ACAT2017 Track-3
Round Table 24 Aug. K.Kato

Toward higher order corrections in HEP...

DCM= regularized integration
+ series extrapolation

- Calculate the integral with finite ε 's ..... $I(\varepsilon_j)$
- Estimate the integral by extrapolation

Assume the integral is analytic in ε
→ Laurent series

$$I = \cdots + C_{-2}\frac{1}{\varepsilon^2} + C_{-1}\frac{1}{\varepsilon} + C_0 + C_1\varepsilon + \cdots$$

$$n = 4 - 2\varepsilon \qquad p^2 - m^2 + i\varepsilon$$

Talk 22 Aug. K.Kato
24 Aug. H.Daisaka
Also 22 Aug. E.de Doncker

Muon g-2 , 2-loop full EW corrections

By N. Nakazawa, T. Ishikawa, Y.Yasui

Started: N. Nakazawa , T.Kaneko

1995 Pisa AIHENP → ACAT series

Resumed the work a few years ago and they almost finished the calculation

Table 1. Theoretical predictions of muon (g-2) [10]

Type of correction	Numerical Value (unit 10^{-11})	Error
QED up to tenth-order	116584718.95	(0.08)
Leading Order Hadronic Vac.Pol.	6923	(42)(3)
NLO Vac.Pol.+Hadronic LBL	7	(26)
ELWK (one-loop)	195.82	(0.02)
ELWK (two-loop)	- 41.2	(1.0)
Theory total	116591803	(1)(42)(26)
Experimental Value	116592091	(54)(33)

[3] Kukhto T V, Kuraev E A, Schiller A and Silagadze Z K 1992 *Nucl.Phys.* **B371** 567

[4] Kaneko T and N Nakazawa 1995 *Proc.of AIHENP*

[6] Czarnecki A, Krause B and Maciano W J 1995 *Phys.Rev.* **D52** R2619
Czarnecki A, Krause B and Marciano W J 1996 *Phys.Rev.Lett.* **76** 3267
Czarnecki A, Marciano W J and Vainshtein A 2003 *Phys.Rev.* **D67** 073006 **D73** 119901

[7] Gribouk T and Czarnecki A 2005 *Phys.Rev.* **D72** 053016

GRACE(w NLG)   1780 two-loop, 70 1-loop C.T.

Diagnostics

UV cancellation

IR cancellation

NLG invariance

(some) Alternate method

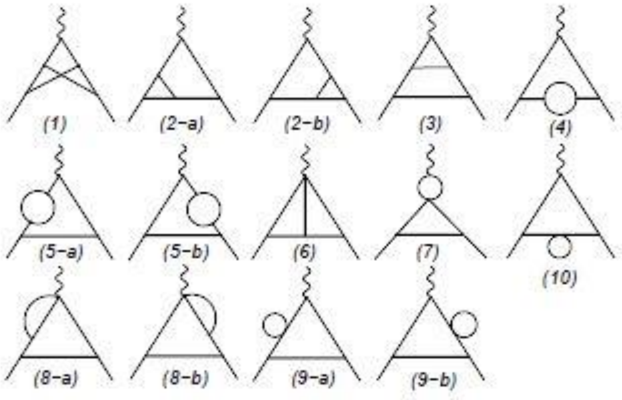

Figure 1. Types of topology

$$I = C_{-1}\frac{1}{\varepsilon} + C_0 + C_1\varepsilon + \cdots$$

In the course of calculation, they also found that the numerical extrapolation is better than the analytical handling of terms to avoid complicated lengthy formula and also human error assuming the integral can be computed with high accuracy.

extrapolation

$\{\varepsilon_j\} \quad (j = 1,..n)$ → Integration → $\{I(\varepsilon_j)\}$

Wynn's algorithm

$$a(j,k+1) = a(j+1,k-1) + \frac{1}{a(j+1,k) - a(j,k)}$$

Input

$$a(j,0) = I(\varepsilon_j), \quad a(j,-1) = 0$$

Linear solver

$$\varepsilon_j{}^K I(\varepsilon_j) = C_0 + C_1\varepsilon_j + C_2\varepsilon_j{}^2 + \cdots + C_n\varepsilon_j{}^n$$

Larger *n* is NOT always good, but an appropriate *n* seems to exist.

NLG(non-linear gauge fixing)

$$\mathscr{L}_{GF} = -\frac{1}{\xi_W}F^+F^- - \frac{1}{2\xi_Z}(F^Z)^2 - \frac{1}{\xi}(F^A)^2 \tag{11}$$

where

$$\begin{aligned}
F^{\pm} &= \left(\partial^\mu \mp ie\tilde{\alpha}A^\mu \mp i\frac{ec_W}{s_W}\tilde{\beta}Z^\mu\right)W^{\pm}_\mu + \xi_W\left(M_W\chi^{\pm} + \frac{e}{2s_W}\tilde{\delta}H\chi^{\pm} \pm i\frac{e}{2s_W}\tilde{\kappa}\chi_3\chi^{\pm}\right) \\
F^Z &= \partial^\mu Z_\mu + \xi_z\left(M_Z\chi_3 + \frac{e}{2s_Wc_W}\tilde{\varepsilon}H\chi_3\right) \\
F^A &= \partial^\mu A_\mu
\end{aligned} \tag{12}$$