Robust circle reconstruction with the Riemann fit

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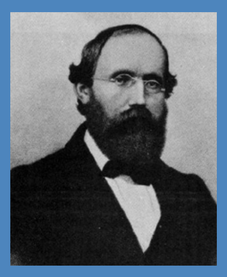


Introduction

- Fitting circular arcs or full circles is important in HEP, for example:
  - Tracks in a homogeneous magnetic field
  - Circles of photons in a Ring Imaging Cherenkov (RICH) detector
- **Significant background** can be expected in both cases
- In drift chambers mirror points are present
- **Robustification** is required

Mapping to the Riemann paraboloid

- Points in the (x, y)-plane:

$$(u_i, v_i), \quad i = 1, \ldots, N$$

- Mapping to the Riemann paraboloid:

$$x_i = u_i, \quad y_i = v_i, \quad z_i = x_i^2 + y_i^2$$

- If the points lie on the circle

$$(u - u_0)^2 + (v - v_0)^2 = \rho^2$$

  the mapped points lie on the plane

$$z - 2xu_0 - 2yv_0 = \rho^2 - u_0^2 - v_0^2$$

- The circle fit is transformed into **fitting a plane** to the mapped points

Robust regression

- Use Least Median of Squares (LMS) regression to fit the plane $\boldsymbol{n}\boldsymbol{r} + c = 0$ to the mapped points
- Instead of minimizing the sum of the squared distances, their **median** is minimized:

$$(\boldsymbol{n}_{\mathrm{LMS}}; c_{\mathrm{LMS}}) = \arg\min_{(\boldsymbol{n};c)} \mathrm{med}_{i=1}^{N}\, d_i^2$$

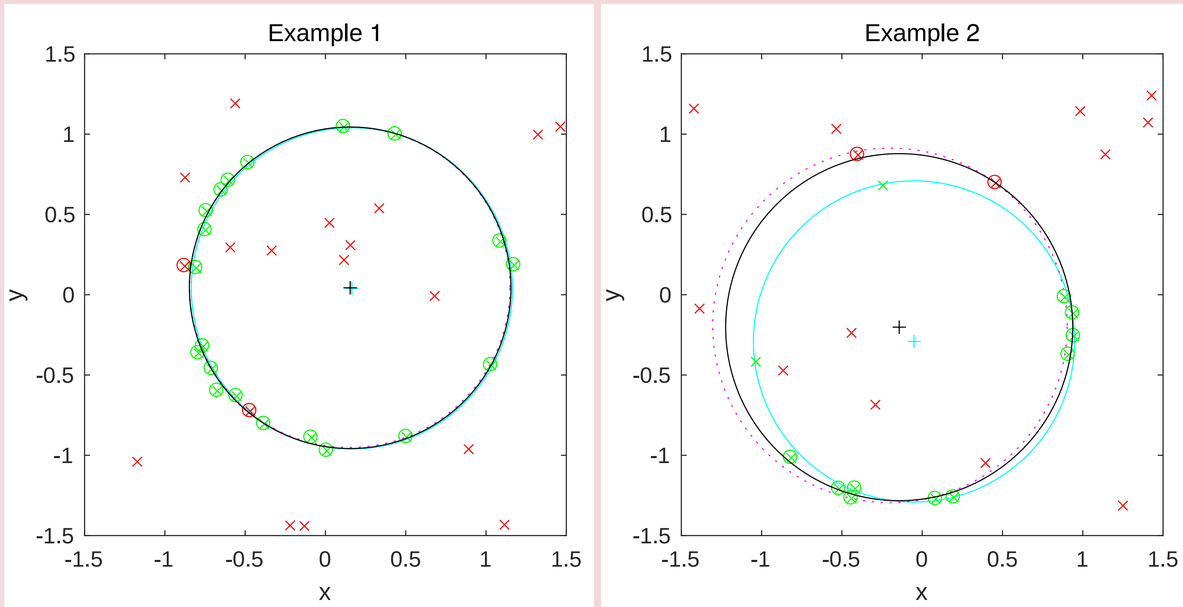
  where d_i is the (weighted) distance from the point $\boldsymbol{r} = (x_i, y_i, z_i)^{\mathrm{T}}$ to the plane
- The center and radius of the circle is computed as in the least-squares Riemann fit
- The final weights of the points are determined by an **M-estimator**

Three applications

- **Full circles with background**
- **Overlapping circles with background**
- **Tracks with mirror points**

Full circles with background

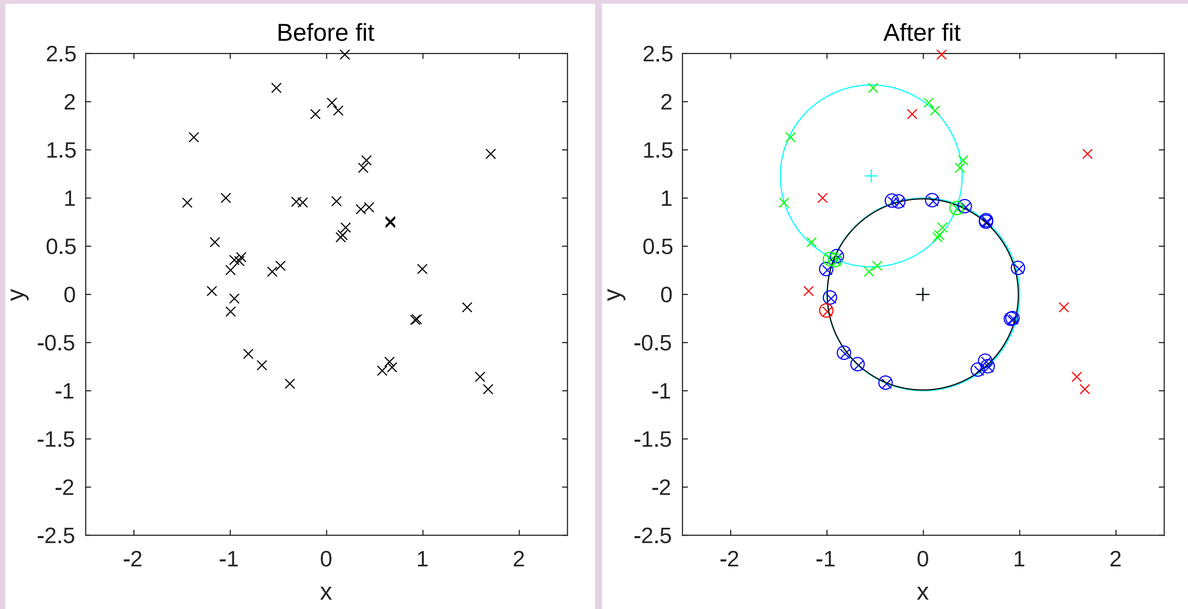
- Simulation of 5000 circles with superimposed background
- Number of signal points drawn from Poisson(20), uniform along the circle
- Number of background points drawn from Poisson(15), uniform in a window around the circle
- Random shift of all points with $\sigma_x = \sigma_y = 0.02$
- In 22.3% of the events at least as many background points as signal points
- Average **purity**: 94.6%
- Average **completeness**: 98.8%
- **Efficiency** of the circle finding: 98.0%
- Two examples: successful fit on the left, biased fit on the right



Cyan: true circle with true center, green: signal points, red: background points, circled: points tagged as inliers by the M-estimator, magenta: LMS circle, black: final circle with final center

Overlapping circles with background

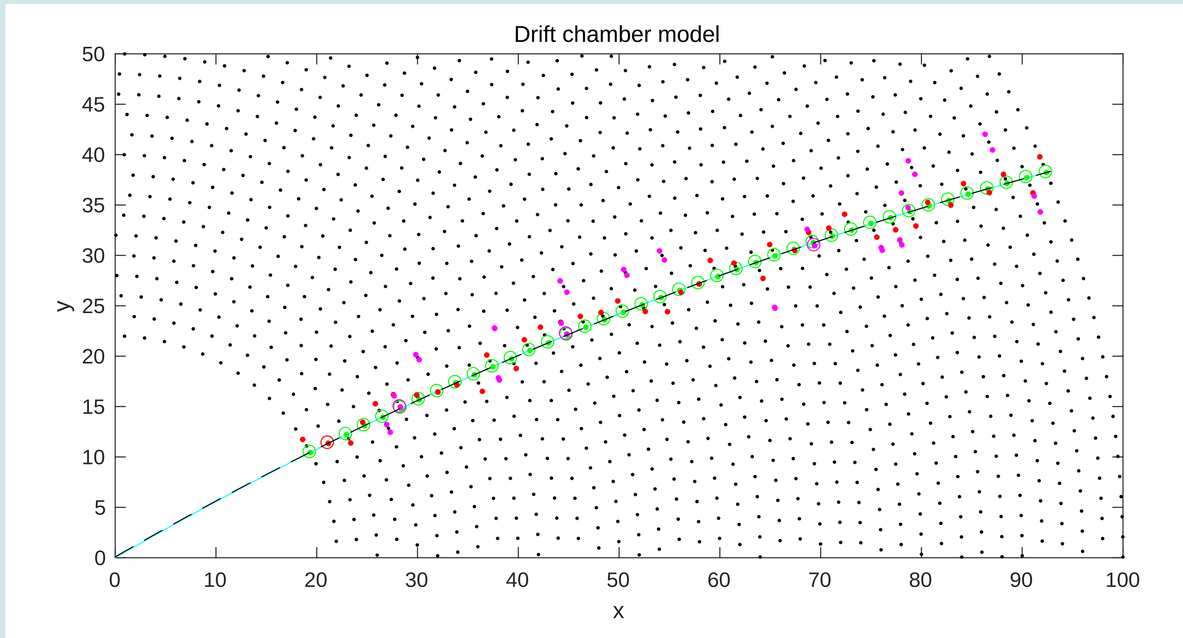
- Simulation of 5000 pairs of circles with superimposed background
- Number of signal points on the primary circle drawn from Poisson(20)
- Number of signal points on the secondary circle drawn from Poisson(15)
- Number of background points drawn from Poisson(10), uniform in the window
- The found circle is compared to both true circles
- Average **purity**: 94.2%
- Average **completeness**: 98.7%
- **Efficiency** of the circle finding: 94.2%
- Example: all points on the left, found circle on the right



Cyan: true circles with true centers, blue: signal points of the primary circle, green: signal points of the secondary circle, red: background points, circled: points tagged as inliers by the M-estimator, black: final circle with final center

Tracks with mirror points

- Simulation of 5000 tracks in a drift chamber model, similar to CDC of Belle II
- 40 layers of sense wires, cell size 2 cm, point resolution 0.150 mm
- Additional noise points in each layer close to the track with probability $p_{\mathrm{N}} = 0.0, 0.2, 0.4$
- Average **purity**: 0.994, 0.966, 0.942
- Average **completeness**: 0.994, 0.998, 0.998
- Example of a track at noise level $p_{\mathrm{N}} = 0.4$: 17 noise points and their mirror points. In layer 2 the signal point is missing and its mirror point is included. Three noise points are included in layers 6, 15 and 28.



Cyan: true track, green: true points, red: mirror points, magenta: noise points, circled: points included in the track, black: final track.

References

[1] A. Strandlie, J. Wroldsen, R. Frühwirth, B. Lillekjendlie, *Particle tracks fitted on the Riemann sphere*, Comp. Phys. Commun. 131 (2000) 95–108
[2] A. Strandlie, R. Frühwirth, *Exploration and extension of an improved Riemann track fitting algorithm*, Nucl. Instrum. Meth. A867 (2017) 72–77
[3] P.J. Rousseeuw, A.M. Leroy, *Robust Regression and Outlier Detection*, Wiley & Sons (1987)