R^* and five loop calculations

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Physics of the pole parts

Interesting physics just from the pole parts of diagrams:

- Anomalous dimensions (beta function, etc.)
- Splitting functions
- Decay rates (Higgs decay, etc.)

Pole parts are easier to compute than the finite parts

Goal

Compute poles of five loop diagrams

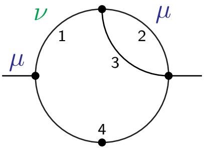
R^* operation

- The poles parts come from the divergent momentum configurations
- The recursive R^* operation takes care of combinatorics of subdivergences [Chetyrkin, Smirnov '83]
- We have extended R^* to Feynman integrals with arbitrary numerator structure [Herzog,Ruijl '17]

UV counterterm operation

$\Delta(G) =$ poles of G when all momenta go to ∞ with all contributions from subdivergences subtracted

Identifying divergent (sub)diagrams



$$= \int d^D p_1 \int d^D p_2 \frac{Q \cdot p_2 \, p_1^{\nu}}{{p_1}^2 {p_2}^2 {p_3}^2 {p_4}^4}$$

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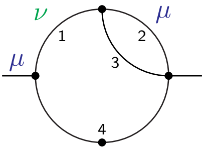

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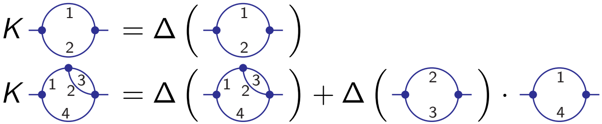
- Get degree of divergence through power counting
- Each loop contributes $+4$ due to the measure
- All momenta $\to \infty$: $8 + 1 + 1 - 2 - 2 - 2 - 4 = 0$ (log)
- $p_2, p_3 \to \infty$: $4 + 1 - 2 - 2 = 1$ (linear)
- $p_4 \to 0$: $4 - 4 = 0$ (log IR)

R^*-operation by example

$$K\left(\frac{2}{\epsilon^2} + 4 + 2\varepsilon\right) \equiv \frac{2}{\epsilon^2}$$



Consider all sets of non-overlapping divergent subdiagrams

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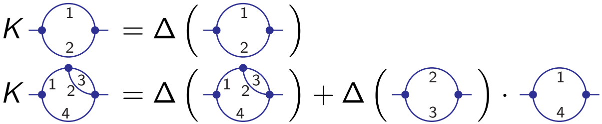

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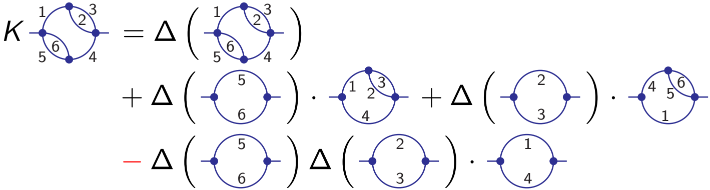
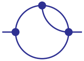
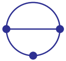
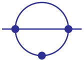


Counterterm operation Δ

For log diagrams, Δ does not depend on external momenta or masses!

$\Delta\Big($  $\Big) = \Delta\Big($  $\Big) = \Delta\Bigg($  $\Bigg)$

Infrared rearrangement (IRR)

Rearrange diagrams to simpler ones we can compute (always possible)

How to compute Δ?

- We use its definition:

$$\Delta(G) \overset{\text{IRR}}{=} \underbrace{\Delta(G')}_{\text{Simpler than G}} = K(G') - \underbrace{\text{subdivergences}(G')}_{\text{Lower-loop diagrams}}$$

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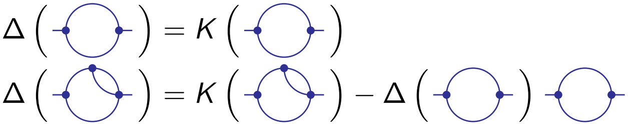
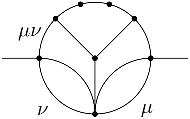
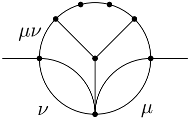
- Recursive application:

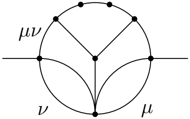


- FORCER can compute all rearranged diagrams [Ruijl,Ueda,Vermaseren '17]

Five-loop example

K 

$= \Delta$ 

$+$ subdiv 

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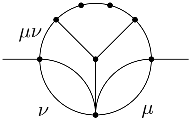
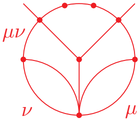
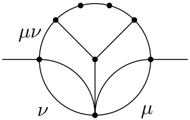
K 

$= \Delta$ 

$+$ subdiv 

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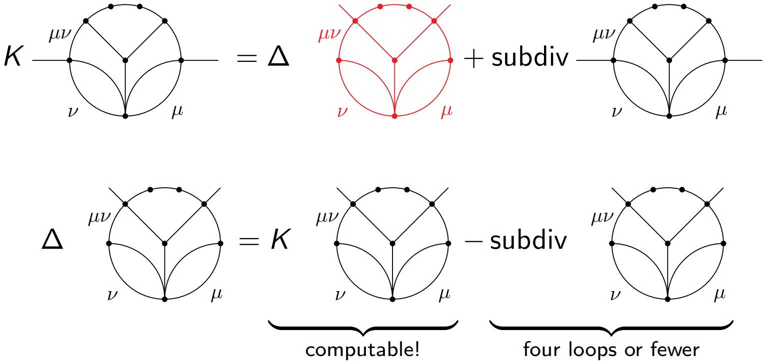
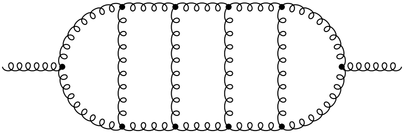


R^*-operation overview

1. Make input diagram logarithmic [creates a lot of terms]
2. Infrared rearrange diagrams
3. Identify counterterms
4. Make each counterterm log [millions of counterterms]
5. Tensor reduce the diagrams
6. Repeat process for new counterterms

Computational blow-up

- Troublesome five loop diagrams:



- Represents 12 029 521 scalar integrals!
- Computation may require a terabyte of disk space
- Time-consuming integral reductions
- R^* is slow (subgraph finding, tensor reduction)

Computational blow-up

- Every triple gluon vertex creates 6 terms:

$$v_{3g}(p_1^{\mu,a}, p_2^{\nu,b}, p_3^{\rho,c}) = -if^{abc}\Big[(p_1 - p_2)^\rho g_{\mu\nu} + (2p_2 + p_1)^\mu g_{\nu\rho} + (-2p_1 - p_2)^\nu g_{\mu\rho}\Big]$$

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- Subgraph finding and Taylor expansions are invariant under momentum contraction, so rewrite the rule:

$$v_{3g}(p_1^{\mu,a}, p_2^{\nu,b}, p_3^{\rho,c}) = -if^{abc}\left[p_1^\sigma t_3{}^{\sigma\nu\rho\mu} - p_2^\sigma t_3{}^{\sigma\mu\rho\nu}\right]$$
$$t_3{}^{\mu\nu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - 2g_{\mu\nu}g_{\rho\sigma} \ .$$

- Only 1024 terms for time-consuming functions

Tensor reductions

- We encounter rank 10 tensor integrals
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- Using group theory: 46 305 terms (still too much)

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- Naive Passarino-Veltman reduction requires 843 908 625 terms
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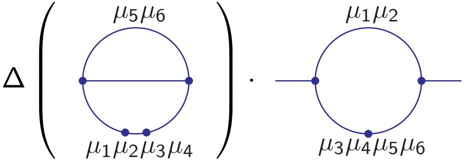
- For rank 6 in general:

$$c_1 \cdot g^{\mu_1\mu_2} g^{\mu_3\mu_4} g^{\mu_5\mu_6} + c_2 \cdot g^{\mu_1\mu_3} g^{\mu_2\mu_4} g^{\mu_5\mu_6} + \ldots \text{(13 more)}$$

- 15 coefficients and 15 tensor structures

Tensor reductions: symmetries

- In practice we have symmetries:



- Symmetric in exchanges of $\mu_1, \ldots, \mu_4$ and μ_5, μ_6 inside Δ
- Symmetric in μ_1, μ_2 and $\mu_3, \ldots, \mu_6$ outside Δ

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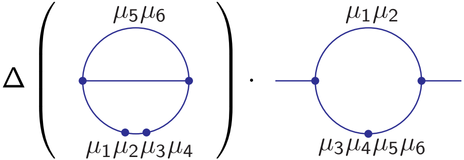


- Symmetric in exchanges of $\mu_1, \ldots, \mu_4$ and μ_5, μ_6 inside Δ
- Symmetric in μ_1, μ_2 and $\mu_3, \ldots, \mu_6$ outside Δ
- We thus see:

$$c_1 \cdot (g^{\mu_1\mu_2} g^{\mu_3\mu_4} g^{\mu_5\mu_6} + 2g^{\mu_1\mu_3} g^{\mu_2\mu_4} g^{\mu_5\mu_6}) +$$
$$c_3 \cdot (2g^{\mu_1\mu_2} g^{\mu_3\mu_5} g^{\mu_4\mu_6} + 10g^{\mu_1\mu_5} g^{\mu_2\mu_6} g^{\mu_3\mu_4})$$

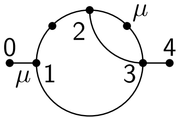
- Only 2 coefficients and 4 terms

Canonical labeling of counterterms

- Prevent double work by canonicalizing counterterms
- Extend the notion of an edge with additional information:

e(first vertex, second vertex, propagator power, indices)

- For example:



$$= e(0,1,1,\mu)e(1,2,2)e(2,3,1)$$
$$e(2,3,2,\mu)e(1,3,1)e(3,4,1)$$

- Use McKay's canonicalization algorithm

Results at five loops

- Computed five loop beta function for general colour group
  - Verified QCD result of [Baikov,Chetrykin,Kühn '16]
  - Took 6 days on a pc with 32 cores
- Recomputed $H \to b\bar{b}$, R-ratio
  - Easy: took a few hours on one pc
- Computed $H \to gg$ [HRUVV '17]
  - Quartically divergent diagrams...
  - Hard: took two months

Acknowledgements

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