

Heavy baryons based on pion mean fields

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In Collaboration with
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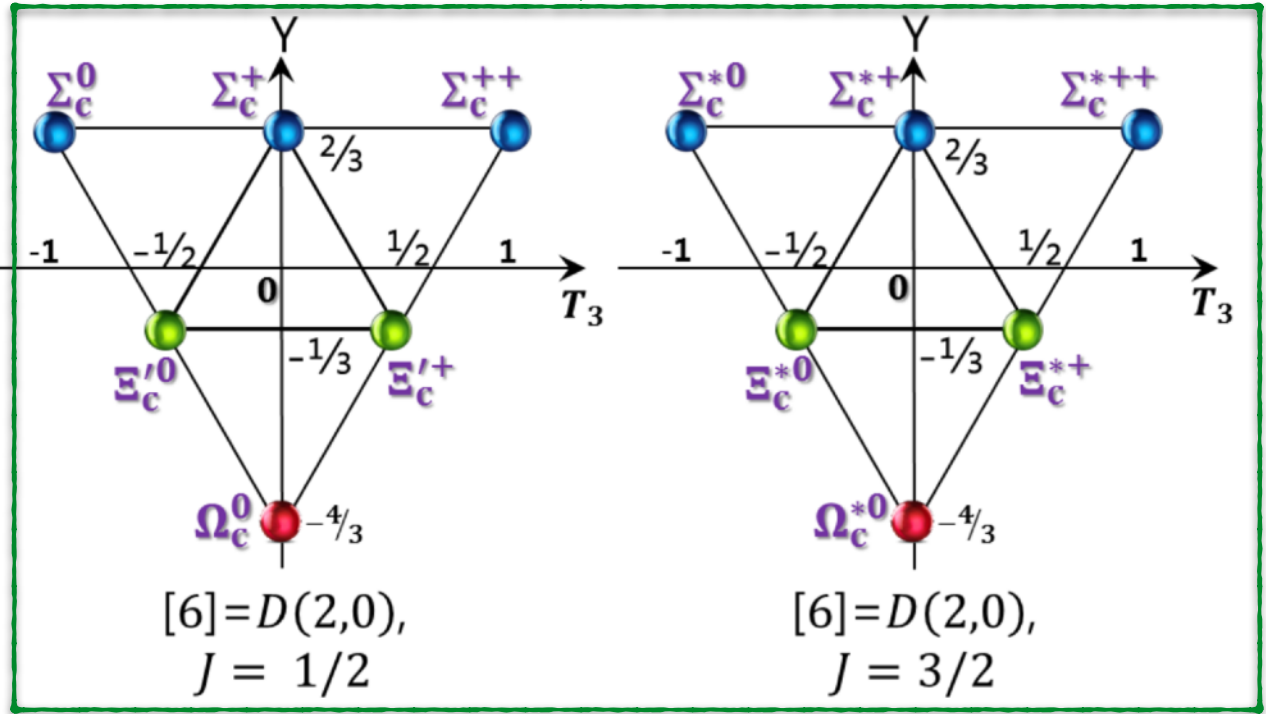
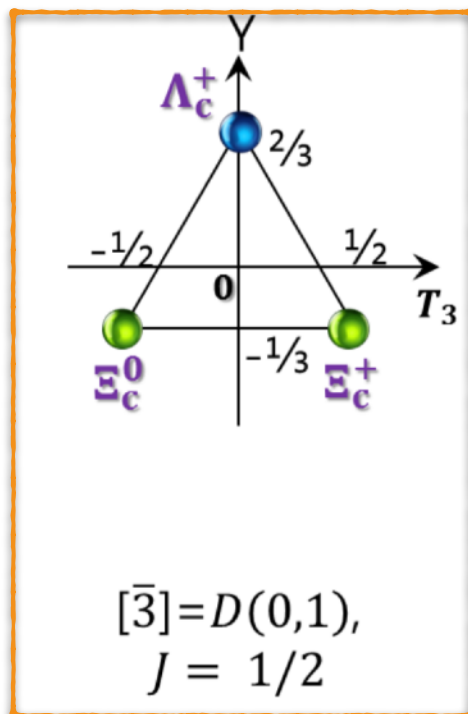
Heavy baryon:
One static color source
+
 $N_c - 1$ quarks

SU(3) representation for heavy baryons

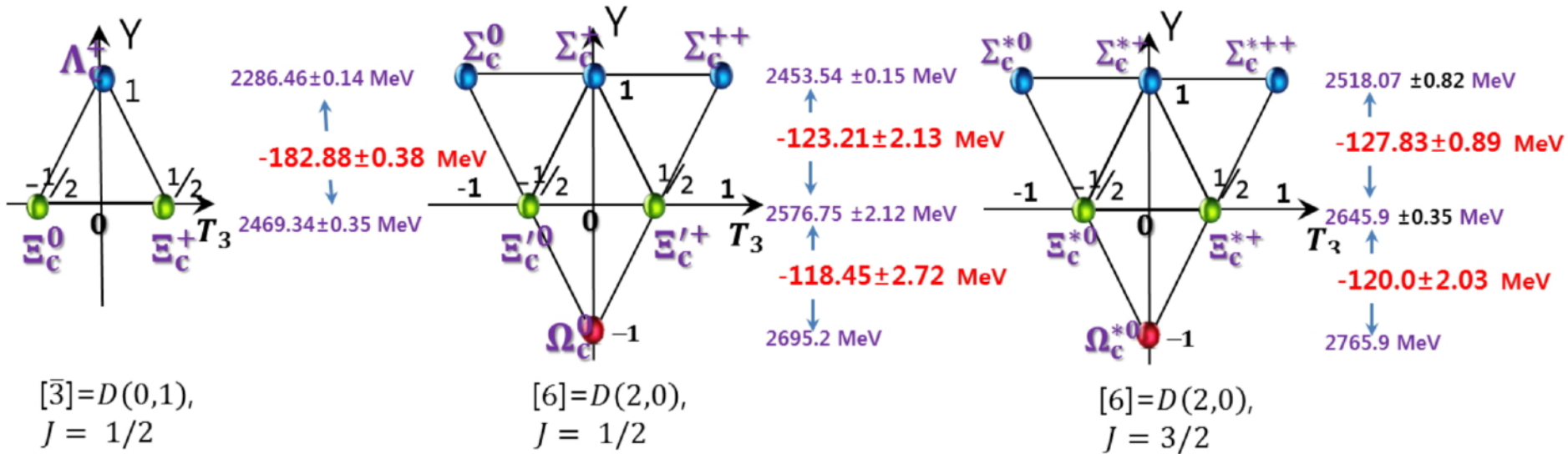


- In the heavy quark mass limit, a heavy quark spin is conserved, so light-quark spin is also conserved.
- In this limit, a heavy quark can be considered as **a color static source**.
- Dynamics is governed by light quarks.

$$3 \otimes 3 = \bar{3} \oplus 6$$

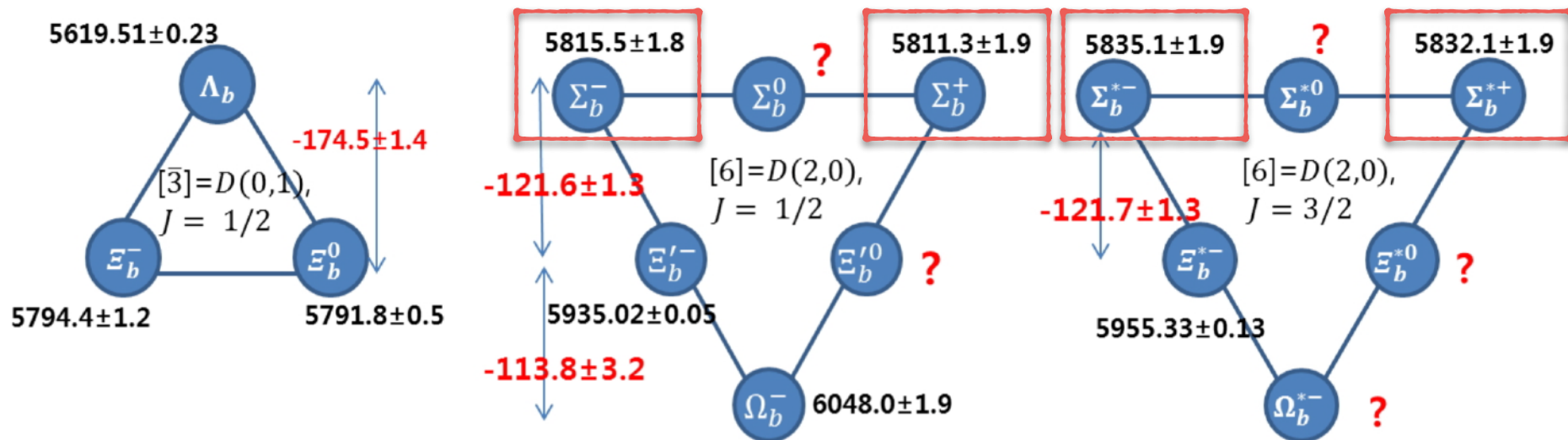


Experimental status: Charmed baryons



- The masses of charmed baryons are all well known experimentally.
- They can be used to check the validity of the present approach.

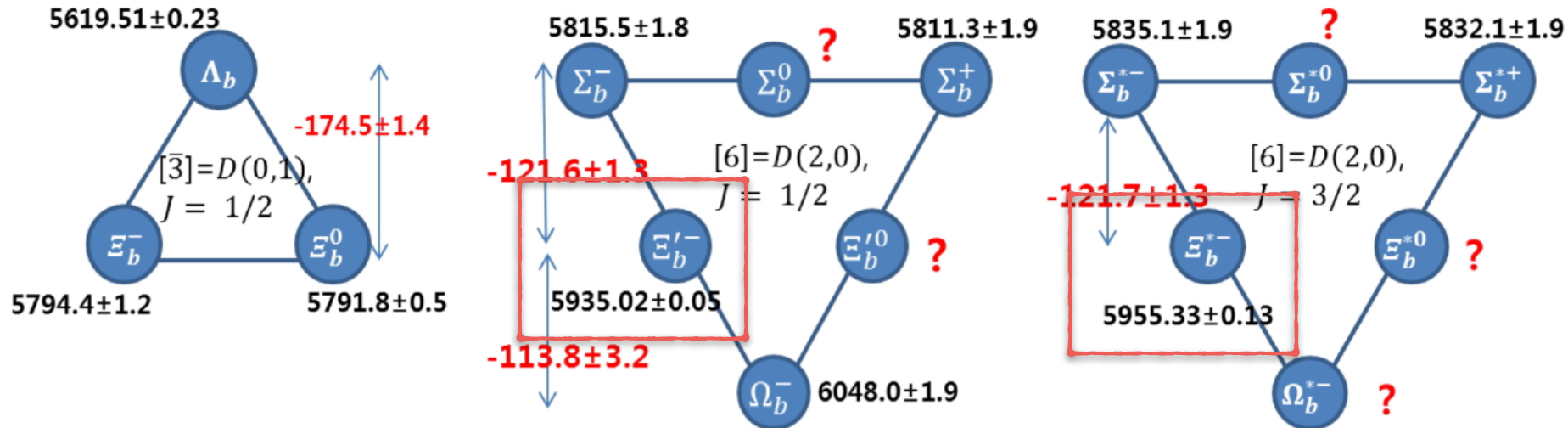
Experimental status: Bottom baryons



CDF, PRD85, 092011 (2012)

$$\begin{aligned}
 M_{\Sigma_b^+} &= (5811.3_{-0.8}^{+0.9} \pm 1.7) \text{ MeV}, & M_{\Sigma_b^-} &= (5815.5_{-0.5}^{+0.6} \pm 1.7) \text{ MeV} \\
 M_{\Sigma_b^{*+}} &= (5832.1 \pm 0.7_{-1.8}^{+1.7}) \text{ MeV}, & M_{\Sigma_b^{*-}} &= (5835.1 \pm 0.7_{-1.7}^{+1.8}) \text{ MeV}
 \end{aligned}$$

Experimental status: Bottom baryons



CMS, PRL 108, 252002 (2012)

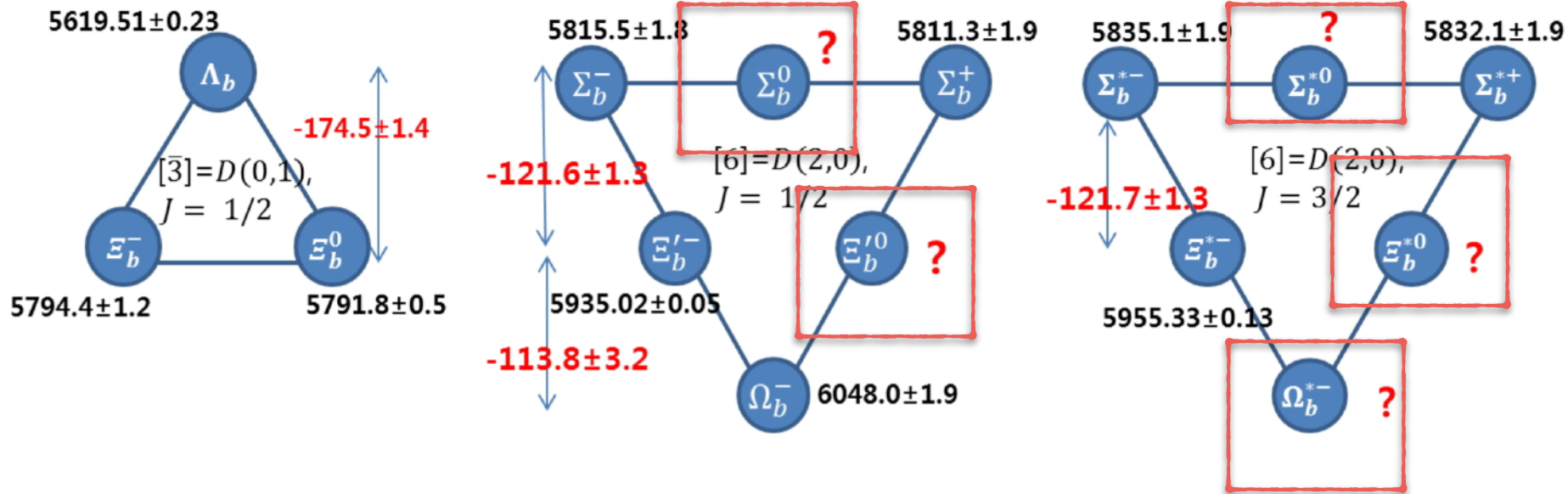
$$M_{\Xi_b} = (5948.9 \pm 0.8 \pm 1.2) \text{ MeV}$$

LHCb, PRL 114 062004 (2015)

$$M_{\Xi_b'} = (5935.02 \pm 0.02 \pm 0.05) \text{ MeV},$$

$$M_{\Xi_b^*} = (5955.33 \pm 0.12 \pm 0.05) \text{ MeV}$$

Experimental status: Bottom baryons



- Masses of the five bottom baryons are still unknown.

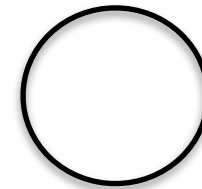
Ω_b^{*-} can soon be measured at the LHC.

Light baryons in pion mean fields

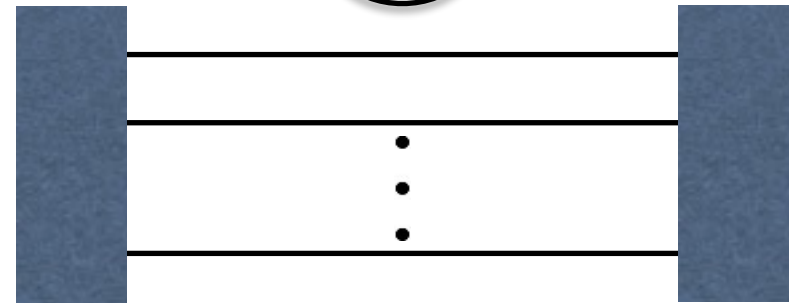
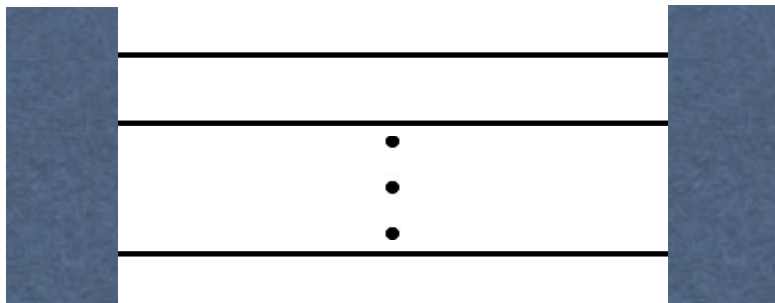


Classical solitons (Chiral Quark-soliton model)

$$\langle J_N(\vec{x}, T) J_N^\dagger(\vec{y}, -T) \rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\text{val}} + E_{\text{sea}})T]}$$



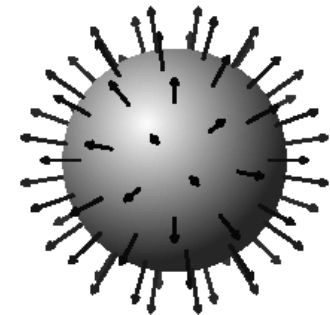
Vacuum
Polarisation
or Mean fields



$$\frac{\delta}{\delta U}(N_c E_{\text{val}} + E_{\text{sea}}) = 0 \rightarrow M_{\text{cl}} = N_c E_{\text{val}}(U_c) + E_{\text{sea}}(U_c)$$

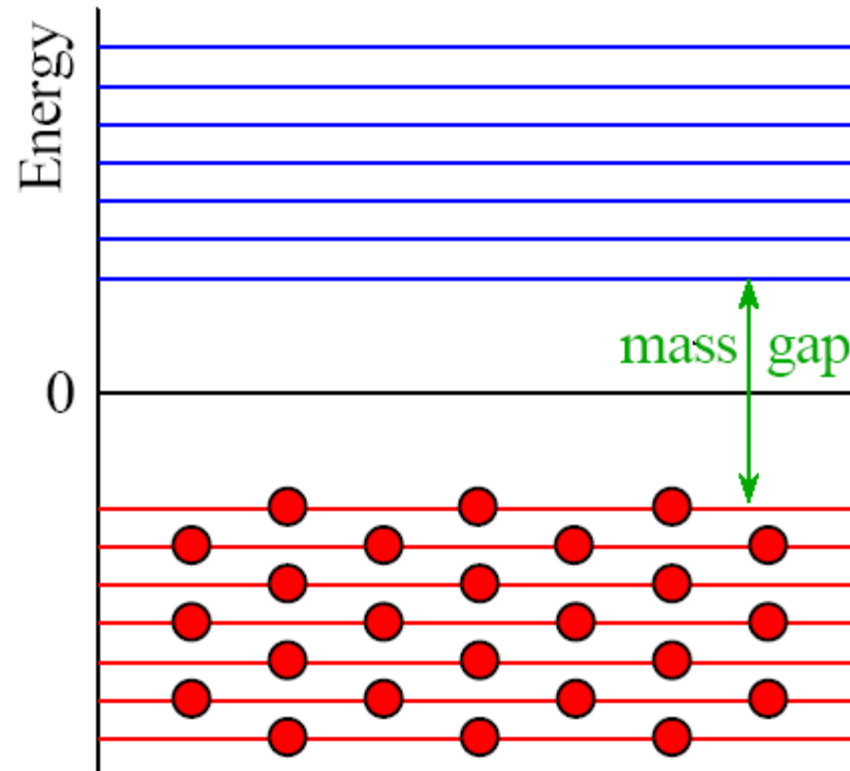
Hedgehog Ansatz:

$$U_{\text{SU}(2)} = \exp[i\gamma_5 \mathbf{n} \cdot \boldsymbol{\tau} P(r)]$$



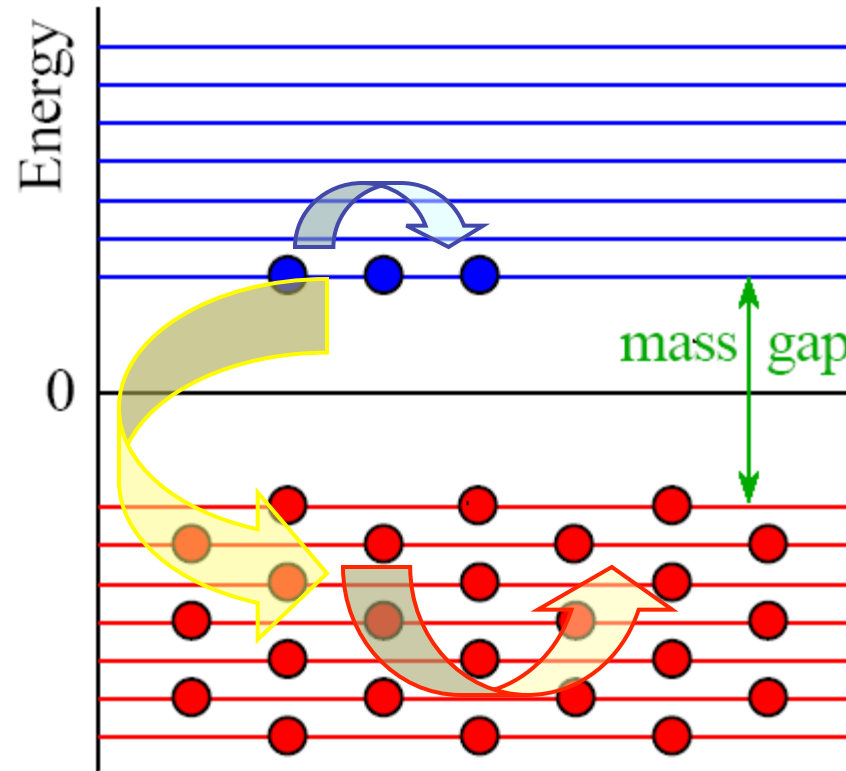
hedgehog

Light baryons in pion mean fields



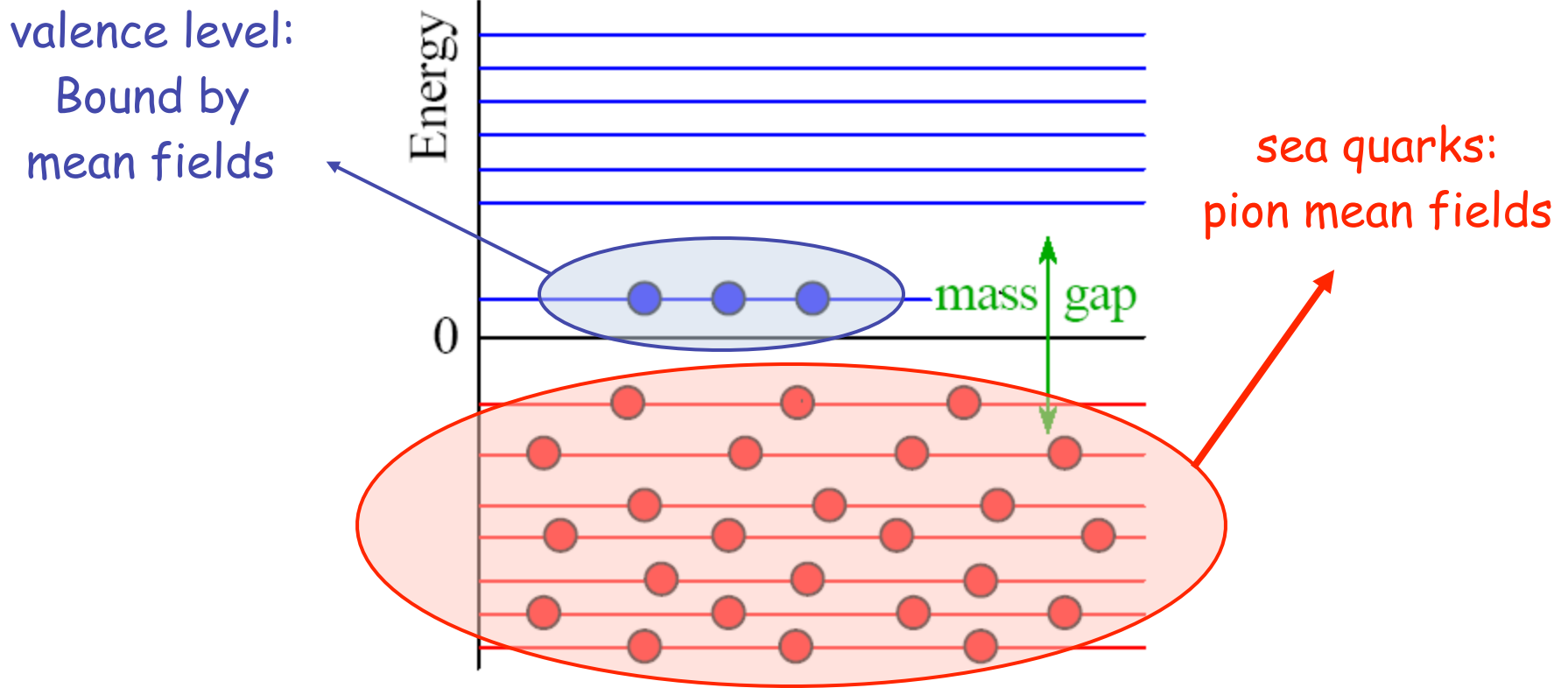
Spontaneous chiral symmetry breaking

Light baryons in pion mean fields



The presence of the N_c valence quarks creates the pion mean fields.

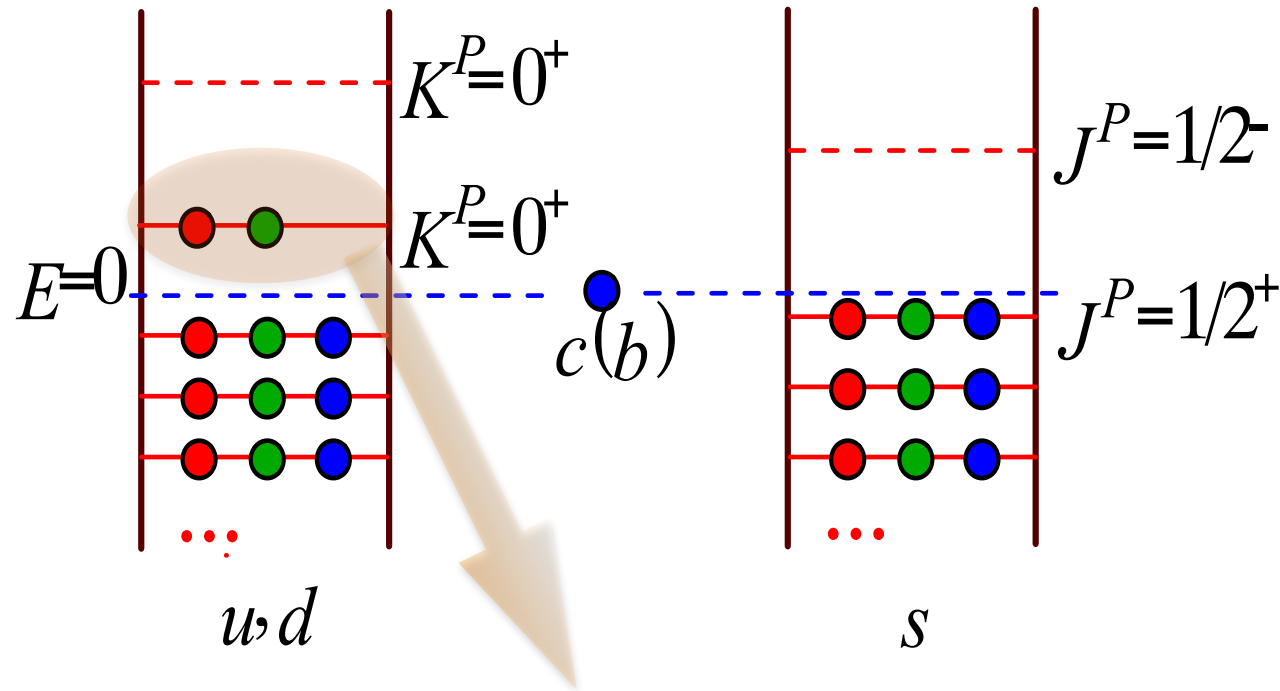
Light baryons in pion mean fields



The valence quarks produce the mean field in the large N_c limit and are self-consistently bound by it!

Heavy baryons

- Valence quarks are bound by the pion mean field.
- Light quarks govern a heavy-light quark system.
- Heavy quarks can be considered as merely **static color sources**.



Meson mean field by N_c-1 valence quarks


Collective Hamiltonian




Collective rotational Hamiltonian

$$H_{(p,q)}^{\text{rot}} = M_{\text{sol}} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{a=4}^7 \hat{J}_a^2$$

$$\mathcal{E}_{(p,q)}^{\text{rot}} = \boxed{M_{\text{sol}}} + \frac{J(J+1)}{2I_1} + \frac{C_2(p,q) - J(J+1) - 3/4}{2I_2} \boxed{Y'^2}$$

classical nucleon mass


$$Y' = \frac{N_c}{3}$$

Right hypercharge: Constraint on the quantization of the chiral soliton. This constraint selects a tower of the allowed rotational excitations of the SU(3) hedgehog.

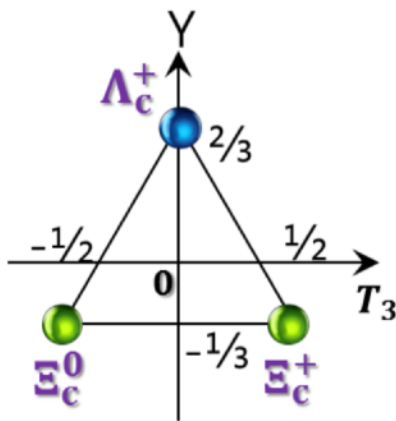
Collective Hamiltonian



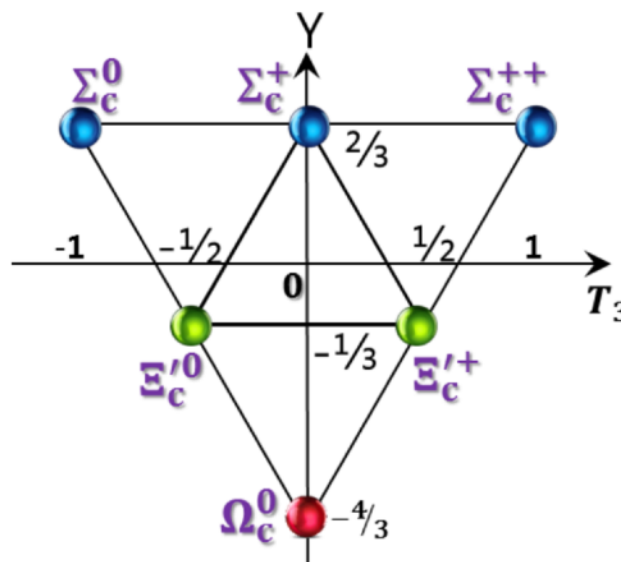
In the case of a heavy baryon, we have $N_c - 1$ light quarks.

$$Y' = \frac{N_c - 1}{3}$$

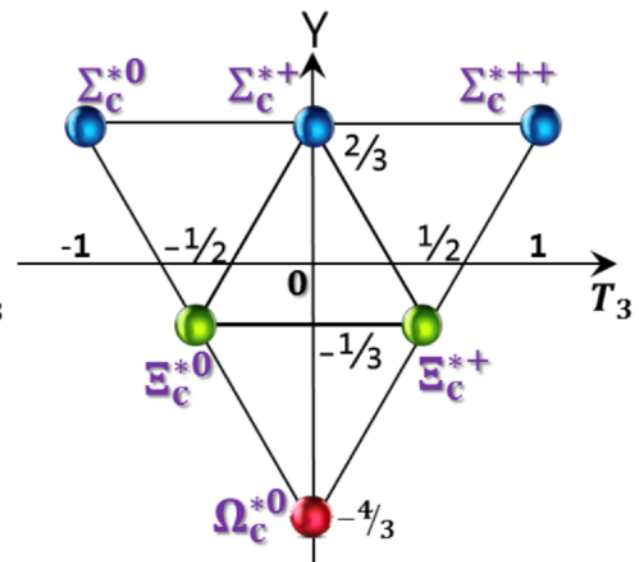
- The lowest rotational excites states
 - $(p, q) = (0, 1) = \bar{\mathbf{3}}$ with $S_L = 0$
 - $(p, q) = (2, 0) = \mathbf{6}$ with $S_L = 1$



$$[\bar{\mathbf{3}}] = D(0, 1), \\ J = 1/2$$



$$[\mathbf{6}] = D(2, 0), \\ J = 1/2$$



$$[\mathbf{6}] = D(2, 0), \\ J = 3/2$$

Collective Hamiltonian



Modifying Collective rotational Hamiltonian

$$H_{(p,q)}^{\text{rot}} = M_{\text{sol}} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{a=4}^7 \hat{J}_a^2$$

$$\mathcal{E}_{(p,q)}^{\text{rot}} = \boxed{M_{\text{sol}}} + \frac{J(J+1)}{2I_1} + \frac{C_2(p,q) - J(J+1) - 3/4}{2I_2} \boxed{Y'^2}$$

Nc-I soliton mass (B=2/3)

 $Y' = \frac{N_c - 1}{3}$

Moments of Inertia and Sigma pi-N term: sum over valence quark states:

$$I_{1,2}, K_{1,2}, \Sigma_{\pi N} \longrightarrow \left(\frac{N_c - 1}{N_c} \right) I_{1,2}, \left(\frac{N_c - 1}{N_c} \right) K_{1,2}, \left(\frac{N_c - 1}{N_c} \right) \Sigma_{\pi N},$$

SU(3) symmetry breaking



- The collective Hamiltonian for SU(3) symmetry breaking

$$H_{\text{br}} = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} J_i$$

In the light-quark sector, we fix these dynamical parameters as

$$\alpha = -\frac{2m_s}{3}\sigma - \beta Y' = -(255.03 \pm 5.82) \text{ MeV}$$

$$\beta = -\frac{m_s K_2}{I_2} = -(140.04 \pm 3.20) \text{ MeV}$$


$$\gamma = \frac{2m_s K_1}{I_1} + 2\beta = -(101.08 \pm 2.33) \text{ MeV}$$

$$\alpha \rightarrow \bar{\alpha} = \frac{N_c - 1}{N_c} \alpha$$

Masses of the heavy baryons



- Mass expressions

$$M_{B,\mathcal{R}}^Q = M_{\mathcal{R}}^Q + \delta_{\mathcal{R}} Y$$


Center values of the heavy baryon masses

$$M_{\mathcal{R}}^Q = m_Q + \mathcal{E}_{(p,q)}^{\text{rot}}$$

$$M_{\frac{3}{2}}^Q = m_Q + \left(\frac{N_c}{N_c - 1} \right) \frac{1}{2I_2}$$
$$M_6^Q = m_Q + \left(\frac{N_c}{N_c - 1} \right) \frac{1}{2I_1}$$

Modified mean fields

Center values of the masses



$$M_{\frac{3}{2}}^Q = \frac{M_{\Lambda_Q} + 2M_{\Xi_Q}}{3}$$

$$M_6^Q = \frac{M_{\Sigma_Q} + 2M_{\Sigma_Q^*} + 2M_{\Xi_Q'} + 4M_{\Xi_Q^*}}{9}$$

For the charmed sector

$$M_{\frac{3}{2}}^c = (2408.7 \pm 0.2) \text{ MeV}$$

$$M_6^c = (2580.8 \pm 0.5) \text{ MeV}$$

For the bottom sector

$$M_{\frac{3}{2}}^b = (5735.2 \pm 0.4) \text{ MeV}$$

$$M_6^b = (5908.0 \pm 0.3) \text{ MeV}$$

Sum rule for $M_{\Omega_Q^*}$



$$M_{\Omega_Q^*} = 2M_{\Xi'_Q} + M_{\Sigma_Q^*} - 2M_{\Sigma_Q}$$

Using this sum rule, we obtain

$$M_{\Omega_c^*} = (2764.5 \pm 3.1) \text{ MeV}$$

$$M_{\Omega_c^*} = (2765.9 \pm 2.0) \text{ MeV} \quad \text{Experiment}$$


$$M_{\Omega_b^*} = (6076.8 \pm 2.25) \text{ MeV}$$

Experimentally unknown!

Masses of the heavy baryons



- Mass expressions

$$M_{B,\mathcal{R}}^Q = M_{\mathcal{R}}^Q + \delta_{\mathcal{R}} Y$$


SU(3) symmetry breaking

$$\delta_{\bar{3}} = \frac{3}{8} \bar{\alpha} + \beta$$

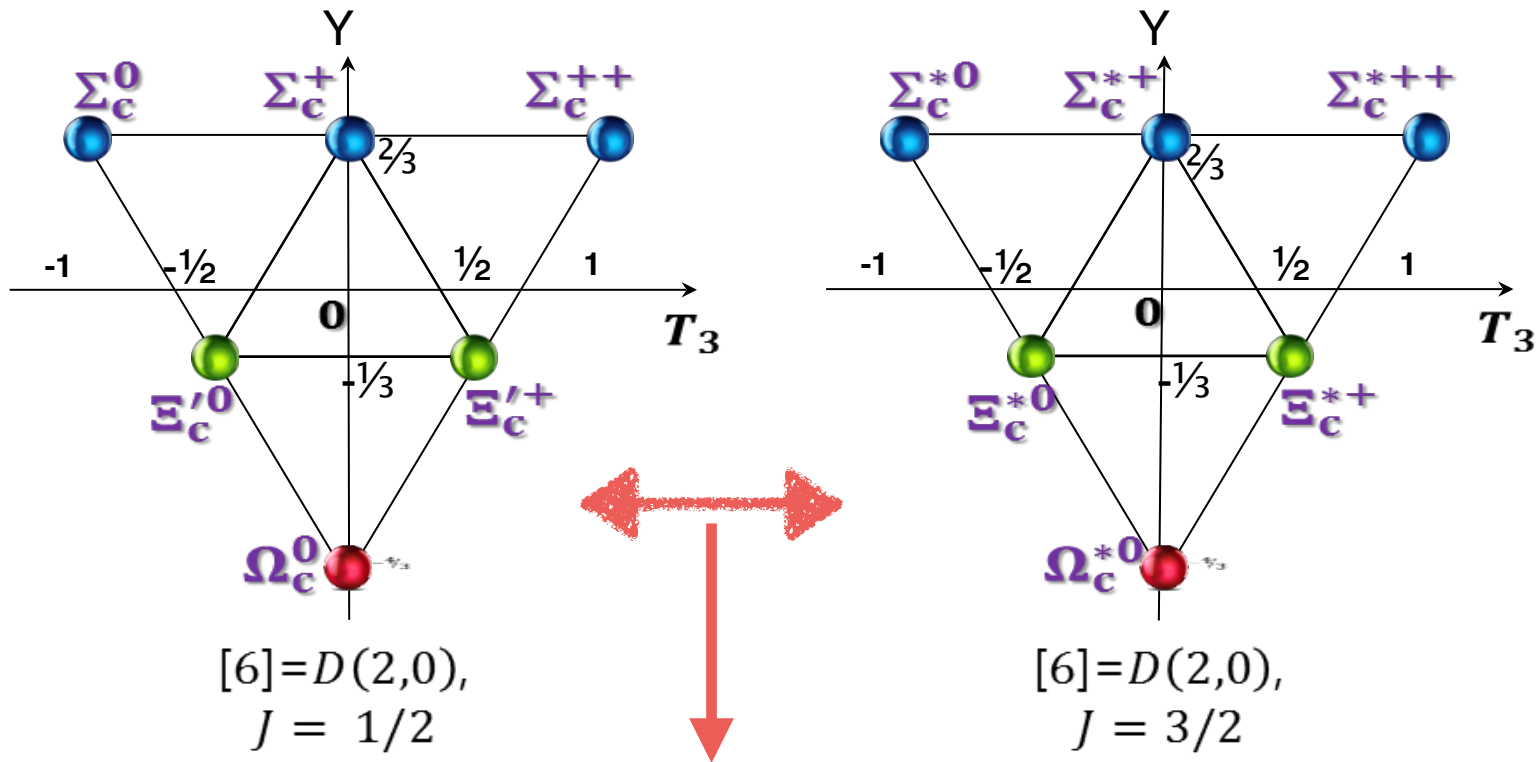
$$\delta_{\bar{3}} = (-203.8 \pm 3.5) \text{ MeV}$$

$$\delta_6 = \frac{3}{20} \bar{\alpha} + \beta - \frac{3}{10} \gamma$$

$$\delta_6 = (-135.2 \pm 3.3) \text{ MeV}$$

Modified mean fields

Hyperfine mass splittings



Hyperfine splitting between different spin states

$$H_{LQ} = \frac{2}{3} \frac{\kappa}{m_Q M_{\text{sol}}} \mathbf{S}_L \cdot \mathbf{S}_Q = \frac{2}{3} \boxed{\frac{\kappa}{m_Q}} \mathbf{S}_L \cdot \mathbf{S}_Q$$

The ratio can be determined by the center values of the sextet masses

Expressions for the masses



$$\frac{\kappa}{m_c} = (68.1 \pm 1.1) \text{ MeV}$$

$$\frac{\kappa}{m_b} = (20.3 \pm 1.0) \text{ MeV}$$

\mathcal{R}_J	B_Q	T	Y	M_{B_Q}
$\bar{\mathbf{3}}_{1/2}$	Λ_Q	0	$\frac{2}{3}$	$\frac{2}{3}\delta_{\bar{\mathbf{3}}} + M_{\bar{\mathbf{3}}}^Q$
	Ξ_Q	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\bar{\mathbf{3}}} + M_{\bar{\mathbf{3}}}^Q$
$\mathbf{6}_{1/2}$	Σ_Q	1	$\frac{2}{3}$	$\frac{2}{3}\delta_{\mathbf{6}} - 2\kappa/3m_Q + M_{\mathbf{6}}^Q$
	Ξ'_Q	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\mathbf{6}} - 2\kappa/3m_Q + M_{\mathbf{6}}^Q$
	Ω_Q	0	$-\frac{4}{3}$	$-\frac{4}{3}\delta_{\mathbf{6}} - 2\kappa/3m_Q + M_{\mathbf{6}}^Q$
$\mathbf{6}_{3/2}$	Σ_Q^*	1	$\frac{2}{3}$	$\frac{2}{3}\delta_{\mathbf{6}} + \kappa/3m_Q + M_{\mathbf{6}}^Q$
	Ξ_Q^*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\mathbf{6}} + \kappa/3m_Q + M_{\mathbf{6}}^Q$
	Ω_Q^*	0	$-\frac{4}{3}$	$-\frac{4}{3}\delta_{\mathbf{6}} + \kappa/3m_Q + M_{\mathbf{6}}^Q$

Results for the charmed baryon masses



\mathcal{R}_J^Q	B_c	Mass	Experiment [17]	Deviation ξ_c
$\overline{\mathbf{3}}_{1/2}^c$	Λ_c	2272.5 ± 2.3	2286.5 ± 0.1	-0.006
	Ξ_c	2476.3 ± 1.2	2469.4 ± 0.3	0.003
$\mathbf{6}_{1/2}^c$	Σ_c	2445.3 ± 2.5	2453.5 ± 0.1	-0.003
	Ξ'_c	2580.5 ± 1.6	2576.8 ± 2.1	0.001
	Ω_c	2715.7 ± 4.5	2695.2 ± 1.7	0.008
$\mathbf{6}_{3/2}^c$	Σ_c^*	2513.4 ± 2.3	2518.1 ± 0.8	-0.002
	Ξ_c^*	2648.6 ± 1.3	2645.9 ± 0.4	0.001
	Ω_c^*	2783.8 ± 4.5	2765.9 ± 2.0	0.006

$$\xi_c = (M_{\text{th}}^{B_c} - M_{\text{exp}}^{B_c})/M_{\text{exp}}^{B_c}$$

Results for the bottom baryon masses



\mathcal{R}_J^Q	B_b	Mass	Experiment [17]	Deviation ξ_b
$\bar{\mathbf{3}}_{1/2}^b$	Λ_b	5599.3 ± 2.4	5619.5 ± 0.2	-0.004
	Ξ_b	5803.1 ± 1.2	5793.1 ± 0.7	0.002
$\mathbf{6}_{1/2}^b$	Σ_b	5804.3 ± 2.4	5813.4 ± 1.3	-0.002
	Ξ'_b	5939.5 ± 1.5	5935.0 ± 0.05	0.001
	Ω_b	6074.7 ± 4.5	6048.0 ± 1.9	0.004
$\mathbf{6}_{3/2}^b$	Σ_b^*	5824.6 ± 2.3	5833.6 ± 1.3	-0.002
	Ξ_b^*	5959.8 ± 1.2	5955.3 ± 0.1	0.001
	Ω_b^*	6095.0 ± 4.4	—	—

Prediction from the present work

$$\xi_b = (M_{\text{th}}^{B_b} - M_{\text{exp}}^{B_b})/M_{\text{exp}}^{B_b}$$

Summary & Outlook

- We aimed at investigating the masses of heavy baryons, based on pion mean fields.
- The light quarks govern heavy-baryon systems, while a heavy quark plays a role of a mere static color source.
- $N_c - 1$ valence light quarks produce the pion mean field, in which the light quarks and a heavy quark are bound.
- We introduce a phenomenological spin-spin interaction that accounts for hyperfine mass splittings.

What we can predict in the next



- Doubly charmed (bottom) heavy baryons:

$N_c - 2$ light-quark system

QQq

Qualiton

- A tetraquark system:

$\bar{Q}\bar{Q}qq \rightarrow (\bar{Q}\bar{Q})(qq)$

A kind of diquark colored soliton

What we can predict in the next



- Strong decays of heavy baryons
- Magnetic moments of heavy baryons
- Strong axial-vector coupling constants
- Vector coupling constants
- Form factors and GPDs

What we need is to compute three-point correlation functions within the present method:

$$\langle B_Q(\mathcal{R}_1) | \mathcal{O}^{\mathcal{R}} | B'_Q(\mathcal{R}_2) \rangle$$

*Though this be madness,
yet there is method in it.*

Hamlet Act 2, Scene 2

Thank you very much!