Package-X 2.1 and CollierLink

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ACAT Track 3

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Released in 2015:

Package-X is a Mathematica package to (mainly) calculate one-loop integrals analytically.

Calculations carried out in 3 steps:

$$\int rac{d^dk}{(2\pi)^d} rac{k^\mu k^
u}{\left[(k+p)^2-m^2
ight]k^2} ext{ at } p^2=0$$

- 1. LoopIntegrate $[\mathbf{k}_{\mu} \, \mathbf{k}_{\nu}, \, \mathbf{k}, \, \{\mathbf{k} + \mathbf{p}, \, \mathbf{m}\}, \, \{\mathbf{k}, \, \mathbf{0}\}]$ $p_{\mu} \, p_{\nu} \, PVB[0, \, 2, \, p.p, \, 0, \, \mathbf{m}] + g_{\mu,\nu} \, PVB[1, \, 0, \, p.p, \, 0, \, \mathbf{m}]$
- 2. %/. $\mathbf{p}.\mathbf{p} \to \mathbf{0}$ $\mathbf{p}_{\mu} \ \mathbf{p}_{\nu} \ PVB[0, 2, 0, 0, m] + \mathbf{g}_{\mu,\nu} \ PVB[1, 0, 0, 0, m]$
- 3. % // LoopRefine $\left(\frac{1}{9} + \frac{1}{3} \left(\frac{1}{\epsilon} + \text{Log}\left[\frac{\mu^2}{m^2}\right]\right)\right) p_{\mu} p_{\nu} + \left(\frac{3 m^2}{8} + \frac{1}{4} m^2 \left(\frac{1}{\epsilon} + \text{Log}\left[\frac{\mu^2}{m^2}\right]\right)\right) g_{\mu,\nu}$

Important goal: **EASY TO USE**

Version 2 Released in 2016:

A) boxes

```
LoopIntegrate[1, k, {k, m}, {k+p1, m}, {k-p3+p1, m}, {k-p2, m}]

PVD[0, 0, 0, 0, p1.p1, p3.p3, p1.p1+2p1.p2-2p1.p3+p2.p2-2p2.p3+p3.p3, p2.p2, p1.p1-2p1.p3+p3.p3, p1.p1+2p1.p2+p2.p2, m, m, m, m]
```

LoopRefine[%] // DOExpand

$$\begin{aligned} & \text{ConditionalExpression} \Big[\frac{1}{s\sqrt{1 - \frac{4s^2}{s} - \frac{4s^2}{t}}} + 2 \left[-\frac{\pi^2}{2} - \text{Log} \Big[\frac{\left(1 + \sqrt{1 - \frac{4s^2}{s}}\right) + \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \right]^2 - \text{Log} \Big[\frac{s \left(1 + \sqrt{1 - \frac{4s^2}{t}}\right) \left(\sqrt{1 - \frac{4s^2}{t}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \Big]^2 + \\ & \text{Log} \Big[-\frac{t \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)^2}{4 \, \text{m}^2} \Big] \text{Log} \Big[-\frac{s \left(\sqrt{1 - \frac{4s^2}{t}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)^2}{4 \, \text{m}^2} \Big] + 2 \, \text{Log} \Big[-\frac{s \left(\sqrt{1 - \frac{4s^2}{s}} + \sqrt{1 - \frac{4s^2(s+t)}{s}}\right) \left(-\sqrt{1 - \frac{4s^2}{t}} + \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \Big]^2 - \\ & 2 \, \text{PolyLog} \Big[2, -\frac{\left(-1 + \sqrt{1 - \frac{4s^2}{s}}\right) s \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \Big] + 2 \, \text{PolyLog} \Big[2, -\frac{\left(-1 + \sqrt{1 - \frac{4s^2}{s}}\right) t \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \Big] + \\ & 2 \, \text{PolyLog} \Big[2, -\frac{s \left(-1 + \sqrt{1 - \frac{4s^2}{t}}\right) \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}{4 \, \text{m}^2} \Big] - 2 \, \text{PolyLog} \Big[2, -\frac{\left(-1 + \sqrt{1 - \frac{4s^2}{s}}\right) t \left(\sqrt{1 - \frac{4s^2}{s}} - \sqrt{1 - \frac{4s^2(s+t)}{s}}\right)}}{4 \, \text{m}^2} \Big] + \\ & A.I. \, Davydychev \\ & arXiv: \, 9307323 \end{aligned}$$

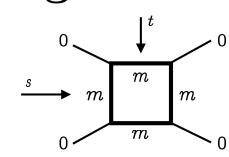


m

Version 2 Released in 2016:

B) series expansions

Large mass expansion $(m \sim \infty)$



LoopRefineSeries[%%, {m, Infinity, 12}]

$$\frac{1}{6 \, \, \text{m}^4} \, + \, \frac{\text{s+t}}{60 \, \, \text{m}^6} \, + \, \frac{2 \, \, \text{s}^2 \, + \, \text{s} \, \, \text{t} \, + \, 2 \, \, \text{t}^2}{840 \, \, \text{m}^8} \, + \, \frac{3 \, \, \text{s}^3 \, + \, \text{s}^2 \, \, \text{t} \, + \, \text{s} \, \, \text{t}^2 \, + \, 3 \, \, \text{t}^3}{7560 \, \, \text{m}^{10}} \, + \, \frac{12 \, \, \text{s}^4 \, + \, 3 \, \, \text{s}^3 \, \, \text{t} \, + \, 2 \, \, \text{s}^2 \, \, \text{t}^2 \, + \, 3 \, \, \text{s} \, \, \text{t}^3 \, + \, 12 \, \, \text{t}^4}{166 \, \, 320 \, \, \text{m}^{12}} \, + \, O \left[\frac{1}{\text{m}} \right]^{13} \, + \, \frac{1}{100 \, \, \text{m}^{10}} \, + \,$$

Forward limit $(t \sim 0)$

LoopRefineSeries[%%%, {t, 0, 2}]

$$\left(\frac{2}{m^2 s} + \frac{\text{DiscB[s, m, m]}}{m^2 s} \right) + \left(-\frac{4 \left(3 m^2 - s \right)}{9 m^4 s^2} - \frac{\left(4 m^2 - s \right) \text{DiscB[s, m, m]}}{6 m^4 s^2} \right) t + \\ \left(\frac{240 m^4 - 140 m^2 s + 23 s^2}{225 m^6 s^3} + \frac{\left(4 m^2 - s \right)^2 \text{DiscB[s, m, m]}}{30 m^6 s^3} \right) t^2 + O[t]^3$$

precision -0.0174953803351620551279058 + 0.01010581673617592175607057 i

C) improved numerics

machine ScalarD0[5.4, 1.2, 5.4, 1.2, 54.1, -12, 1.1, 1.1, 1.1, 2.3]

precision -0.0174954 + 0.0101058 i

arbitrary ScalarD0[5.4^20, 1.2^20, 5.4^20, 1.2^20, 54.1^20, -12^20, 1.1^20, 1.1^20, 1.1^20, 2.3^20]

Version 2 Released in 2016:

D) Dirac algebra

Calculate traces (already in v1.0)

Spur[k.
$$\gamma$$
+m1, γ_{μ} , (k+p). γ +m1, γ_{ν}]

8
$$k_{\mu}$$
 k_{ν} + 4 k_{ν} p_{μ} + 4 k_{μ} p_{ν} + 4 m^2 $g_{\mu,\nu}$ - 4 $k.k$ $g_{\mu,\nu}$ - 4 $k.p$ $g_{\mu,\nu}$

Put open fermion lines in canonical form:

$$\langle u[p2, m], p1.\gamma, \gamma_{\mu}, p2.\gamma, u[p1, m] \rangle // FermionLineExpand$$

$$-2 \text{ im } \langle u[p2, m], \sigma_{\mu, \{-p1+p2\}}, u[p1, m] \rangle + \langle u[p2, m], \gamma_{\mu}, u[p1, m] \rangle (3 \text{ m}^2 - 2 \text{ p1.p2})$$

Compute a Feynman diagram (QED vertex function):

LoopIntegrate [
$$\langle u[p', m], \gamma_{\vee}, (p', \gamma - k, \gamma + m 1), \gamma_{\mu},$$

$$(p.\gamma - k.\gamma + m1)$$
, γ_{\vee} , $u[p, m]$, k , $\{k-p, m\}$, $\{k-p, m\}$,

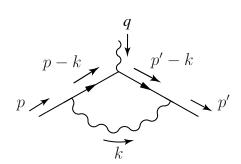
$$\{k, 0, 1\}$$
] /. $\{p.p \rightarrow m^2, p.p. \rightarrow m^2, p.p. \rightarrow -q.q/2 + m^2\}$ //

LoopRefine

$$\frac{2 \text{ im DiscB[q.q, m, m] } \langle u[p, m], \sigma_{\mu, \{-p+p,\}}, u[p, m] \rangle}{4 m^2 - q.q} +$$

$$\langle u[p, m], \gamma_{\mu}, u[p, m] \rangle \left(-\frac{\text{DiscB}[q,q,m,m]}{4 m^2 - q,q} + \frac{4 m^2 - q,q}{4 m^2 - q,q} + \frac{1}{4 m^2 - q,q} + \frac{1}{4$$

$$\left(1 - \frac{2 \operatorname{DiscB}[q.q, m, m] \left(2 m^2 - q.q\right)}{4 m^2 - q.q}\right) \left(\frac{1}{\epsilon} + \operatorname{Log}\left[\frac{\mu^2}{m^2}\right]\right) + 2 \left(2 m^2 - q.q\right) \operatorname{ScalarC0IR6}[q.q, m, m]$$





Version 2 Released in 2016:

E) Intuitive and readable user interface

Input and Output in StandardForm

```
In[5]:= LDot[a, b]

Out[5]= a.b

In[4]:= \langle u[\mathbf{p}, \mathbf{m}], \gamma_{\mu}, v[\mathbf{p}, \mathbf{m}] \rangle // InputForm

FermionLine[{1, p, m}, {-1, p, m}, DiracMatrix[LTensor[DiracG, \mu]]]
```

Traditional form also available!

DiracMatrix
$$\left[\gamma_{\mu}, (\mathbf{k} + \mathbf{p}).\gamma, \gamma_{\nu}, \mathbb{P}\mathbf{L}\right] //$$

TraditionalForm

$$\gamma^{\mu} (\not k + \not p) \gamma^{\nu} P_{L}$$
 $\uparrow \cdot p$

F) Integrated documentation

```
General Tensor Operations and Symbols
LTensor (\Box_{\Box}), LDot (\Box.\Box), LScalarQ — Lorentz tensors and scalar products
Contract — contract tensors with repeated indices
Transverse, Longitudinal — project 2nd or higher rank Lorentz tensors
MandelstamRelations — express scalar products in terms of Mandelstam invariants
g_{\square,\square} = \varepsilon_{\square,\square,\square,\square} — metric tensor, and Levi-Civita symbol
Computing One-Loop Integrals
LoopIntegrate, LoopRefine — routines for one loop integrals
LoopRefineSeries — compute series expansions of loop integrals
d = ε = μ − dimensional regularization symbols
Dirac Algebra and Fermion Spinors
Spur — compute traces over product of Dirac matrices
Projector — project fermion self energy and vertex functions onto form factors
1 • \gamma_{\square} • \gamma5 • PL • PR • \sigma_{\square,\square} — objects in spinor space
DiracMatrix, FermionLine (u, v), FermionLineProduct, FermionLineExpand — open fermion chains
Generated Symbols
PVA = PVB = PVC = PVD = PVX - symbolic Passarino-Veltman functions
Special Functions and Abbreviations »
Kallenλ · DiscB · ScalarCO · ScalarDO · ...
```



Scalar_{D0}



Ferm

Details ar

An inter

ScalarD0 [s_1 , s_2 , s_3 , s_4 , s_{12} , s_{23} , m_0 , m_1 , m_2 , m_3] gives the scalar Passarino-Veltman four-point function n_1

external invariants s_1 ,

LoopRefine

expar Details and Options

▼ Examples (9)

The inp	•	Basic Examples
them.		Evaluate nun

In[1]:= ScalarD0 [1.2

Evaluate numeri

Evaluate symbol

- Out[1]= -0.230987 + 0
- Fermio The precision of matrice In[2]:= ScalarD0[1.2
- For exp replace: Out[2]= -0.230986611
- Fermio
- Additior algebra

Dii

GOI

- algebra In[1]:= ScalarDO [0, 0]

 Ch: Out[1]= $\frac{1}{m^2 M^2 M^4} \frac{Lc}{(m^2 M^2 M^4)}$
 - In[2]:= ScalarD0 [M²,
 - Out[2]= $\frac{2 M^2 \text{ Disc}}{m^2 (-m^2 + 4 M^2)}$

-point function

LoopRefine [expr]
converts the Passarino-Vel
evaluated.

TATE

Details and Options

Examples (25)

▼ Basic Examples (2)

Compute
$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d}$$

Out[1]=
$$p_{\mu} p_{\nu} PVB [0, 2, s]$$

Out[2]=
$$\left(\frac{-12 \, m^2 + 13 \, s}{18 \, s} + \frac{1}{18} \, \left(\frac{1}{18} \, \left(21 \, m^2 - 4 \, s \right) \right) \right)$$

Interpretation:

$$\mu^{2} \leftarrow \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k\mu}{\left[k^{2}-m^{2}\right]\left[k\right]} \frac{i}{\left[k\right]} \frac{i}{16\pi^{2}} \left[\left(\frac{-12m^{2}+13s}{18s} + \frac{1}{18s}\right) + \frac{1}{18s} + \frac{1}{18s}$$

LoopRefineSeries

LoopRefineSeries
$$[f, \{s, s_0, n\}]$$

generates a Taylor series expansion of f containing Passarino–Veltman functions about the point $s=s_0$ to order $(s-s_0)^n$.

in terr

ig to

LoopRefineSeries[
$$f$$
, { s , s 0, n s}, { t , t 0, n t}, ...]

successively finds Taylor series expansions with respect to s, then t, etc.

Details and Options

▼ Examples (11)

▼ Basic Examples (1)

Apply LoopRefineSeries in place of LoopRefine to make a Taylor series expansion of a loop integral:

$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\left\lceil k^2 - m^2 \right\rceil \left\lceil (k-p)^2 - m^2 \right\rceil}$$

$$In[1]:= LoopRefineSeries[LoopIntegrate[1, k, {k, m}, {k-p, m}], {p.p, 0, 2}]$$

Out[1]=
$$\left(\frac{1}{\epsilon} + \text{Log}\left[\frac{\mu^2}{m^2}\right]\right) + \frac{p \cdot p}{6m^2} + \frac{(p \cdot p)^2}{60m^4} + O[p \cdot p]^3$$

Convert to a normal expression:

Out[2]=
$$\frac{1}{\epsilon} + \frac{\mathbf{p} \cdot \mathbf{p}}{6 \, \mathbf{m}^2} + \frac{(\mathbf{p} \cdot \mathbf{p})^2}{60 \, \mathbf{m}^4} + \text{Log} \left[\frac{\mu^2}{\mathbf{m}^2} \right]$$

Interpretation

$$\approx \frac{i}{16\pi^2} \left[\left(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln\left(\frac{\mu^2}{m^2}\right) \right) + \frac{p^2}{6m^2} + \frac{(p^2)^2}{60m^4} + O(p^2)^3 \right]$$

LoopIntegrate[
$$num$$
, k , $\{k + p_0, m_0, w_0\}$, $\{k + p_1, m_1, w_1\}$, ...]

$$= \frac{1}{C_{\epsilon}} \, \mu^{2\epsilon} \, \int \! \frac{d^d k}{(2\pi)^d} \, \frac{num}{\left[(k+p_0)^2 - m_0^2 + i\varepsilon \right]^w 1 \, \left[(k+p_1)^2 - m_1^2 + i\varepsilon \right]^{w_2} \, \dots}$$

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Selected features of Package-X

Package-X can generate analytic expressions for arbitrarily high rank tensor integrals with up to four distinct propagators, each with arbitrary integer weight, giving UV and IR divergent, and finite parts at arbitrary (real-valued) kinematic points, and can can construct multivariable Taylor series expansions near arbitrary (non-singular) kinematic point to arbitrary order.

Package-X can calculate traces of products of gamma matrices, and perform tensor algebraic operations on open fermion lines.

Package-X can numerically evaluate scalar basis functions with either machine precision (fast) and arbitrary precision (slow) at any kinematic point.



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Applications: anything... except fully differential cross sections

- shift in vacuum expectation values
- shift in pole mass
- wavefunction renormalization
- electroweak oblique parameters
- particle electromagnetic moments (EDMs, polarizability, ...)
- 2 body decays, 3 body decays
- counterterms
- wilson coefficients
- cross sections at threshold

. . .

Motivation:

Until *Package*-X, there was no comprehensive one-loop package available. There are semi-analytic packages (like FeynCalc,...) and numerical ones (like LoopTools,...)

- Sometimes difficult to use
- Does not always get an answer, like at vanishing Gram determinant, etc...



New in Package-X 2.1

A) Command-line readiness

Before, names of keywords were the symbols

```
Eμ,ν,ρ,ς gρ,ν DiracMatrix[γν, σρ,ς] // FullForm

Times[DiracMatrix[LTensor[\[Gamma], \[Nu]],
   LTensor[\[Sigma], \[Rho], \[FinalSigma]]],

LTensor[\[DoubleStruckG], \[Nu], \[Rho]],
   LTensor[\[CurlyEpsilon], \[Mu],
   \[Nu], \[Rho], \[FinalSigma]]]
```

```
Now, keywords have proper full names,
and parse into full names by front-end.

ε<sub>μ,ν,ρ,ς</sub> g<sub>ρ,ν</sub> DiracMatrix [γ<sub>ν</sub>, σ<sub>ρ,ς</sub>] // FullForm

Times [DiracMatrix [LTensor [DiracG, \[Nu]],
LTensor [DiracS, \[Rho], \[FinalSigma]]],
LTensor [MetricG, \[Nu], \[Rho]], LTensor [
LeviCivitaE, \[Mu], \[Nu], \[Rho], \[FinalSigma]]]
```

All old front-end notebooks continue to work, but now also convenient to use command-line and edit .m (.wl) files:

```
■ Square amplitude

applyPolarizationCompleteness[expr_, List[p_,0,idx_]]:=

Replace[expr,{
    HoldPattern[rest_. Power[LTensor[MetricG, PatternSequence[idx,a_]|PatholdPattern[rest_. LTensor[v_,idx]^2]:> (-LDot[v,v]) rest,
    HoldPattern[rest_. LTensor[MetricG, PatternSequence[idx,a_]|PatternSequence[rest_. LTensor[MetricG, PatternSequence[idx,a_]|PatternSequence[rest_. LTensor[MetricG, idx, idx]]:> rest (_Dim),
    HoldPattern[rest_. LTensor[v_,idx]*LTensor[w_,idx]]:> (-LDot[v,w])

expr:> ReplaceOnce[expr,LTensor[vec_,l___,idx,r___]:>-LTensor[vec,l,]];
```

```
Mathematica 10.3.1 for Mac OS X x86 (64-bit)
Copyright 1988-2015 Wolfram Research, Inc.

In[1]:= <<X`
Package-X v2.1.0 [developer version], by Hiren H. Patel

In[2]:= FermionLine[{-1,p,m},{1,p,m},DiracMatrix[LTenson[DiracG,mu]]]*LTensor[q,mu]

Out[2]= < v[p, m], y(mu), u[p, m] > q(mu)

In[3]:= %//Contract

Out[3]= < v[p, m], y.q, u[p, m] >

In[4]:=
```



New in Package-X 2.1

B) Calculate the discontinuity across normal threshold cut for any one-loop integral:

Set option Part→Discontinuity[s] to LoopRefine

$$\int rac{d^d k}{(2\pi)^d} rac{k^\mu k^
u}{[(k+p)^2-m^2]\,k^2}$$

- 1. LoopIntegrate $[k_{\mu} k_{\nu}, k, \{k+p, m\}, \{k, 0\}]$ $p_{\mu} p_{\nu} PVB[0, 2, p.p, 0, m] + g_{\mu,\nu} PVB[1, 0, p.p, 0, m]$
- 2. % /. $\mathbf{p}.\mathbf{p} \to \mathbf{t}$ $\mathbf{p}_{\mu} \ \mathbf{p}_{\nu} \ PVB[0, 2, t, 0, m] + \mathbf{g}_{\mu,\nu} \ PVB[1, 0, t, 0, m]$
- 3. LoopRefine[%, Part \rightarrow Discontinuity[t]] $\frac{2 i \pi (-m^2 + t)^3 \text{ HeavisideTheta}[-m^2 + t] p_{\mu} p_{\nu}}{3 t^3} \frac{i \pi (-m^2 + t)^3 \text{ HeavisideTheta}[-m^2 + t] g_{\mu,\nu}}{6 t^2}$

Applications:

- Compute tree-level cross sections (optical theorem)
- Leptogenesis: only need imaginary parts of 1-loop diagrams

Important!

Discontinuity

```
Discontinuity [s]
is a setting for Part to LoopRefine to compute the discontinuity across the normal threshold s.

Discontinuity [s, t]
```

computes the Mandelstam double spectral function in overlapping channels s and t.

▶ Details and Options

▼ Examples (7)

▼ Basic Examples (1)

Calculate the discontinuity across the normal threshold cut of the scalar two-point function



In[1]:= LoopIntegrate[1, k, {k+p, m}, {k, m}]
 LoopRefine[%, Part → Discontinuity[p.p]]
Out[1]= PVB[0.0.p.m.m]



April 2016, COLLIER released!



- Numerical library of Passarino-Veltman functions (scalar *and* tensor integrals)
- Arbitrarily high rank *and* arbitrarily many legs
- Dimensional regularization *and* mass regularization
- Complex internal masses
- Dedicated expansions near vanishing Gram/Cayley determinants
- **Two** independent implementations (and nothing to do with FF library)

Bring to Mathematica! \implies CollierLink



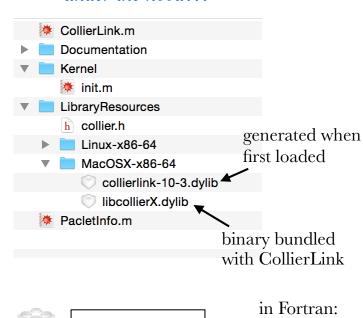
Three features -

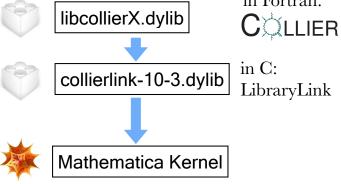
Indeterminate

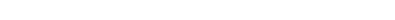
1. Direct numerical evaluation of Passarino-Veltman functions

```
<< X`
<< CollierLink`
Package-X v2.1.0 [ALPHA], by Hiren H. Patel
For more information, see the guide
CollierLink v1.0.0 [ALPHA], by Hiren H. Patel
         \mathbf{C}_{0011}
PVC[1, 2, 0, 0, 0, 3.2, 1.0 - .2 I, 1.0 - .2 I, 2.0]
-0.0155049 + 0.010052 i
                                    complex
                                     masses
Plot[{Re[PVC[0, 0, 0, 5.3, 12.1, s, 1.1, 1.2, 3.0]],
  Im[PVC[0, 0, 0, 5.3, 12.1, s, 1.1, 1.2, 3.0]]}, {s, -20, 50}]
-20
    -10
               10
       -1.0
       -1.5
PVD[0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.0]
COLLIER::nocase: Error flag -10: Unable to
    evaluate PVD[0, 0, 0, 0, 1., 2., 3., 4., 5., 6., 7., 8., 9., 10.];
    case is not supported or implemented.
```

under the hood...









Three features -

2. Automatic code generation, compilation, and relinking

$$\int rac{d^d k}{(2\pi)^d} rac{\left(k^2 + k.p + k.q + p.q
ight)^2}{\left(k + p
ight)^2 \left[(k + q)^2 - m^2
ight]k^2}$$

$$\left\{ p.p \to m^2, q.q \to 0, p.q \to \frac{1}{2} (m^2 - t) \right\}$$

$$\frac{3}{4} PVA[0, 0] + \frac{1}{4} PVA[0, m] + \left(\frac{m^2}{4} + \frac{1}{2} (m^2 - t)\right) PVB[0, 0, 0, 0, m] +$$

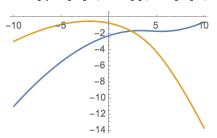
$$\left(-\frac{m^2}{4} + \frac{1}{2} (m^2 - t)\right) PVB[0, 0, m^2, 0, 0] + \frac{1}{4} (m^2 - t) PVB[0, 1, 0, 0, m] +$$

$$\frac{1}{4} (m^2 - t) PVB[0, 1, m^2, 0, 0] + \frac{1}{4} (m^2 - t)^2 PVC[0, 0, 0, m^2, t, 0, 0, 0, m]$$
pretend this is

f = CollierCompile[{t, m}, Evaluate[myIntegral]]

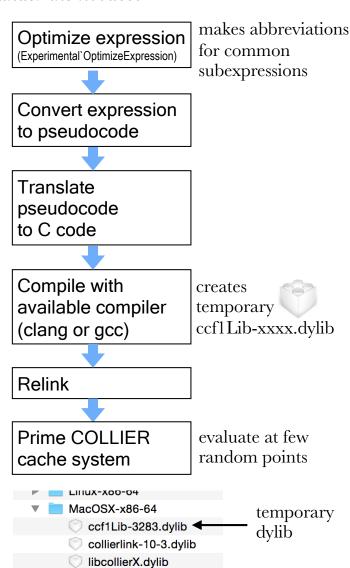


Plot[{Re[f[t, 2.1]], Im[f[t, 2.1]]}, {t, -10, 10}]



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under the hood...

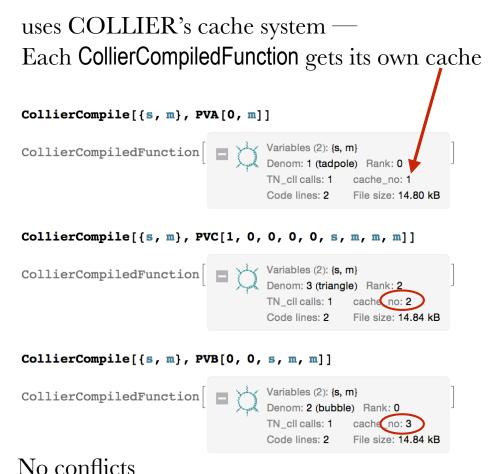




M Deeletlefe --

Three features -

2. Automatic code generation, compilation, and relinking



And CollierCompiledFunction can be parallelized!

CollierCompiledFunction

```
CollierCompiledFunction[args, ...]
represents compiled code for evaluating a compiled function using the COLLIER library.

Compile the infrared divergent scalar triangle function:

In[1]:= myfun = CollierCompile[{qSq}, PVC[0, 0, 0, 0, 0, qSq, 0, 0, 0]]

Out[1]= CollierCompiledFunction

Variables (1): {qSq}
Denom: 3 (triangle) Rank: 0

Load CollierLink on all parallel kernels with ParallelNeeds, and distribute the definition of myFunto parallel kernels:

In[2]:= ParallelNeeds ["CollierLink`"]
DistributeDefinitions [myfun]

Out[2]= {myfun}

Compute a table of values in parallel:

In[3]:= ParallelTable [myfun[qSq], {qSq, 5.1, 10.3}]

Out[3]= {-0.868639 - 1.00361 i, -0.675789 - 0.931296 i, -0.540322 - 0.867299 i, -0.440657 - 0.811331 i, -0.364729 - 0.762362 i, -0.305283 - 0.719311 i}
```

Each parallel kernel gets its own copy of dylib



Three features -

!Evaluation code

localVar1 = (inputVar2**2)
localVar2 = (-inputVar1)

localVar5 = (localVar3**2)

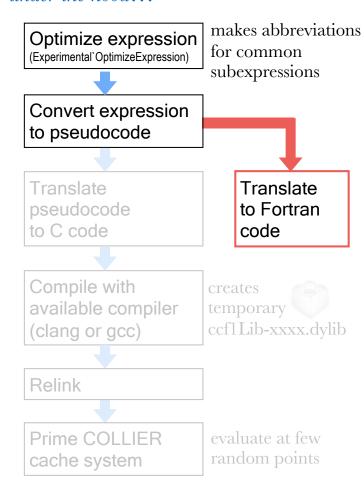
localVar3 = (localVar1+localVar2)
localVar4 = ((1.d0/2.d0)*localVar3)

3. Automatic Fortran code generation

```
CollierCodeGenerate[{t, m}, Evaluate[myIntegral]]
```

```
This code was automatically created using Package-X v2.1.0 CollierLink
  routine CollierCodeGenerate[] by Hiren H. Patel, and requires the
  Fortran library COLLIER v1.1 by A. Denner, S. Dittmaier, L. Hofer
  for numerical evaluation.
                                                                           1.1
                                                                           1.1
  Language: Fortran
                                     Creation date: Aug-21-2017 21:15
  Function name: generatedFunction Arguments (2): {t,m}
 Author: hhpatel
                                                                           1.1
  Initialization requirement: Init_cll(3,1,'')
DOUBLE COMPLEX FUNCTION generatedFunction(inputVar1, inputVar2)
 USE COLLIER
  IMPLICIT NONE
 DOUBLE COMPLEX, INTENT(IN) :: inputVar1 ! t
 DOUBLE COMPLEX, INTENT(IN) :: inputVar2 ! m
  !Allocate memory for calculated results of Passarino-Veltman,
     coefficient functions, and their UV divergent parts
  DOUBLE COMPLEX :: pvx1(1), pvx1uv(1) ! PVA[_,0]
 DOUBLE COMPLEX :: pvx2(1), pvx2uv(1) ! PVA[_,m]
 DOUBLE COMPLEX :: pvx3(2), pvx3uv(2) ! PVB[_,_,0,0,m]
  DOUBLE COMPLEX :: pvx4(2), pvx4uv(2) ! PVB[_,_,Compile`optVar5,0,0]
  DOUBLE COMPLEX :: pvx5(3), pvx5uv(3) ! PVC[_,_,_,Compile`optVar5,t,0,0,0,m]
  !Local variables
  DOUBLE COMPLEX :: localVar1
                              ! optVar5
 DOUBLE COMPLEX :: localVar2
 DOUBLE COMPLEX :: localVar3
                              ! optVar8
 DOUBLE COMPLEX :: localVar4
                               ! optVar9
 DOUBLE COMPLEX :: localVar5
                               ! optVar21
```

under the hood...



Idea is to be able to easily plug this code into popular event generators MCFM, MadGraph, ...



Summary

- ★ Introduced *Package*-X, described its salient features
- ★ Described updates in upcoming release *Package*-X v2.1
- ★ Introduced new package, CollierLink
 - 1. Direct numerical evaluation of Passarino-Veltman functions
 - 2. Automatic code generation, compilation, and relinking
 - 3. Automatic Fortran code generation

Conclusions:

Together with CollierLink, *Package*-X provides a user-friendly yet truly comprehensive tool for the calculation of one loop integrals.

Applications: anything... except fully differential error sections

