

Parallel Random Number Generation for SIMD/SIMT



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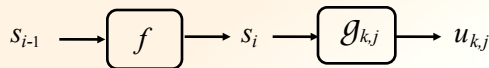
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Abstract

Efficient random number generation with high quality statistical properties is one of important components of Monte Carlo simulation in many areas of computational science. As high performance parallel computing systems equipped with deeper vector pipelines or many-cores technologies are widely used, a variety of parallel pseudo-random number generators (PRNGs) have been developed for specific hardware architectures such as SIMD or GPU (SIMT). However, concurrent random number services for hybrid computing models are not commonly available. In this report, we present design considerations, implementation details and preliminary computing performance of parallel PRNG engines which support both SIMD and SIMT with multiple independent streams of random number sequences.

Pseudo-Random Number Generation



f (transition function): $\mathcal{S} \rightarrow \mathcal{S}\{s_i\}$ (state space),
 g (output function): $\mathcal{K} \times \mathcal{S} \rightarrow \mathcal{U}\{u_{k,j}\}$ (output space $\in [0, 1)$)
 $\mathcal{K}\{k\}$ (key space), $g_{k,j} = b_k \circ h_j$ (b_k : bijection, h_j : selector)

Algorithm-basis vs. Counter-basis Generators

Ex. \mathbb{F}_2 -Linear modulo m

$$\vec{x}_i = \mathbf{A} \vec{x}_{i-1} \bmod 2$$

$$\vec{y}_i = \mathbf{B} \vec{x}_i \bmod 2$$

$$u_i = \sum_{l=1}^w y_{i,l-1} 2^{-l}$$

LFSR, GFSRMT, WELL, ...

	Algorithm	Counter
f	complicated	simple
b	-	complicated
h	simple	medium

Strong block cipher

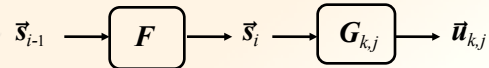
$$L = R \oplus F_k(R) \oplus L, \quad R = R \oplus F_k(L)$$

$$L' = R \oplus (R), \quad R' = F_k(R) \oplus L$$

$$L'' = R \oplus (R), \quad R'' = F_k(R) \oplus L'$$

AES, ARS

Parallel pRNG for SIMD/SIMT



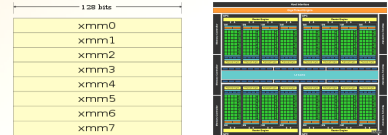
Parallel Random Number Generation

$$\vec{S}_{i-1} = (s^1, s^2, \dots, s^m)_{i-1}$$

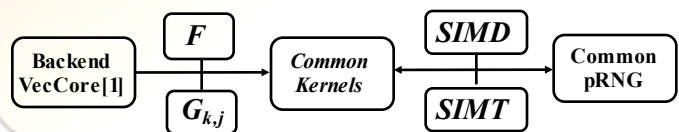
m = SIMD vector, or SIMT threads

$$\vec{u}_{k,j} = (u^1, u^2, \dots, u^m)_{k,j}$$

Vectorization



• Common Kernels are used for both SIMD and SIMT using VecCore[1]



Examples of Implementation

MRG32k3a [2]

Multiple Recursive Generator (MRG) with \mathbb{F}_2 -Linear PRNG State ($s_{i,n}$), Linear recurrence vector (u_i) and Output (u_i)

$$\begin{aligned} s_{1,n} &= (x_{1,n}, x_{1,n+1}, x_{1,n+2}) \\ s_{2,n} &= (x_{2,n}, x_{2,n+1}, x_{2,n+2}) \\ x_{1,n} &= [a_{12}x_{1,n-2} - a_{13}x_{1,n-3}] \bmod m_1 \\ x_{2,n} &= [a_{21}x_{1,n-1} - a_{22}x_{1,n-2}] \bmod m_2 \\ u_n &= [(x_{1,n} - x_{2,n}) \bmod m_1] / (m_1 + 1) \\ \text{period} &= (m_1^2 - 1)(m_2^2 - 1) / 2 \approx 2^{191} \approx 3.1 \times 10^{57} \end{aligned}$$

```
// struct MRG32k3a { random state of MRG32k3a
template <typename Backend>
struct MRG32k3a {
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