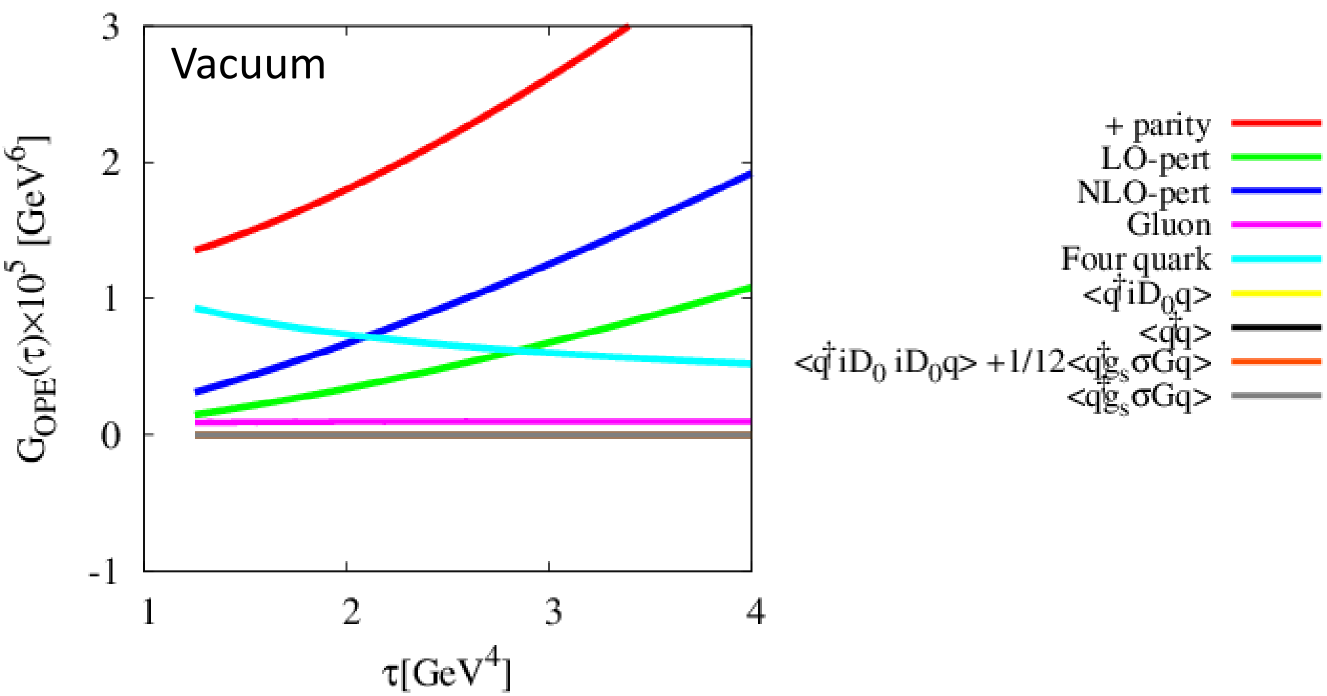
# Future plan

- We will study the effective mass and vector self-energy.

# Backup slides

$$G_{OPE}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_c^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

Density dependence of the $G_{OPE}(\tau)$



# Backup slides

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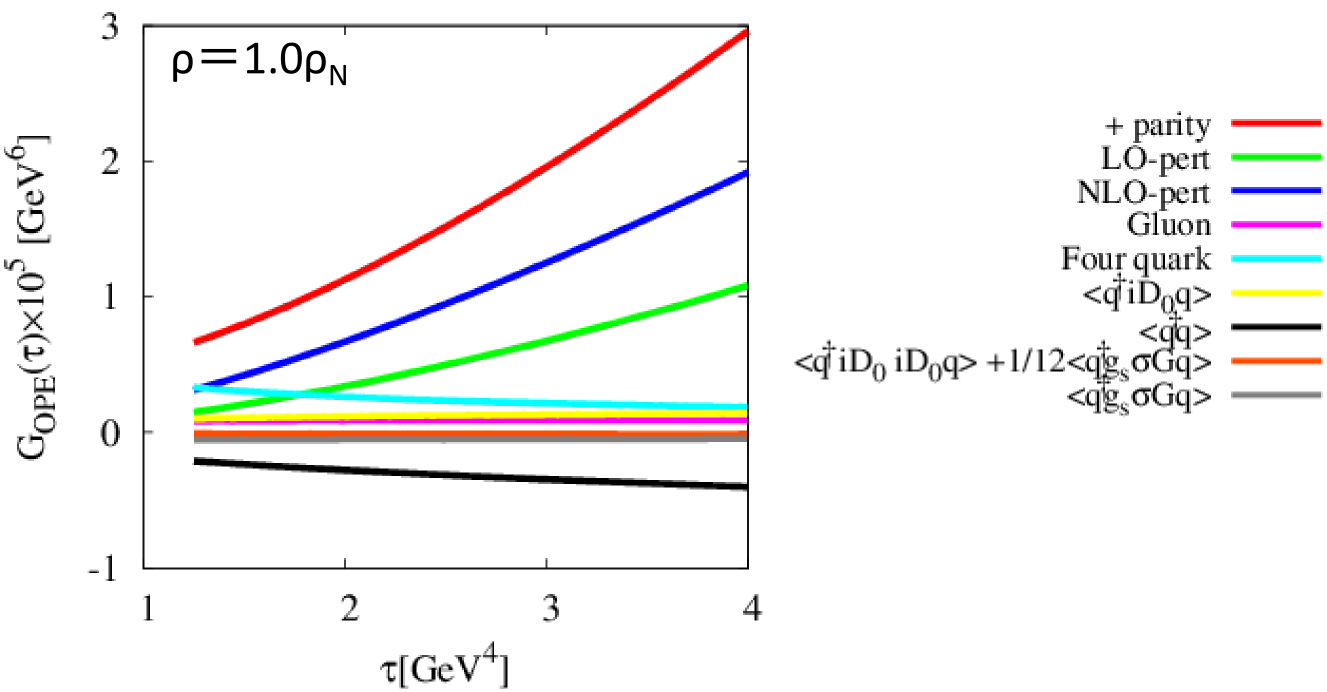

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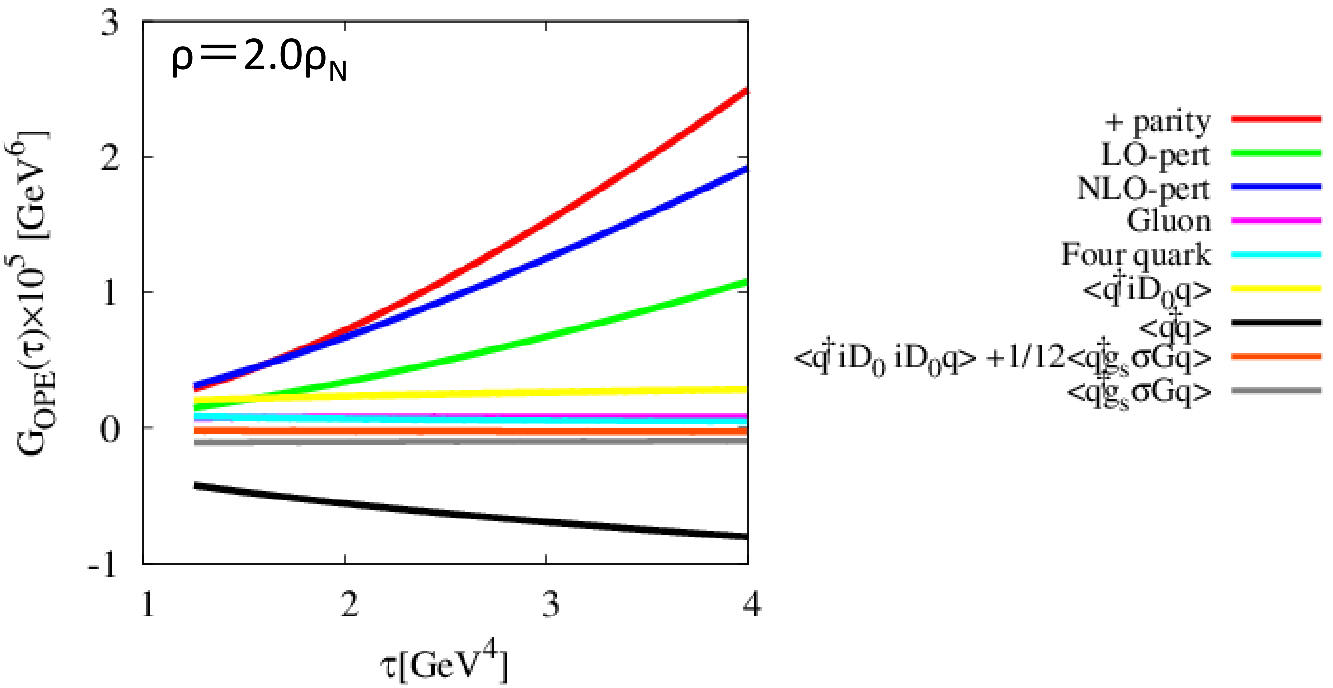
Density dependence of the $G_{OPE}(\tau)$



# Backup slides

$$\langle\overline{q}q\rangle_{\rho_N} = \langle\overline{q}q\rangle_0 + \rho_N\langle\overline{q}q\rangle_N = \langle\overline{q}q\rangle_0 + \rho_N\frac{\sigma_N}{2m_q}$$

$$\langle q^\dagger q\rangle_{\rho_N} = \rho_N\frac{3}{2}$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_{\rho_N} = \langle\frac{\alpha_s}{\pi}G^2\rangle_0 + \rho_N\langle\frac{\alpha_s}{\pi}G^2\rangle_N$$

$$\langle q^\dagger iD_0q\rangle_{\rho_N} = \rho_N\langle q^\dagger iD_0q\rangle_N = \rho_N\frac{3}{8}M_NA_2^q$$

$$\langle q^\dagger iD_0iD_0q\rangle_{\rho_N} + \frac{1}{12}\langle q^\dagger g\sigma\cdot Gq\rangle_{\rho_N} = \left(\langle q^\dagger iD_0iD_0q\rangle_N + \frac{1}{12}\langle q^\dagger g\sigma\cdot Gq\rangle_N\right)\rho_N = \rho_N\frac{1}{4}M_N^2A_3^q$$

$$\langle q^\dagger g\sigma\cdot Gq\rangle_{\rho_N} = \rho_N\langle q^\dagger g\sigma\cdot Gq\rangle_N$$

$m_c^{pole} = 1.67 \pm 0.07$ GeV

$\alpha_s = 0.5$

$\langle\overline{q}q\rangle_0 = -(0.256 \pm 0.002 GeV)^3$

$m_q = 4.75\mathrm{MeV}$

$\sigma_N = 45\mathrm{MeV}$

| | |
|---|---|
| $\langle q^\dagger q\rangle_{\rho_N}$ | $\rho_N\frac{3}{2}$ |
| $\langle\frac{\alpha_s}{\pi}G^2\rangle_0$ | $0.012 \pm 0.0036\mathrm{GeV}^4$ |
| $\langle\frac{\alpha_s}{\pi}G^2\rangle_N$ | $-0.65 \pm 0.15\mathrm{GeV}$ |
| $A_2^q$ | $0.62 \pm 0.06$ |
| $A_2^g$ | $0.359 \pm 0.146$ |
| $A_3^q$ | $0.15 \pm 0.02$ |
| $e_2$ | $0.017 \pm 0.047$ |
| $m_0^2$ | $0.8 \pm 0.2\mathrm{GeV}^2$ |
| $\langle q^\dagger g\sigma\cdot Gq\rangle_N$ | $-0.33\mathrm{GeV}^2$ |

# Backup slides

$$\Pi_{old}(q) = i \int \theta(x_0)\langle T\{j(x)\bar{j}(0)\}\rangle e^{iqx} dx = m\Pi^m_{old}(q_0, |\vec{q}|) + \not{q}\Pi^q_{old}(q_0, |\vec{q}|) + \not{u}\Pi^u_{old}(q_0, |\vec{q}|).$$

$$\rho^i_{old}(q_0, |\vec{q}|) \equiv \frac{1}{\pi} Im[\Pi^i_{old}(q^2)] \quad (i = m, q, u)$$

$$\rho^+_{old\ OPE} = q_0 \rho^q_{old} + m_Q \rho^m_{old} + u_0 \rho^u_{old}$$

$$\int_{-\infty}^{\infty} \rho^+_{old\ OPE}(q_0) W(q_0) dq_0 = \int_0^{\infty} \rho^+_{hadron}(q_0) W(q_0) dq_0$$

$$W(q_0) = \frac{1}{\sqrt{4\pi\tau}} \exp\left( -\frac{(q_0^2 - m_c^2)^2}{4\tau} \right)$$

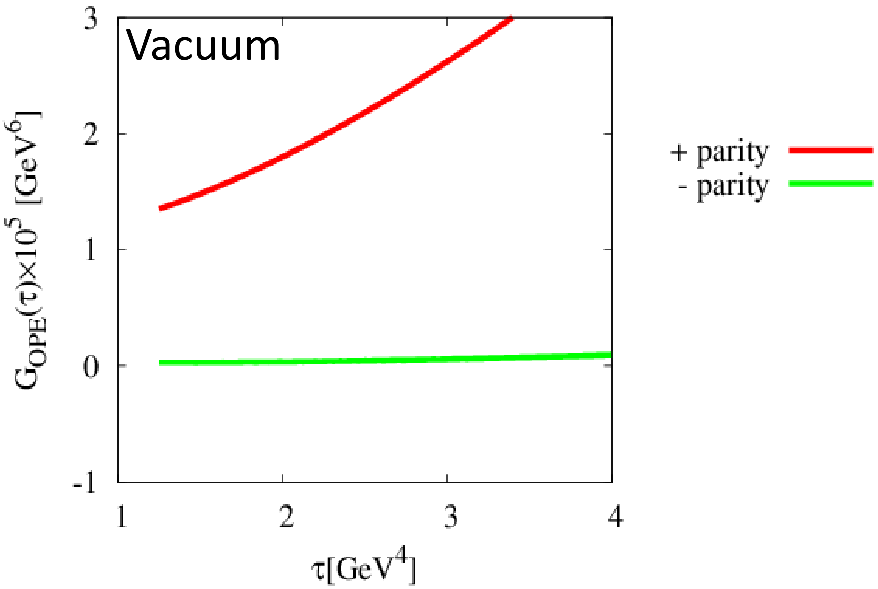
# Backup slides

Negative parity $G_{OPE}(\tau)$

$$\rho^{+}_{old\ OPE} = q_0\rho^{q}_{old} + m_Q\rho^{m}_{old} + u_0\rho^{u}_{old}$$

$$\rho^{-}_{old\ OPE} = q_0\rho^{q}_{old} - m_Q\rho^{m}_{old} + u_0\rho^{u}_{old}$$

$$G_{OPE}(\tau) = \int_{-\infty}^{\infty} \rho_{old\ OPE}(q_0)W(q_0)dq_0$$



# Backup slides

$$\chi^2 = \frac{1}{n_{set} \times n_\tau} \sum_{j=1}^{n_{set}} \sum_{i=1}^{n_\tau} \frac{(G^j_{OPE}(\tau_i) - G^j_{SPF}(\tau_i))^2}{\sigma^j(\tau_i)^2}$$

$$\sigma^j(\tau_i)^2 = \frac{1}{n_{set} - 1} \sum_{j=1}^{n_{set}} (G^j_{OPE}(\tau_i) - \overline{G_{OPE}}(\tau_i))^2$$

$$G_{\mathrm{SPF}}(\tau) = \int_0^\infty \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(q_0^2 - m_Q^2)^2}{4\tau}\right) \rho(q_0) dq_0$$

$n_{set}$ : The number of the condensate sets which are randomly generated with errors

$n_\tau$ : The number of the point $\tau$ in the analyzed $\tau$ region

Error bar: $|\chi^2 - 1| < 0.1$