Higher Curvature Effects in the ADD and RS Models

Work in progress

- ADD + RS (classic) have many common features:

  (i) Localized SM fields on a boundary

  (ii) Bulk has constant curvature :

  ADD (Minkowskian) , RS (AdS_5)

  (iii) Gravity in bulk described by Einstein-Hilbert action

possible bulk cosm. const.

$$S = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} \left\{ R \;(+\text{ constant}) \right\}$$

fundamental scale

Ricci scalar

∴ How is ADD/RS phenomenology altered if we give up (iii) + consider more general actions ?? i.e,

$\boxed{R \longrightarrow F}$ ... a well-behaved function of invariants

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* Why do this ??

- in both models $\sqrt{s} \sim M_{(eff)}$ are probed + we know E.H is an effective theory $< M$ ... so 'correction' terms should be present..

- Strings predict such terms sub-leading in $1/M^2$ etc

- such terms have been considered for other reasons e.g, cosmology/dark energy issues

Here, to be tractable, we restrict ourselves to

$$F(R,P,Q) \; : \quad P \equiv R_{AB} R^{AB} \; ; \; Q \equiv R_{ABCD} R^{ABCD}$$

Ricci tensor   Curvature tensor

.. which has been considered in cosmo studies ...

* What do we want to know ???

→ graviton KK properties : masses, wave functions, matter couplings, propagators, ...

(not, e.g, self-couplings of the gravitons)

Then, to obtain these quantities in a constant curvature background (which we have here)

* It is sufficient to expand F to second order in the invariants:

background value

$$\boxed{F = F_0 + \sum_i (X_i - X_{i0}) F_{X_i} + \frac{1}{2}\sum_{ij}(X_i - X_{i0})(X_j - X_{j0}) \cdot F_{X_i X_j} + \text{higher order dropped terms}}$$

$X_i = (R, P, Q)$

$\partial_{X_i} F \big|_{\text{background}}$

background value

$$\boxed{S_{eff} \to \frac{M^{D-2}}{2}\int d^D x \sqrt{g}\,\{\Lambda + a_1 R + a_2 R^2 + a_3 C + a_4 GB\}}$$

Weyl scalar $\equiv C_{ABCD} C^{ABCD}$

Gauss-Bonnet term

... where $\{\Lambda, a_i\}$ are functions of F_{X_i}, $F_{X_i X_j}$, F_0 +

$$R_0 \left(= \langle R \rangle_{\text{background}}\right) \begin{cases} = 0 & \text{in ADD} \\ = -20k^2 & \text{in interval RS} \end{cases}$$

$$\Rightarrow \boxed{GB \equiv R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \equiv R^2 - 4P + Q}$$

Field Content (in D-dimensions!)

- massless tensor field (usual gravitons etc)
- massive tensor ghost (Yikes!)
- massive scalar (tachyonic?)

.. many ways to see this ...

$\Longrightarrow$ Consider gravitons being exchanged between SM sources (4D) $\boxed{T_{\mu\nu}}$. Then, eg, in ADD (before KK-sums) (n extra dims)

$$\Omega = \text{"}\ \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)}{k^2 - m_n^2}^{**}$$

usual KK masses

this is the usual graviton exchange structure (eg, GRW)

ghost!! wrong sign!

$$- \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)}{k^2 - (m_n^2 + m_T^2)}$$

massive in bulk tensor field

$$+ \frac{T^2/(n+2)(n+3)}{k^2 - (m_n^2 + m_S^2)}\ \text{"}$$

bulk mass for scalar field

- Remove tachyons $\therefore$ $m_S^2 > 0$ (demanded)
- Remove ghosts $\therefore$ $m_T^2 \to \infty$ $\to$ $F(R, P-4Q)$ only
  $\hookrightarrow$ $\boxed{a_3 = 0}$

$**$ $\boxed{T^2 = T_{\mu\nu}T^{\mu\nu}}$

.. a similar requirement in RS : $\boxed{F(R, P-4Q)}$

ADD : $\Lambda = 0 \rightarrow R_0, F_0 = 0$ (flat space)

$$F \rightarrow \underbrace{F_R}_{>0 \text{ (no KK ghosts)}} R + \left\{ -F_Q + \tfrac{1}{2} F_{RR} \right\} R^2 + F_Q \cdot G$$

$$\boxed{m_s^2 = \frac{(n+2)\, F_R}{4(n+3)\, \underbrace{(F_{RR}/2 - F_Q)}_{\geq 0 \text{ (no tachyons)}}}}$$

{ Note in "GB gravity"
$F_{RR}/2 - F_Q = 0$ so
$m_s^2 \to \infty$ (removed) }

- m_s is naturally $O(M)$ so a new KK spectrum of scalars begins at $\sim$TeV (Demir + Tanyildizi '05)

?? effect ??

Small as " $T^2/T_{\mu\nu}T^{\mu\nu}$ " $\sim (m^2_{\text{external}}/s) \ll 1$ at LHC / ILC ...

$\rightarrow$ $\boxed{\bar{M}_{Pl}^2 = V_n M^{n+2} F_R}$ ($F_R > 0$ recall)

.. from the zero mode graviton wavefunction normalization

- KK graviton masses left <u>invariant</u> ($m_{\vec{n}}^2 = \vec{n}^2/R^2$)

- in <u>units of M</u>, graviton <u>emission</u> cross sections are modified:

$$\boxed{d\sigma_{ADD} \to \underline{F_R}^{-1}\, d\sigma_{ADD}(M^2, m_{\vec{n}}^2, s,t,u)}$$

- Similarly (neglecting the new scalars) graviton <u>exchange</u> amplitudes will be rescaled:

$$\boxed{\mathcal{A}^{(grav)}_{ADD} \Rightarrow F_R^{-1}\, \mathcal{A}^{(grav)}_{ADD}}$$

... We expect F_R to be $O(1)$ ... but $F_R = 1$ in many cases since the background metric is flat, e.g,

$\Longrightarrow$ if F is a polynnomial in R $(= R + aR^2 + bR^3 + \ldots)$ then $F_R = 1$ automatically...

The rescaling by F_R^{-1} in $d\sigma_{ADD}$ + $A^{(grav)}_{ADD}$ implies that M is <u>not</u> an observable from this sector <u>alone</u>

only $\boxed{M F_R^{1/4}}$ is!

$\boxed{\text{RS on an interval}}$

Λ_b = bulk cosmo const.

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

↑ warp factor

$\boxed{k \sim M}$

$$\boxed{R_0 = \langle R \rangle_{AdS_5} = -20k^2}$$

$R \to F(R, P-4Q)$

$$\to \boxed{224\,k^4 F_Q + 8k^2 F_R + F_0 = 2\Lambda_b/M^3}$$ constraint

$\hookrightarrow k(M, \Lambda_b)$ is a derived parameter

ex. $\boxed{F = R + \beta R^2/M^2}$

$$\to \quad k^2 = \frac{3M^2}{40\beta}\left\{1 \pm \left(1 + \frac{40\Lambda_b}{9M^5}\beta\right)^{1/2}\right\}$$ two roots!

$\to -\Lambda_b/6M^3$ as $\beta \to 0$ (negative root)

(usual RS relationship)

$$S_{eff} = \int d^5x\,\sqrt{-g}\left\{-\Lambda_b + a_1\frac{M^3}{2}R + \frac{\alpha M}{2}G + \frac{\beta M}{2}R^2\right\}$$

dimensionless coefficients

e.g.,

$$\frac{\beta}{M^2} = \underbrace{-F_Q + \frac{1}{2}F_{RR}}_{\text{ADD result}} - 20k^2 F_{RQ} - 280\,k^4 F_{QQ} \qquad \text{etc}$$

Scalar gets a bulk mass:

$$\boxed{m_s^2 = \frac{3a_1}{16\beta} M^2}$$

$$\simeq \frac{3}{8} \frac{F_R + 20k^2 F_{RR} + 280k^4 F_{RQ}}{F_{RR} - 2f_Q - 40k^2 F_{RQ} - 560k^4 F_{QQ}}$$

KK Scalar spectrum: $(2-\nu) J_\nu(x_{s_n}) + x_{s_n} J_{\nu-1}(x_{s_n}) = 0$

$\nu^2 = 4 + m_s^2/k^2$ is large

$\rightarrow \boxed{m_{s_n} = \underbrace{x_{s_n}} k e^{-\pi k r_c}}$ $\boxed{x_{s_0} = ?}$

- If $\beta/a_1 = 1$, $k/M = 0.05$ $\rightarrow \frac{m_s}{k} \simeq 8.7$ $\rightarrow x_{s_0} \simeq 11$

$(x_1^{grav} = 3.83)$ $\rightarrow \approx$ $\boxed{3\times \text{ heavier}}$ than 1^{st} graviton KK [Fig]

... as in ADD these scalars are more weakly coupled than gravitons by a factor

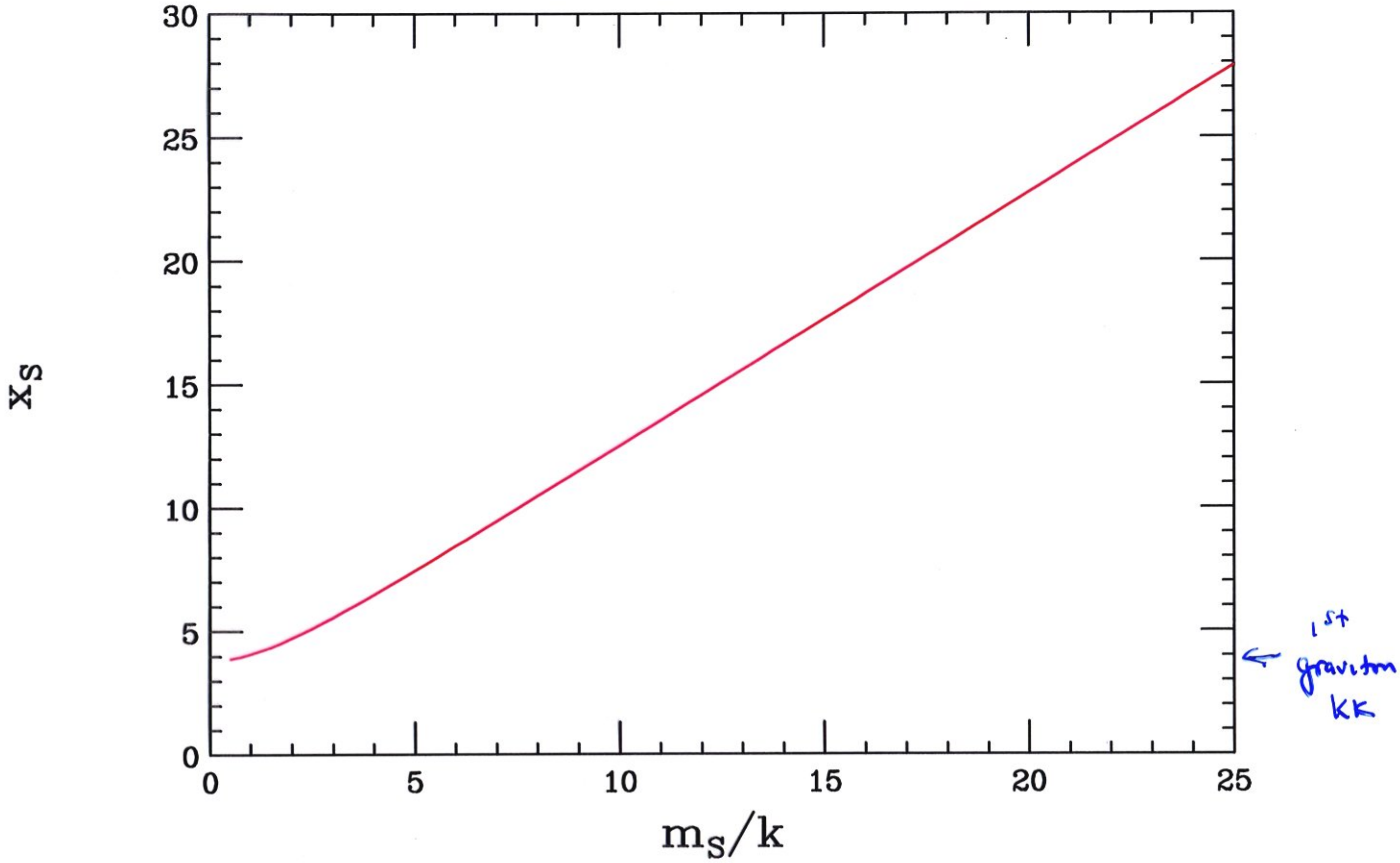
$\sim \left(m_{ext}^2 / 12s \right)$ in amplitude [Fig]

Graviton sector

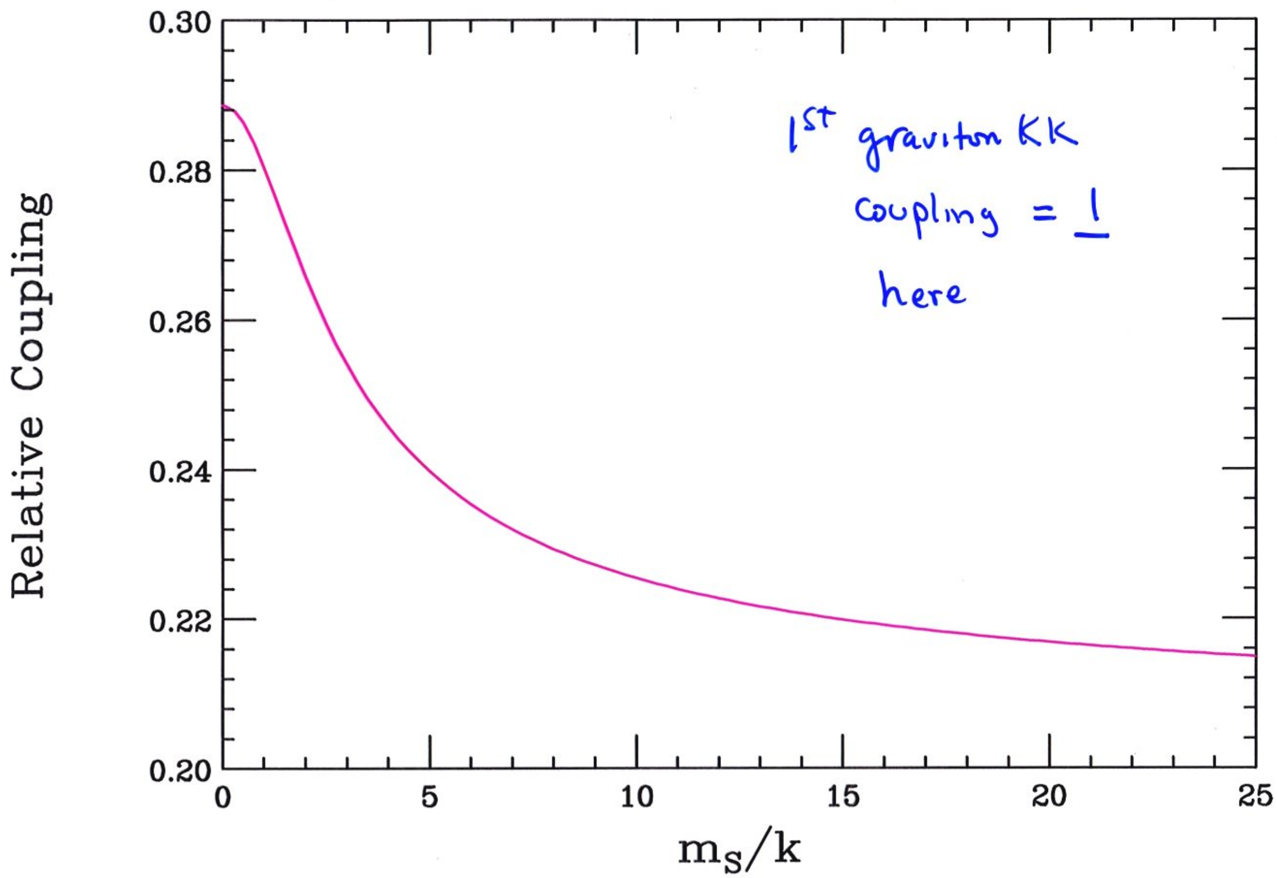
$$\boxed{\bar{M}_{Pl}^2 = \frac{M^3}{k} \cdot \mathcal{H}}$$

$$\boxed{\mathcal{H} = F_R + 36k^2 f_Q + 1000k^4 F_{RQ} + 10080k^6 F_{QQ}}$$

Root For Lightest Scalar State



Bulk mass


0.30
0.28
0.26
0.24
0.22
0.20
0
5
10
15
20
25
Relative Coupling
m_S/k
1st graviton KK
coupling = 1
here

Bulk mass

- KK graviton masses left unaltered as in the ADD case.... (in units of k)**

- In RS, the couplings are unaltered in units of $\Lambda_\pi = \bar{M}_{pl}\, e^{-\pi k r_c}$ ... of couse this has shifted slightly in term of the parameters in the action....

BTW

IF the RS model is S^1/Z_2 w/ 2 branes then $\langle R \rangle = -20k^2$'s + δ-functions + not strictly constant....

.. to eliminate δ', δ'' terms in the EOM @ $R^2_{MNST} = 0$

Furthermore... this is a very strong constraint on F in this case + in fact, Fixes F in the bulk...

$$F = R + \frac{\alpha}{M^2}\, GB$$

uniquely in 5D for RS

the modified δ function coefficients lead to significant pheno alterations in KK masses + couplings for gravitons

- There are no scalars in this case.

** Recall the value of k is shifted as a function of the action parameters (M, Λ_b)

Summary / Conclusions

- It is possible to obtain ADD + RS-like sol's from more general gravity actions

- These lead to subtle alterations in the model predictions → scalar KK towers (not Higgs KK's)

⇒ Besides alterations in model relationships, these include rescaling of "classic" predictions involving graviton KK states

- Experimental observation of any of these effects provides info on a more fundametal theory than EH ...

⇒ Work in progress ...