The Photon Content of (Un)polarized Protons

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Introduction : Equivalent Photon Approximation

- Weizsäcker and Williams independently derived a technique $\rightarrow$ Equivalent Photon Approximation (EPA), for processes involving charged fermions.

- Central idea : cross section approximated by a covolution of an equivalent number of photons in that particle, the real photoproduction cross section : satisfactory result is expected when the interaction of the particle with the target is mediated by virtual photon exchange.

- Probability to find a photon in an untrarelativistic fermion f :

$$
\begin{aligned}
f_{\gamma/E}(x,E) &= \frac{\alpha}{\pi}Q^2\left\{\frac{1+(1-x)^2}{x}\left(ln\frac{E}{m}-\frac{1}{2}\right)\right. \\
&\quad + \frac{x}{2}\left[ln\left(\frac{2}{x}-2\right)+1\right] \\
&\quad\quad \left. + \frac{((2-x)^2}{2x}ln\left(\frac{2-2x}{2-x}\right)\right\}.
\end{aligned}
$$

x is the photon energy in units of E (E $>>$ m), Q: charge of fermion

- Most commonly, log enhanced term is retained.

- In the collission of two charged particles, either one or both of them can be thought of as consisting of photons.

Photon Content of the Nucleon

- Based on the Equivalent Photon Approximation (EPA), originally applied to a charged fermion.
- The EPA can be extended to the nucleon $N = p, n$ (non-pointlike particle) to simplify the calculation of complicated cross sections :

$$(\Delta)\sigma_{NY} \approx (\Delta)\sigma_{NY}^{\mathrm{EPA}} = (\Delta)\gamma \otimes (\Delta)\hat{\sigma}_{\gamma Y}$$

- In $(\Delta)\sigma_{NY}$
  $Y = l, N$ interacts with N via virtual photon
  $Q^2 = -t = -k^2$: virtuality of the photon
  $\rightsquigarrow$ The EPA is a good approx. when $Q^2 \approx 0$
- In $(\Delta)\sigma_{NY}^{\mathrm{EPA}}$
  $(\Delta)\hat{\sigma}_{\gamma Y}$: real photoproduction cross section
  μ^2: momentum scale in $(\Delta)\hat{\sigma}_{\gamma Y}$
  x: fraction of the proton's momentum carried by the (collinear) photon
  $(\Delta)\gamma(x, \mu^2)$: universal, scale dependent, equivalent photon distribution of the nucleon

$$(\Delta)\gamma(x, \mu^2) = (\Delta)\gamma_{e\ell}(x) + (\Delta)\gamma_{ine\ell}(x, \mu^2)$$

Photon Content of the Nucleon

- In terms of the elastic form factors $G_E(t)$, $G_M(t)$ (m : nucleon mass, $\tau = -t/4m^2$):

$$\Delta\gamma_{e\ell}(x) = -\frac{\alpha}{2\pi}\int_{t_{\min}}^{t_{\max}} \frac{dt}{t}\left\{\left[2 - x + \frac{2m^2x^2}{t}\right] G_M^2(t) \right.$$
$$\left. -2\left[1 - x + \frac{m^2x^2}{t}\right] G_M(t)\,\frac{G_M(t) - G_E(t)}{1+\tau}\right\}$$

Glück, Pisano, Reya, PL **B540**, 75 (2002)

$$\gamma_{e\ell}(x) = -\frac{\alpha}{2\pi}x\int_{t_{\min}}^{t_{\max}} \frac{dt}{t}\left\{2\left[\frac{1}{x}\left(\frac{1}{x} - 1\right) + \frac{m^2}{t}\right]\right.$$
$$\left. \times\ \frac{G_E^2(t) + \tau\, G_M^2(t)}{1+\tau} + \ G_M^2(t)\right\}$$

Kniehl, PL **B254**, 267 (1991)

$t_{\min} \approx -\infty, \ \ t_{\max} \approx -m^2x^2/(1-x)$

Photon Content of the proton (contd)

- $(\Delta)\gamma_{inel}$ obeys the LO evolution equation

$$\frac{d}{d\ln\mu^2}(\Delta)\gamma_{inel}(x,\mu^2) = \frac{\alpha}{2\pi}\sum_{q=u,d,s} e_q^2 \int_x^1 \frac{dy}{y}$$
$$\times (\Delta)P_{\gamma q}\left(\frac{x}{y}\right)\left[(\Delta)q(y,\mu^2) + (\Delta)\bar{q}(y,\mu^2)\right]$$

Glück, Stratmann, Vogelsang, PL **B343**, 399 (1995)
Glück, Pisano, Reya, PL **B540**, 75 (2002)

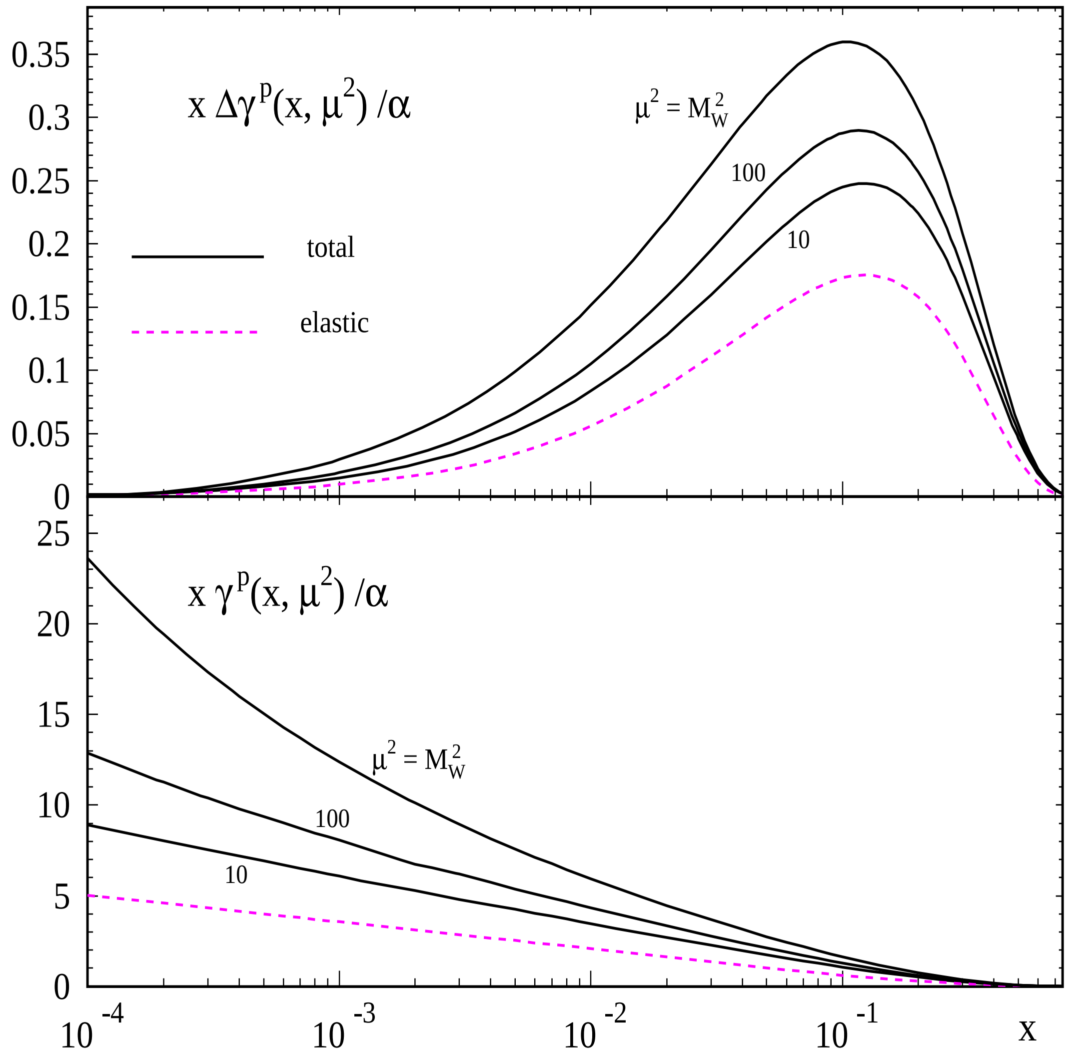
$\Delta P_{\gamma q}(y) = 2 - y$, $P_{\gamma q}(y) = [1 + (1-y)^2]/y$

$\Delta \overset{(-)}{q}$: LO pol. parton distr. GRSV2001 (v.s.)

$\overset{(-)}{q}$: LO unpol. parton distr. GRV98

'minimal' not compelling boundary condition $(\Delta)\gamma_{inel} = 0$ at $\mu_0^2 = 0.26$ GeV^2

Photon Distribution of the Proton



μ^2 is in GeV^2

Glück, Pisano, Reya (2002)

Photon Distribution of the Proton

- $x\,\Delta\gamma^p(x,\mu^2)$ vanishes at small x, $x\,\gamma^p(x,\mu^2)$ increases: $\Delta\gamma^p(x,\mu^2) \ll \gamma^p(x,\mu^2)$, for $x \leq 10^{-3}$

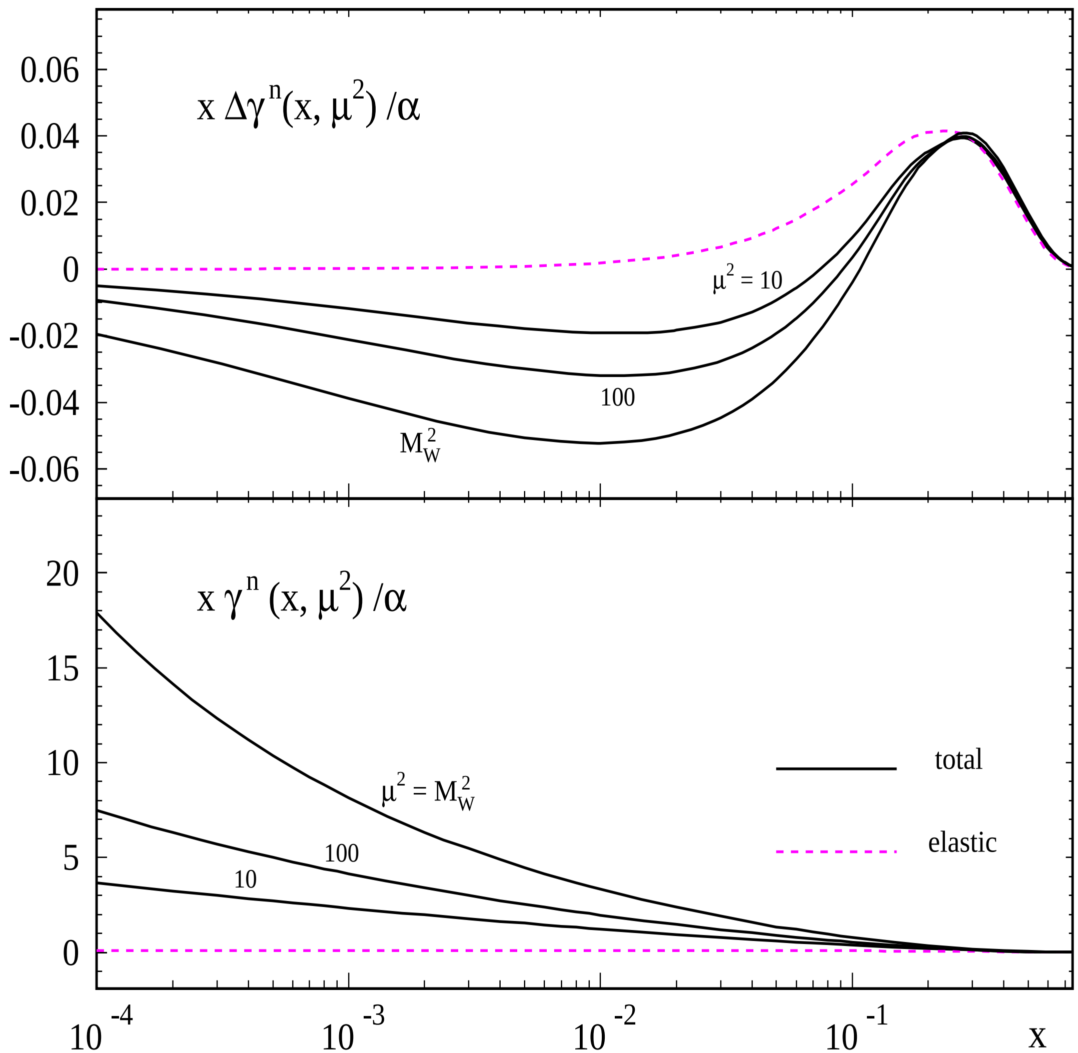
  $x\,\Delta\gamma^p_{e\ell}(x),\; x\,\Delta\gamma^p_{inel}(x,\mu^2) \to 0$ as $x \to 0$

  $x\,\gamma^p_{e\ell}(x),\; x\,\gamma^p_{inel}(x,\mu^2) \to \infty$ as $x \to 0$

  $\Delta\gamma^p_{e\ell}(x)/\gamma^p_{e\ell}(x) \to 1$ as $x \to 1$

- The elastic contribution dominates at moderate values of μ^2: $(\Delta)\gamma^p_{e\ell}(x) \geq (\Delta)\gamma^p_{inel}(x,\mu^2)$, for $\mu^2 \leq 100\,\mathrm{GeV}^2$

Photon Distribution of the Neutron



μ^2 is in GeV^2

Glück, Pisano, Reya (2002)

Photon Distribution of the Neutron

- $\Delta\gamma^n(x,\mu^2)$ is sizeable smaller than $\Delta\gamma^p(x,\mu^2)$

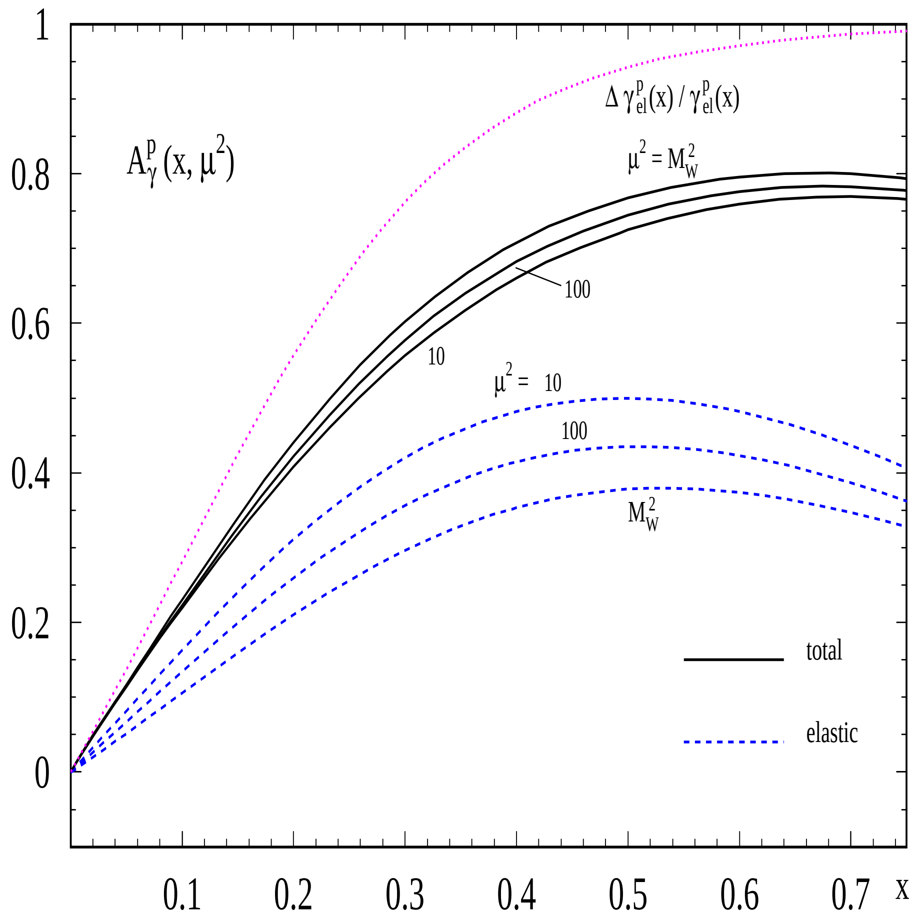
- $\Delta\gamma^n_{in e\ell}(x,\mu^2)$ is marginal for $x \geq 0.2$,

  $\gamma^n_{e\ell}(x)$ is marginal and non-singular as $x \to 0$

- At small x, $(\Delta)\gamma^n(x,\mu^2)$ behaves as $(\Delta)\gamma^p(x,\mu^2)$:

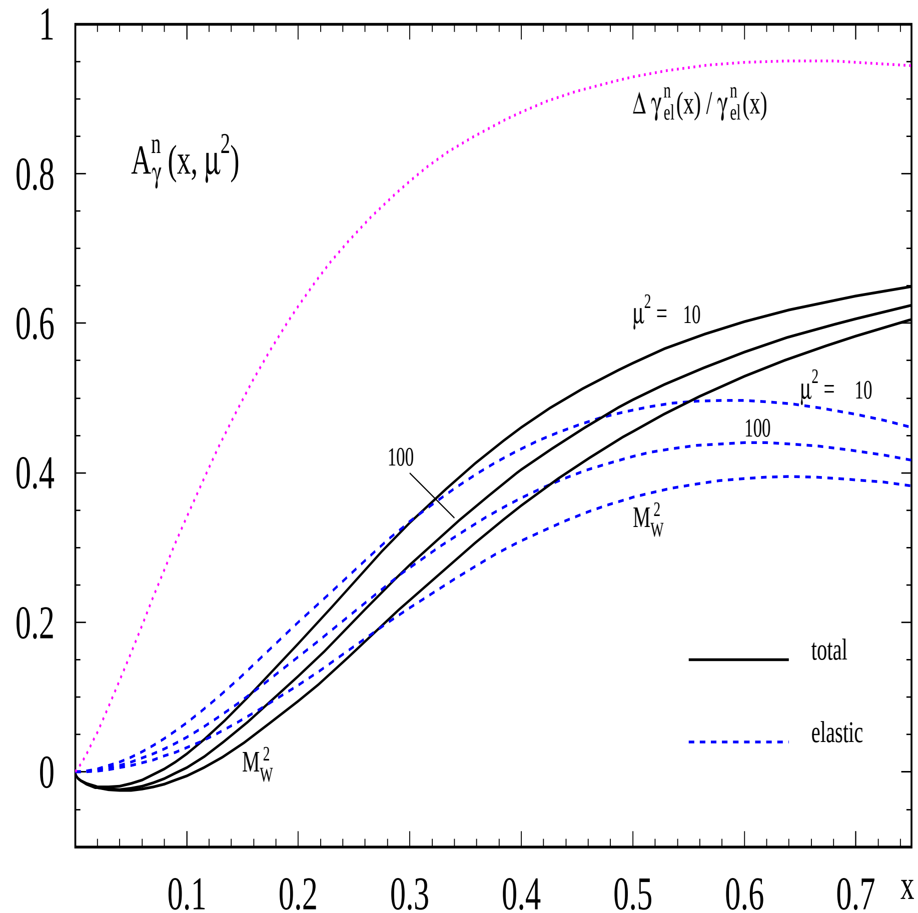
Asymmetries

- $A_\gamma^{p,n}(x,\,\mu^2) \;=\; \frac{\Delta\gamma^{p,n}\,(x,\mu^2)}{\gamma^{p,n}\,(x,\mu^2)}$



Asymmetries

- $A_\gamma^{p,n}(x,\,\mu^2) \;=\; \frac{\Delta\gamma^{p,n}\,(x,\,\mu^2)}{\gamma^{p,n}\,(x,\,\mu^2)}$



To Measure the Photon Distribution of the Proton

- QED Compton process in $ep \to e\gamma X$

  Blümlein, Levman, Spiesberger, JP **G19**, 1695 (1993)

- The exact cross section σ is known

  manifestly covariant AM, Pisano, EPJ **C30**, 477 (2003)
  helicity formalism Courau, Kessler, PR **D46**, 117 (1992)

- $(\Delta)\sigma^{\mathrm{EPA}}$ has also been calculated with $\mu^2 = \hat{s}$

  Glück, Pisano, Reya, Schienbein, EPJ **C27**, 427 (2002)
  De Rújula, Vogelsang PL **B451**, 437 (1999)

- The QED Compton process has been recently analyzed by HERA-H1 and σ compared to σ^{EPA}

QED Compton at HERA

- HERA-H1 cuts: $E'_e, E'_\gamma > 4\,\mathrm{GeV}$, $E'_e + E'_\gamma > 20\,\mathrm{GeV}$,
$$0.06 < \theta_e, \theta_\gamma < \pi - 0.06,$$
$$\Phi = |\pi - |\Phi_e - \Phi_\gamma|| < \pi/4$$

Lendermann, DESY-THESIS-2002-004; Lendermann, Schultz-Coulon, Wegener, DESY-03-85

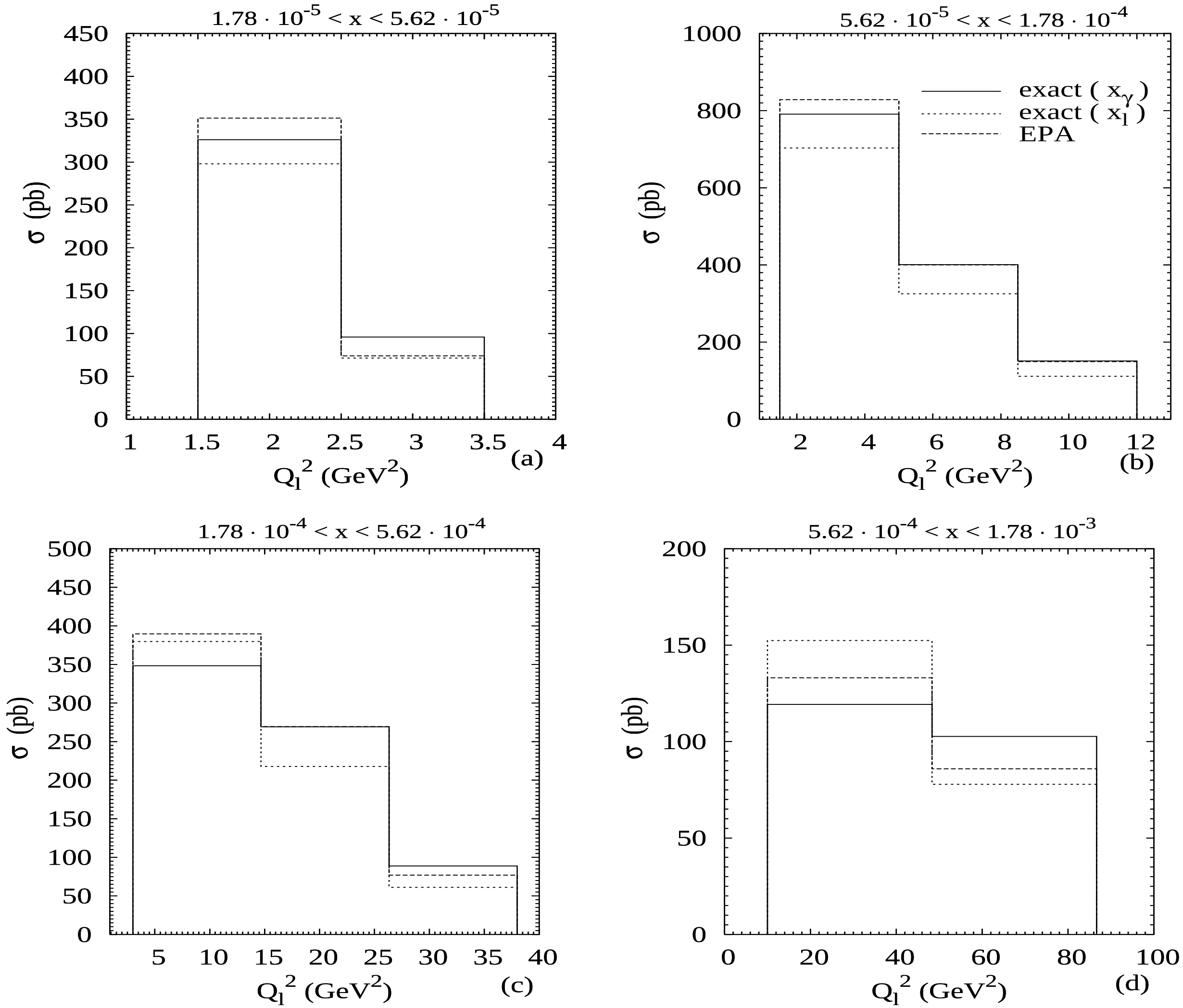
- Exact $\sigma = (\sigma_{e\ell} + \sigma_{ine\ell})$ versus σ^{EPA} and σ^{Len}
The bins are in $Q_l^2 = -\hat{t}$ and x_l, with $x_l = \frac{Q_l^2}{2P\cdot(l-l')}$, $x_\gamma = \frac{l\cdot k}{P\cdot l}$
x_γ is the fraction of longitudinal momentum of the proton carried by the photon
$\rightsquigarrow$ $x_\gamma \simeq x_l \simeq x = \frac{\hat{s}}{S}$ as $Q^2 \simeq 0$
- In σ, σ^{EPA}, : F_2 parametriz. ALLM97 ($Q^2 \to 0$)

$$\frac{|\sigma^{\mathrm{EPA}} - \sigma|}{\sigma} \approx 17\% \qquad \frac{|{\sigma_{e\ell}}^{\mathrm{EPA}} - \sigma_{e\ell}|}{\sigma_{e\ell}} \approx 0.8\%$$

- In x_γ bins:

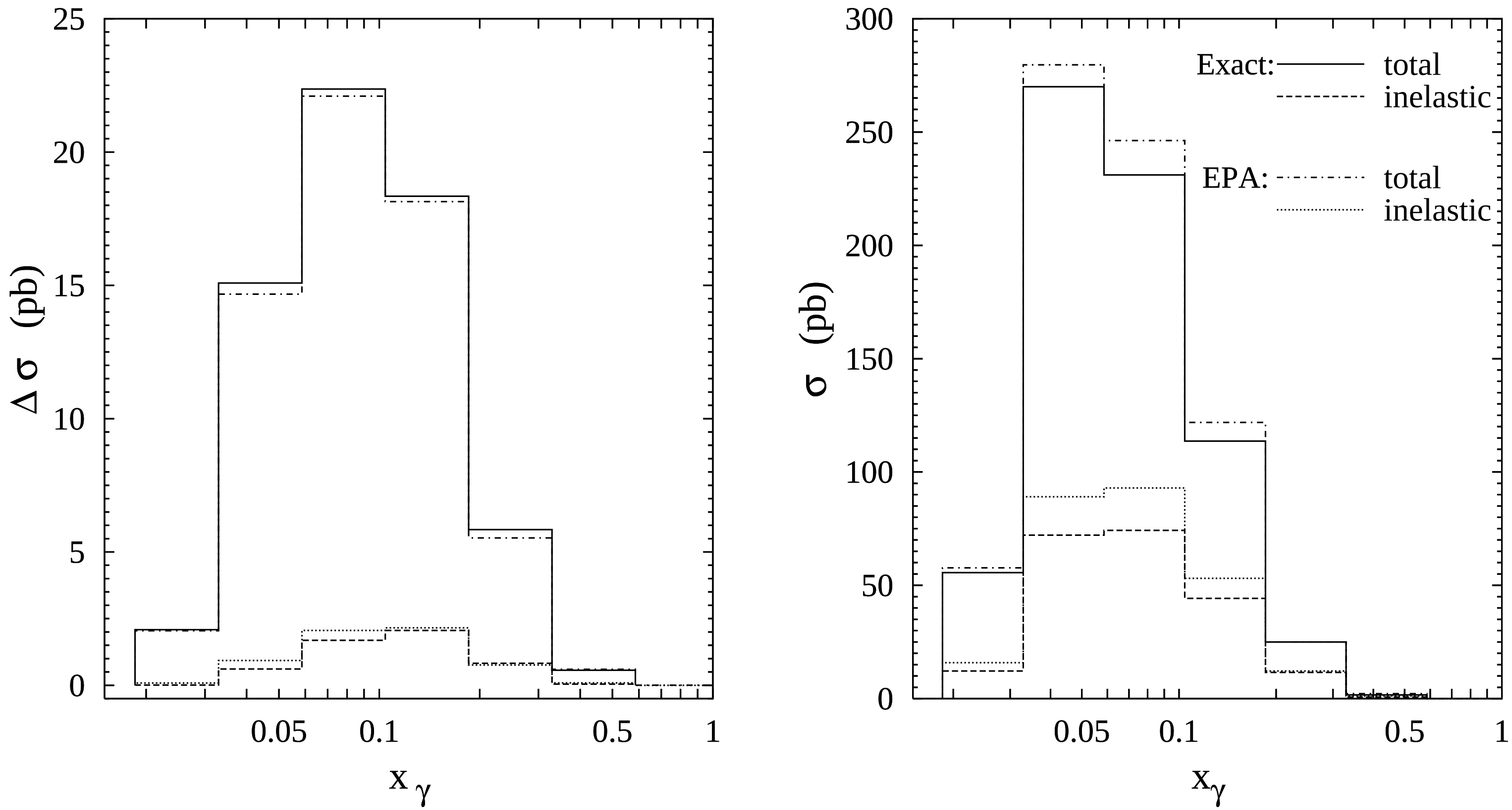
(1) $$\frac{|\sigma^{\mathrm{EPA}} - \sigma|}{\sigma} \approx 9\% \qquad \frac{|{\sigma_{e\ell}}^{\mathrm{EPA}} - \sigma_{e\ell}|}{\sigma_{e\ell}} \approx 0.6\%$$

QED Compton at HERA



QED Compton at HERMES

AM, Pisano, PRD 2004



QED Compton at HERMES

$$\Delta\sigma_{e\ell} = 59.1 \text{ pb} \qquad \sigma_{e\ell} = 4.83 \times 10^2 \text{ pb}$$

$$\Delta\sigma_{ine\ell} = 5.21 \text{ pb} \qquad \sigma_{ine\ell} = 2.15 \times 10^2 \text{ pb}$$

- Kinematical cuts: $E'_e, E'_\gamma > 4\,\text{GeV}, \;\; \hat{s} > 1\,\text{GeV}^2,$
  $0.04 < \theta_e, \theta_\gamma < 0.2, \;\; \Phi < \pi/4$

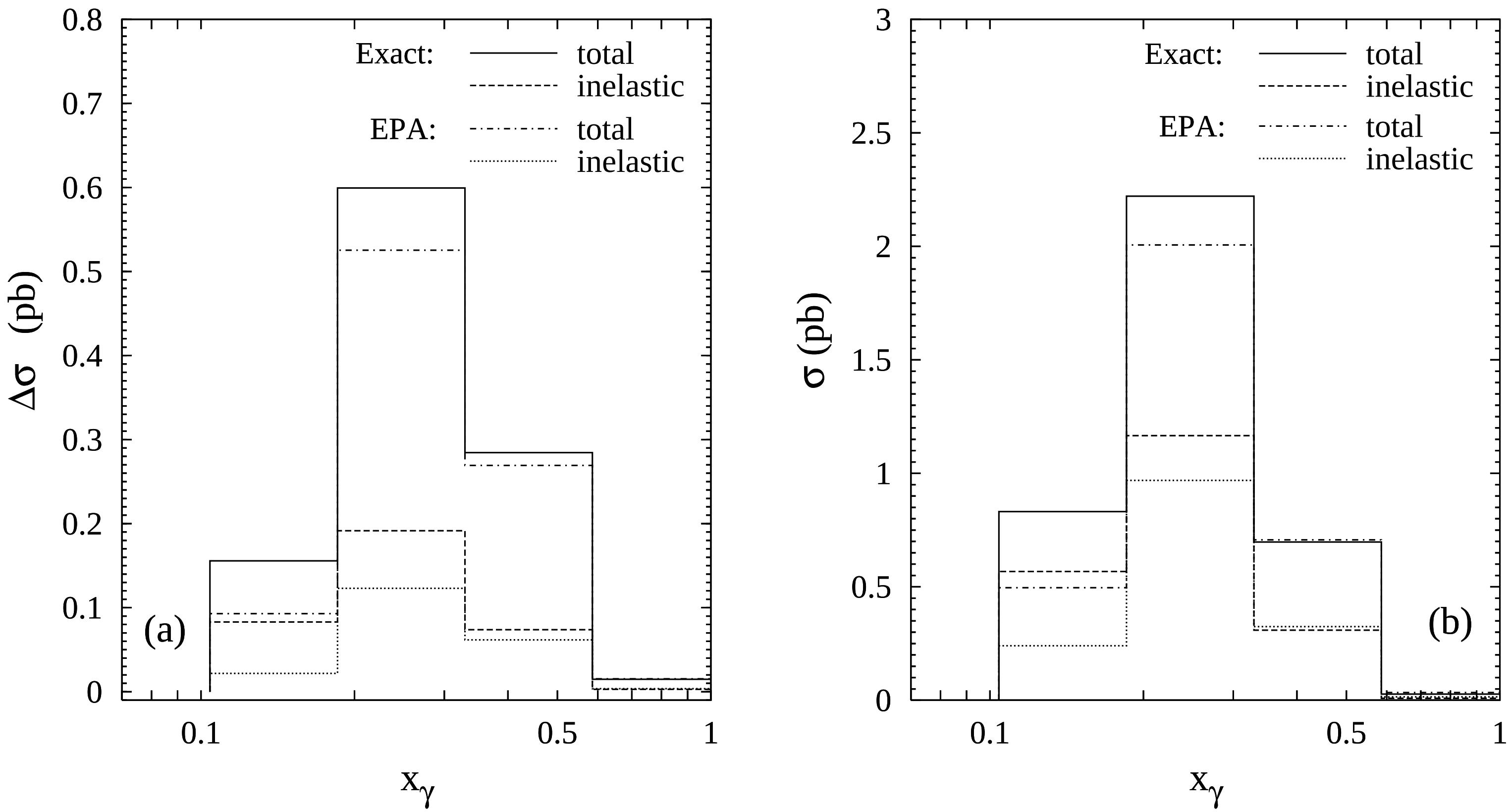
- EPA works better than at HERA: (smaller Q^2)

$$\frac{\Delta\sigma^{\text{EPA}} - \Delta\sigma}{\sigma} = -1.9\%, \qquad \frac{\sigma^{\text{EPA}} - \sigma}{\sigma} = -3.1\%$$

$$\frac{\Delta\sigma^{\text{EPA}}{}_{e\ell} - \Delta\sigma_{e\ell}}{\sigma_{e\ell}} = -3.5\%, \qquad \frac{\sigma_{e\ell}{}^{\text{EPA}} - \sigma_{e\ell}}{\sigma_{e\ell}} = -5.5\%$$

QED Compton at COMPASS

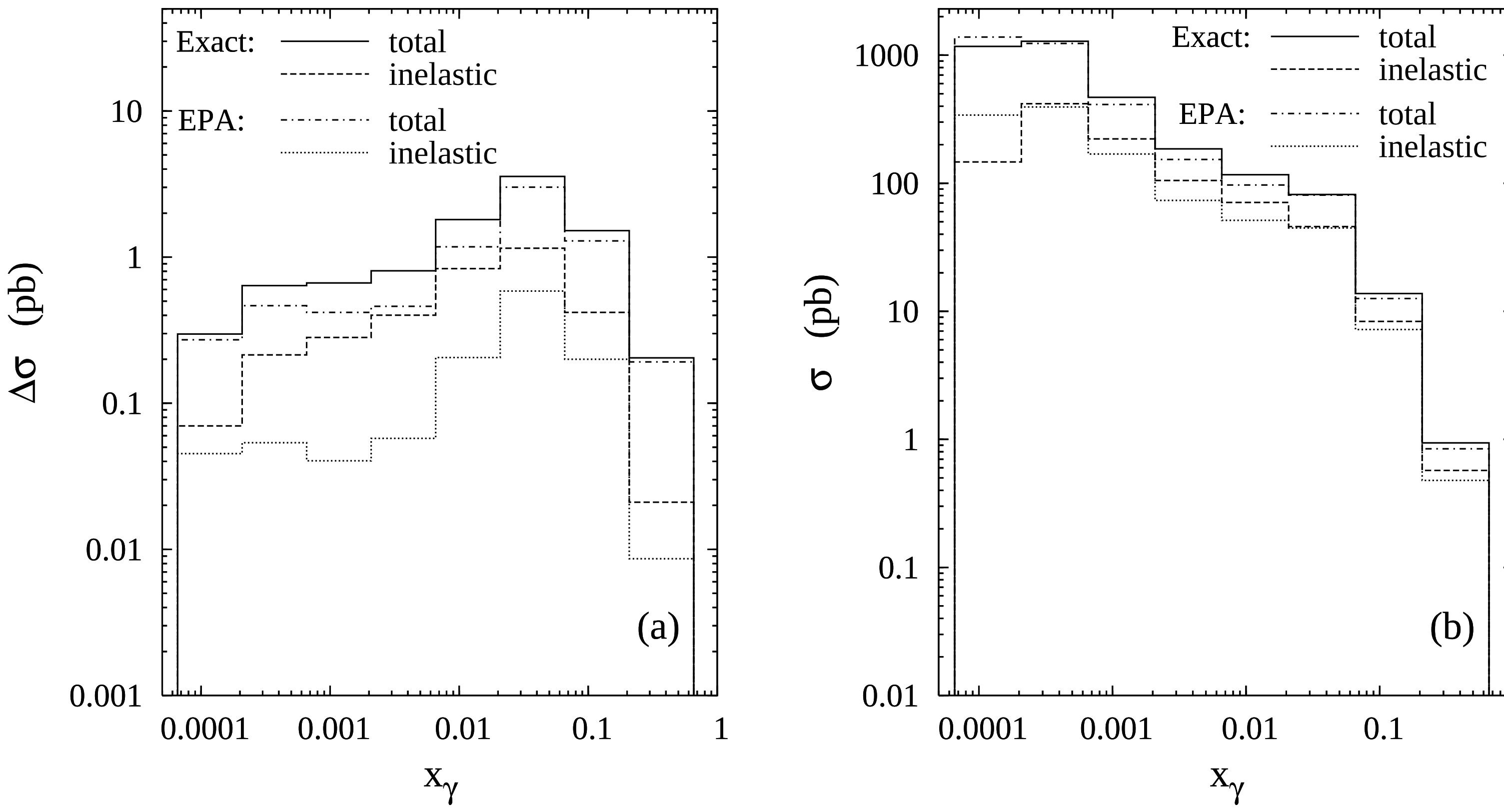
AM, Pisano, PRD 2004



- $E_\mu = 160$ GeV; $0.04 < \theta_\mu, \theta_\gamma < 0.18$; $E'_\mu, E'_\gamma > 4$ GeV
- Background VCS suppressed
- Integrated cross section agrees with EPS (14 %), polarized (15 %) Not as good as HERMES

QED Compton at eRHIC

AM, Pisano, PRD 2004



- $E_e = 10$ GeV, $E_p = 250$ GeV; $0.06 < \theta_\mu, \theta_\gamma < \pi - 0.06$; $E'_\mu, E'_\gamma > 4$ GeV
- Background VCS suppressed
- Integrated cross section agrees with EPS (1.6 %), polarized (9.8 %)

Conclusions

- The photon content of the nucleon $(\Delta)\gamma^N(x, \mu^2)$ evaluated in the EPA allow calculation of photon-induced subprocesses in elastic/deep inelastic ep and hadronic ($pp, ...$) reactions
- Some of these reactions (QED Com process in $ep \to e\gamma p$ and $ep \to e\gamma X$) will provide informations concerning the structure functions $F_{1,2}$ and $g_{1,2}$ in the low Q^2 region
- Kinematical cuts have to be studied in order to extract $(\Delta)\gamma^N(x, \mu^2)$ from experiments and check its range of validity and accuracy
- The photon content of the electron $(\Delta)\gamma^e(x, \mu^2)$ allow calculation of photon-induced subprocesses in e^+e^- and ($e\gamma, \gamma\gamma, ...$) reactions; direct and resolved contributions of the photon
- Unpol. including non-log terms
  Frixione, Mangano, Nason, Ridolfi 93 (HERA)
  Polarized : Florian, Frixione 99