

Discussion Panel: Opening Statement



Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Stephen Jones



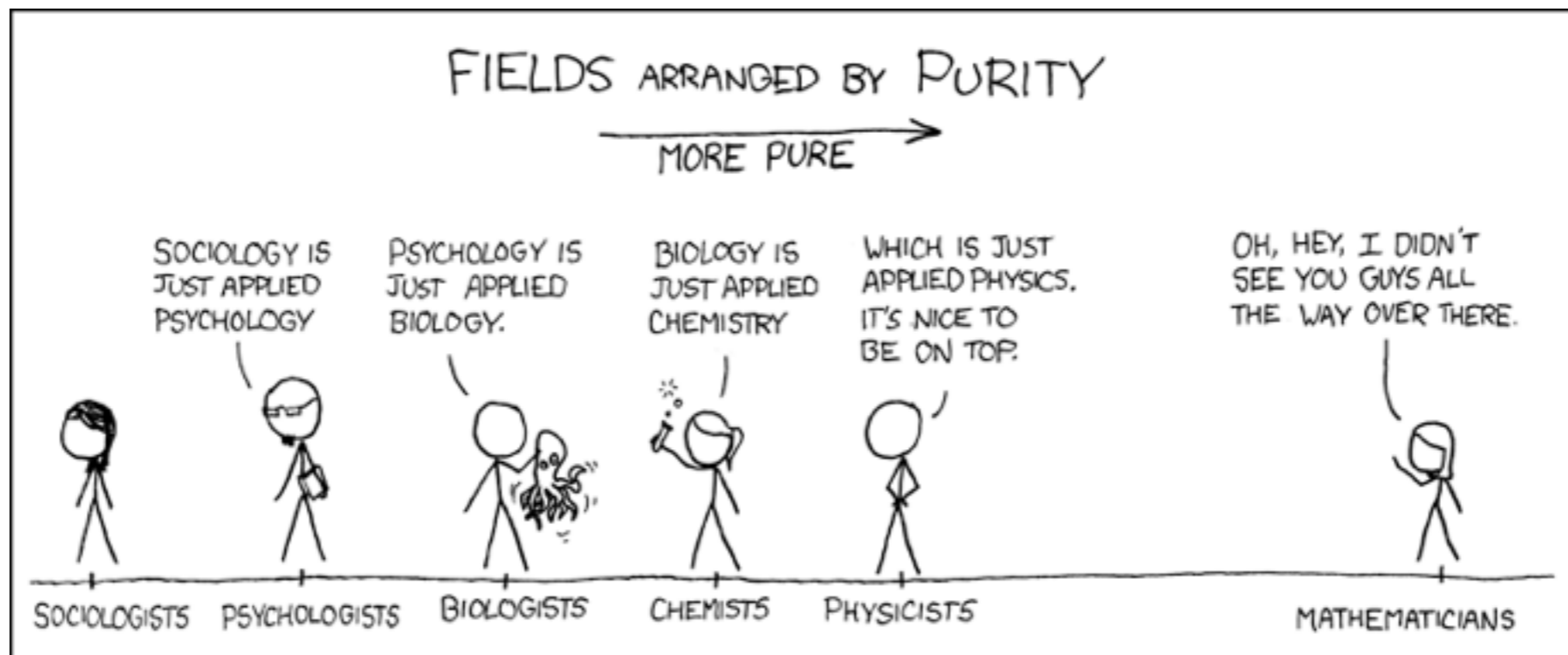
MAX-PLANCK-GESELLSCHAFT



Motivation

Why compute higher order corrections?

- (SM) Provide null hypothesis for our experimental colleagues
- (BSM) Provide information that can be used to optimise experimental searches, interpret data in terms of a theory constraint
- Understand the structure of QFTs (e.g. anomalous dimensions, beta functions, hidden symmetries, connection between renormalisation schemes)
- Love of mathematics, ...



Randall Munroe (xkcd.com/435), CC BY-NC 2.5

Analytic vs. Numerical

	Analytic	Numerical
Exact Pole Cancellation	✓	✗ numerical
Fast Evaluation	✓	✗
Control of Integrable Singularities	✓	Difficult
Extension to More Scales	Difficult	Less Difficult
Automation	Difficult	✓

Table based on: Gudrun Heinrich, MIAPP17

The best of both worlds?

Top-quark pair production

- Integral Reduction: Laporta algorithm (analytic)
- Integrals: Numerical solution of differential equation (handwork + numerical)
Bärnreuther, Czakon, Fiedler 13

4-loop \overline{MS} -on-shell quark mass in QCD & general SU(N)

- Integral Reduction: FIRE, CRUSHER (analytic) A. V. Smirnov; P. Marquard, D. Seidel
- Integrals:
 - Some fully analytic
 - Some Mellin-Barnes (handwork + numerical)
 - Some FIESTA (numerical) A. Smirnov, Tentyukov

Marquard, V. Smirnov, A. Smirnov, Steinhauser, Wellmann 16

NLO HH with full top mass

- Partial Integral Reduction: Reduze (analytic)
- Integrals: SecDec (numerical)

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16

Interesting Example: Finite Basis

Always possible to pick finite basis of integrals, rewrite integrals using:

- Dimension Shifts [Tarasov 96](#); [Lee 10](#)
- Dots

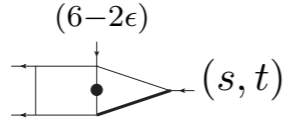
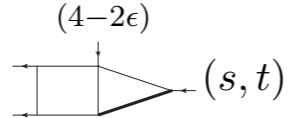
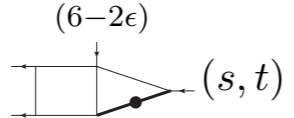
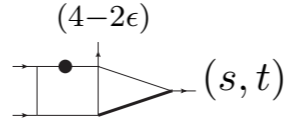
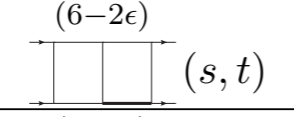
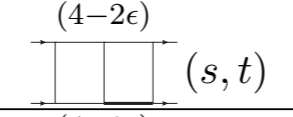
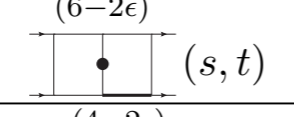
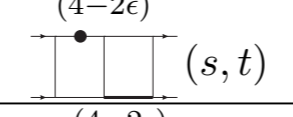
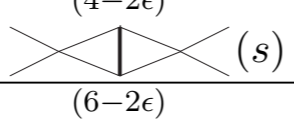
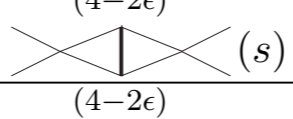
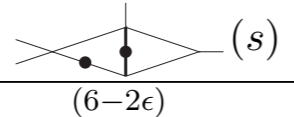
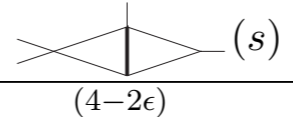


[Panzer 14](#); [von Manteuffel, Panzer, Schabinger 15](#)

**Two-loop
EW-QCD
Drell-Yan**

[von Manteuffel,
Schabinger 17](#)

Finite Basis...

Conventional...

	201 s	2.34×10^{-4}		384 s	8.12×10^{-4}
	150 s	4.83×10^{-4}		56538 s	1.67×10^{-2}
	280 s	1.00×10^{-3}		214135 s	8.29×10^{-3}
	294 s	1.21×10^{-3}		3484378 s	30.9
	91 s	3.76×10^{-4}		87 s	3.76×10^{-4}
	17 s	5.15×10^{-4}		20 s	1.95×10^{-4}
	119 s	2.32×10^{-3}		118 s	2.12×10^{-3}
Total/Max:	3995 s	5.84×10^{-3}	Total/Max:	5136862 s	30.9

← Rel.
Err.

Huge decrease in time to numerically integrate and relative error

Summary

In defence of numerical results:

- Robust & automated, existing algorithms can deal with many loops/legs/scales with limited human input
- Allow us to explore beyond where we can calculate analytically
- Techniques are evolving (see talks of Kato, de Doncker, Freitas)
- Are capable of achieving the goals outlined in my motivation (sometimes with some help e.g... grids)
- Benefit from analytic insights