Accelerator Physics Exercises

Answers to be handed in before lecture on *2 November 2016*

1. The lectures on longitudinal dynamics imply that two equations can be applied as the particle passes on each turn through the RF cavity:

$$\delta E = V_0 \left(\sin \phi - \sin \phi_s \right)$$

$$\delta \phi == -\frac{2\pi h \eta}{E_0 \beta^2 \gamma} \Delta E$$

where

$$V_0 = \text{peak gap voltage of the cavity}$$

$$E_0 = \mathrm{m_0 c^2} = \text{rest energy of the proton}$$

These can be used to follow a trajectory in the longitudinal phase space

$$(\Delta E, \phi)$$

on each turn around the machine at 2 GeV.

(a) Write a simulation program that assumes an initial point of these coordinates

$$(0, \phi) \quad \text{in the range } 0 < \phi < \pi - \phi_s$$

and that calculates and applies the small changes

$$(\delta E, \delta \phi)$$

to the current coordinate

$$(\Delta E + \delta E, \phi + \delta \phi)$$

counting the turns around the machine.

Hand in a listing of the program with an example of the output.

(b) Change the initial phase to be a degree or so less than

$$\pi - \phi_s$$

and thus trace out the maximum bucket size.

HINTS: Remember that the slip factor is positive (below transition) and that the order of applying the increments is to (1) use the last turn's value of phase to calculate the energy increment, then (2) calculate the energy increment to be applied to the current turn before calculating the phase advance for the next turn (just imagine you were a particle!)

(c) Compare the results with the tables below from page 36 of:

http://doc.cern.ch/archive/electronic/other/preprints//CM-P/cm-p00047617.pdf

Bucket area	Bucket (half) height	Coordinates*
$(heV)^{1/2}\,\alpha(\Gamma)\,(16\gamma/h)\,(2\pi E\lvert\eta\rvert)^{-1/2}$	$(heV)^{1/2}Y(\Gamma)(\gamma/h)\,(\pi E\lvert\eta\rvert)^{-1/2}$	$(\Delta p/m_0c) - \varphi$
$(heV)^{1/2}\,\alpha(\Gamma)\,(16\beta/h)\,[E/(2\pi\lvert\eta\rvert)]^{1/2}$	$(heV)^{1/2}Y(\Gamma)(\beta/h)[E/(\pi\lvert\eta\rvert)]^{1/2}$	$(\Delta E) - \varphi$
$(heV)^{1/2}\alpha(\Gamma)[16\alpha_p R/(h\beta)](2\pi E\lvert\eta\rvert)^{-1/2}$	$(heV)^{1/2}Y(\Gamma)[R/(\gamma_{tr}^{2}h\beta)](\pi E\lvert\eta\rvert)^{-1/2}$	$(\Delta R) - \varphi$
$(heV)^{1/2}\,\alpha(\Gamma)[16\beta/(h^2\Omega)][E/(2\pi\lvert\eta\rvert)]^{1/2}$	$(heV)^{1/2}Y(\Gamma)[\beta/(h^2\Omega)][E/(\pi\lvert\eta\rvert)]^{1/2}$	$(\Delta E/h\Omega) - \varphi$

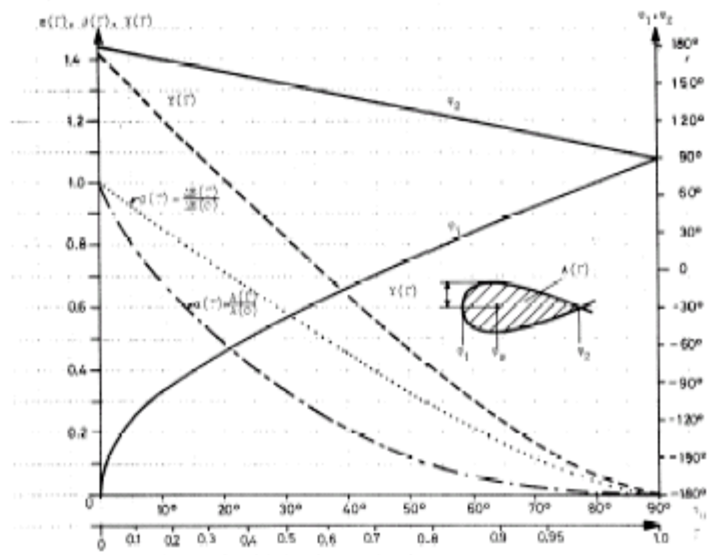
For $\alpha(\Gamma)$ see below and Appendix C; for η see Section 3.1 on preceeding page.

$$Y = Y(\Gamma) = \dot{\varphi}_{max}/(\sqrt{2}\,2\pi\,\nu_0)_{\varphi_s=0} = \dot{\varphi}_{max}(heV)^{-1/2}(\beta/\Omega)(\pi E\lvert\eta\rvert)^{1/2}$$

*[E and eV in keV, φ in rad, ΔR in cm]

Ideal adiabatic trapping of a linac beam with $\pm\,\Delta E_L$ leads to a minimum bucket (half) height $\Delta E = (\pi/2)\Delta E_L$

b) Bucket width, normalised (half) height and area (see Appendix C for numbers)



2. A 10 GeV (kinetic energy) synchrotron has a magnetic field that rises to 1.5 T in 1 s. Given that the mass of the proton is 0.9383 GeV:

(a) What is the momentum at 1.5 T?
(b) What is $B\rho$?
(c) If 2/3 of the circumference is bending magnets, what are ρ and R, the mean radius?
(d) What is the revolution frequency at 10 GeV?
(e) What is the revolution frequency at 1 GeV?
(f) Assuming the revolution frequency at 1 GeV, calculate the voltage per turn necessary to provide a linear rate of rise of 9 GeV/s. If $\sin \phi_s = \sin 45°$, what is the peak voltage necessary in the cavity?

3. For the above synchrotron, if the mean dispersion around the ring is 9 m, what is **(a)** γ_{tr}? **(b)** momentum at transition? and **(c)** η at 1 GeV and at 10 GeV?

4. For the above synchrotron calculate the following:

(a) If the harmonic number is 10, what is the synchrotron frequency at 1 GeV?

(b) Write a small computer program to plot f_s in steps of 0.05 GeV from 1 to 2 GeV.