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The stimulated Breit-Wheeler process as a source of background e^+e^- pairs at the international linear collider

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Abstract. Passage of beamstrahlung photons through the bunch fields at the interaction point of the ILC determines background pair production. The number of background pairs per bunch crossing due to the Breit-Wheeler, Bethe-Heitler and Landau-Lifshitz processes is well known. However the Breit-Wheeler process also takes place in and is modified by the bunch fields. A full QED calculation of this Stimulated Breit-Wheeler process reveals cross section resonances due to the virtual particle reaching the mass shell. The one loop electron self energy in the bunch field is also calculated and included as a radiative correction. The bunch field is considered to be a contant crossed electromagnetic field with associated bunch field photons. Resonance is found to occur whenever the energy of contributed bunch field photons is equal to the beamstrahlung photon energy. The stimulated Breit-Wheeler cross section exceeds the ordinary Breit-Wheeler cross section by several orders of magnitude and a significantly different pair background may result.

1. INTRODUCTION

The design of the International linear Collider includes intense energetic electron and positron bunch collisions in order to maximise centre of mass energy and collider luminosity. Associated with these intense bunches are strong electromagnetic fields which affect the physics processes at and near the interaction point. Of particular concern is unwanted pair production which causes background radiation in detectors and other collider components. The production of background pairs from two real beamstrahlung photons is included in background calculations. However at present the effect of the strong bunch fields on this process is ignored. This paper seeks to address that issue.

In order to perform a full QED calculation of the Stimulated Breit-Wheeler cross section, a semi-classical approach is adopted in which the Dirac equation is solved exactly in a classical external plane wave field. Both the fermion field operators and the electron propagator are modified. The Stimulated Breit-Wheeler process is a second order intense field QED process, and it is known that this class of processes contains resonant cross-sections [1,2]. In order to calculate the contribution from the resonances radiative corrections have to be included. The persistence of the bunch fields is very short being of order 10^{-14} s. However the resonances are introduced in the virtual particle exchange which is of order 10^{-21} s. There is more than enough time for the resonant background pair process to occur and the calculation presented here is pertinent to all intense charged bunch collisions.

2. DIRAC EQUATION SOLUTIONS IN THE BUNCH FIELDS

The electron and positron bunches at the ILC are ultra-relativistic and the electromagnetic fields produced by ultra-relativistic beams approximate well to a constant, crossed plane wave propagating, say, along the x axis with 4-momentum k and described by 4-potential $a_\mu(k.x)$. For the default Scheme 1 set of ILC bunch parameters including dimensions $\sigma_x\sigma_y\sigma_z$, the number of charges N, relativistic parameters $\gamma, \beta)$ [3] and with B_c being the Schwinger critical field, the numerical values of the bunch field intensity ν^2 and bunch field photon energy ω can be established

$$\begin{aligned} \frac{\omega}{m} &= \frac{\hbar}{mc^2}\frac{e}{m}\frac{5}{6}B_c\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x+\sigma_y)} = 0.0603 \\ \nu^2 &= \frac{e^2}{\omega^2 m^2\beta}\frac{5}{6}B_c\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x+\sigma_y)} = 1.000 \end{aligned} \tag{1}$$

Dirac equation solutions in plane electromagnetic waves are well known [4]. The solutions are a product of a bispinor u_p and an exponential function of the bunch field 4-potential. The fourier transform of this solution yields a set of r Volkov field operators of the fermion 4-momentum p each corresponding to a contribution of r bunch field photons. The 4-potential of a contant crossed electromagnetic field introduces Airy functions into the Volkov solution as long as there is azimuthal symmetry (which is assumed here) about the axis of propagation direction of the bunch fields.

$$\begin{aligned} \Psi^V_{p,r}(x) &= E_p(x)\ u_p\ e^{-i(p+Q)x-irkx} \\ E_p(x) &= [1+\tfrac{e}{2(kp)}\not{k}\not{a}(kx)]Q^{-1/3}Ai(z) \\ \text{where}\quad Q &= \tfrac{e^2a^2}{2(kp)}k \quad\text{and}\quad z=-Q^{-1/3}(r+Q) \end{aligned} \tag{2}$$

3. ELECTRON SELF ENERGY IN THE BUNCH FIELDS

The class of second order external field QED processes are known to have resonant cross sections due to the virtual particle reaching the mass shell. Therefore radiative corrections to the Stimulated Breit-Wheeler cross section calculation considered in this paper are probably necessary. The only radiative correction considered in this instance is the Electron Self Energy which also occurs in the presence of the bunch field (Fig. 1).

The Electron Self Energy in a constant crossed field $\Sigma(p)$ has been calculated before, and contains the usual UV and IR divergences. In addition, an extra pole is introduced in the limit of vanishing bunch field intensity [5]. The UV divergence appears in the photon propagator which is unmodified by the bunch field. The regularisation and renormalisation procedures are therefore the same as the non external field case and the corrected electron propagator to be used for the Stimulated Breit-Wheeler scattering matrix is

$$[\not{p}-m-\Sigma(p)+\Sigma(p\to m)]^{-1} \tag{3}$$

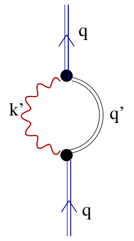

Figure 1. Electron Self Energy Feynman diagrams.

The Electron Self Energy can be calculated after inclusion of the electron propagator in the external field. This is the non-external field electron propagator sandwiched between Volkov E_p functions. The integrations over internal 4-momenta k' and q' are transformed using Heaviside step functions and a shift to 4 dimensional spherical coordinates. The imaginary and real parts are then separated using dispersion relations [6]. Renormalisation introduces an infinitesimal shift to the lower bounds of the remaining integrations and the, thus avoided, IR divergences are removed by further transformations in integration variables. The imaginary part of the Self Energy finally is rendered as

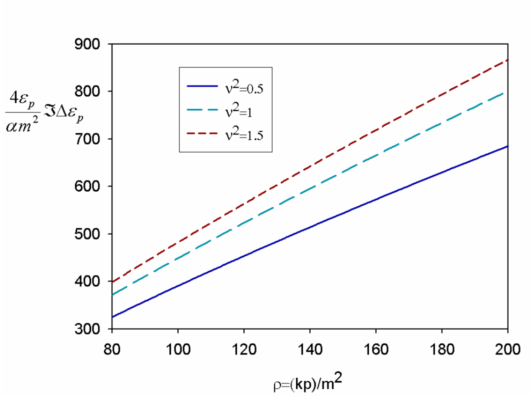

Figure 2. imaginary part of the Electron Self Energy in the bunch field.

$$
\begin{aligned}
&\Im\Delta\varepsilon_p^R(\rho) = \frac{e^2m^2}{16\pi\varepsilon_p}\int_{0+\epsilon}^{\infty}dr\int_{0+\epsilon}^{u_r}\frac{3du}{2(\rho+u^3r^{3/2})^2} \\
&\times\left[-4\rho ru^2Ai(z)^2 - 2\nu^2(1-u)\left(2\rho+\frac{r^3u^6}{\rho+u^3r^{3/2}}\right)\right. \\
&\times\left.\left(ruAi(z)^2+\frac{1}{1-u}Ai'(z)^2\right)\right] \text{ where } \rho=\frac{\nu^2}{(kp)}
\end{aligned} \tag{4}
$$

The final integrations are straightforward and the imaginary part of the Self Energy bears an almost linear relationship with the parameter ρ which is a function of the scalar product of virtual particle and bunch field photon 4-momenta and the bunch field intensity ν^2 (see Figure 2).

4. STIMULATED BREIT-WHEELER CROSS-SECTION

The cross section of the Stimulated Breit-Wheeler process is calculated from the matrix element whose form is written down with the aid of the Feynman diagrams (Figure 3 and Equation 5).

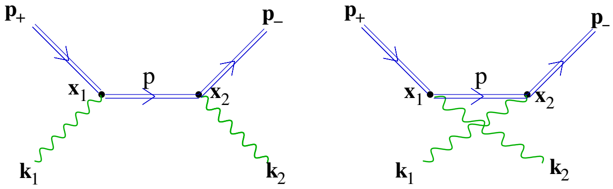

Figure 3. Stimulated Breit-Wheeler Feynman diagrams.

$$\begin{aligned} S^{e}_{fi} &= -\frac{e^2}{2}\int_{-\infty}^{\infty} d^4x_1 d^4x_2 d^4q \; \frac{1}{p^2-m^2}\bar{u}(p_-) \\ &\quad \times \left(\mathcal{A}\rceil^{-\rangle\|_\infty \S_\in -\rangle\|_\in \S_\infty} + \mathcal{B}\rceil^{-\rangle\|_\in \S_\in -\rangle\|_\infty \S_\infty}\right) v(p_+) \\ \text{with}\mathcal{A} &= \overline{E}_{\surd^-}(\S_\in)\gamma_\mu \mathcal{E}_{\surd}(\S_\in)(\not{p} + \Updownarrow)\overline{E}_{\surd}(\S_\infty)\gamma_\nu \mathcal{E}_{-_{\surd^+}}(\S_\infty) \\ \text{and } \mathcal{B} &= \overline{E}_{\surd^-}(\S_\in)\gamma_\nu \mathcal{E}_{\surd}(\S_\in)(\not{p} + \Updownarrow)\overline{E}_{\surd}(\S_\infty)\gamma_\mu \mathcal{E}_{-_{\surd^+}}(\S_\infty) \end{aligned} \tag{5}$$

Squaring the matrix element yields ~ 100000 scalar product terms and four integrations over contributions from bunch field photons. Simplifications of the problem were sought. Beamstrahlung photons are usually emitted in forward directions, so all photons in the pair production process were considered collinear. Furthermore, a centre of mass-like reference frame (Equation 6) was chosen which permitted scalar products of one of the beamstrahlung photons and bunch field photons to be zero. Two of the integrations over bunch field photons were so eliminated and the square of the matrix element contained products of two Airy functions.

$$\begin{aligned} \underset{\sim}{k}_1 + \underset{\sim}{k}_2 + l\underset{\sim}{k} &= \underset{\sim}{q}_- + \underset{\sim}{q}_+ = 0 \\ \omega_1 + \omega_2 + l\omega &= \varepsilon_{q_-} + \varepsilon_{q_+} \end{aligned} \tag{6}$$

The full cross-section is too bulky to write down here. Nevertheless, numerical results were obtained using the Feyncalc Mathematica package and custom written FORTRAN routines.

The appearance of cross-section resonances was established by examining the propagator denominator. With θ_f being the angle which the produced electron makes with the propagation direction of the bunch field photons, and ω, ω_1 being the beamstrahlung photon and bunch field photon energy respectively, resonance was found to occur whenever the total energy of bunch field photons contributed to the process was the same as the beamstrahlung photon energy

$$(\omega_1 - r\omega)(\cos\theta_f + \omega_1 - r\omega) \tag{7}$$

The numerical value of the resonant differential cross section was thereby established (Figure 4) and integration over all production angles gave the full cross section (Figure 5). Comparison was made with the ordinary, non external field Breit-Wheeler cross-sections The differential cross section for the collinear case of the Stimulated Breit-Wheeler process is peaked in longitudinal directions, as is the ordinary Breit Wheeler process. However the overall values are greatly enhanced. Indeed for the beamstrahlung photon energies expected at ILC collisions the enhancement is between five and six orders of magnitude.

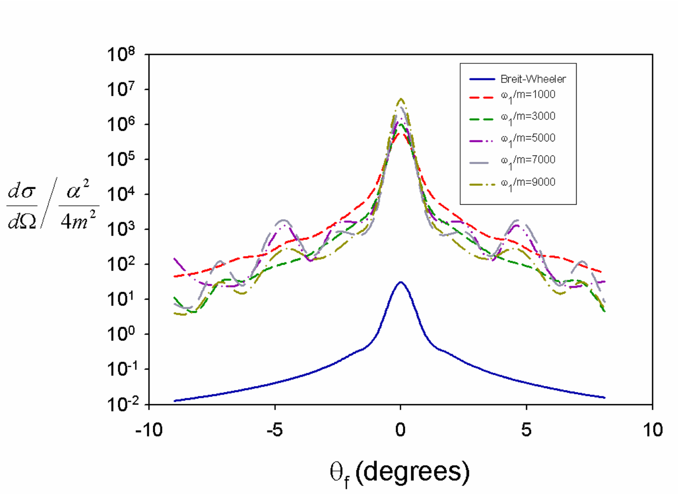

Figure 4. The Stimulated Breit-Wheeler differential cross-section peak.

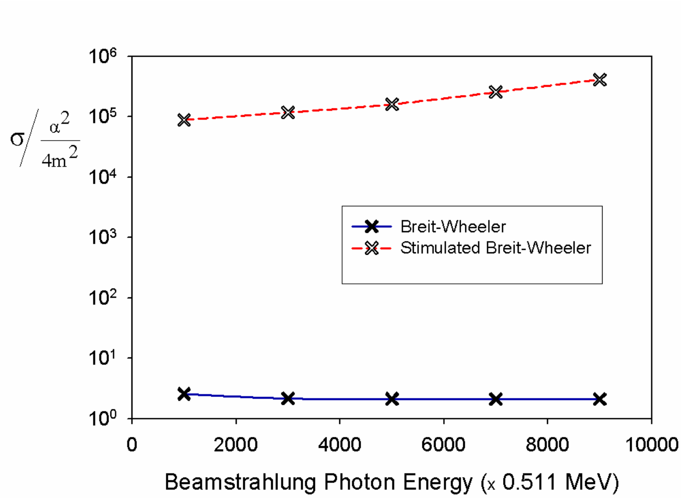

Figure 5. The full cross-section of the Stimulated Breit-Wheeler process.

5. CONCLUSION

The cross section of the Stimulated Breit-Wheeler process was calculated using solutions of the Dirac equation in a constant crossed electromagnetic field and including as a radiative correction, the one loop Electron Self Energy in the bunch field. For default accelerator parameters at the ILC, the Breit-Wheeler cross section used in background pair calculations is underestimated by several orders of magnitude. Background pair generators used in simulation studies need to be reconfigured in order to included the amended cross section. The extra background pairs that can be expected will be produced longitudinally and will be twice as energetic due to the contribution from the bunch fields. The theoretical calculation presented here can be further refined by allowing for non-azimuthal symmetry in which the pair production takes place at the periphery of the bunches rather than on axis.

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