Round table discussion

Analytical vs. numerical methods for NNLO+ computations for LHC
Analytical vs. Numerical

Numerical methods:
- Sector decomposition
- Mellin integral
- Differential equation
- Direct computation method

Analytical methods?
To keep diversity, I will go for the 'maximally' analytical way
Analytical vs. Numerical

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Maximally analytical method?

(I was told that) any loop integrals can be expressed by GKZ-hypergeometric functions [Gel'fand, Graev, Zelevinski˘ı] (but not sure how it works).
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[Gel’fand, Graev, Zelevinskii]
Maximally analytical method?

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Once Wolfram implements such functions and $\epsilon$-expansions

\begin{verbatim}
In[1]:= Series[GKZHypergeometricFunction[{1+ep,...},...], {ep, 0, 6}]
Out[1]= 1/ep + GKZPolyLog[{1,...},...] + ...
In[2]:= % // Normal // N
Out[2]= 1/ep + 1.39446 + ...
\end{verbatim}

(Scientific fiction)

Done! We will lose our jobs
Maximally analytical method?

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Reality: up to MPL. Still we have jobs

Mathematica 11.0.1 for Linux x86 (64-bit)
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In[1]:= ?SpecialFunctions`*
SpecialFunctions`BernoulliBModPrime
SpecialFunctions`BesselKRatio
SpecialFunctions`BesselPolynomialY
SpecialFunctions`ChebyshevV
SpecialFunctions`ChebyshevW
SpecialFunctions`ClearMathieuCache
SpecialFunctions`FunctionExpandHarmonicPolyLog
SpecialFunctions`GammaOver24
SpecialFunctions`GammaR
SpecialFunctions`GammaRatio
SpecialFunctions`GammaS
SpecialFunctions`HarmonicPolyLog

SpecialFunctions`InverseGammaApprox
SpecialFunctions`InverseLogGammaApprox
SpecialFunctions`MultipleFiniteHarmonicSumS
SpecialFunctions`MultiplePolyLog
SpecialFunctions`MultipleZetaValue
SpecialFunctions`Probit
SpecialFunctions`QHypergeometricPFQPolynomial
SpecialFunctions`ShuffleProductExpand
SpecialFunctions`SphericalHarmonicYTriangularArray
SpecialFunctions`StuffleProductExpand
SpecialFunctions`UnderOverflowedQ
How far can we go?

In these two decades, many problems have been successfully solved with harmonic/multiple polylogarithms
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What’s the next?
How far can we go?

In these two decades, many problems have been successfully solved with harmonic/multiple polylogarithms. What’s the next? Elliptic generalizations? Or...?
ML for Feynman integrals?

Many presentations with ML in ACAT 2017
ML for Feynman integrals?

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Speculative/wishful idea: ML also for Feynman Integrals?
ML for Feynman integrals?

Many presentations with ML in ACAT 2017

Speculative/wishful idea: ML also for Feynman Integrals?

Example: Integration-by-parts reduction ‘IBP mining’ for reduction rules: to find good linear combinations?