Physics & Computing Challenges in Lattice QCD

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Original Invitation

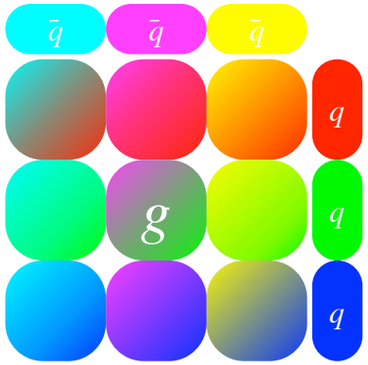
- Talk about “Machine Learning Challenges in Complex Multiscale Systems”
- Title of a TUM Institute for Advanced Study "Focal Period"
  - (I helped organized)
- Alas, you will hear about lattice QCD instead.

Outline

- Nonperturbative QCD & some lattice QCD results
- Basics of lattice gauge theory
- Software
- QCD random number generators
- Solvers for quarks
- Bayesian curve fitting
- Upcoming physics challenges

☛ some ML speculation

Emergent Phenomena in QCD & Some Lattice QCD Highlights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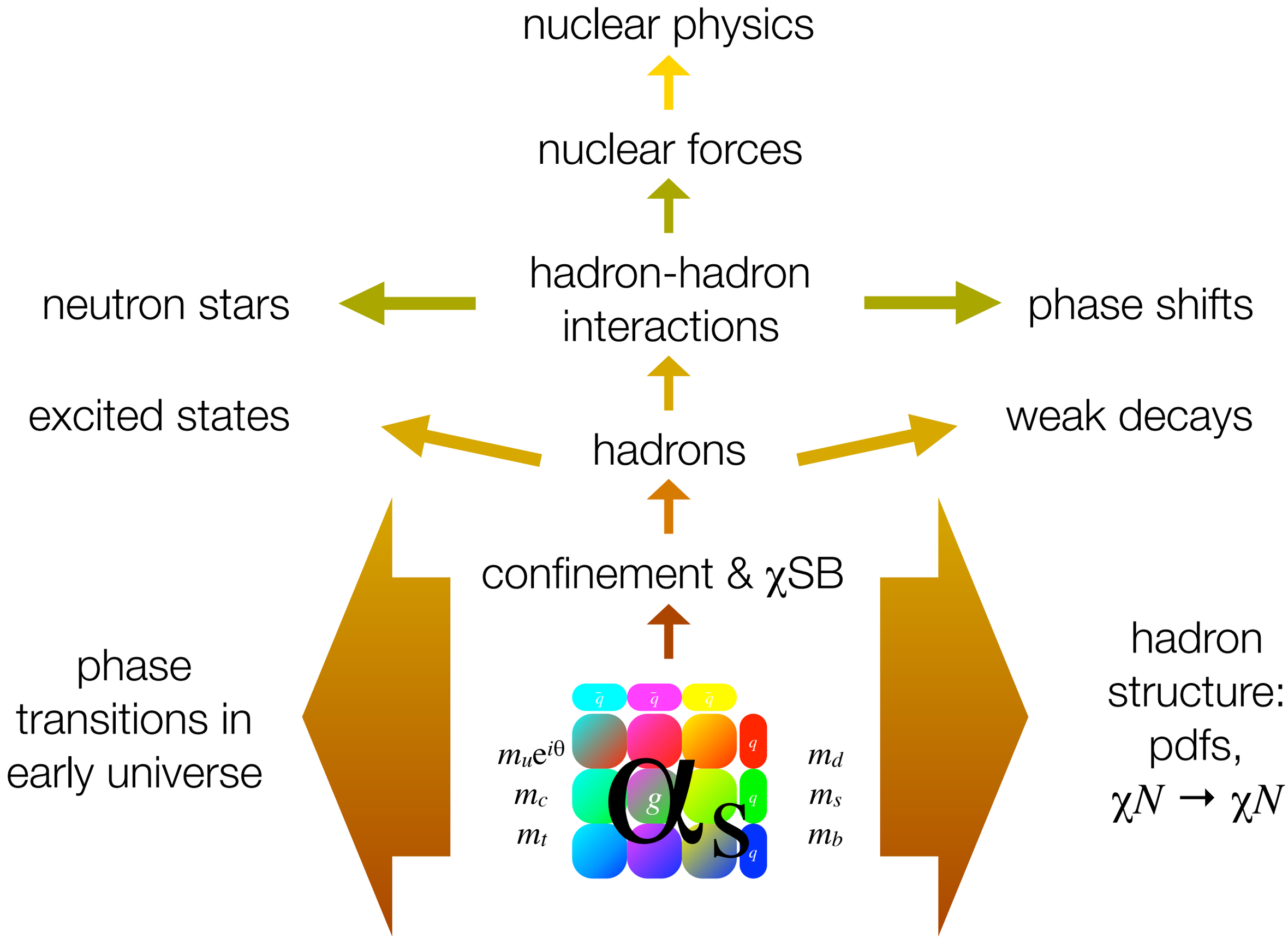
nuclear physics
nuclear forces
hadron-hadron interactions
neutron stars
phase shifts
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weak decays
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confinement & χSB
phase transitions in early universe
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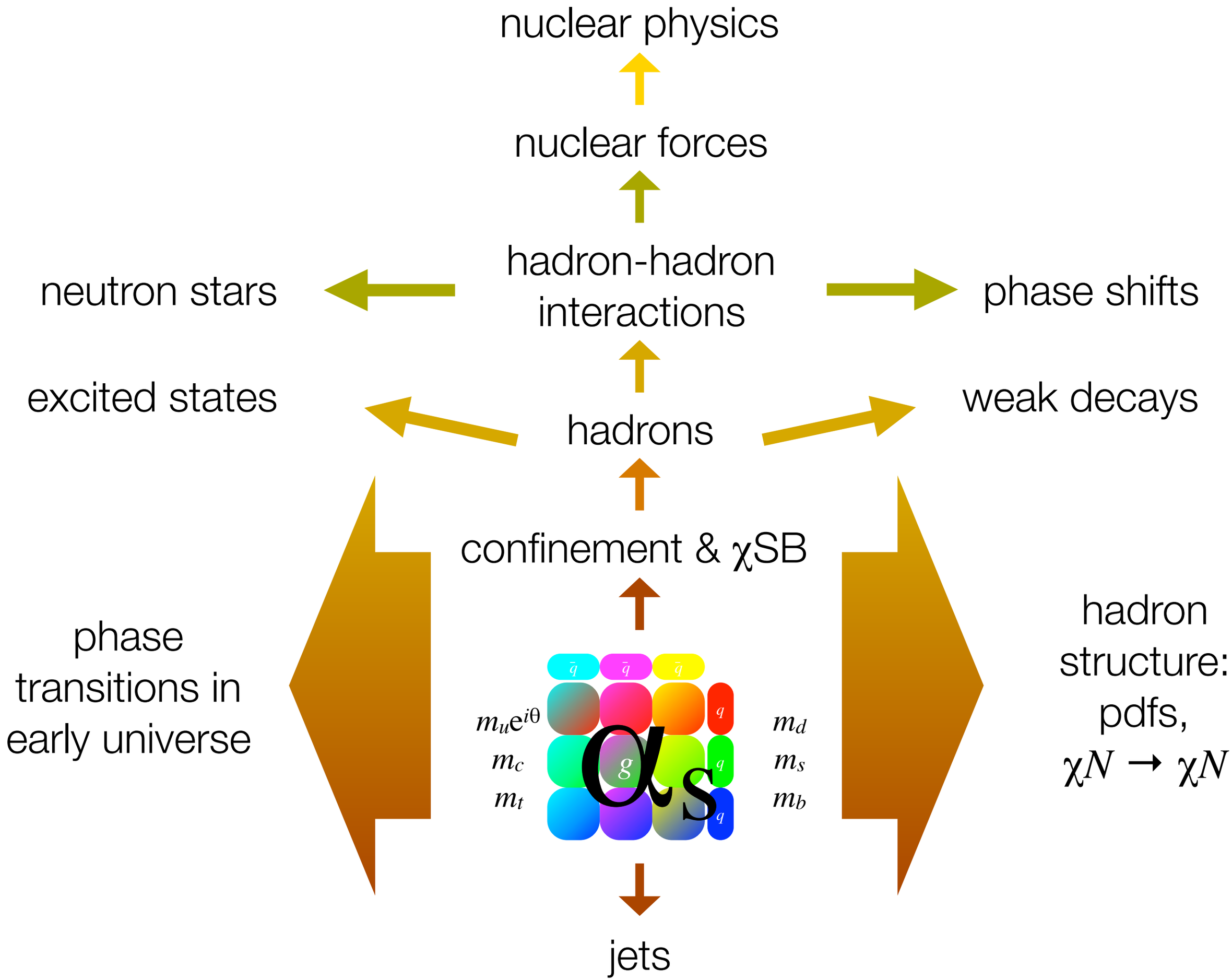
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jets

Topology

- Non-Abelian gauge fields have nontrivial topology (in $d = 4$).
- Suppose, as $x \to \infty$, $A_\mu(x) \to g^{-1}(x)\partial_\mu g(x)$. Then

$$Q = \frac{1}{24\pi^2}\int_{S^3_\infty} d^3\sigma_\mu \varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[(g^{-1}\partial_\nu g)(g^{-1}\partial_\rho g)(g^{-1}\partial_\sigma g)] \in \mathbb{Z}$$

$$= \frac{1}{32\pi^2}\int_{\mathbb{R}^4} d^4x\,\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}], \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu + [A_\mu, A_\nu]$$

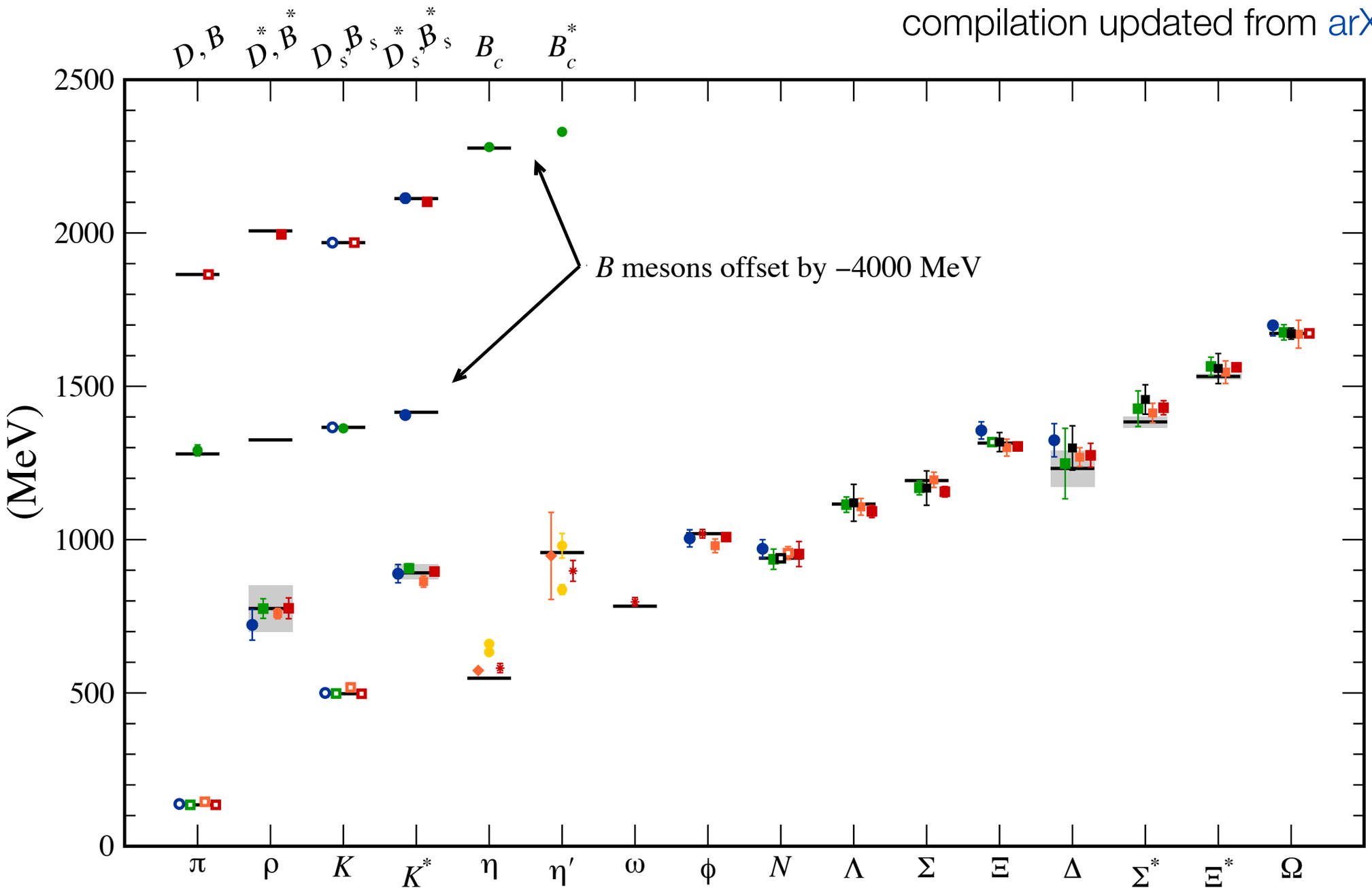
- Leads to the possibility of strong *CP* violation, if $\theta \neq 0$ and $m_u \neq 0$*;
  - lattice QCD: $m_u = 2.16(11)\ \mathrm{MeV}$, so $m_u = 0$ is disfavored;
  - neutron EDM : $\theta \equiv \theta_{\mathrm{QCD}} + \arg M_{\mathrm{quark}} < 10^{-11}$, which is mind boggling.

* Yes, Mike, I know.

QCD Hadron Spectrum

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler&Woloshyn

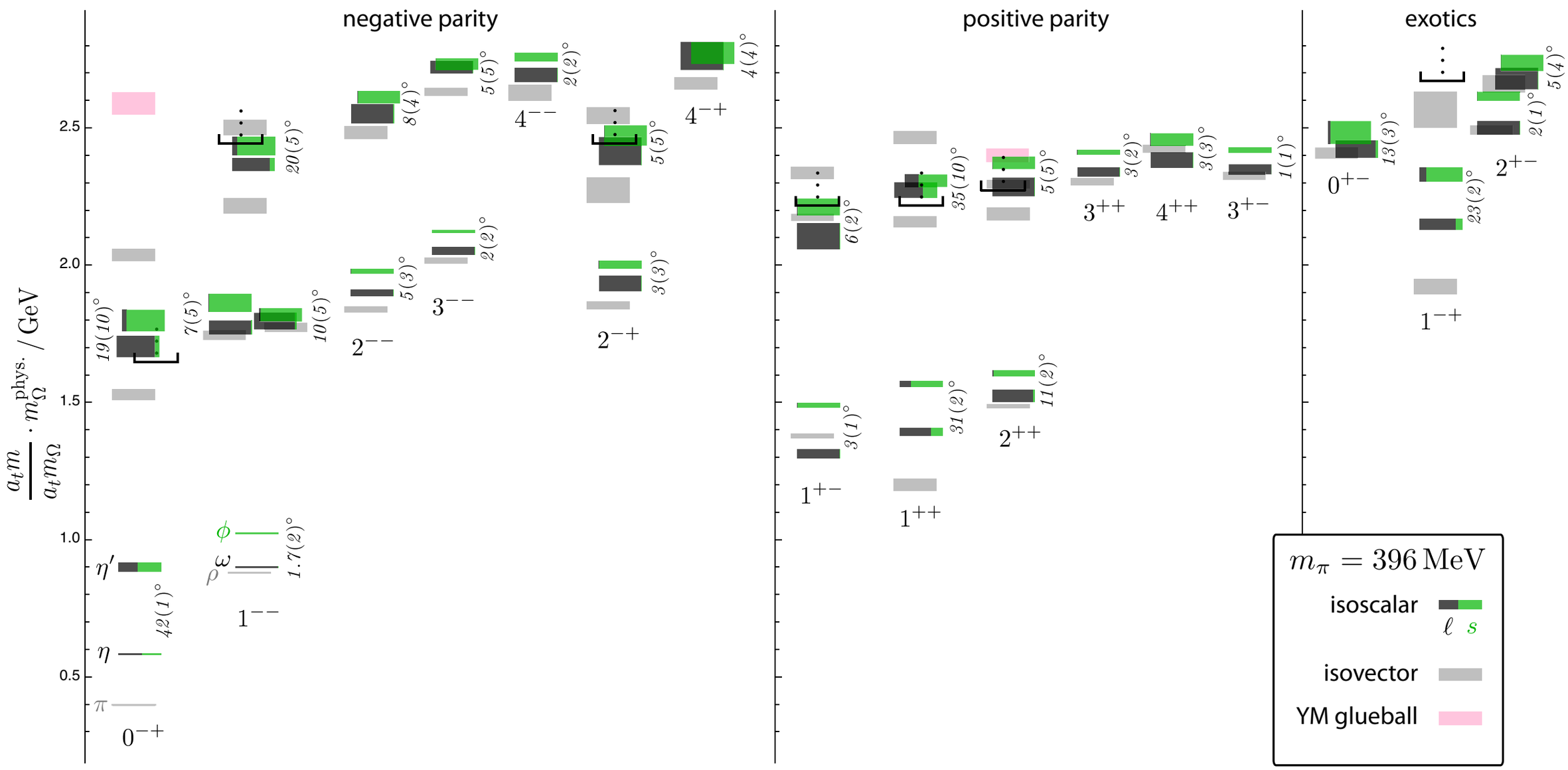
compilation updated from arXiv:1203.1204



numerous quarkonium omitted

Excited States

e.g., Hadron Spectrum Collaboration, *PRD* **83** (2011) 111502

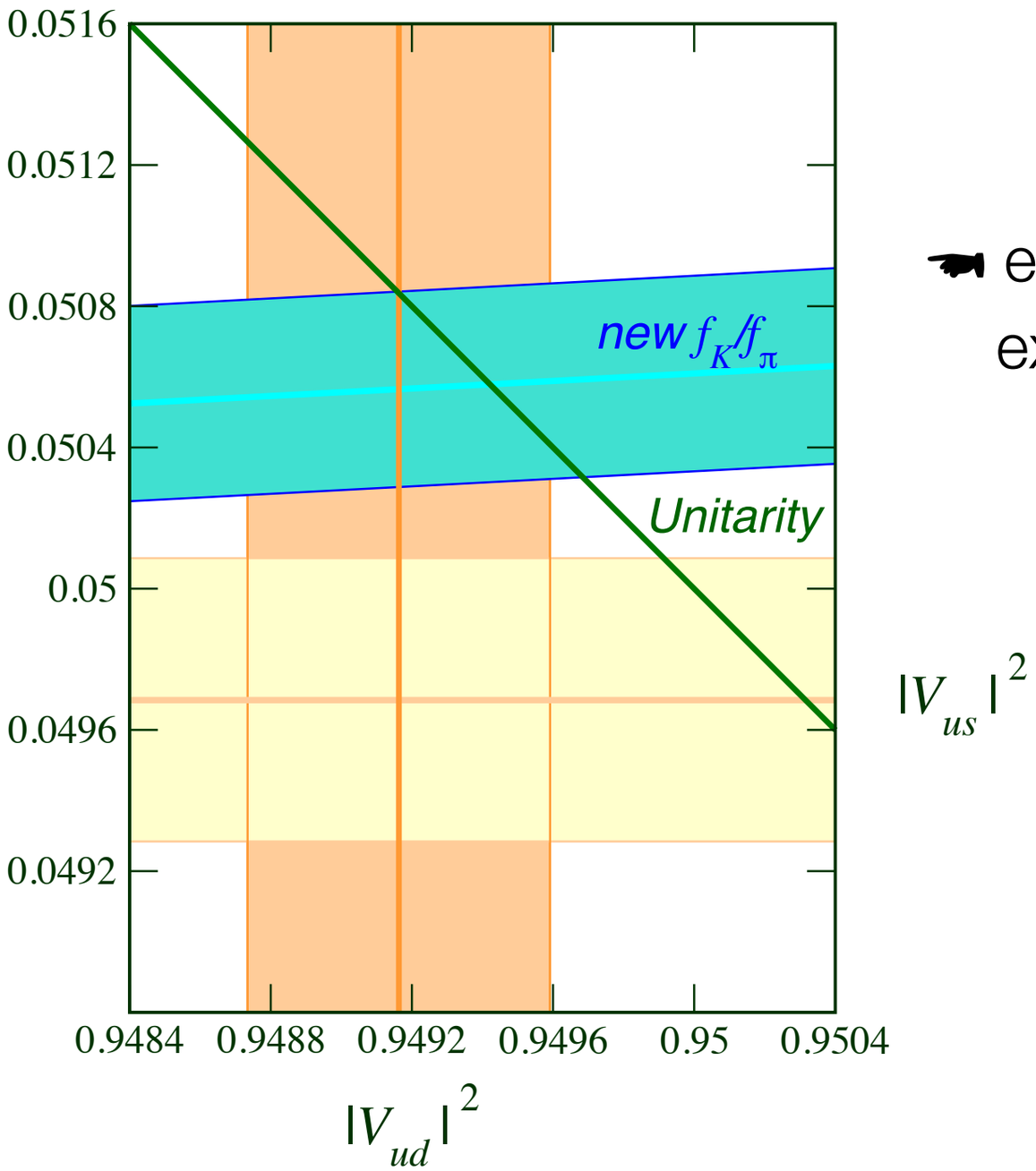


- Future applications to glueball spectra, hybrids, excited baryons, & mixing.

CKM Unitarity:

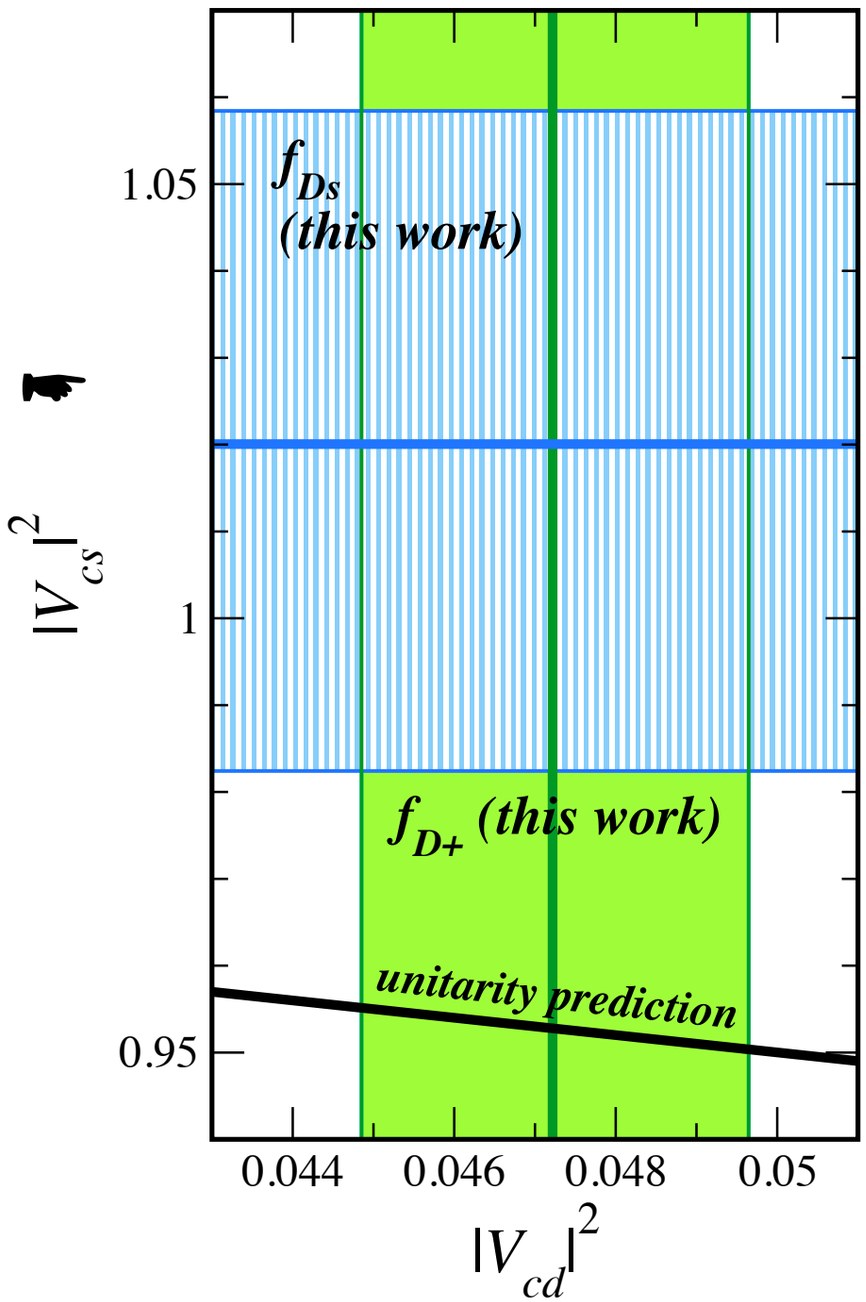
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 \stackrel{?}{=} 1$$



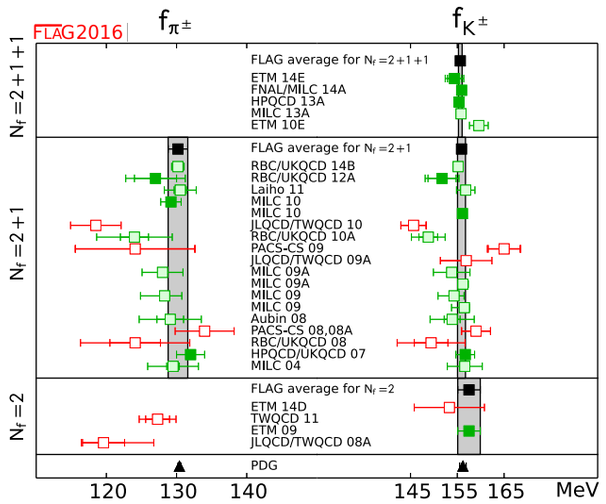
☛ expt. ahead

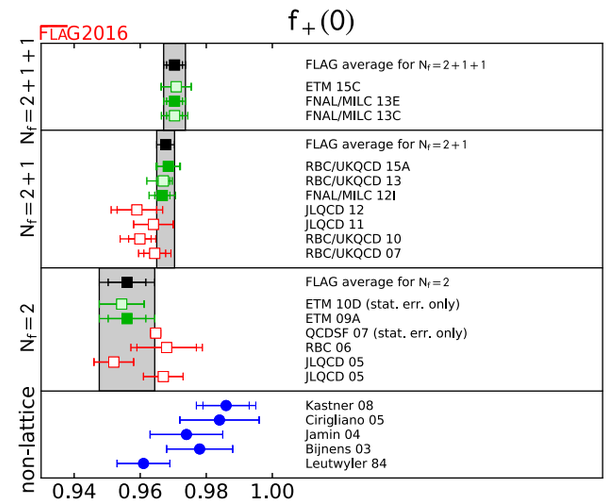
expt. behind ☛

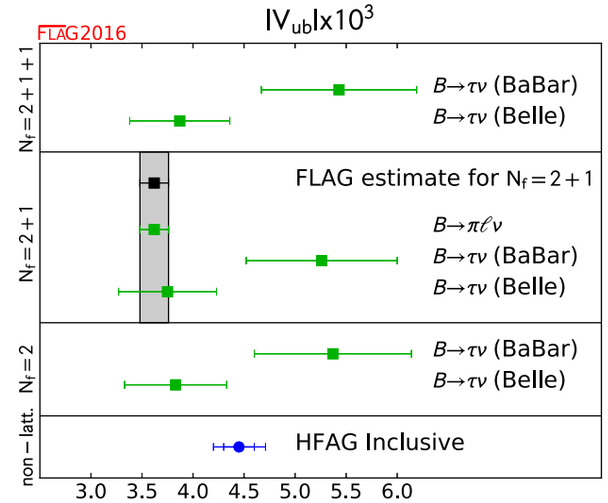


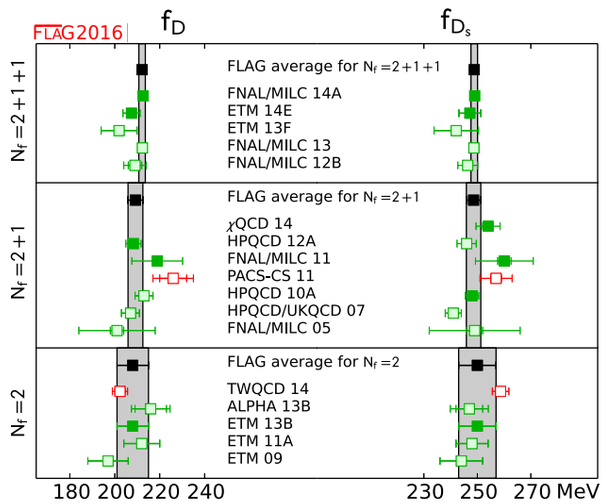
FLAG2016

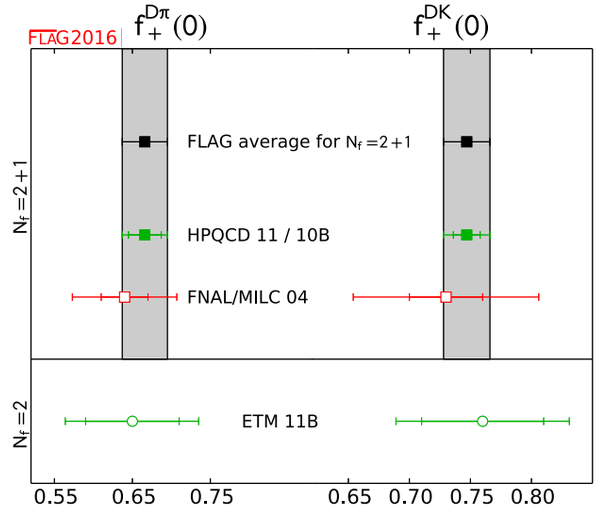
arXiv:1607.00299

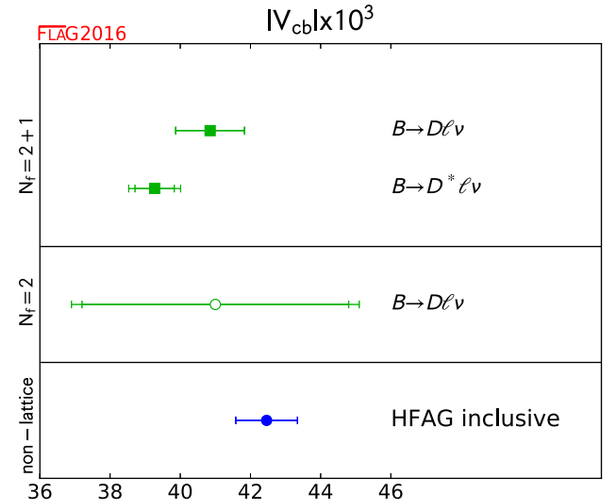


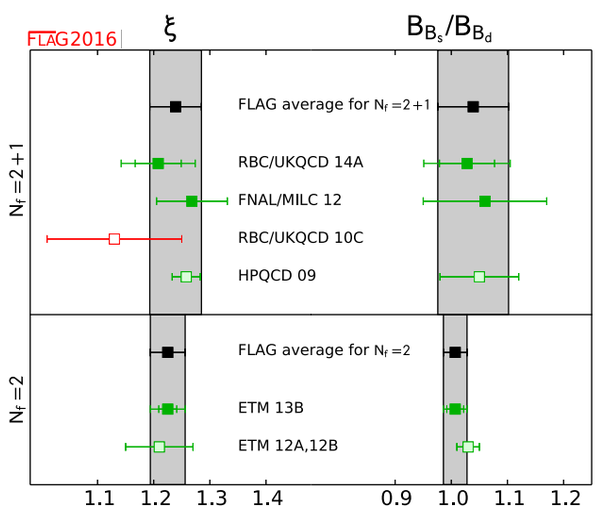


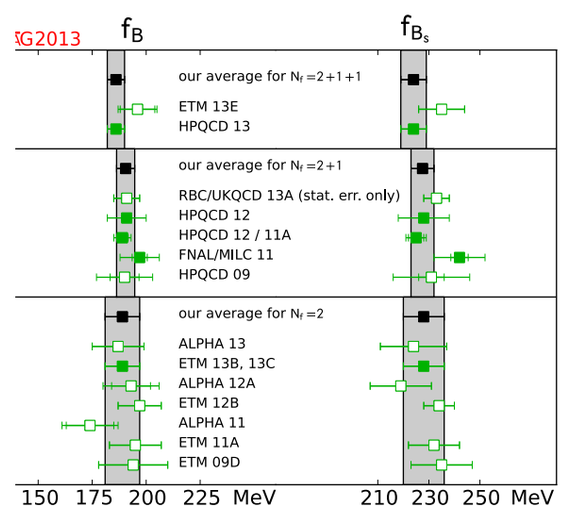


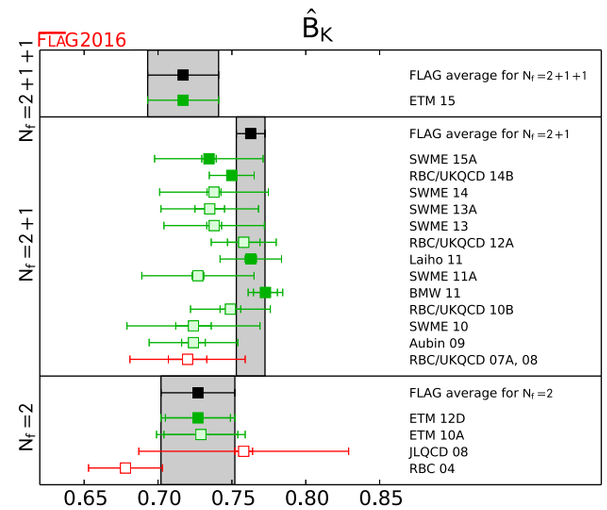












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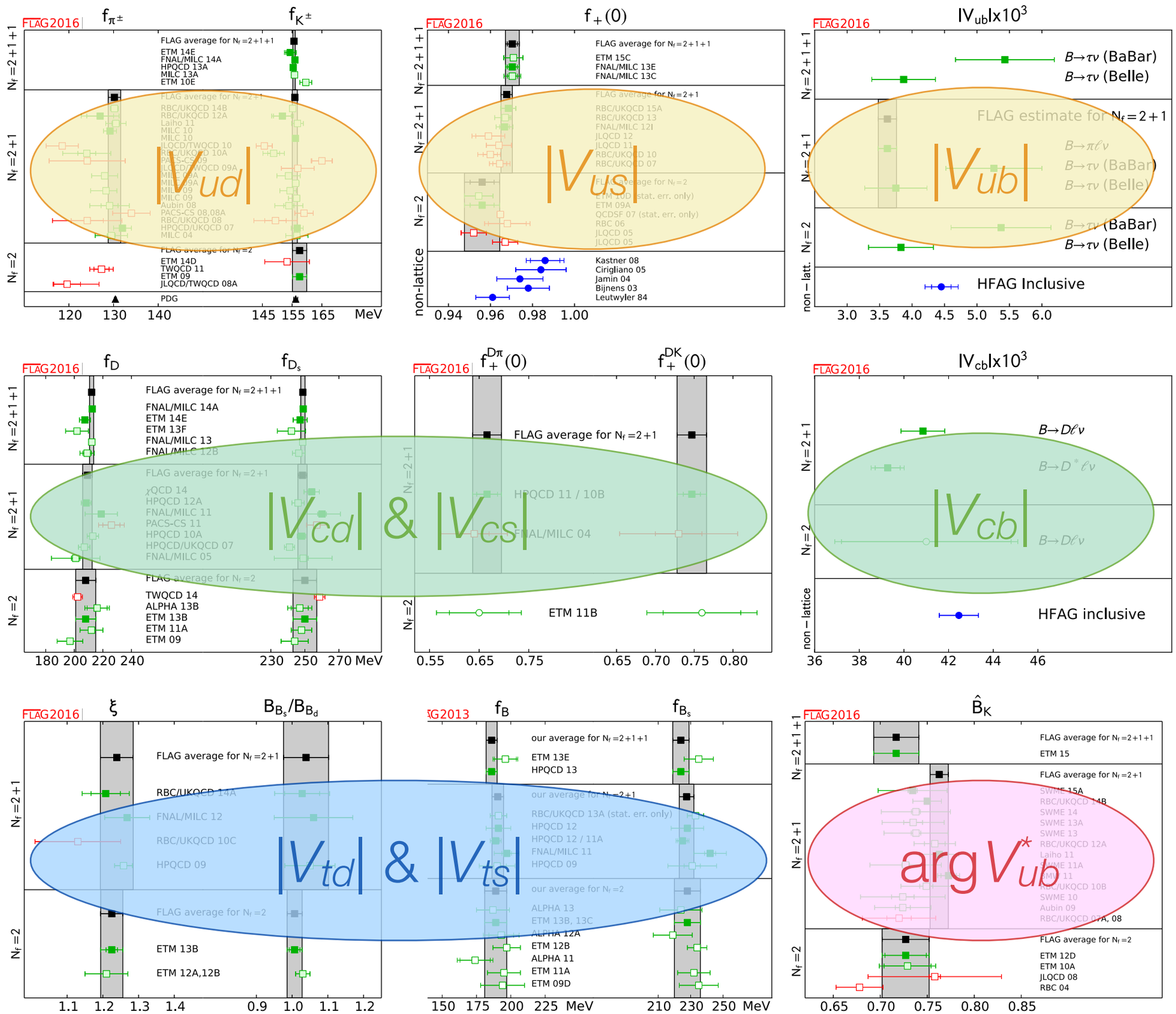

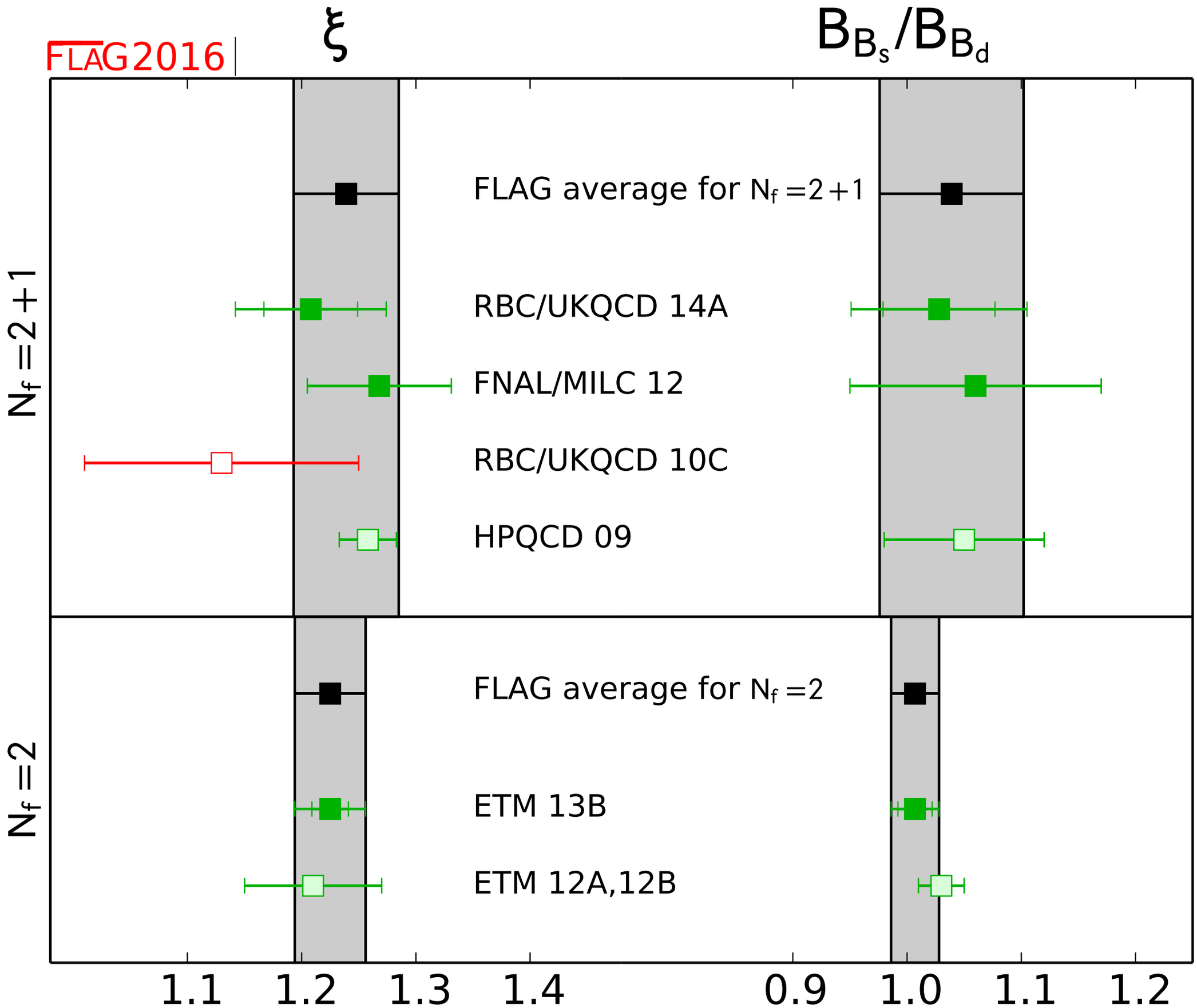
FLAG2016
ξ
B_{B_s}/B_{B_d}
$N_f = 2+1$
FLAG average for $N_f = 2+1$
RBC/UKQCD 14A
FNAL/MILC 12
RBC/UKQCD 10C
HPQCD 09
$N_f = 2$
FLAG average for $N_f = 2$
ETM 13B
ETM 12A,12B
1.1
1.2
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1.4
0.9
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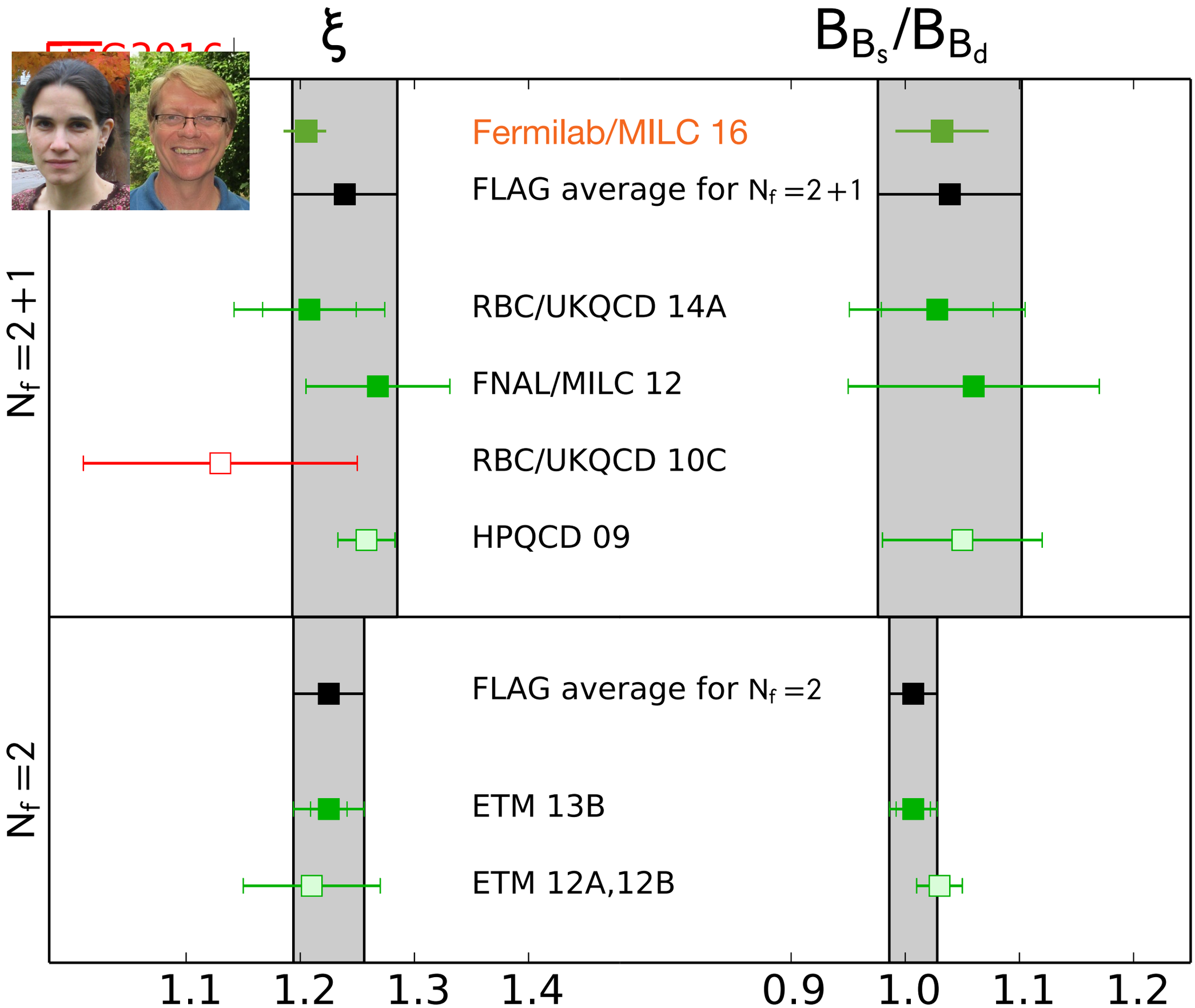
1.1
1.2


ξ
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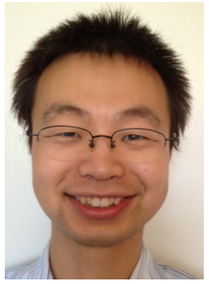
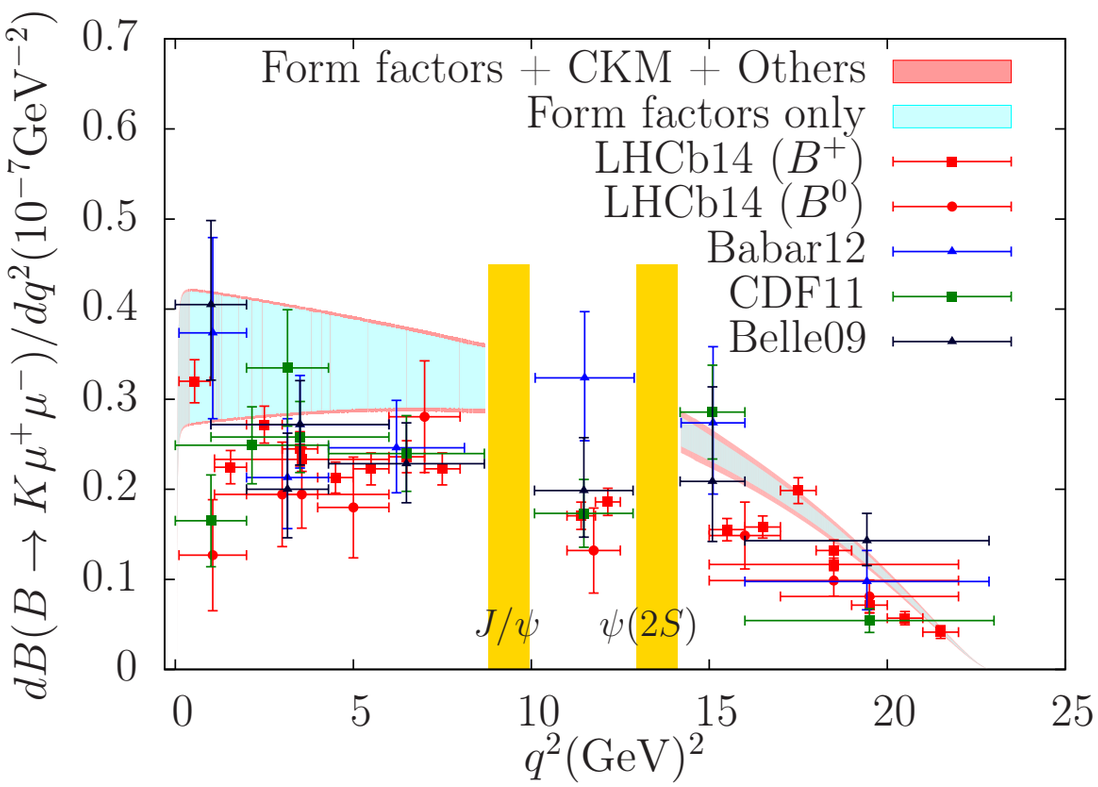
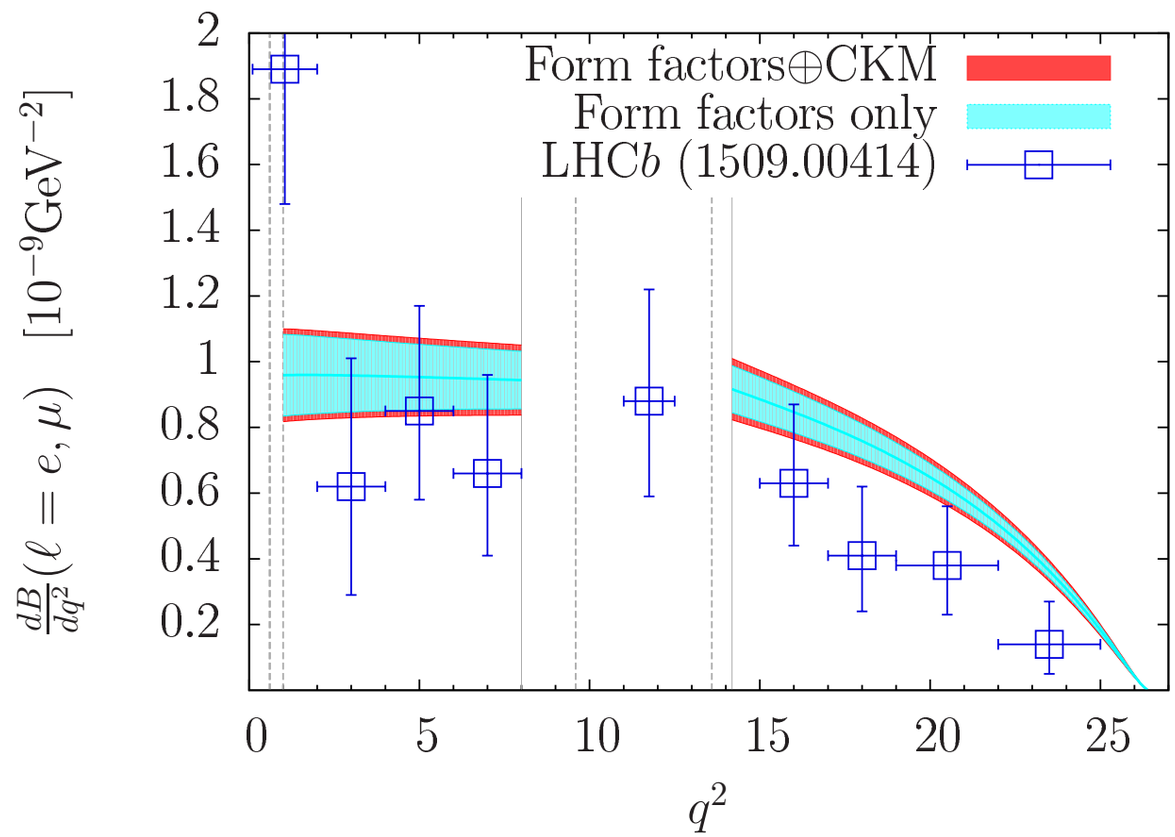
Kinematic Distributions



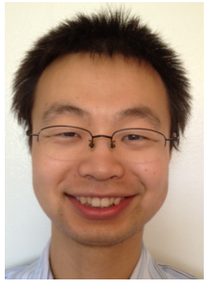
- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot's theory **preceded** measurement:





- arXiv:1510.02349 also contains predictions for the ratio, the flat terms, etc.

Kinematic Distributions



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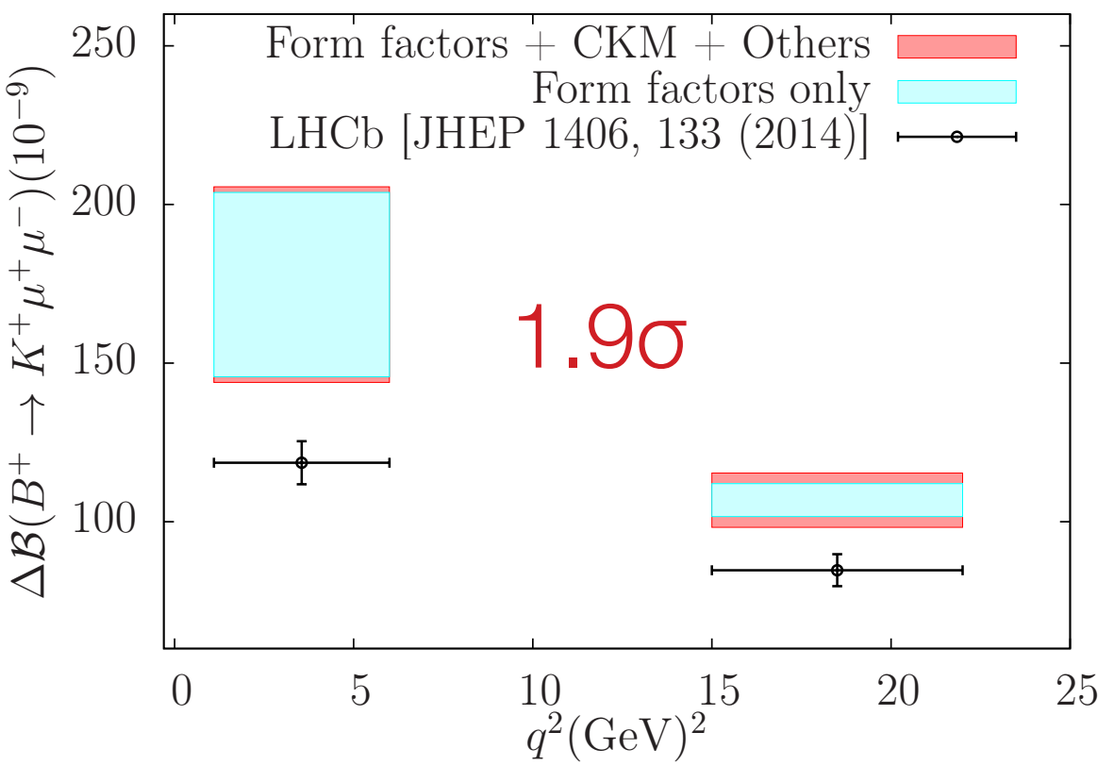

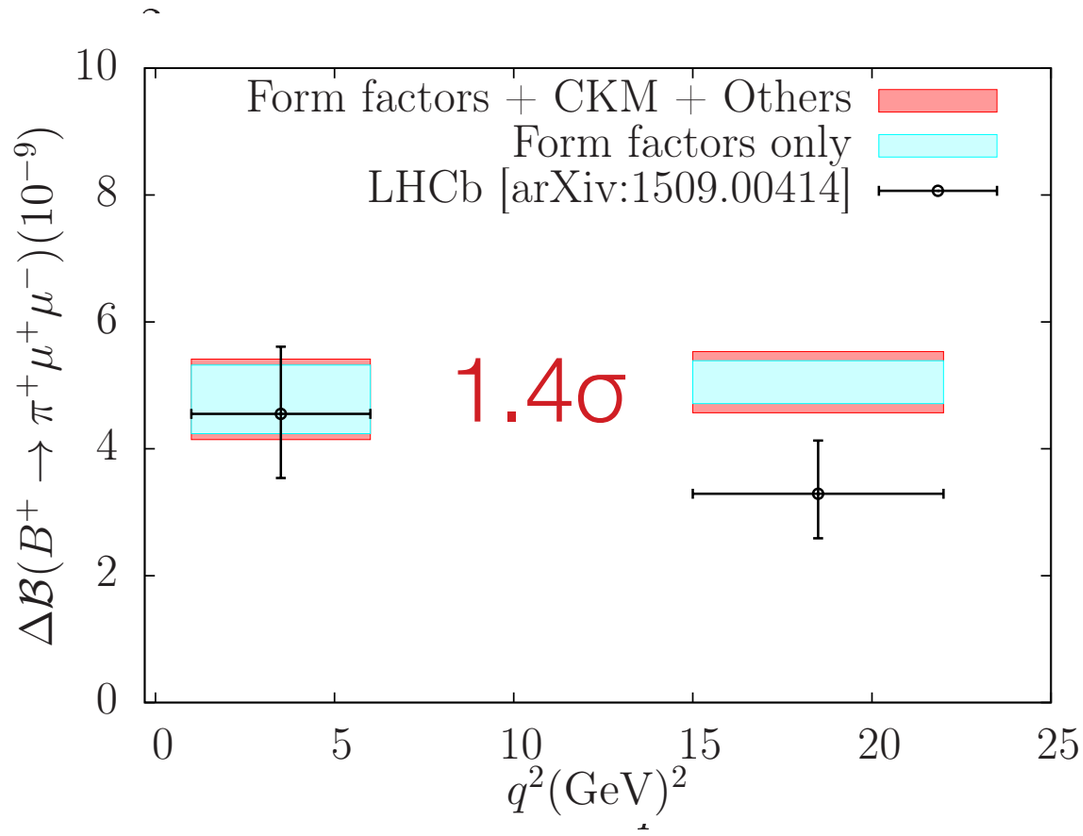

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Synopsis

- Nonperturbative QCD is rich: return to some current challenges at end.
- Low-lying hadron spectrum agrees with experiment at 2% level & better.
- Excited states are more challenging, but progress recently is rapid.
- Quark flavor physics, with sub- to few-percent uncertainties:
  - includes several validated predictions—$D_{(s)}$ decay constants & form factors, B_c mass ☚ over 12 years ago!
  - and B_c^* mass, yet to be seen in experiments.
- Lattice QCD is now a reliable tool for precision physics.

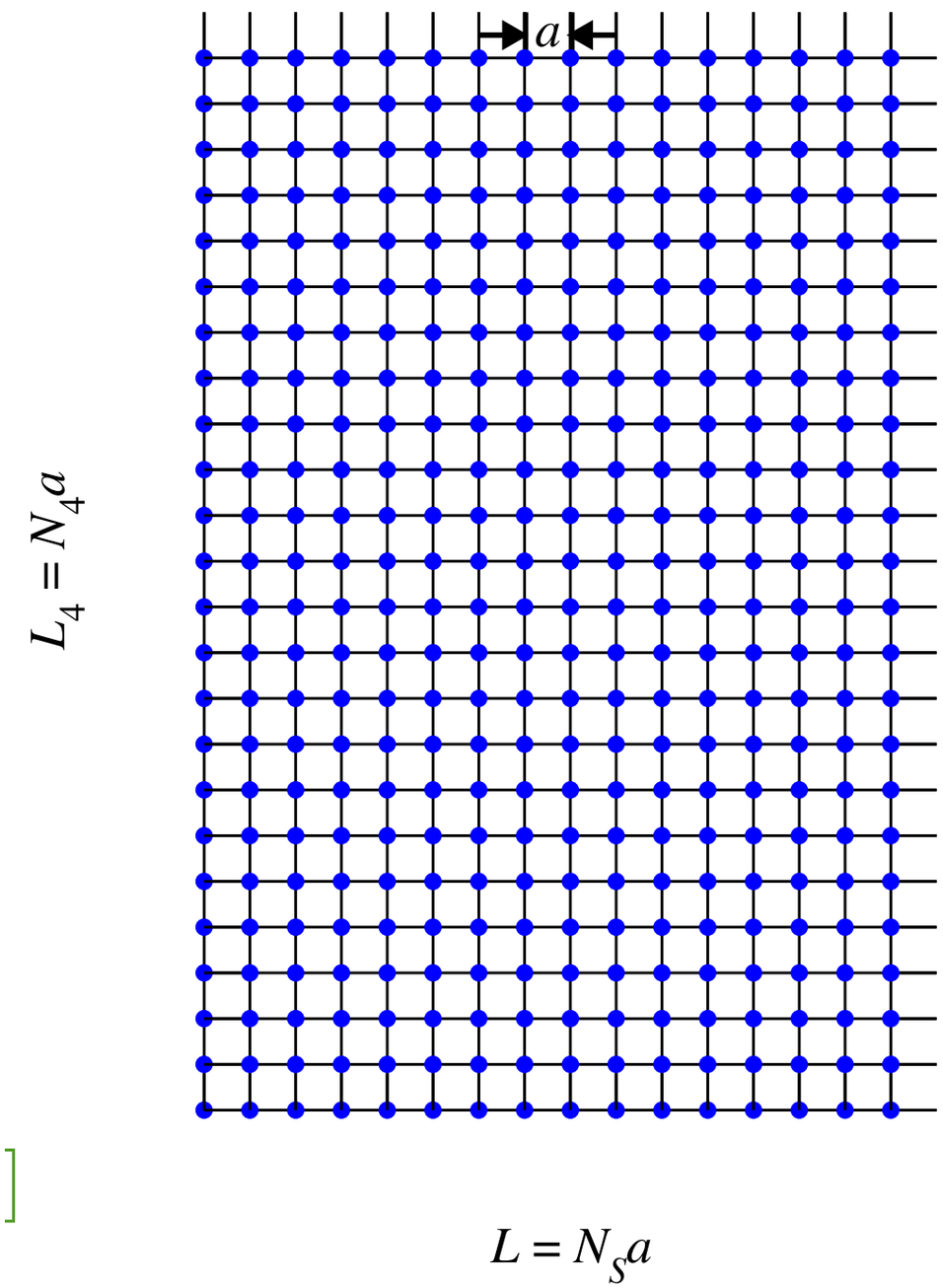
Lattice Gauge Theory

Lattice Gauge Theory

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^9, so compute integrals numerically.

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathscr{D}U \mathscr{D}\psi \mathscr{D}\bar{\psi} \exp(-S)[\bullet]$$

$$= \frac{1}{Z} \int \mathscr{D}U \, \mathrm{Det}(\not{D} + m) \exp(-S_{\mathrm{gauge}})[\bullet']$$



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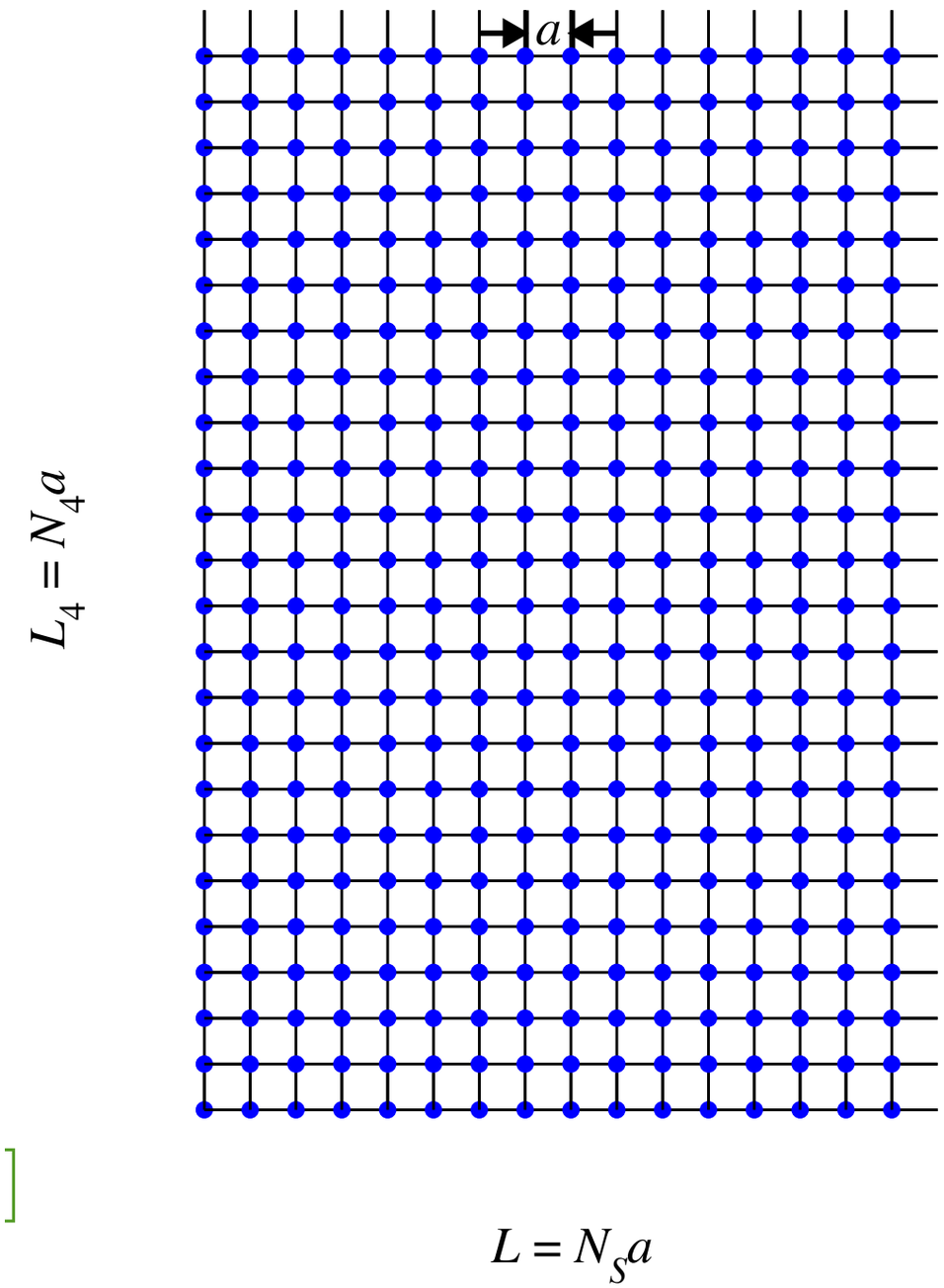

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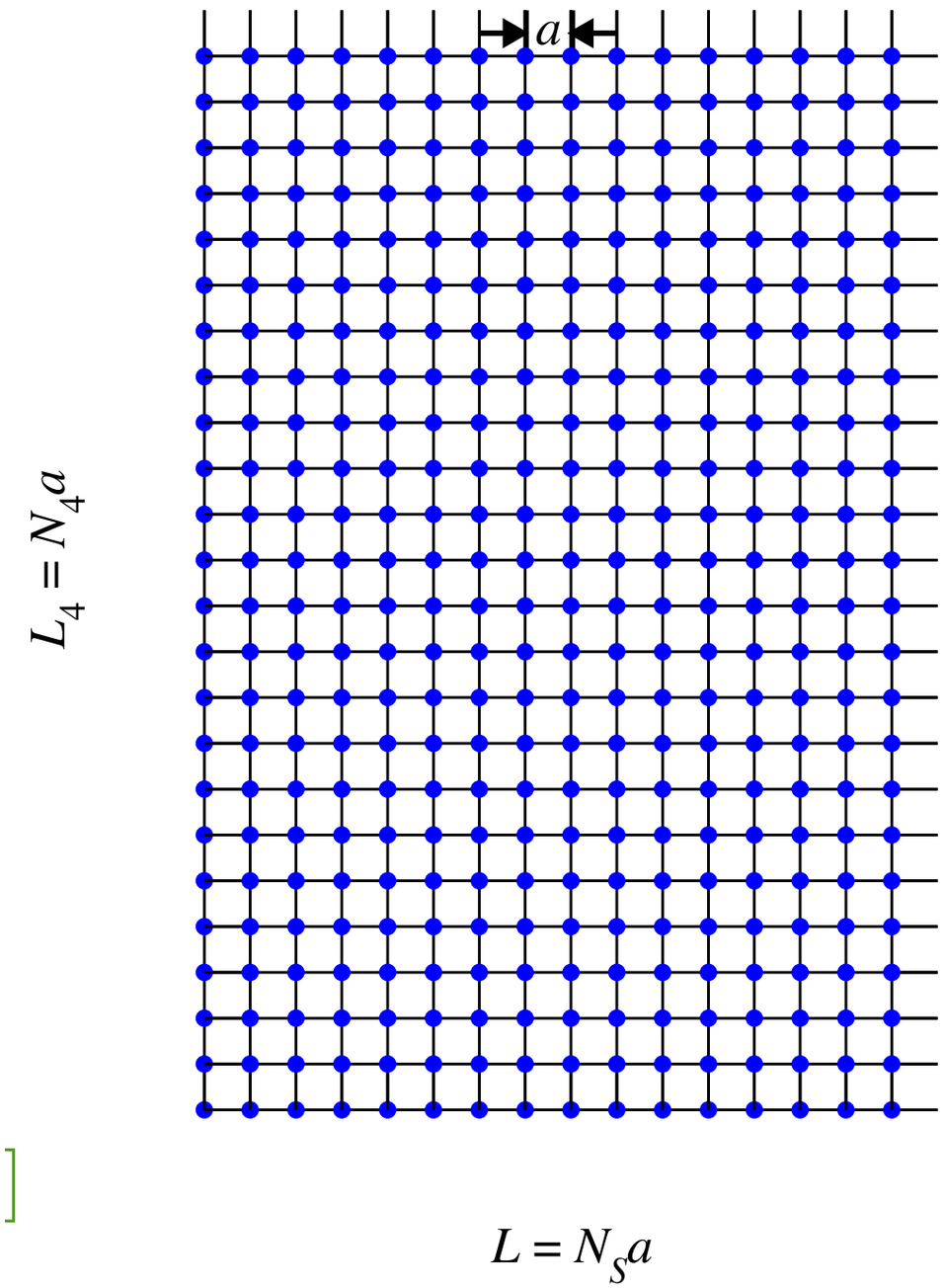


Algorithmic Issues

e.g., ASK, hep-lat/0205021

- lattice $N_S^3 \times N_4$, spacing a
- size $L_S = N_S a$, $L_4 = N_4 a$
- memory $\propto N_S^3 N_4 = L_S^3 L_4 / a^4$
- dimension of spacetime = 4
- $\tau_g \propto a^{-(4+z)}$, $z = 1$ or 2
- critical slowing down
- $\tau_q \propto (m_q a)^{-p}$, $p = 1$ or 2
- especially dire w/ sea quarks
- Imaginary time, $x_4 = it$:
  - static quantities 😀
- thermodynamics: $\Theta = 1/N_4 a$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathscr{D}U \mathscr{D}\psi \mathscr{D}\bar{\psi} \exp(-S)[\bullet]$$

$$= \mathrm{Tr}\{\bullet\, e^{-\hat{H}/\Theta}\} / \mathrm{Tr}\{e^{-\hat{H}/\Theta}\}$$

Computing Hadronic Matrix Elements

- Find masses and "legs" from two-point correlation functions:

$$\langle B(x_4)B^\dagger(0)\rangle = \sum_n |\langle B_n|\hat{B}^\dagger|0\rangle|^2\, e^{-M_{B_n}x_4}$$

$$\langle A_4(x_4)B^\dagger(0)\rangle = \sum_n \langle 0|\hat{A}_4|B_n\rangle\langle B_n|\hat{B}^\dagger|0\rangle\, e^{-M_{B_n}x_4}$$

  where $x_4 = it$ is "Euclidean time". Then get matrix elements from three-point functions:

$$\langle B^\dagger(x_4)\mathscr{O}_i(0)B^\dagger(-x_4')\rangle = \sum_{m,n} \langle 0|\hat{B}^\dagger|B_m\rangle\langle \bar{B}_m|\hat{\mathscr{O}}_i|B_n\rangle\langle B_n|\hat{B}^\dagger|0\rangle\, e^{-M_{B_n}(x_4+x_4')}$$

  in which desired matrix element lies in the middle (of leading term).

- Use supercomputer to generate LHS; hire superhumans to analyze RHS.

The Steps

- Use random number generator to create lattice gauge fields distributed with the weight e^{-S}.
- Solve $(\not{D} + m)_{xz} G_{zy} = b_x$ for quark propagators in these gauge fields.
- Fit correlation functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.

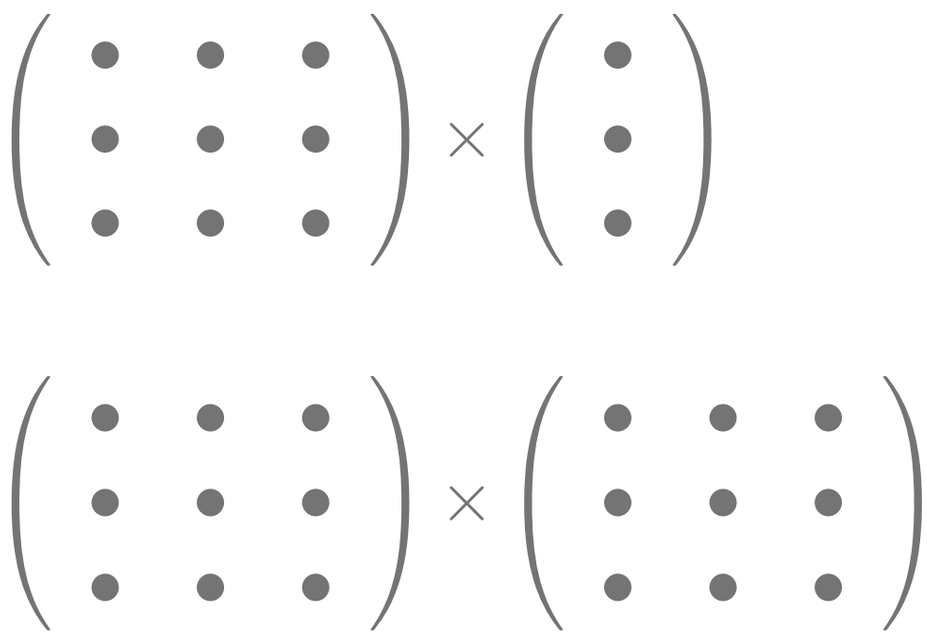
Software (USQCD Example)

- The problem naturally splits into two parts:
  - generating $\{U^{(c)}\}$ requires capability machines, e.g., BlueGene/Q or Cray XK7;
  - working out the $\bullet'$ and the curve fitting requires capacity machines.

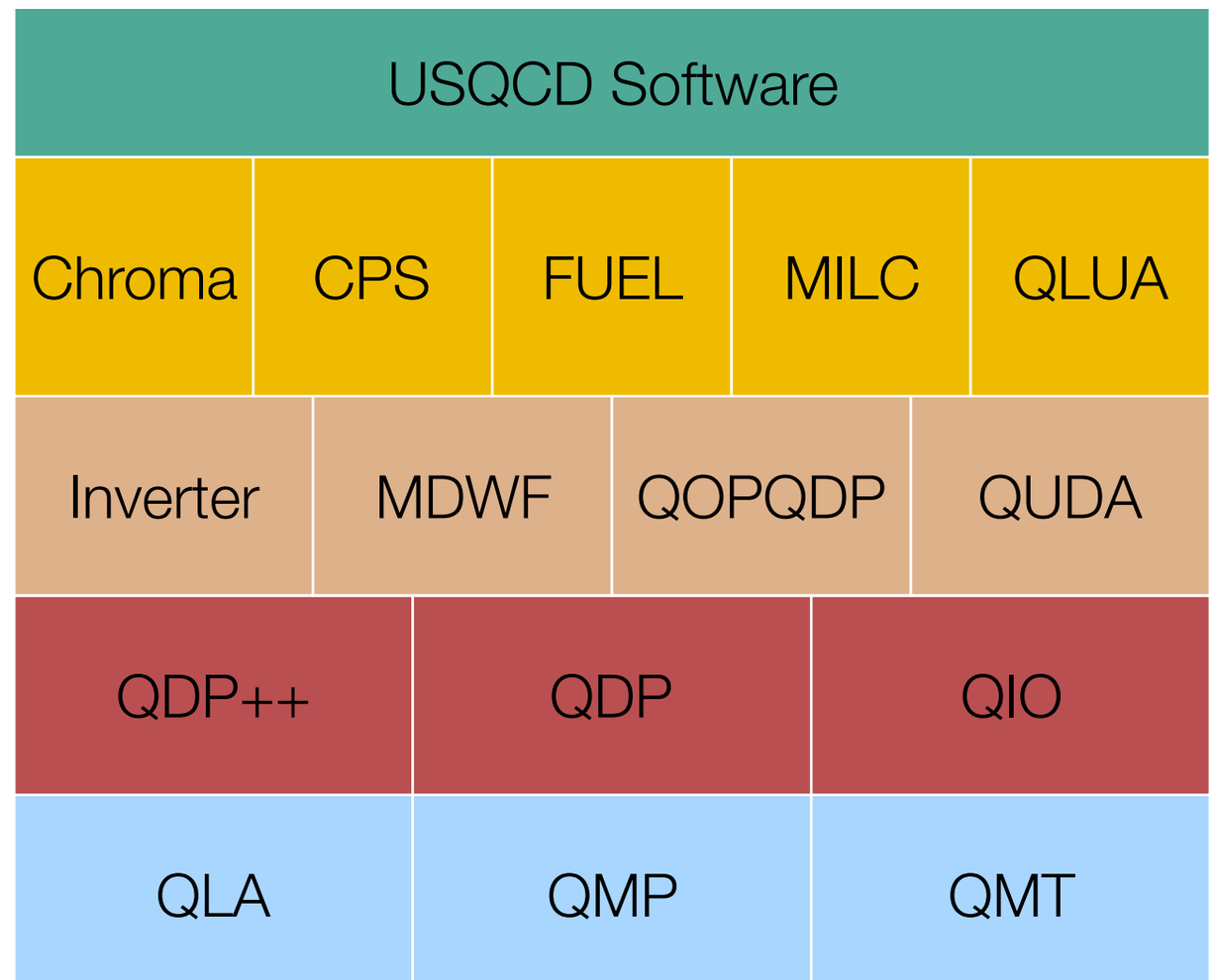
Heterogenous Platforms

- Lattice QCD runs on everything: Intel/AMD in your laptop (i.e., clusters), GPUs, Xeon Φ, Knight's Landing, BlueGene/Q, Cray XK7, etc.

- Lowest-level computations:



optimize these, and a few other things, on any chip.



Remarks

- Lattice QCD was the(?) first community to use GPUs:
  - by now, a few lattice-QCD algorithm experts work for NVIDIA.
- Software development (in US) supported by SciDAC (essential) & industry.
- Other notable efforts (presumably a partial list):
  - European Twisted-Mass Collab: tmLQCD and related packages;
  - Martin Lüscher *et alia*: openQCD (∃ branch for C-periodic boxes)
  - Guido Cossu *et alia* (KEK): Irolro++;
  - Peter Boyle: BAGEL code generator; multithreading Grid library.

Generating Lattice Gauge Fields

Functional Integral

- Objective is to compute many integrals of the form

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathscr{D}U \, \mathrm{Det}(\not{D} + m) \exp(-S_{\mathrm{gauge}}) [\bullet']$$

  with billions and billions of integration variables.

- Action is “extensive”, so the integrand varies over a huge range.

- Only feasible approach is Monte Carlo integration with importance sampling:

$$\langle \bullet \rangle = \frac{1}{C} \sum_{0}^{C-1} \bullet' \left[\{U\}^{(c)} \right]$$

  with each field $\{U\}^{(c)}$ generated with probability $\mathrm{Det}(\not{D} + m) \exp(-S_{\mathrm{gauge}})$.

Field Theory is Local

- … so local MCMC (Metropolis, heat-bath, ...) algorithms work with S_{gauge}:
  - order variables in some nice way, $U_n^{(c)} \to U_n^{(c+1)}$, $\forall\ n = 0, \ldots, N-1$.
- But $\mathrm{Det}(\not{D} + m) = \exp[-\mathrm{Tr}\ \ln(\not{D} + m)]$ leads to nonlocal $S_{\mathrm{F}} = \mathrm{Tr}\ \ln(\not{D} + m)$.
- Led (mid 1980s) to search for global algorithms, $\{U\}^{(c)} \to \{U\}^{(c+1)}$:
  - molecular dynamics: use Hamilton's equations—$z = 1$, but ergodic?;
  - Langevin equation: use random noise ⊕ diffusion—ergodic but $z = 2$;
  - hybrid molecular dynamics: noise ⊕ Hamilton—$z = 1$ (?), ergodic.

Hybrid Molecular Dynamics (HMD)

- These algorithms, or variants, are used in machine learning, for example to examine a posterior probability distribution.

- Consider a potential QFT action, energy, cost function, … $V(\phi) = \ln \mathbb{P}(\phi)$.

- Stat mech of real molecules actually needs $H(\phi, \pi) = \frac{1}{2}\pi \cdot \pi + V(\phi)$:

$$\dot{\pi}_i = -\frac{\partial H}{\partial \phi_i} \qquad \Rightarrow \qquad \pi_i^{(t+1)} = \pi_i^{(t)} - \delta \frac{\partial V}{\partial \phi_i}^{(t)}$$

$$\dot{\phi}_i = +\frac{\partial H}{\partial \pi_i} \qquad \Rightarrow \qquad \phi_i^{(t+1)} = \phi_i^{(t)} + \delta \pi_i^{(t)}$$

- For our purpose, a danger arises for nearly periodic modes.

- Introduce Gaussian random noise ξ_i into momenta every so often:

$$\left.\begin{aligned} \pi_i^{(\lambda,0)} &= \xi_i^{(\lambda)} \\ \pi_i^{(\lambda,1/2)} &= \pi_i^{(\lambda,0)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,0)} \end{aligned}\right\} \quad \text{refresh \& half step}$$

$$\left.\begin{aligned} \phi_i^{(\lambda,n+1)} &= \phi_i^{(\lambda,n)} + \delta \pi_i^{(\lambda,n+1/2)} \\ \pi_i^{(\lambda,n+1/2)} &= \pi_i^{(\lambda,n-1/2)} - \delta \frac{\partial V}{\partial \phi_i}^{(\lambda,n)} \end{aligned}\right\} \quad N-1 \text{ steps}$$

$$\left.\begin{aligned} \phi_i^{(\lambda,N)} &= \phi_i^{(\lambda,N-1)} + \delta \pi_i^{(\lambda,N-1/2)} \\ \pi_i^{(\lambda,N)} &= \pi_i^{(\lambda,N-1/2)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,N)} \end{aligned}\right\} \quad \text{finish } \phi \text{ \& half step } \pi$$

- Introduce Gaussian random noise ξ_i into momenta every so often:

$$\pi_i^{(\lambda,0)} = \xi_i^{(\lambda)}$$

$$\pi_i^{(\lambda,1/2)} = \pi_i^{(\lambda,0)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,0)}$$

refresh & half step

$$\phi_i^{(\lambda,n+1)} = \phi_i^{(\lambda,n)} + \delta \pi_i^{(\lambda,n+1/2)}$$

$$\pi_i^{(\lambda,n+1/2)} = \pi_i^{(\lambda,n-1/2)} - \delta \frac{\partial V}{\partial \phi_i}^{(\lambda,n)}$$

anything reversible

$$\phi_i^{(\lambda,N)} = \phi_i^{(\lambda,N-1)} + \delta \pi_i^{(\lambda,N-1/2)}$$

$$\pi_i^{(\lambda,N)} = \pi_i^{(\lambda,N-1/2)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,N)}$$

finish ϕ & half step π

- Introduce Gaussian random noise ξ_i into momenta every so often:

$$\left.\begin{aligned} \pi_i^{(\lambda,0)} &= \xi_i^{(\lambda)}, \quad \boxed{\phi_i^{(\lambda,0)} = \phi_i^{(\lambda-1,N)}} \\ \pi_i^{(\lambda,1/2)} &= \pi_i^{(\lambda,0)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,0)} \end{aligned}\right\} \text{refresh \& half step}$$

$$\left.\begin{aligned} \phi_i^{(\lambda,n+1)} &= \phi_i^{(\lambda,n)} + \delta \pi_i^{(\lambda,n+1/2)} \\ \pi_i^{(\lambda,n+1/2)} &= \pi_i^{(\lambda,n-1/2)} - \delta \frac{\partial V}{\partial \phi_i}^{(\lambda,n)} \end{aligned}\right\} N-1 \text{ steps}$$

$$\left.\begin{aligned} \phi_i^{(\lambda,N)} &= \phi_i^{(\lambda,N-1)} + \delta \pi_i^{(\lambda,N-1/2)} \\ \pi_i^{(\lambda,N)} &= \pi_i^{(\lambda,N-1/2)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,N)} \end{aligned}\right\} \text{finish } \phi \text{ \& half step } \pi$$

trajectory

- Introduce Gaussian random noise ξ_i into momenta every so often:

$$\left.\begin{aligned} \pi_i^{(\lambda,0)} &= \xi_i^{(\lambda)}, \quad \boxed{\phi_i^{(\lambda,0)} = \phi_i^{(\lambda-1,N)}} \\ \pi_i^{(\lambda,1/2)} &= \pi_i^{(\lambda,0)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,0)} \end{aligned}\right\}$$

refresh & half step

Hybrid (aka Hamiltonian) Monte Carlo:
accept $\phi_i^{(\lambda,N)} \longrightarrow \phi_i^{(\lambda+1,0)}$ à la Metropolis w/ $e^{-\Delta H}$

$$\left.\begin{aligned} \phi_i^{(\lambda,N)} &= \phi_i^{(\lambda,N-1)} + \delta \pi_i^{(\lambda,N-1/2)} \\ \pi_i^{(\lambda,N)} &= \pi_i^{(\lambda,N-1/2)} - \tfrac{1}{2}\delta \frac{\partial V}{\partial \phi_i}^{(\lambda,N)} \end{aligned}\right\}$$

finish ϕ & half step π

Step-size Errors

$O(\delta^{n>2})$ methods:
Omelyan, Mryglod, Folk

- The discrete Hamiltonian evolution conserves energy only up to $O(\delta^2)$, but "shadow Hamiltonian" [*e.g.*, Kennedy, Silva, Clark (2012)] is conserved:

$$\tilde{H} = H - \frac{\delta^2}{24}\left(\{V,\{V,T\}\} - 2\{T,\{T,V\}\}\right) + O(\delta^4)$$

  where $\{\ ,\ \}$ is a Poisson bracket (last seen in undergrad mechanics).

- Consider spectrum $\{\omega \mid \omega_{\min} < \omega < \omega_{\max}\}$ ~ gradients of $V(\phi) = \ln \mathbb{P}(\phi)$.

- How fast is it? What is z?

  - Trajectory length $\tau = N\delta$: optimized at $\tau_{\text{opt}} \sim \pi/2\omega_{\max}$, in which case slow (not so steep) modes move only $\tau\omega_{\min} \sim \omega_{\min}/\omega_{\max}$:

  - need many trajectories to get anywhere and, effectively, $z = 2$ not 1 😖

Stability

- Short trajectory length is needed because if $\tau > 2\pi/\omega_{\max}$, then there are some modes that end up close to where they started (~cycle)
  - assumes V has dense spectrum below $\omega_{\max}$.
- Way out is known but usually forgotten: refresh with probability $\omega_{\min}\delta$.
- Then some modes traverse a cycle sometimes, but none cycles always.
- Average trajectory has $\langle\tau\rangle \sim 1/\omega_{\min}$, so all modes evolve rapidly; $z = 1$ 😃
- *cf*. Duane (1985), Mackenzie (1989), (review) ASK (1992).

Sea Quarks

- Remember QCD?
- And the nonlocal action of quarks on the gauge field $S_{\mathrm{F}} = \mathrm{Tr}\,\ln(\not{D} + m)$.
- In the global-update algorithms, we need (∂ in Lie groups, no worries):

$$\frac{\partial S_{\mathrm{F}}}{\partial U_\mu(x)} = \mathrm{Tr}[(\not{D} + m)^{-1}] = \left\langle \xi^\dagger (\not{D} + m)^{-1} \xi \right\rangle_\xi$$

  so we only need to solve $(\not{D} + m)G[\xi] = \xi$ for $G[\xi]$ [Batrouni et al. (1985)].
- This solve must be carried out at each time step along the way:
  - thus, the most demanding part of lattice gauge theory.

Quark Propagators

On the Turing Completeness of
Microsoft PowerPoint

By Tom Wildenhain

Solvers

- Also need to solve $(\not{D} + m)G[b] = b$ for valence quarks with "source" b, out of which hadron correlation functions are constructed.

- All lattice formulations of $\not{D}$ yield a sparse matrix ⇒ Krylov solvers.

- Solve slows down as $\omega_{\rm min}/\omega_{\rm max} \sim ma \to 0$.

- The lowest modes are very nonperturbative: (near) zero modes of $\not{D}$, associated with gauge-field topology.

- Solution called "deflation": project out the first few-to-several modes:

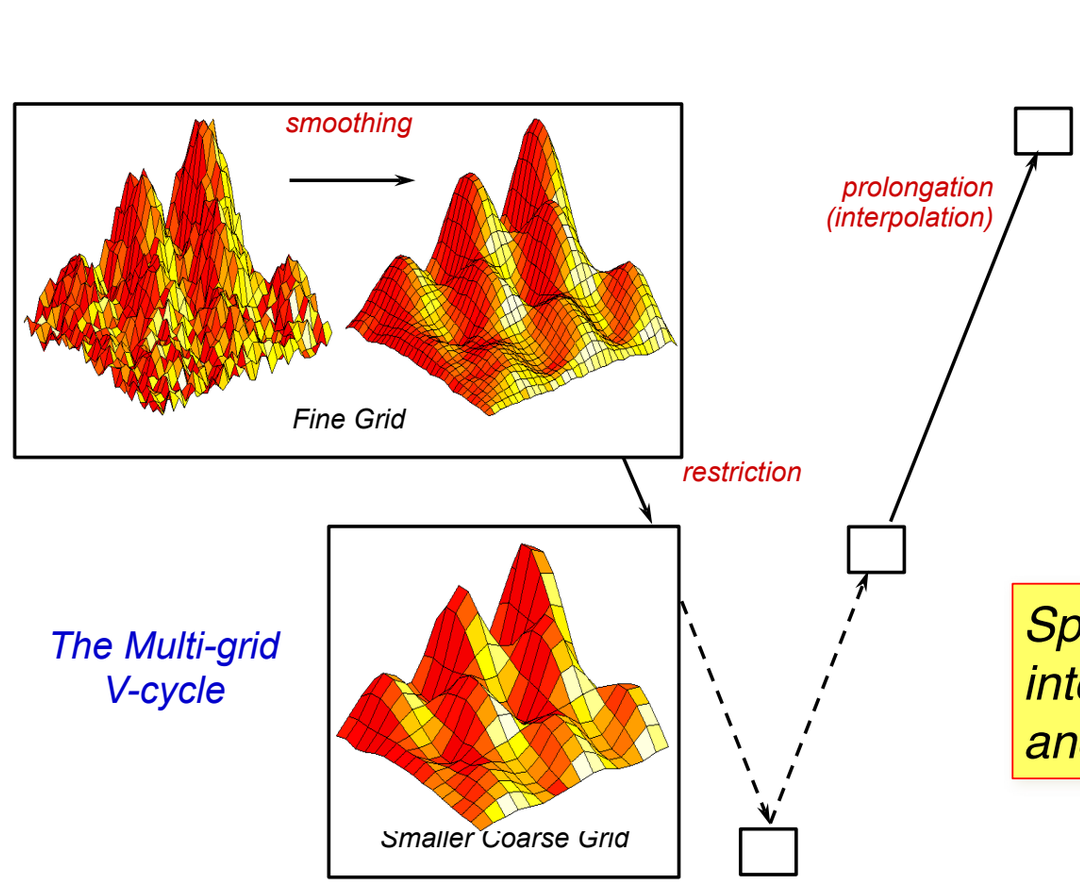
$$(\not{D} + m)^{-1} = \left[\not{D} + m - \sum_{|\lambda|<\lambda_{\rm del}} \lambda |\psi_\lambda\rangle\langle\bar{\psi}_\lambda|\right]^{-1} + \sum_{|\lambda|<\lambda_{\rm del}} \lambda^{-1} |\psi_\lambda\rangle\langle\bar{\psi}_\lambda|$$

Example: Multigrid

- Asaam
- Brannick et al. (2008)
- Babich et alia (2010)
- Frommer et al. (2013)
- Everything has to be gauge covariant.
- Slide from Brower at TUM Focal Period; graphic from Kahl (?).

Adaptive Smooth Aggregations Algebraic MultiGrid

Slow convergence of Dirac solver is due small eigenvalues for vectors in near null subspace: S .



$$D : S \simeq 0$$

Spilt the vector space into near null space S *and the complement* $S_\perp$

Rampant Speculation

- HMC, in practice, has several adjustable choices:
  - trajectory length, details of integrator, etc.
- Quark solver also has adjustable choices:
  - various preconditioners (in any method);
  - how many modes to deflate (any methods);
  - smoothing & how to cycle in multigrid.
- Every Markov chain goes on for millions of iterations (lasting years):
  Can we learn how to accelerate the simulation while it runs?

Curve Fitting

Fitting Correlation Functions

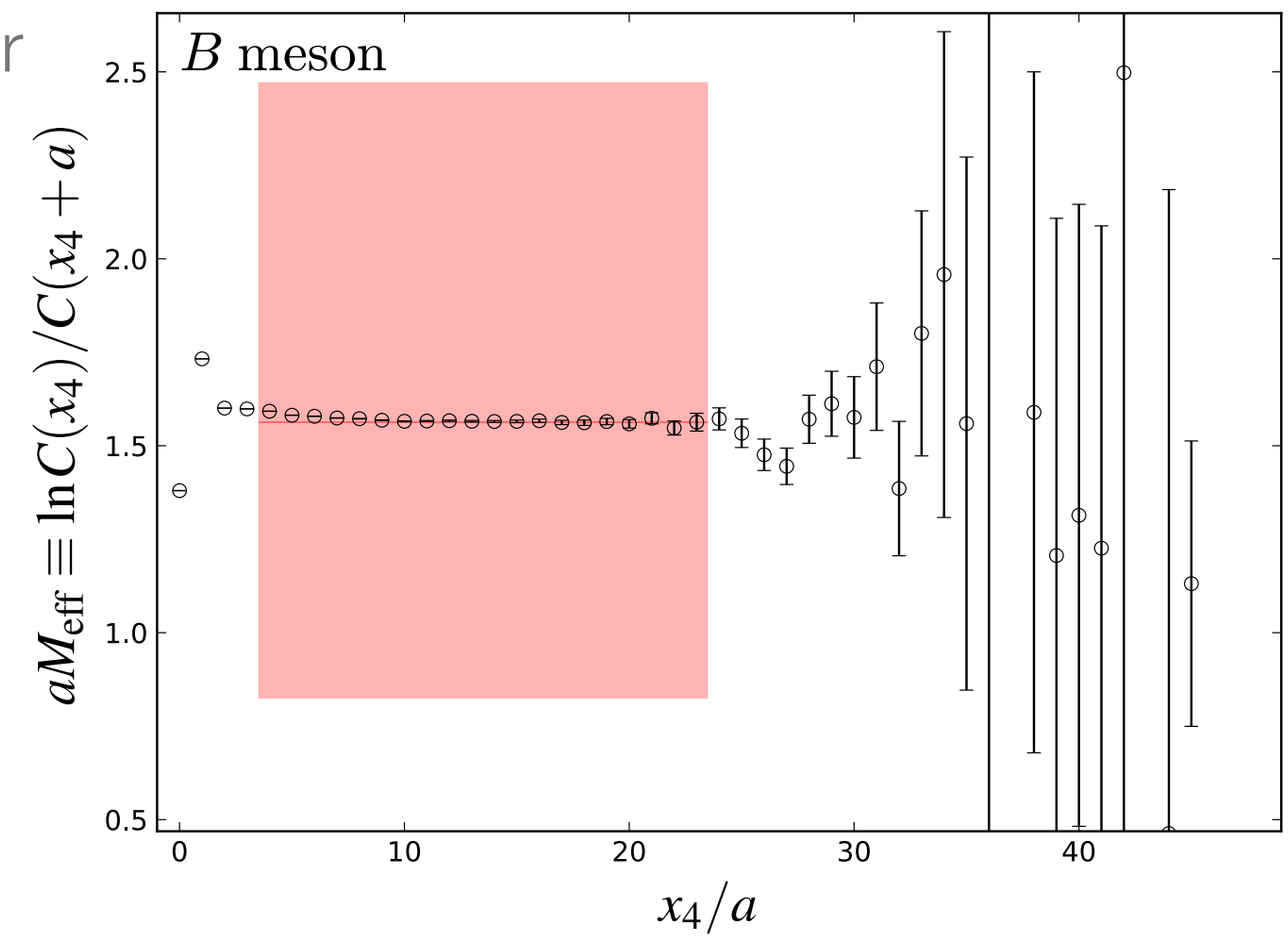
- Our hired superhumans' task is to fit a sum of exponentials:

$$C_{ij}(x_4) = \left\langle B_i(x_4) B_j^\dagger(0) \right\rangle = \sum_{n=0} \langle 0|\hat{B}_i|B_n\rangle\langle B_n|\hat{B}_j^\dagger|0\rangle e^{-M_n x_4} + \mathrm{bc}_4$$

- Saved by the fact that the MC data are highly correlated:
  - roughly, principle components are height, slope, curvature, ….
- Lowest-lying state dominates when x_4 is large, but …
  - … statistical errors grow (exponentially) with x_4 (except pion).
- Augment least-squares cost function with Gaussian priors on M_0, $\ln \Delta M_n$, $Z_n^{(i)} = \langle 0|B^{(i)}|B_n\rangle$.

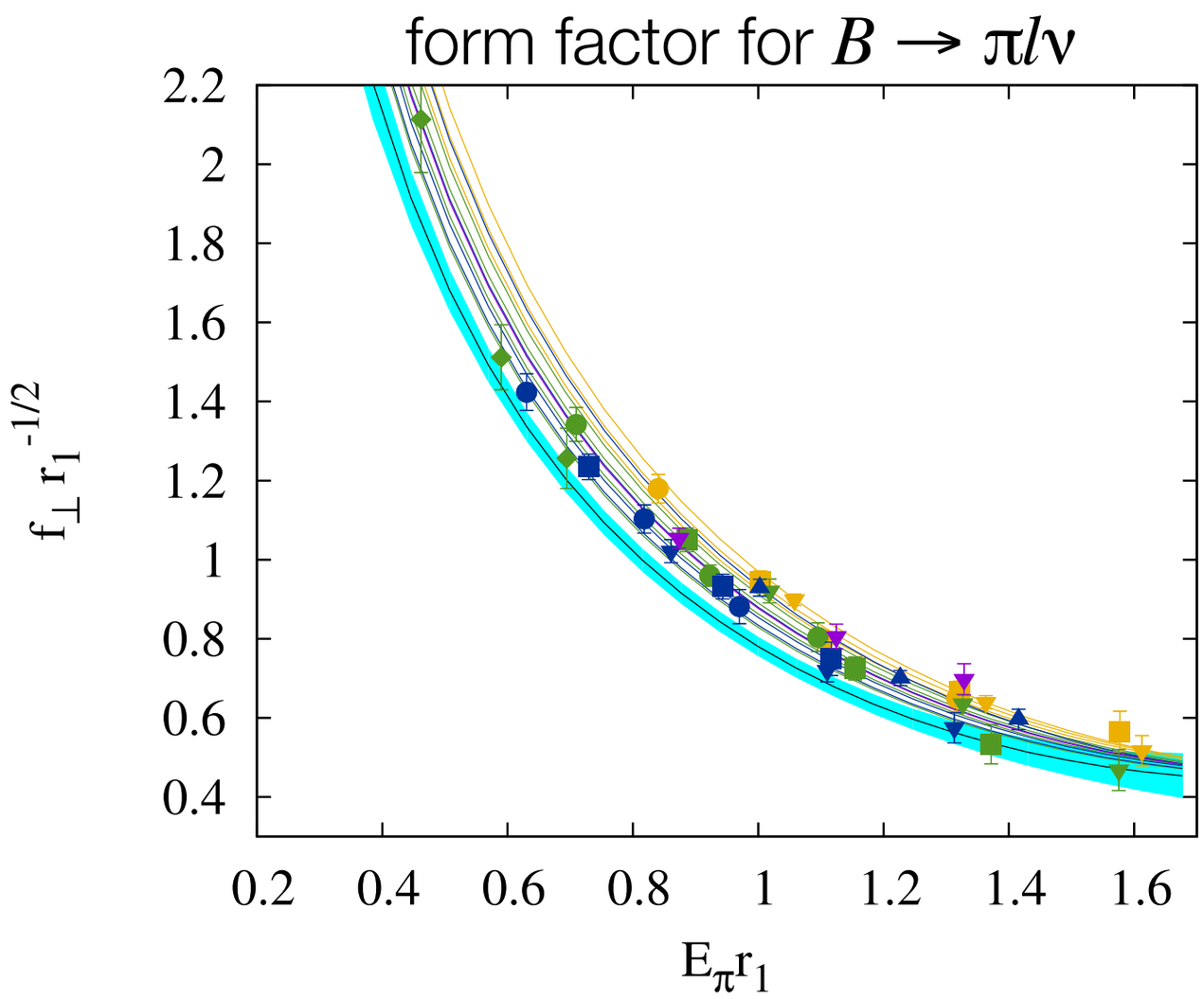
Constrained Curve Fitting

- In principle, we cannot think about extracting more than $N \lesssim \frac{1}{2}N_4$ states.
- Thus, priors are necessary: *e.g.*, truncate with $Z_n^{(i)} \sim \delta(0)$, for n too large.
- Less extreme is a Gaussian prior of sensible width:
  - we can look at large x_4 data (and then omit them from further analysis);
  - we know mass splittings are $400 \overset{\times}{\div} 2$ MeV.



Chiral-Continuum Limit

- After extracting masses & matrix elements for a range of the 1 + n_f (+ 1) parameters, the data are combined with effective field theories for
  - lattice-spacing dependence;
  - quark-mass dependence.
- More curve fitting.
- As before we know the model, just not where to truncate:
  - Gaussian priors again.
- Output is a curve!



Example Fit Function (Schematic)

nonanalytic terms from NLO HMrSχPT aka "chiral logs"

heavy-quark discretization effects (derived in HQET)

fine tune c-quark hopping parameter

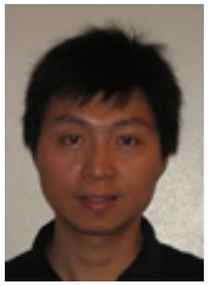
$$F_i = F_i^{\text{logs}} + F_i^{\text{analytic}} + F_i^{\text{HQ disc}} + F_i^{\alpha_s a^2 \text{ gen}} + F_i^{\kappa} + F_i^{\text{renorm}}$$

analytic terms in N^{n}LO χPT: base fit $n = 2$

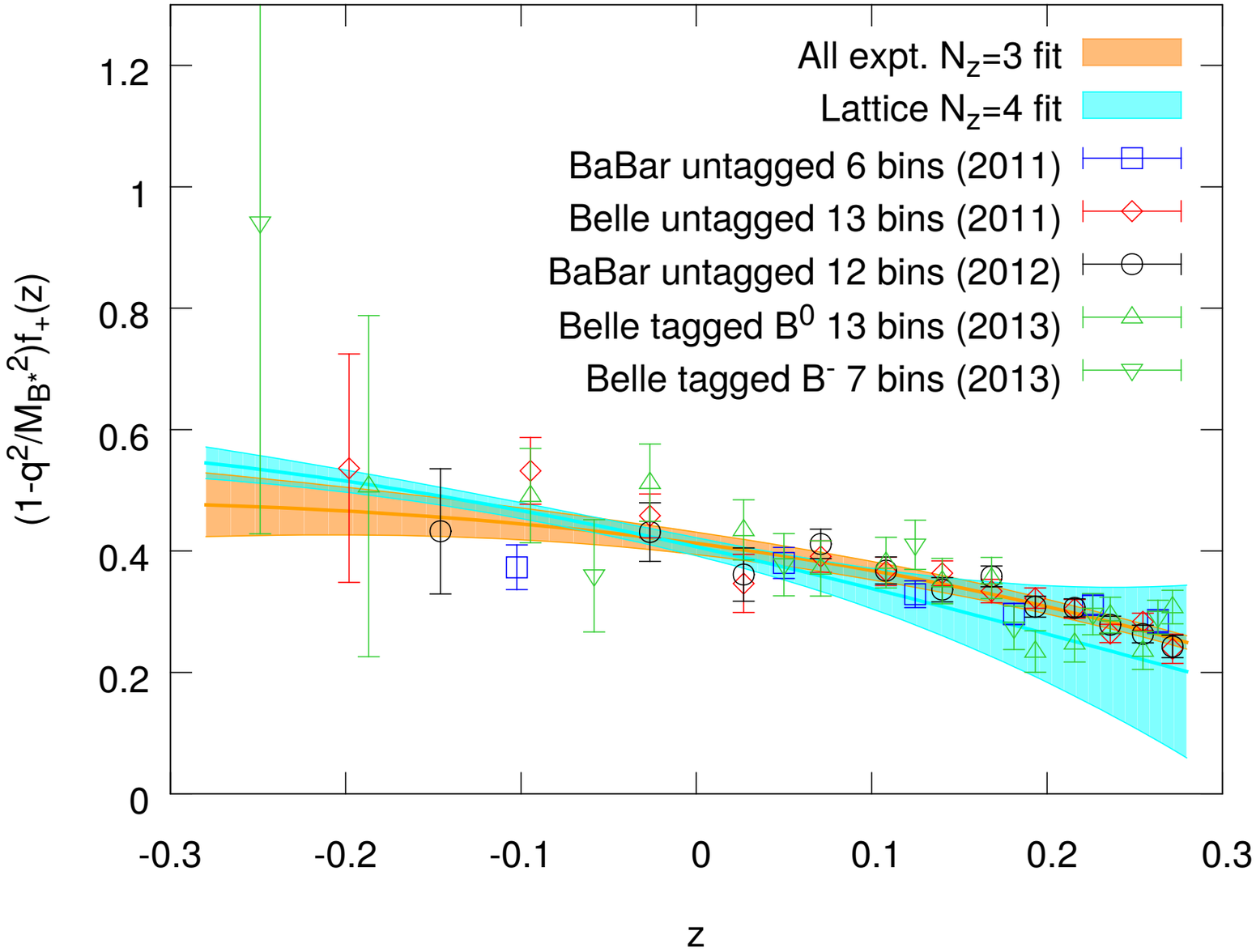
gluon & light-quark cutoff effects à la Symanzik

fit $\alpha_s^2 \rho_{ij}^{[2]}$ (alternatively $\alpha_s^3 \rho_{ij}^{[3]}$)

Combining Lattice QCD with Experiment



- $B \to \pi l \nu$, Fermilab/MILC, arXiv:1503.07839.



- Fit curve from QCD with experimental data:

$$10^3|V_{ub}| = 3.72(16)$$

- BTW, this superhuman is now in data science 😞

Future Physics Challenges in QCD

Scope and Precision

- The precision on some meson matrix elements is sub-percent:

$$f_{B^+} = 186(4)\ \mathrm{MeV} \qquad \Rightarrow 189.6(9)\ \mathrm{MeV}$$

FLAG 2016

Fermilab-MILC 2017

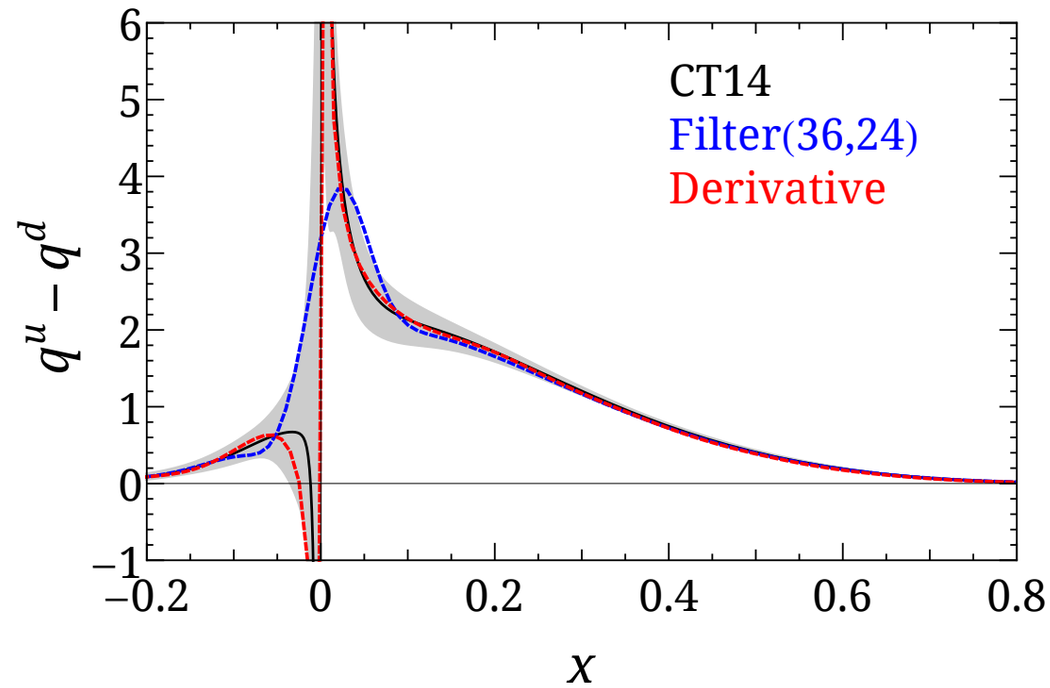
$$f_{B^+} = 224(5)\ \mathrm{MeV} \qquad \Rightarrow 231.1(8)\ \mathrm{MeV}$$

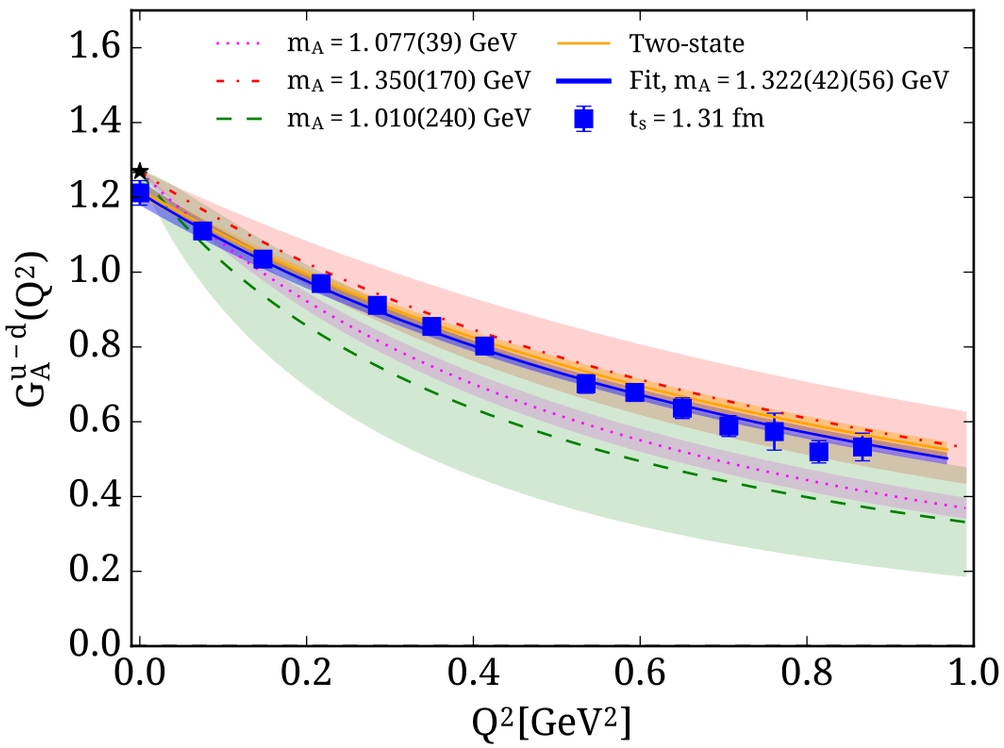
  - Future work will need isospin effects: $m_u \neq m_d$ & QCD+QED.

- Success attracts demands:
  - many new needs in particle physics;
  - even more in nuclear physics.

Nucleon Matrix Elements

- Hadron structure:
  - key in hadronic physics;
  - also in LHC physics ☛





☛ and neutrino physics.

- Numerous nucleon form factors needed so experiment can confront nuclear models on sound footing.

Muon Anomalous Magnetic Moment

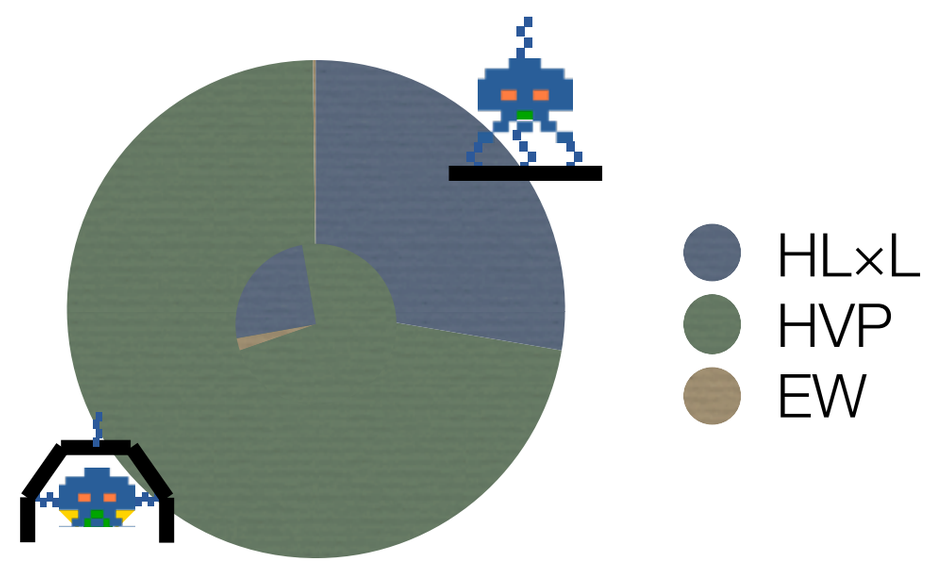
- Fermilab E989 is being mounted to explore a well-known tension between BNL E821 & SM.

$$10^{10} a_\mu = \begin{cases} 11659208.9(6.3) & \text{expt} \\ 11659180.2(4.9) & \text{SM with HVP from } e^+e^- \end{cases}$$

$$a = \mu \frac{2m}{e\hbar} - 1 = \frac{1}{2}(g-2)$$

28.7(6.3)(4.9) is a huge difference!

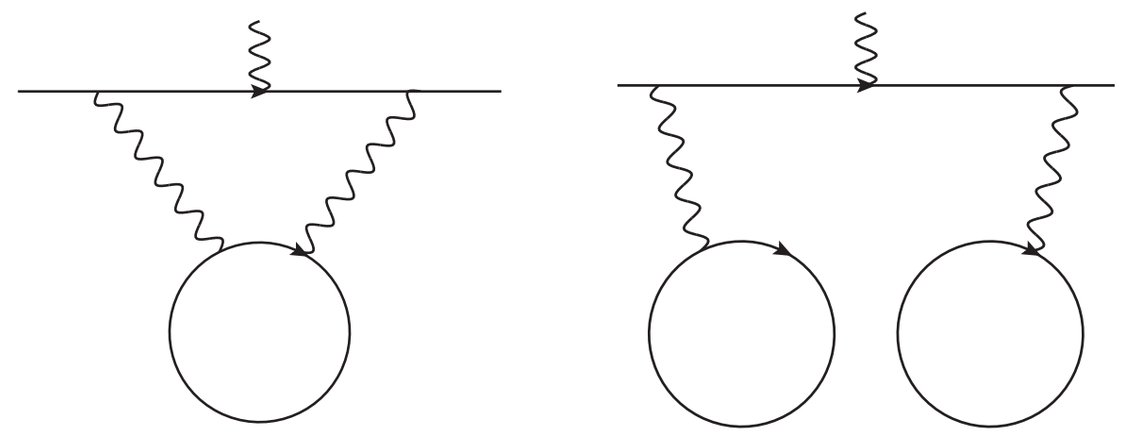
BNL E821 → FNAL E989



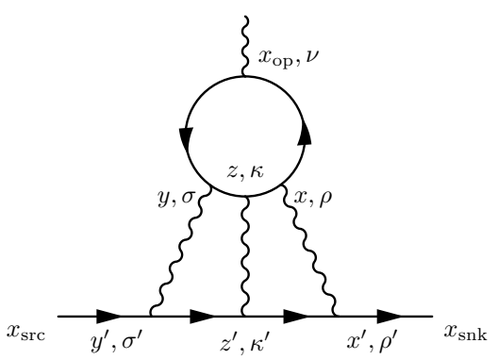
Standard Model Calculation

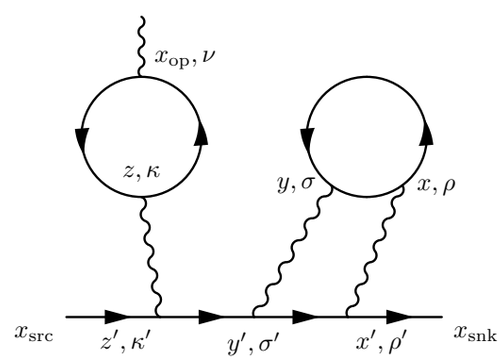
Hadronic Contributions

- Two kinds of hadronic contributions:
  - vacuum polarization:
    - all gluons implied but quarks & photons are connected or disconnected



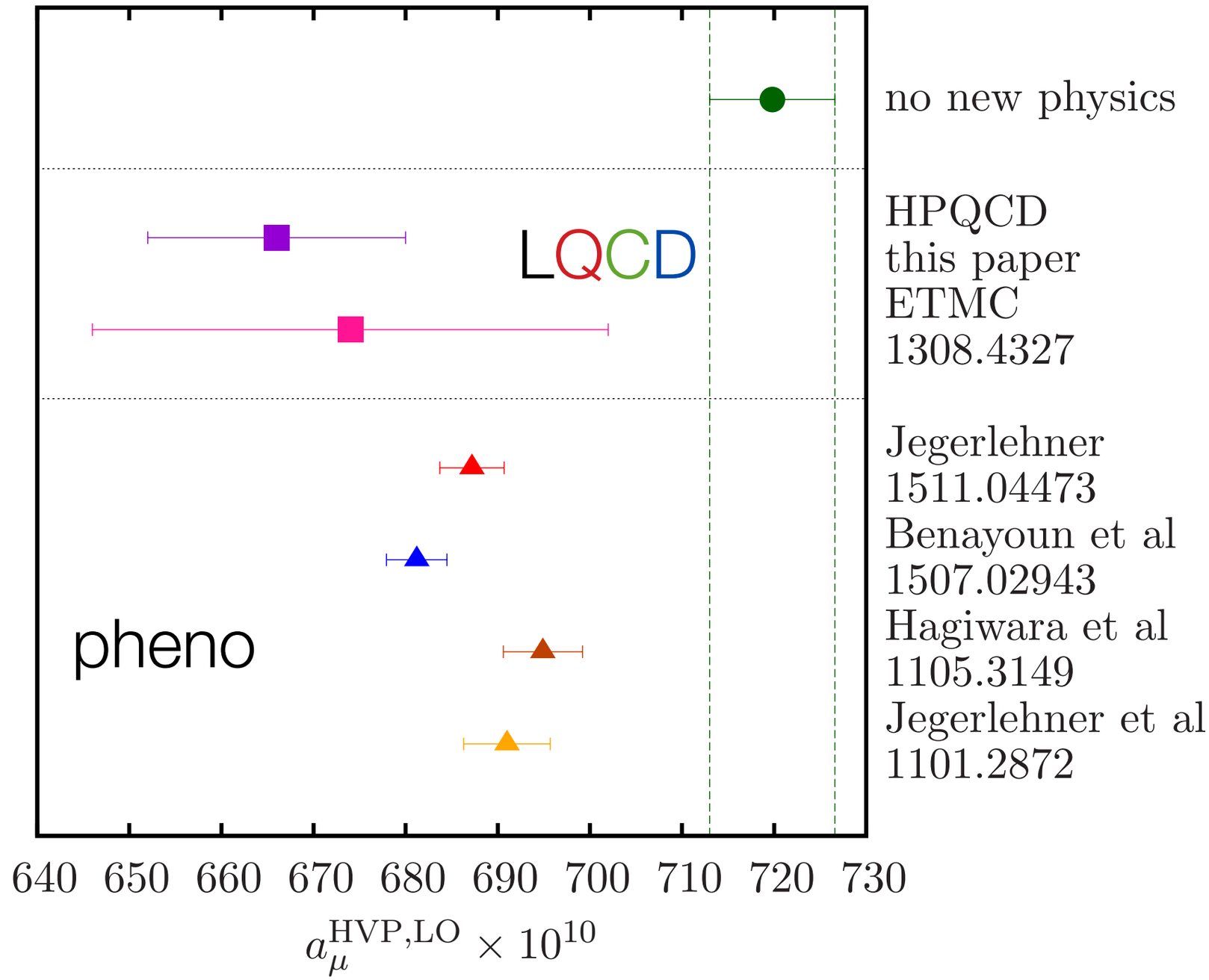
  - "light by light scattering"





HPQCD, arXiv:1601.03071

- Second full result from lattice QCD.
- Uses MILC's 2+1+1 ensembles.
  - 3 lattice spacings;
  - 3 volumes;
  - 3 quark masses, including physical.
- 3σ from exp't.

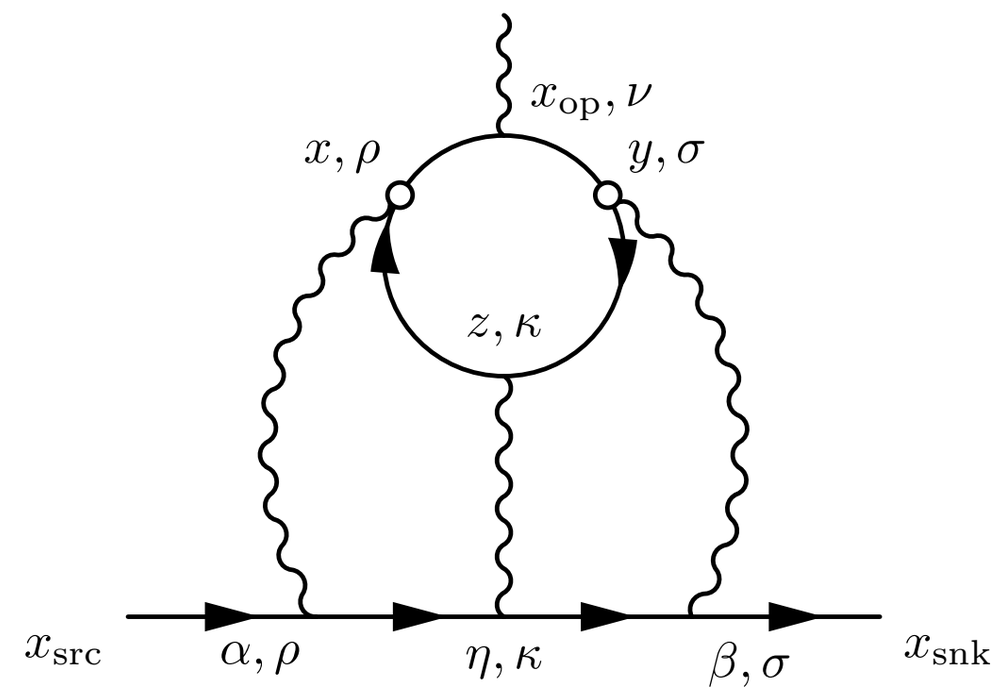


HLbL Progress from RBC

- Compute in coordinate space [arXiv: 1510.07100]

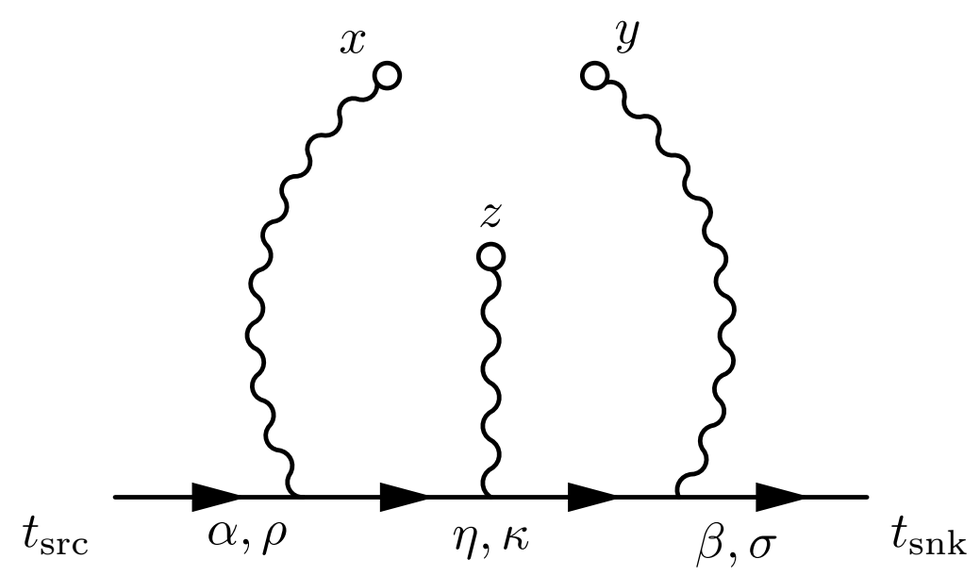
- finite-volume lattice QCD-QED [arXiv: 1610.04603]:

$$10^{10}\, a_\mu^{\mathrm{HLbL}} = 5.35(1.35)$$



- New idea: attach blob to infinite volume QED kernel: sum x, y randomly but wisely; sum x_{op}, y completely.

- Test with QED (instead of QCD) blob [arXiv:1705.01067].



Conclusions and Outlook

Remarks

- Numerical lattice QCD could be a great element of this conference:
  - “machines and algorithms” sub-community is large & includes applied mathematicians;
  - pioneering work on algorithms (*e.g.*, HMC);
  - pioneering work on hardware (*e.g.*, GPUs);
  - probably also a lot for lattice-QCD practitioners to learn: How can advanced ML techniques help us?
- Our work is not going to get easier: success fosters expectations.

Thank you!