Extra dimension searches at hadron collider to NLO-QCD

Prakash Mathews

Saha Institute of Nuclear Physics

- TeV scale gravity models
- Need to go beyond Leading Order
- NLO QCD results for Drell-Yan at TeV scale
- PDF and Scale uncertainties
- Summary

Extra Spacial Dimensions

The gauge hierarchy problem has been one of the main motivation for physics beyond the SM

- Apparent weakness of gravity accounted for by
  - Large extra dimensions (ADD)
  - warped extra dimension (RS1)
- Only gravity allowed to propagate the compact extra spacial dimensions, SM constrained on the 3 brane
- In 4-dim, for ADD and RS model, leads to very distinct KK spectrum and their effective interaction with SM particles
- Interaction of KK tower with SM fields on the Brane

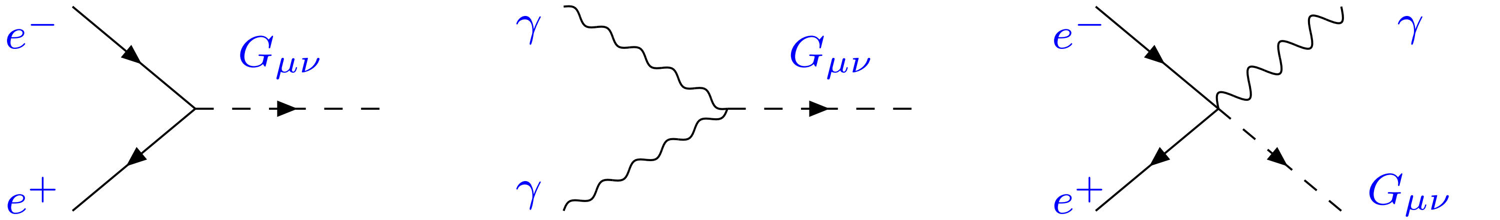
ADD $\quad \mathcal{L} \sim -\frac{1}{M_P} T^{\mu\nu} \sum_{n=0}^{\infty} G_{\mu\nu}^{(n)}$

RS $\quad \mathcal{L} \sim -\frac{1}{M_P} T^{\mu\nu} G_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} G_{\mu\nu}^{(n)}$

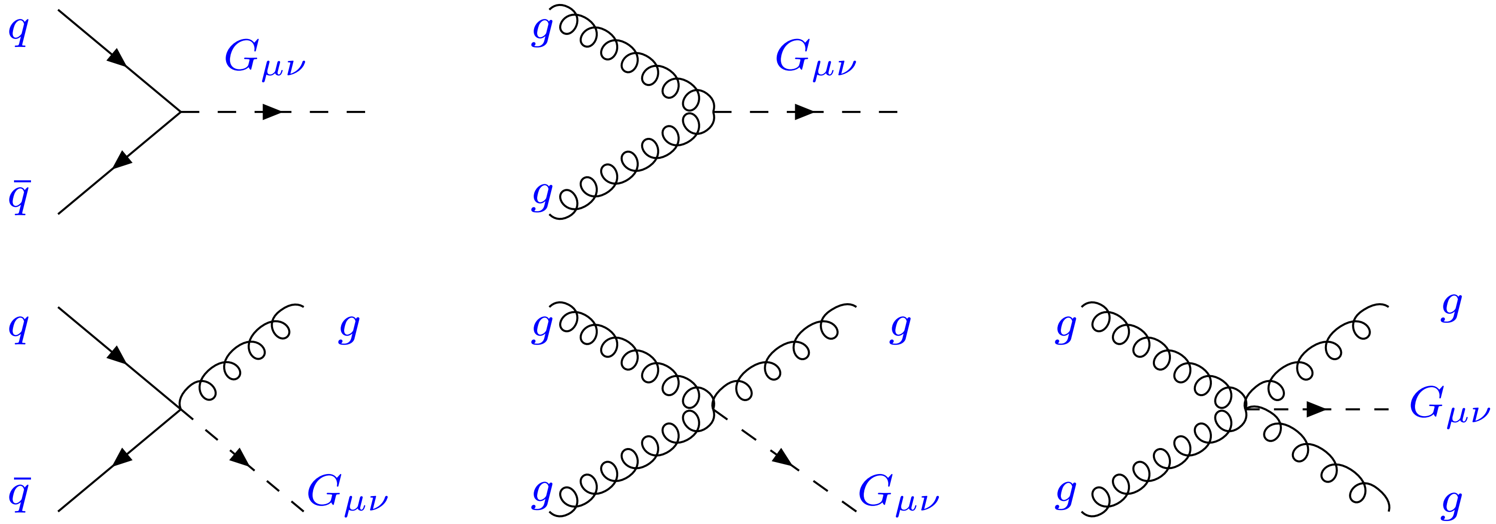
RULE OF THUMB: ATTACH A GRAVITON TO ANY SM LEG OR VERTEX

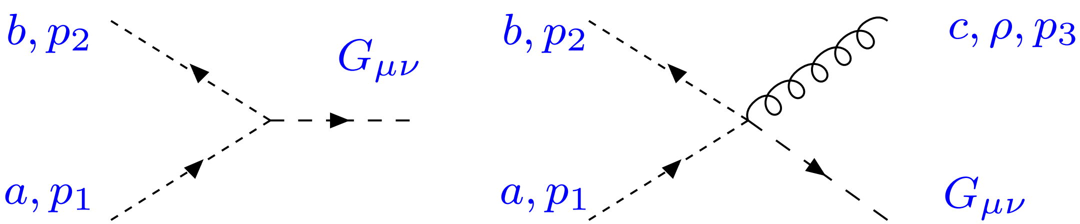
Feynman Rules

- QED



- QCD





Ghost Couplings

with V Ravindran et. al. JHEP 0408 (2004) 048

Large Extra dimension

Physics of extra dim is the physics of the KK spectrum and in the ADD case we have to sum over the tower of KK modes to get an observable effect

- Massless graviton and KK modes couple with SM fields with coupling M_P^{-1}
- Effects of KK modes
  - Real KK modes emission
  - Virtual KK modes exchange
- Since the KK modes are M_P^{-1} suppressed one has to sum over the tower of KK modes to get observable effect— an individual ADD KK mode can not be detected
  - Real case $\Rightarrow$ Inclusive production cross section of KK mode

  Phase space enhancement compensates M_P^{-1} suppression for production of a single KK mode. All states upto $m_n = \sqrt{s}$ can be emitted— integral cut off by kinematics

  - Virtual case $\Rightarrow$ contact interaction

  In contrast to real KK emission the summation of virtual KK modes depends on the UV cutoff

Virtual Exchange

- Being virtual all states in fact contribute, not kinematically bound— but bounded by the validity of the effective theory
- KK density of state

$$\rho(m_{\vec{n}}) = \frac{R^d m_{\vec{n}}^{d-2}}{(4\pi)^{d/2}\Gamma(d/2)}$$

- Sum over KK mode propagator

$$\sum_{\vec{n}} \frac{1}{s - m_{\vec{n}}^2 + i\epsilon} = \int_0^\infty dm_{\vec{n}}^2 \, \rho(m_{\vec{n}}) \, \frac{1}{s - m_{\vec{n}}^2 + i\epsilon}$$

- Dominated by UV contribution:

$d = 1$ convergent

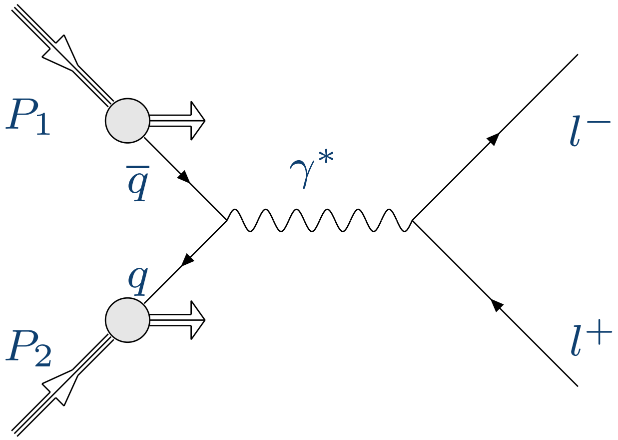
$d = 2$ $\ln(\frac{s}{\Lambda_c^2})$

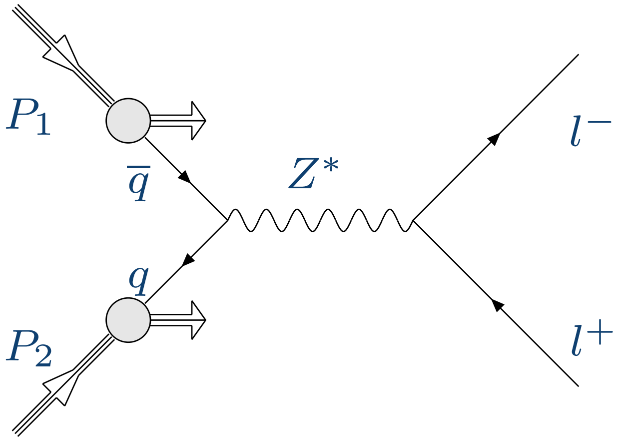
$d > 2$ $\frac{\Lambda_c^{d-2}}{M_D^{d+2}} \Rightarrow \frac{1}{M_D^4}$

LOOKED FOR EXTRA DIMENSIONS @ LEP2
SEARCHES ON @ TEVATRON & FUTURE IS @ LHC/LC

Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$

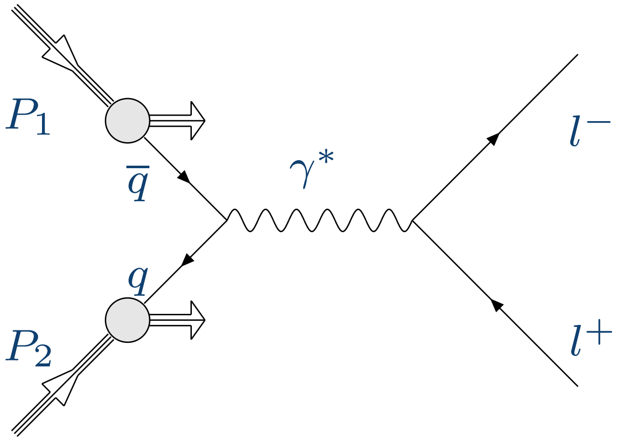


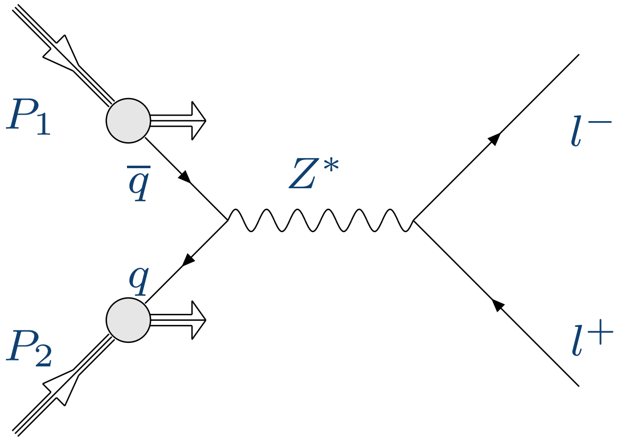


SM

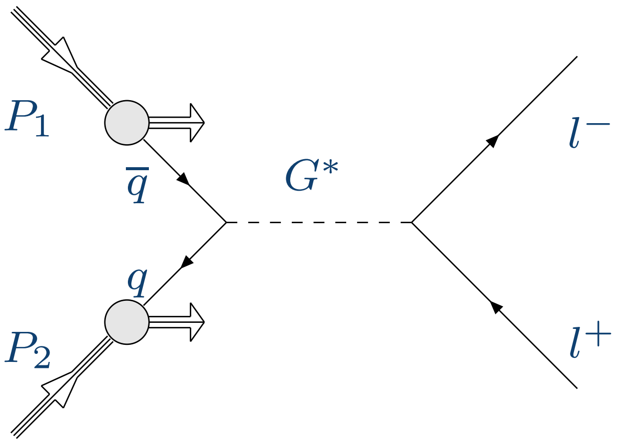
Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$



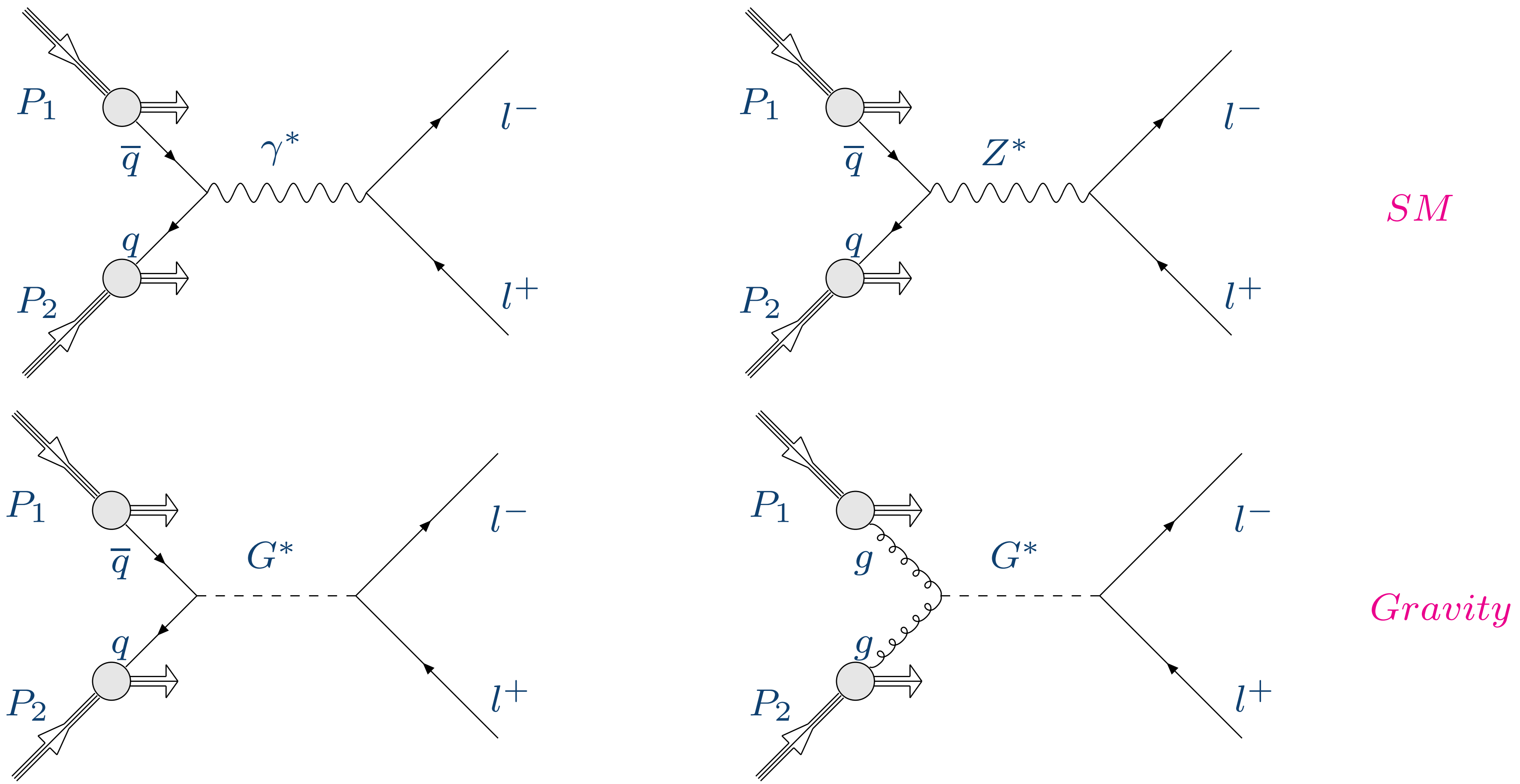


SM



Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \rightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$



Parton Model

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, Q^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, Q^2, \mu_F\right)$$

- Partonic cross section perturbatively calculable:

$$d\hat{\sigma}^{ab}\left(z, Q^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R^2)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, Q^2, \mu_F, \mu_R\right)$$

- Non-perturbative partonic flux:

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

- $f_a^{P_1}(x, \mu_F)$ are Parton distribution functions, x is the partonic momentum fraction
- μ_R is the Renormalisation scale
- μ_F is the Factorisation scale

Source of Theoretical Uncertainties

- Renormalisation scale:
  Due to UV divergence at beyond Leading Order

$$\alpha_s \longrightarrow \alpha_s(\mu_R^2)$$

- Factorisation scale:
  Originate from light quarks and massless gluon. Parton distribution functions are renormalised at the factorisation scale μ_F

$$f_a(x) \longrightarrow f_a(x, \mu_F^2) \qquad\qquad a = q, \bar{q}, g$$

- Parton Distribution Functions:
  Not calculable but extracted from experiments in some factorisation scheme by various groups by global fits to available data on DIS, DY and other hadronic process
- Observables are "free" of μ_R and μ_F
- "Fixed order" perturbative results depend on μ_R and μ_F
- Can in principle give large uncertainties

IT IS HENCE IMPORTANT FOR EXTRA DIMENSION SEARCHES TO HAVE BETTER CONTROL OVER THE THEORETICAL UNCERTAINTIES

Mass Factorisation

Divergent NLO correction involve loop and phase space integrals— regularised using dimensional regularisation $n = 4 + \epsilon$

- IR divergences:
  IR divergences appears when $|k| \to 0$, cancels between virtual and real diagrams (Bloch-Nordsieck Theorem)
- Collinear divergences:
  • Final state collinear: singularities cancel because we are considering a final state inclusive process summing over all experimentally indistinguishable final states (KLM Theorem)

  • Initial state collinear: singularities are left over— leads to mass factorisation

Mass Factorisation Theorem:

- Collinear divergences can be factored out of the sub process cross section
- Mass singularities encountered in QCD are process independent

Universality of the collinear singularities enables a process independent way to absorb them into parton distributions

Politzer NPB 129 (1977) 310 $\cdots$

$\cdots$ Collins, Sopper, Sterman

Natural question

- Is this leading order result stable in the perturbation theory?
- Why should we ask this question at all here?
- Because we are dealing with partons such as quark and gluons at the initial state which are sensitive to Factorisation scale even at Leading order

$$d\sigma^{PP}\left(x,Q^2\right) = \sum_{ab}\int_x^1 \frac{dz}{z}\Phi_{ab}^{(0)}\left(z,Q^2,\mu_F^2\right)\sigma_{ab}^{(0)}\left(\frac{x}{z},Q^2,M_S^2\right) + \cdots$$

Leading Partonic cross section is "independent" of μ_F

- Uncertainty can come from Factorisation scale μ_F through the LO flux $\Phi_{ab}^{(0)}\left(z,\mu_F\right)$
- How serious is it?

Next-to-Leading Order process involving Gravity

- Compute NLO QCD corrections to LO DY processes

$$d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}^{(0)}_{ab}(\hat{s}, Q^2, \mu_F^2)\left[1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\Delta^{(1)}_{ab}(\hat{s}, Q^2, \mu_F^2, \mu_R^2)\right]$$

- Soft and collinear divergences are regulated in dimensional regularisation $n = 4 + \varepsilon$
- Collinear mass factorisation is done in $\overline{MS}$ scheme
- Coefficient functions to NLO evaluated for

$$\frac{d\sigma(Q)}{dQ} \qquad \frac{d\sigma(Q,Y)}{dQ\ dY} \qquad \frac{d\sigma(Q,\cos\theta)}{dQ\ d\cos\theta}$$

with V. Ravindran et. al. NPB713 (2005) 333
with V. Ravindran hep-ph/0507250

- PDF and scale dependence

with M. C. Kumar et. al. in preparation

Contributing Subprocess

Leading Order:

Standard Model	Gravity
$q + \bar{q} \to \gamma/Z$	$q + \bar{q} \to G$ $g + g \to G$

Next-to-Leading Order:

Standard Model	Gravity
$q + \bar{q} \to \gamma/Z + q$, $q + \bar{q} \to \gamma/Z$ + one loop $q + g \to \gamma/Z + q$, $\bar{q} + g \to \gamma/Z + \bar{q}$	$q + \bar{q} \to G + q$, $q + \bar{q} \to G$ + one loop $q + g \to G + q$, $\bar{q} + g \to G + \bar{q}$ $g + g \to G + g$, $g + g \to G$ + one loop

Mass Factorisation

Drell-Yan coefficient function ($\overline{MS}$ scheme)

$$\bar{\Delta}^i_{ab}(z, Q^2, \frac{1}{\varepsilon}) \;=\; \sum_{c,d} \Gamma_{ca}(z, \mu_F^2, \frac{1}{\varepsilon}) \otimes \Gamma_{db}(z, \mu_F^2, \frac{1}{\varepsilon}) \otimes \Delta^i_{cd}(z, Q^2, \mu_F^2)$$

(**Bare**) (**Mass Factorised**)

$$\Delta^i_{ab} = \Delta^{(0),i}_{ab} + \frac{\alpha_s(\mu_R^2)}{4\pi}\,\Delta^{(1),i}_{ab}$$

$$\Gamma_{cd}(z, \mu_F) \;=\; \delta_{cd}\delta(1-z) + \frac{\alpha_s(\mu_R^2)}{4\pi}\,\frac{1}{\varepsilon}\left(\frac{\mu_F^2}{\mu_R^2}\right)^{\varepsilon/2} P^{(0)}_{cd}(z)$$

$P^{(0)}_{cd}(z)$ LO Altarelli-Parisi splitting functions

Δ^i_{ab} TO BE EVALUATED ORDER BY ORDER IN PERTURBATION THEORY

COEFFICIENT FUNCTIONS INDEPENDENT OF ADD OR RS MODEL

Observables

Distributions:

$$\frac{d\sigma(Q)}{dQ} \qquad \left.\frac{d\sigma(Q,Y)}{dQ\;dY}\right|_{Q_0} \qquad \left.\frac{d\sigma(Q,\cos\theta)}{dQ\;d\cos\theta}\right|_{Q_0}$$

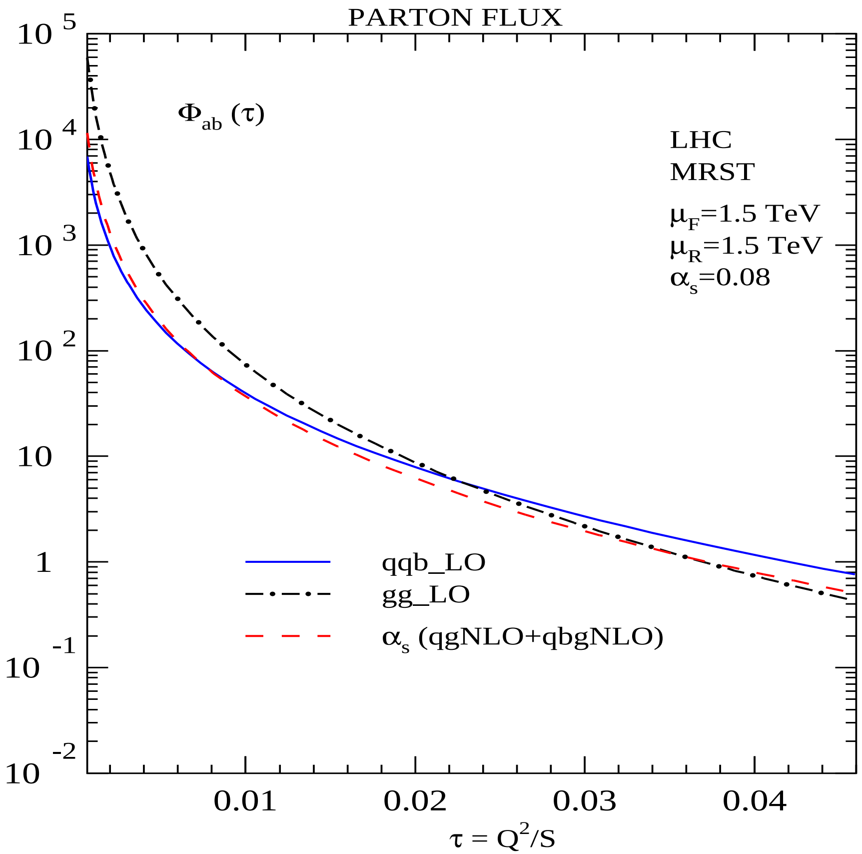
K-Factor:

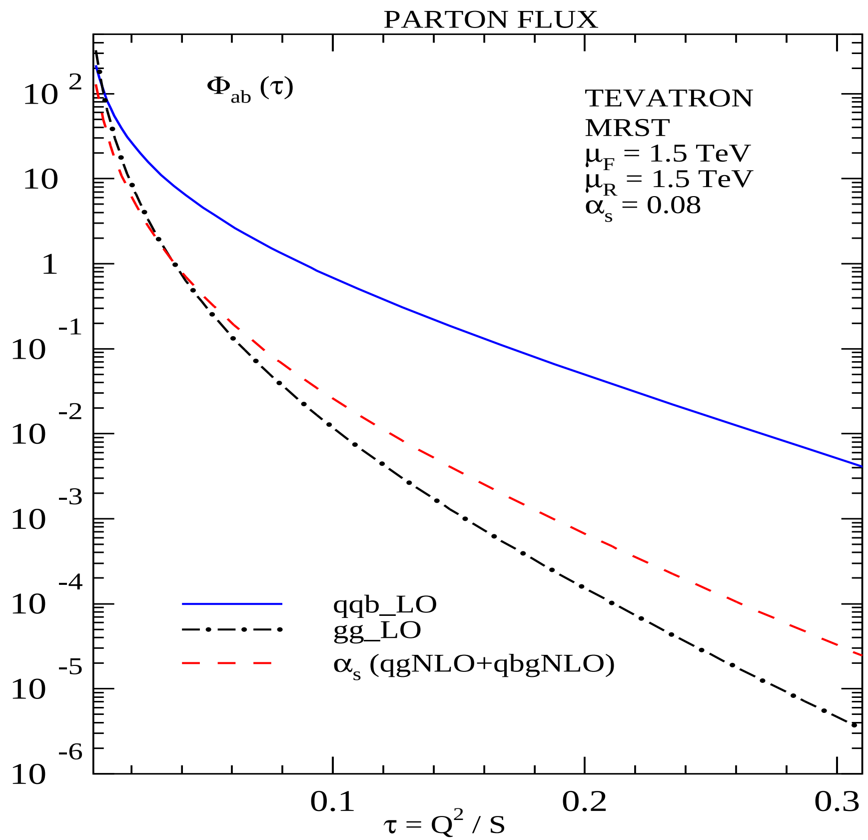
$$K = \left[\frac{d\sigma_{LO}(Q)}{dQ}\right]^{-1}\left[\frac{d\sigma_{NLO}(Q)}{dQ}\right]$$

- Scale dependence (μ_F & μ_R) variation of above observables in going from LO to NLO
- Dependence of above observables on the various PDFs

Flux at LHC and Tevatron

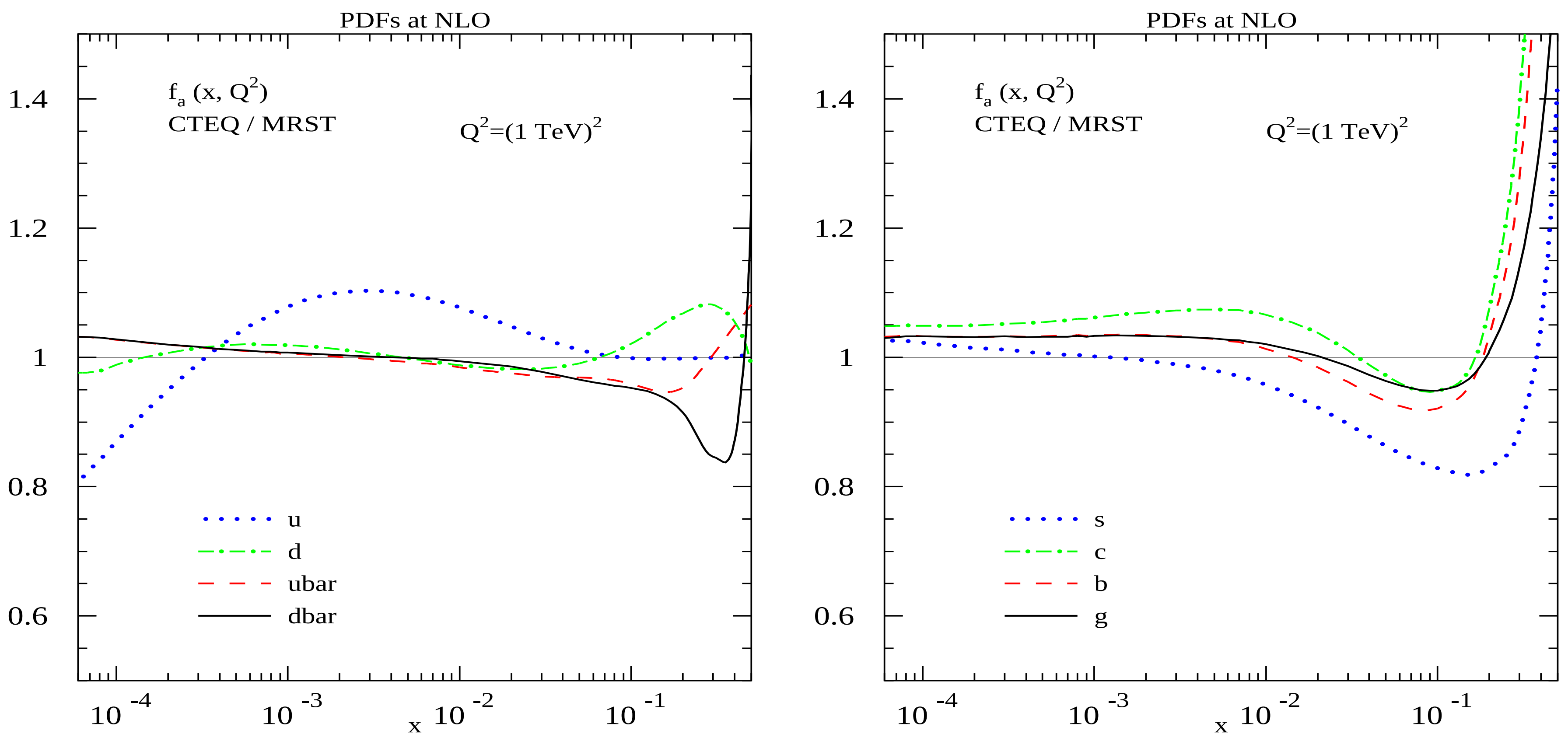
$$\Phi_{ab}(\tau,\mu_F) \quad = \quad \int_\tau^1 \frac{dz}{z} f_a\left(z,\mu_F\right) f_b\left(\frac{\tau}{z},\mu_F\right) \qquad \tau = \frac{Q^2}{S}$$



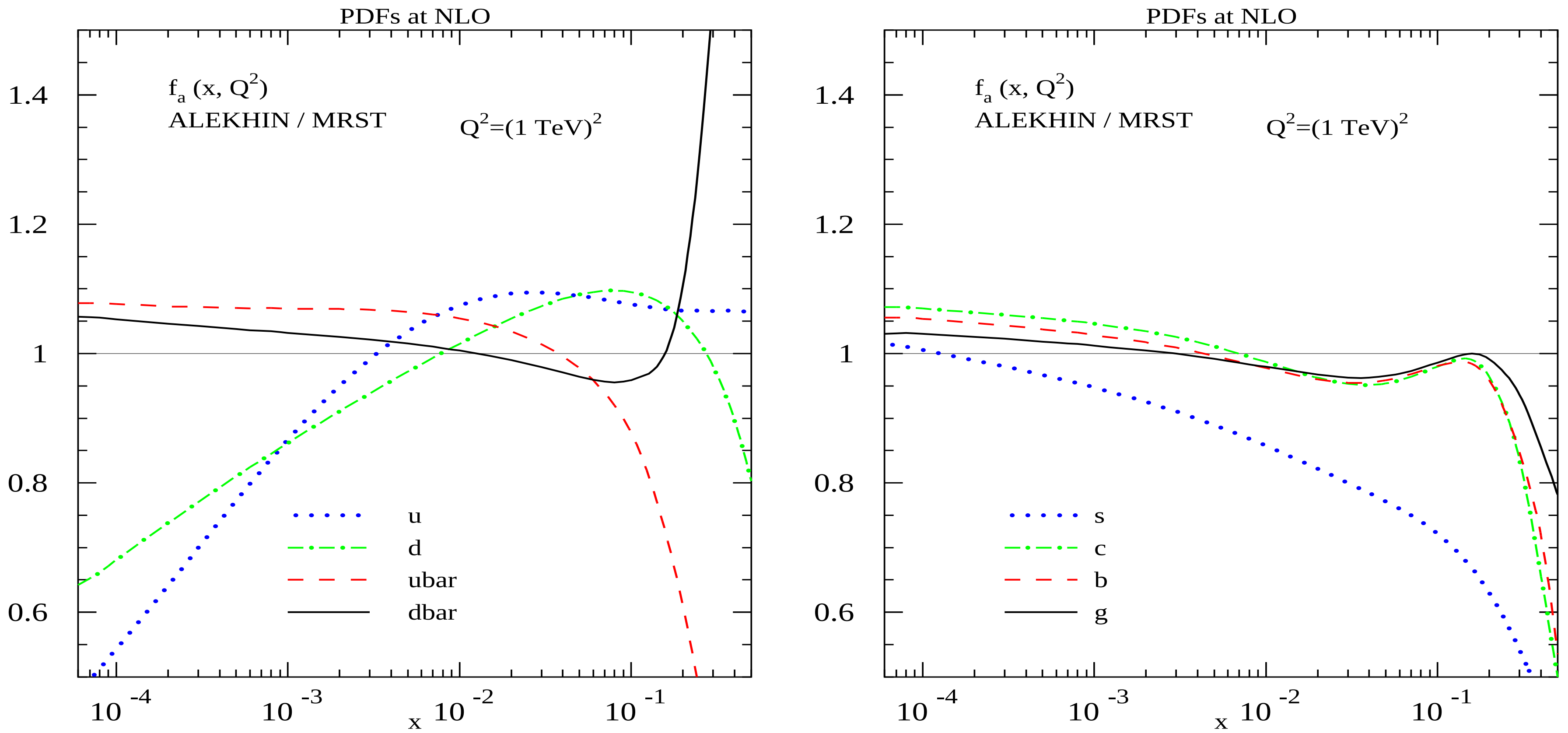


- Gluon flux dominates at LHC
- $q\ \bar{q}$ flux is largest at Tevatron

CTEQ/MRST

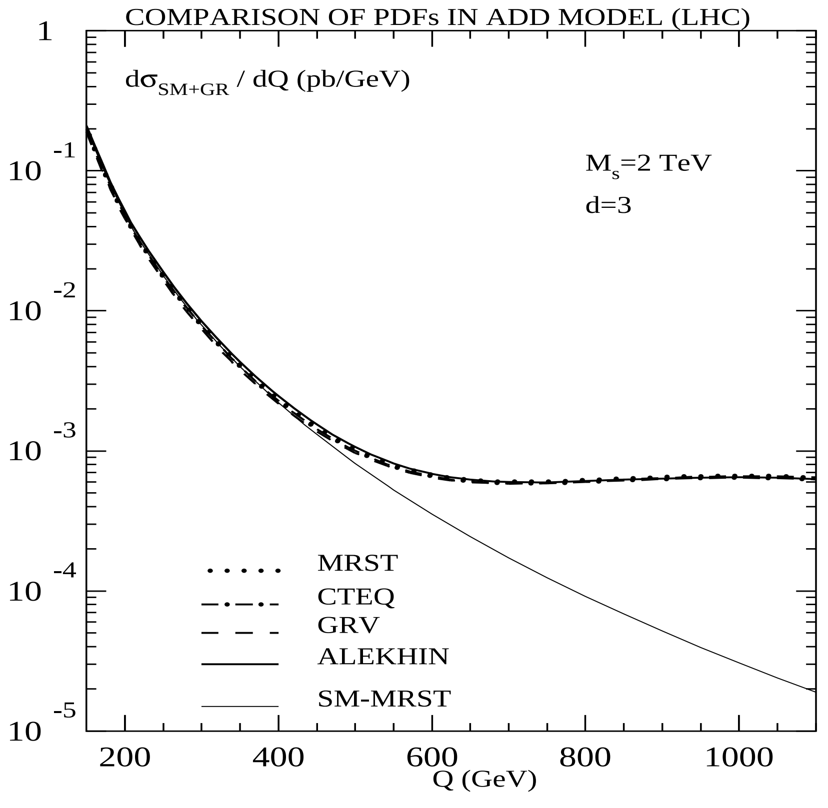

PDFs at NLO
$f_a\,(x, Q^2)$
CTEQ / MRST
$Q^2=(1\ \text{TeV})^2$
1.4
1.2
1
0.8
0.6
u
d
ubar
dbar
10^{-4}
10^{-3}
x
10^{-2}
10^{-1}
PDFs at NLO
$f_a\,(x, Q^2)$
CTEQ / MRST
$Q^2=(1\ \text{TeV})^2$
1.4
1.2
1
0.8
0.6
s
c
b
g
10^{-4}
10^{-3}
x
10^{-2}
10^{-1}

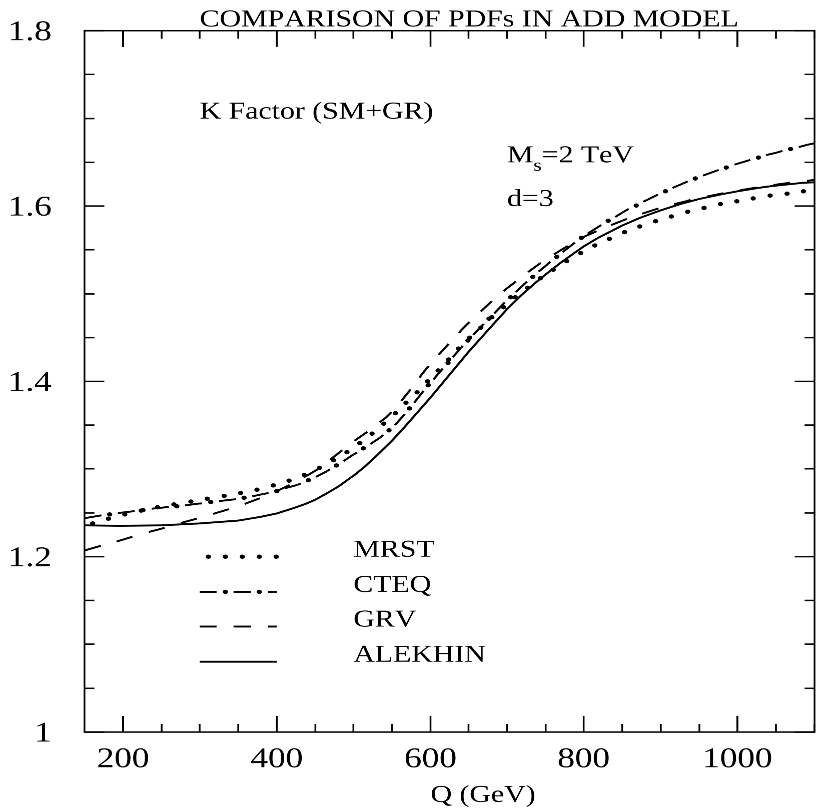
ALEKHIN/MRST



Invariant lepton pair mass Q distributions

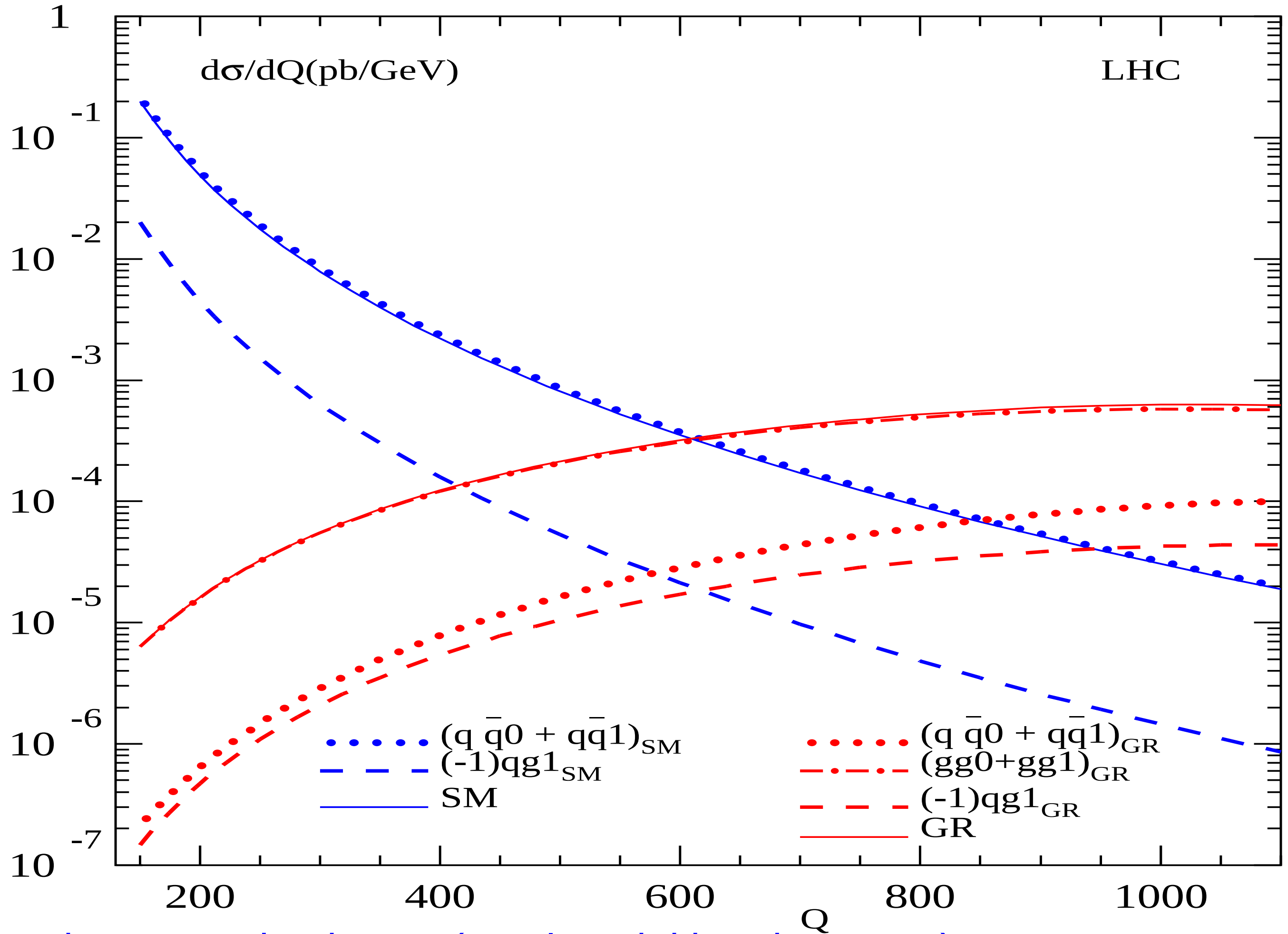
$$\frac{d\sigma(Q)}{dQ}$$





- LHC: ADD invariant mass distribution and K-factor for various PDFs

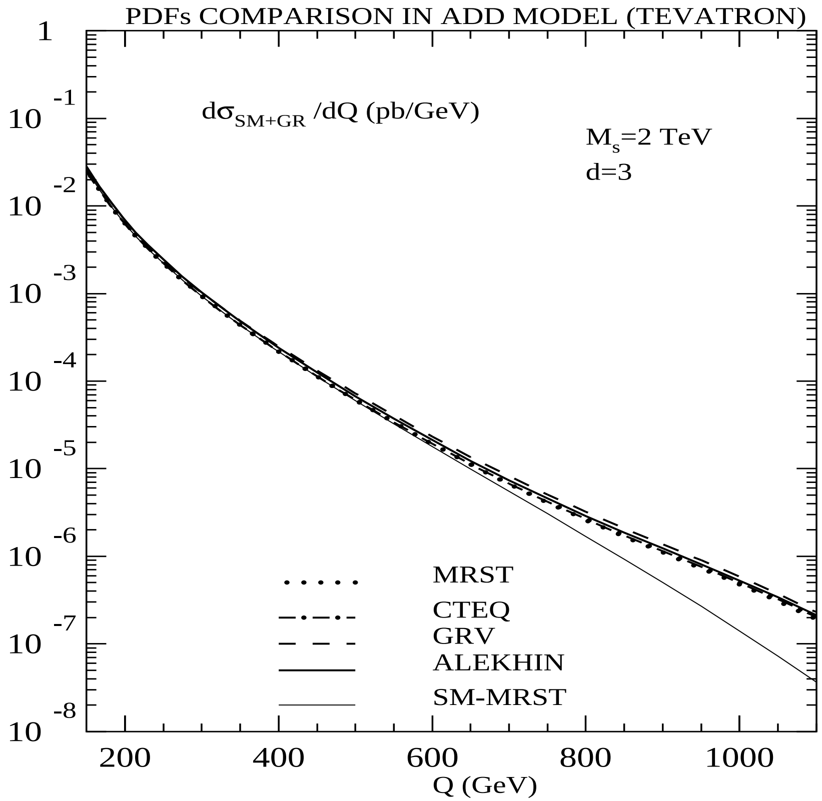
Contributions at LHC

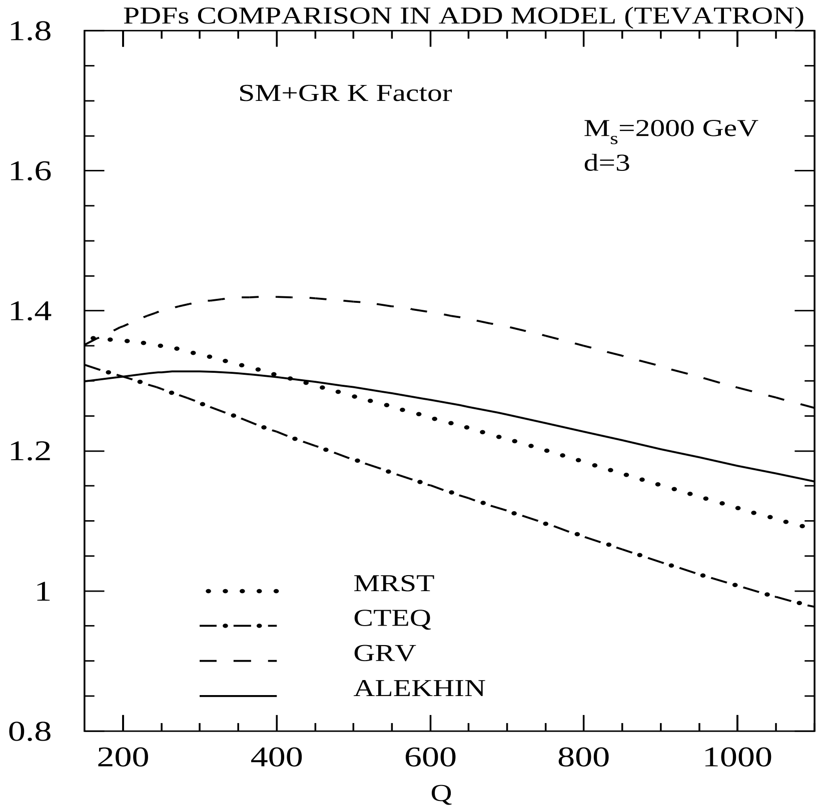


- SM the $q\bar{q}$ subprocess dominates (no gluon initiated process)
- Gravity mediated process gg sub process initiated process dominates and substantially contributes to the cross section at large Q^2

Invariant lepton pair mass Q distributions

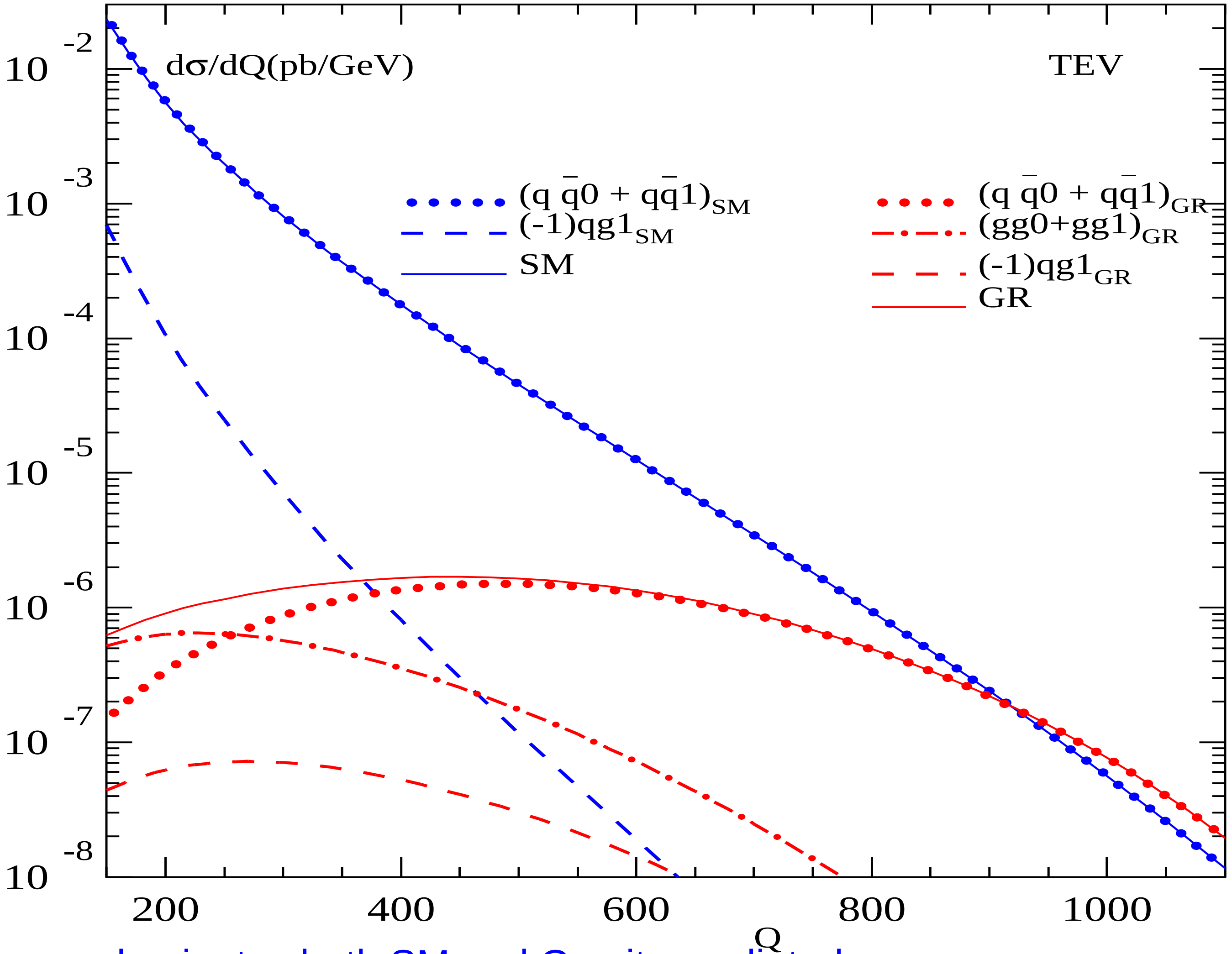
$$\frac{d\sigma(Q)}{dQ}$$





- TEVATRON: ADD invariant mass distribution and K-factor for various PDFs

Contributions at Tevatron

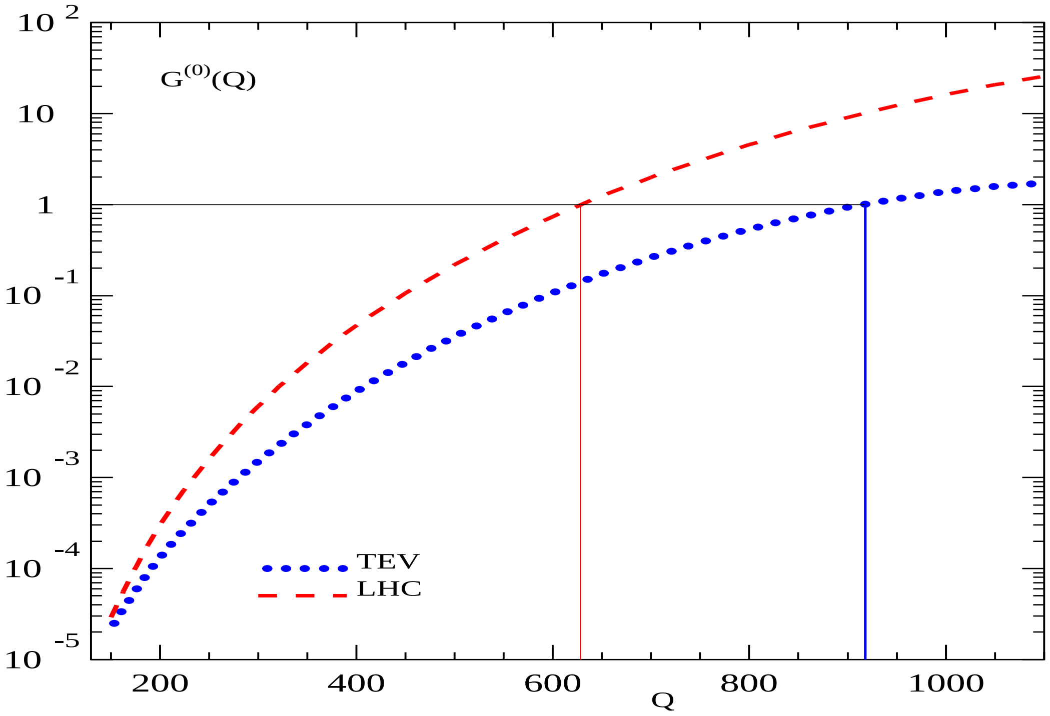


- $q\bar{q}$ sub process dominates both SM and Gravity mediated process

K-Factor

$$K^{(SM+GR)}(Q) \quad = \quad \frac{K^{SM} + K^{GR}\, G^{(0)}}{1 + G^{(0)}}$$

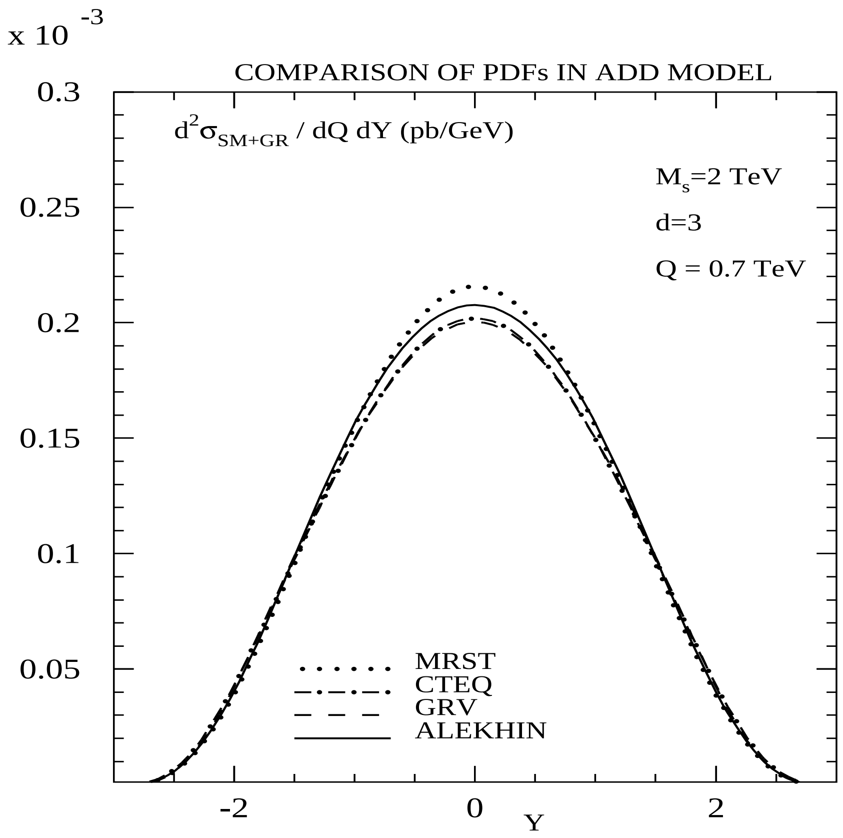
$$G^{(0)}(Q) = \left[\frac{d\sigma_{LO}^{SM}(Q)}{dQ}\right]^{-1} \left[\frac{d\sigma_{LO}^{GR}(Q)}{dQ}\right]$$

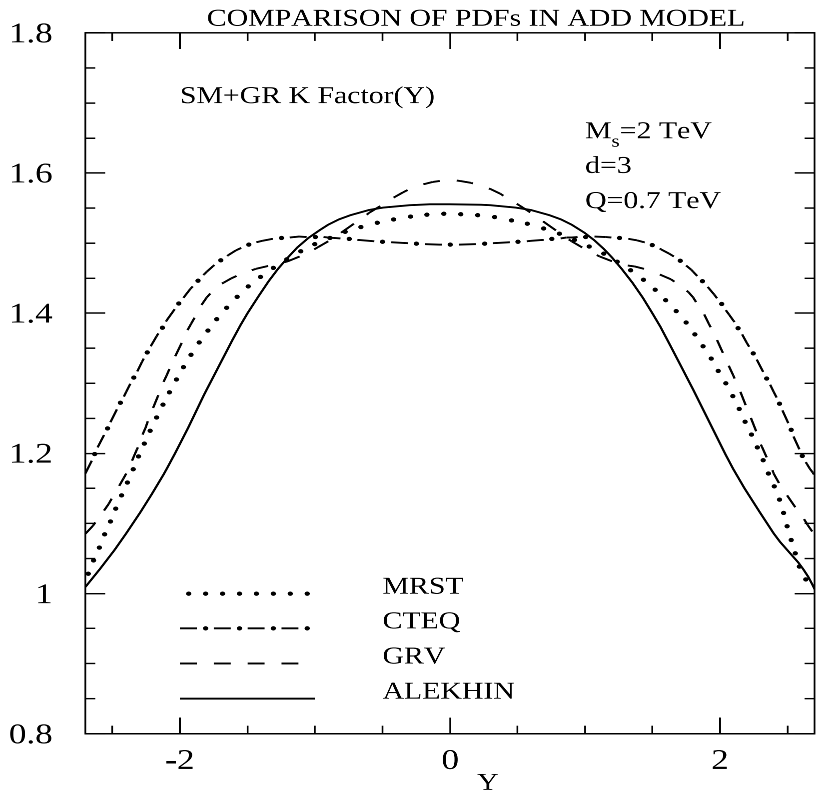


- $G^{(0)}(Q)$ behavior is governed by a competing 'couplings' and PDF flux at LHC and TEV
- At high Q when Gravity contribution becomes comparable to SM, the PDF flux dictates the proceedings

Rapidity distribution

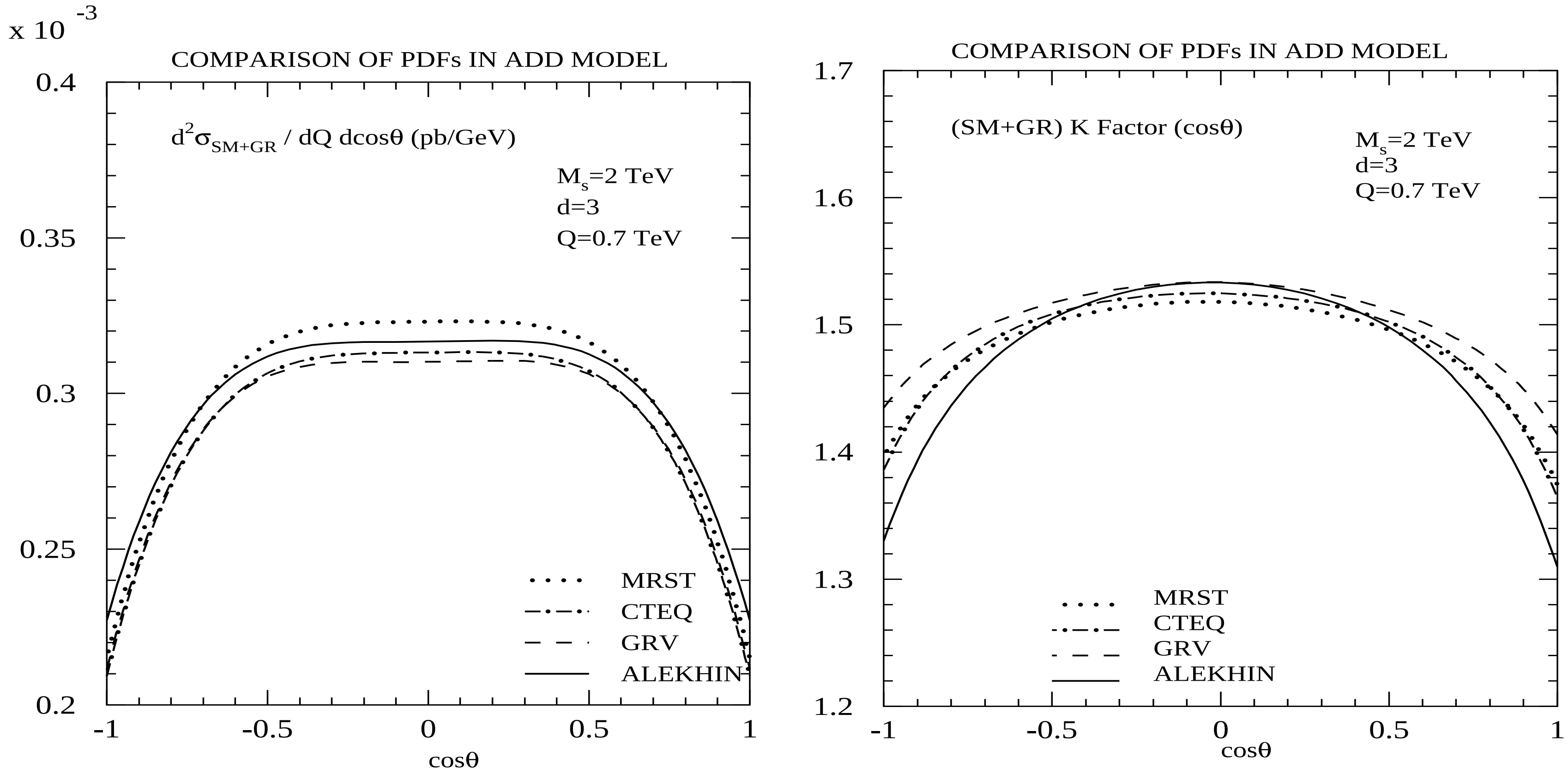
$$\left.\frac{d\sigma(Q,Y)}{dQ\,dY}\right|_{Q_0}$$





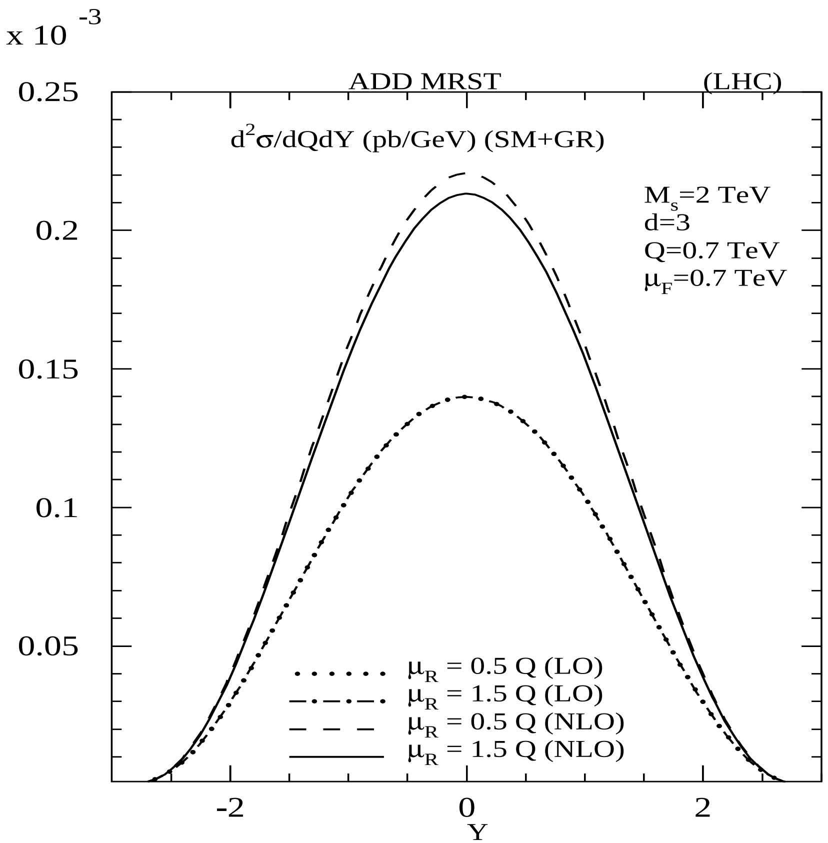
Angular distribution

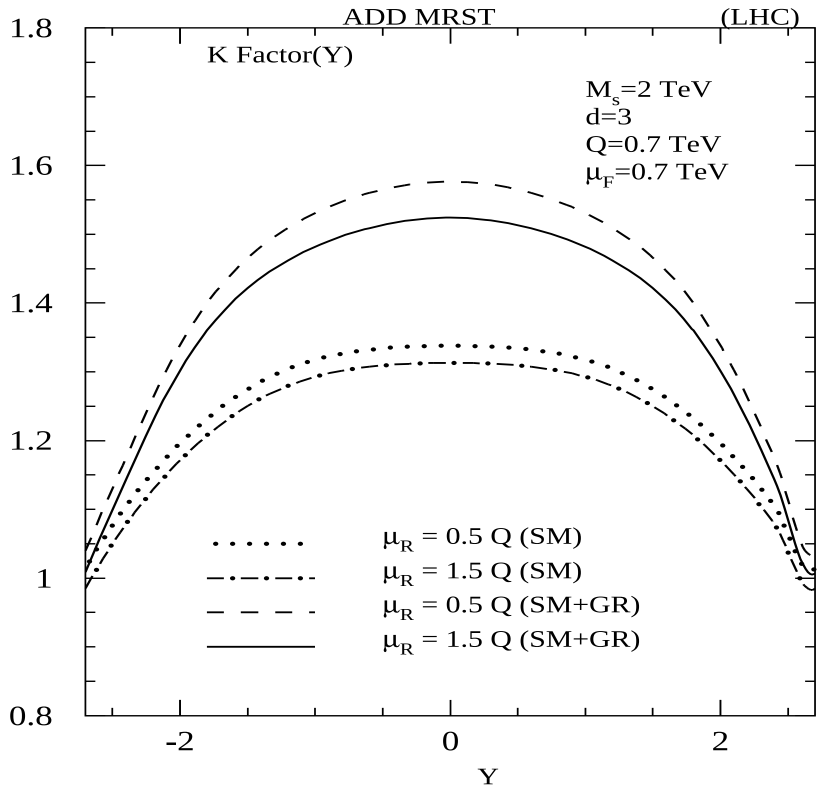
$$\left.\frac{d\sigma(Q,\cos\theta)}{dQ\;d\cos\theta}\right|_{Q_0}$$



Renormalisation scale

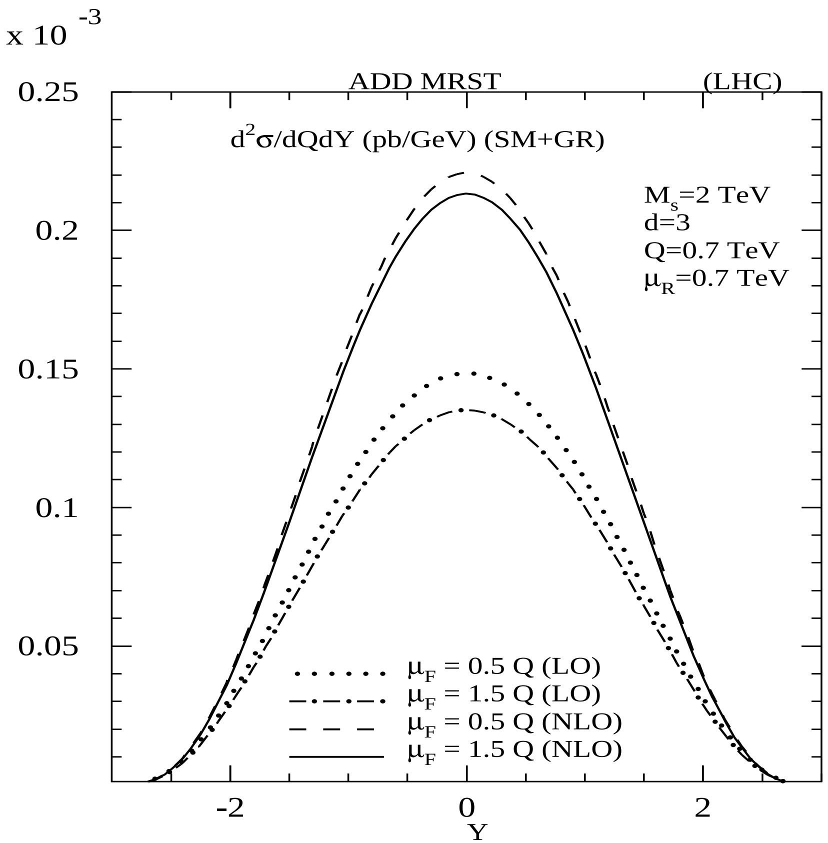
$$\left.\frac{d\sigma(Q,Y)}{dQ\,dY}\right|_{Q_0}$$

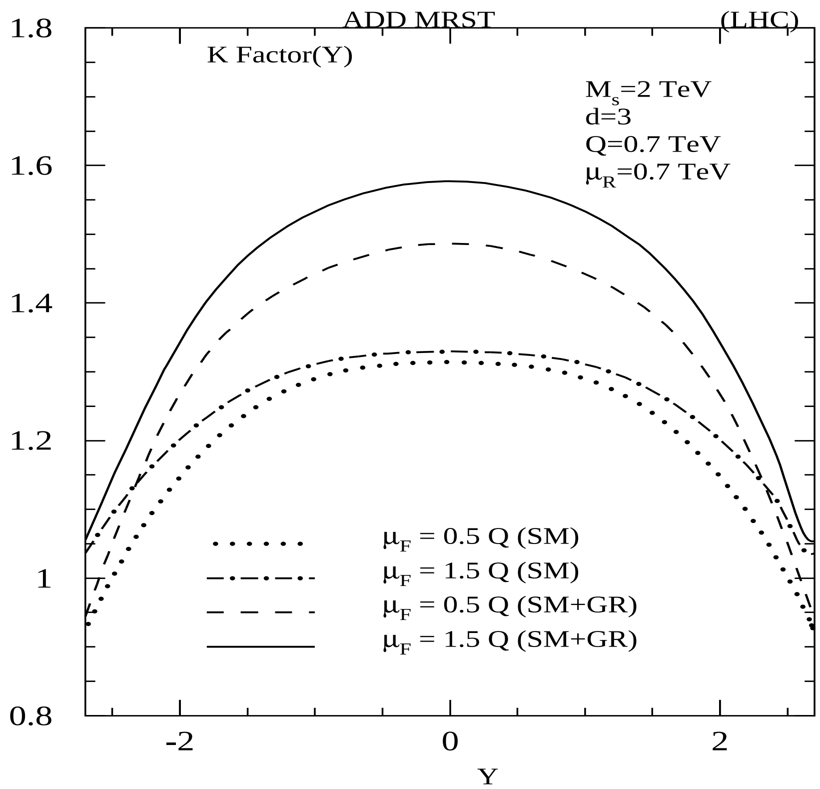




Factorisation scale

$$\left.\frac{d\sigma(Q,Y)}{dQ\,dY}\right|_{Q_0}$$





Warped Extra Dimension

- Non-Factorisable geometry, 5-dim AdS space— constant negative curvature

$$ds^2 = e^{-2\mathcal{K}r_c|\phi|}\ \eta_{\mu\nu}\ dx^\mu dx^\nu + r_c^2\ d\phi^2$$

- Interaction of RS KK tower with SM fields on the TEV Brane

$$\mathcal{L} \quad \sim \quad -\frac{1}{M_P} T^{\alpha\beta}\ G^{(0)}_{\alpha\beta} - \frac{1}{\Lambda_\pi} T^{\alpha\beta} \sum_{n=1}^{\infty} G^{(n)}_{\alpha\beta}$$

$$\Lambda_\pi \quad \sim \quad M_P\ e^{-\mathcal{K}r_c\pi} \sim \mathcal{O}(\mathrm{TeV})$$

- Zero mode decouples (massless graviton) Newtonian Gravity intact M_P^{-1}
- Excited massive KK modes couple to SM with TeV^{-1} suppression

$$M_n = x_n\ \mathcal{K}\ e^{-\mathcal{K}r_c\pi} \equiv x_n\ m_0$$

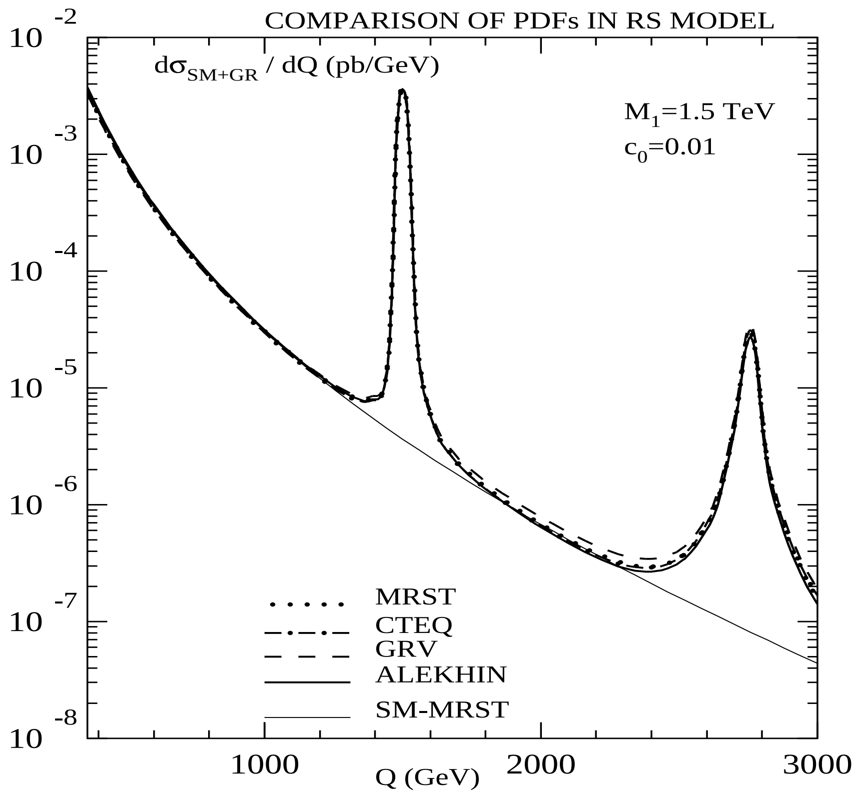
- Two basic parameters of the RS model are

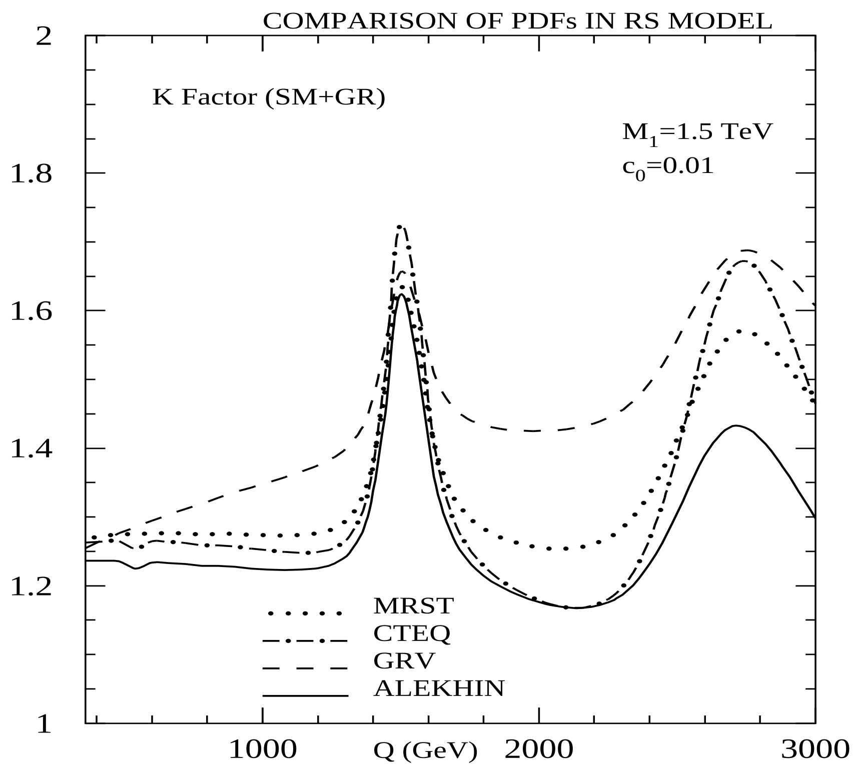
$$m_0 = \mathcal{K}\ e^{-\mathcal{K}r_c\pi} \qquad\qquad c_0 = \frac{\mathcal{K}}{M_P}$$

PHENOMENOLOGICAL IMPLICATION VERY DISTINCTIVE COMPARED TO ADD

RS Scenario Results

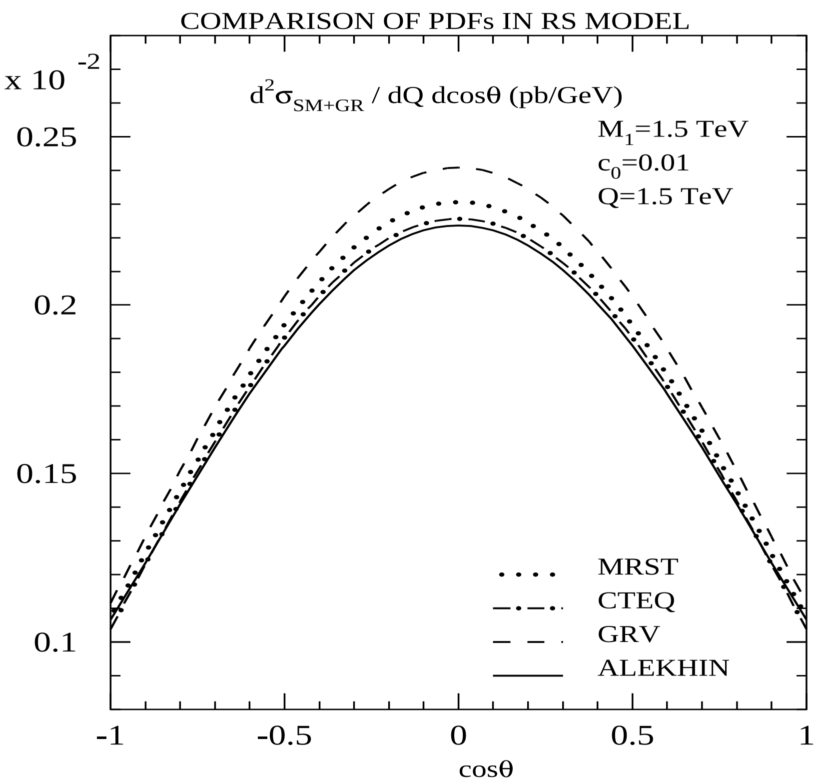
$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) \quad = \quad \frac{c_0^2}{m_0^2}\sum_{n=1}^{\infty}\frac{1}{s - M_n^2 + iM_n\Gamma_n} \quad \equiv \frac{c_0^2}{m_0^4}\lambda\left(\frac{Q}{m_0}\right)$$

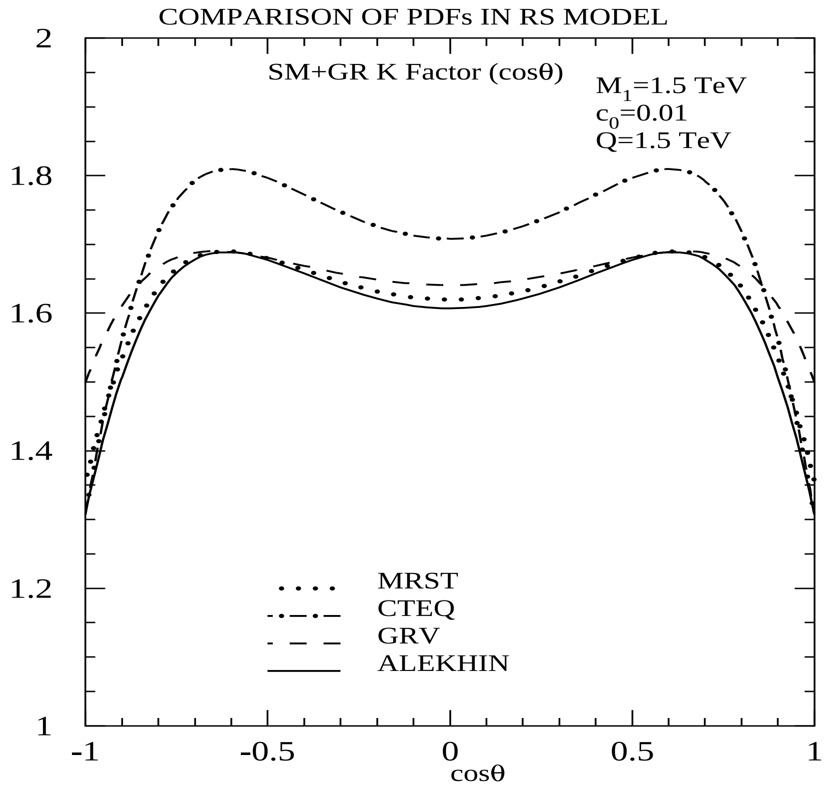




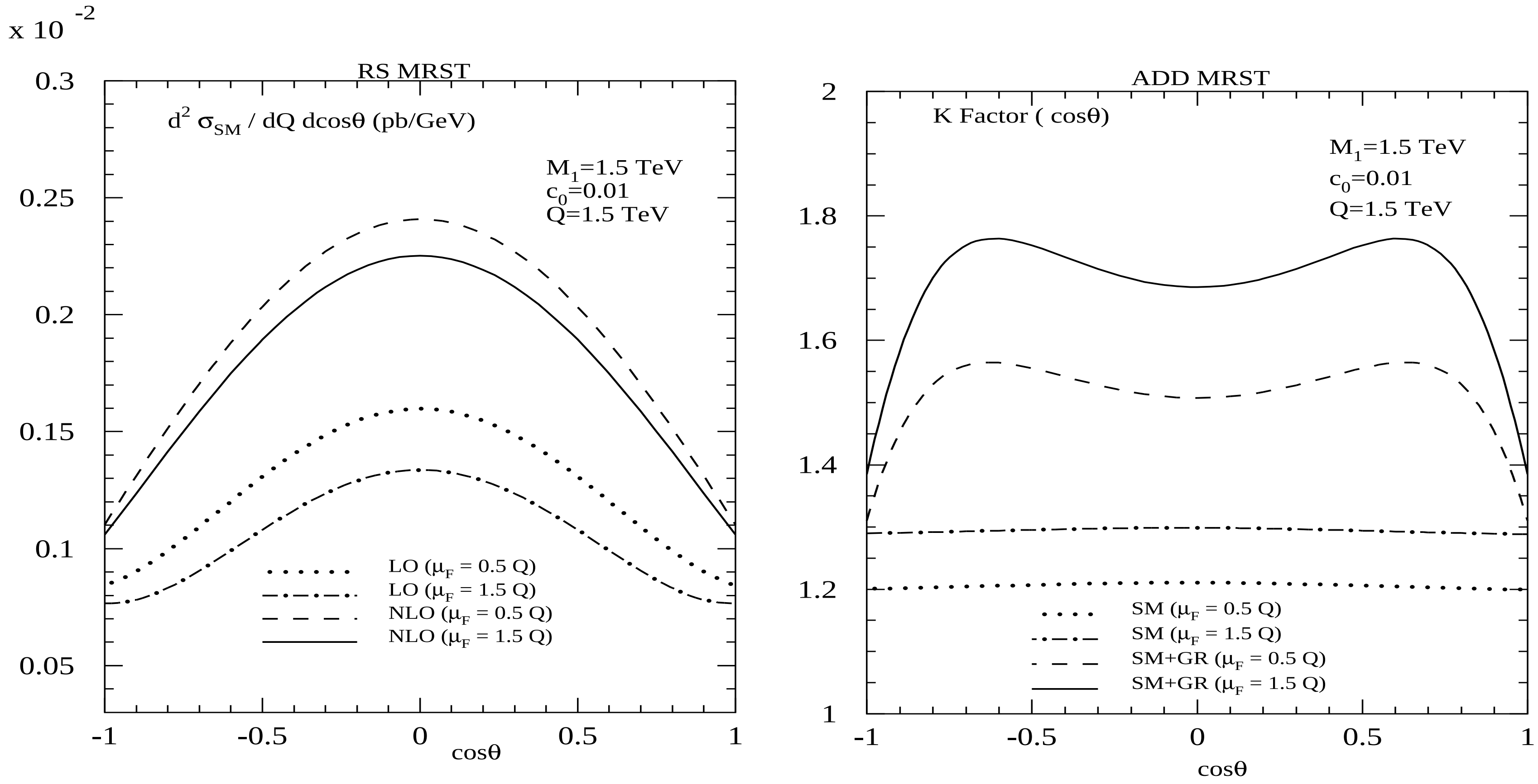
- Away from the resonance region gravity contribution is negligible
- K-Factor behavior can be understood from the $G^{(0)}$ behavior for the RS model

Angular distribution:





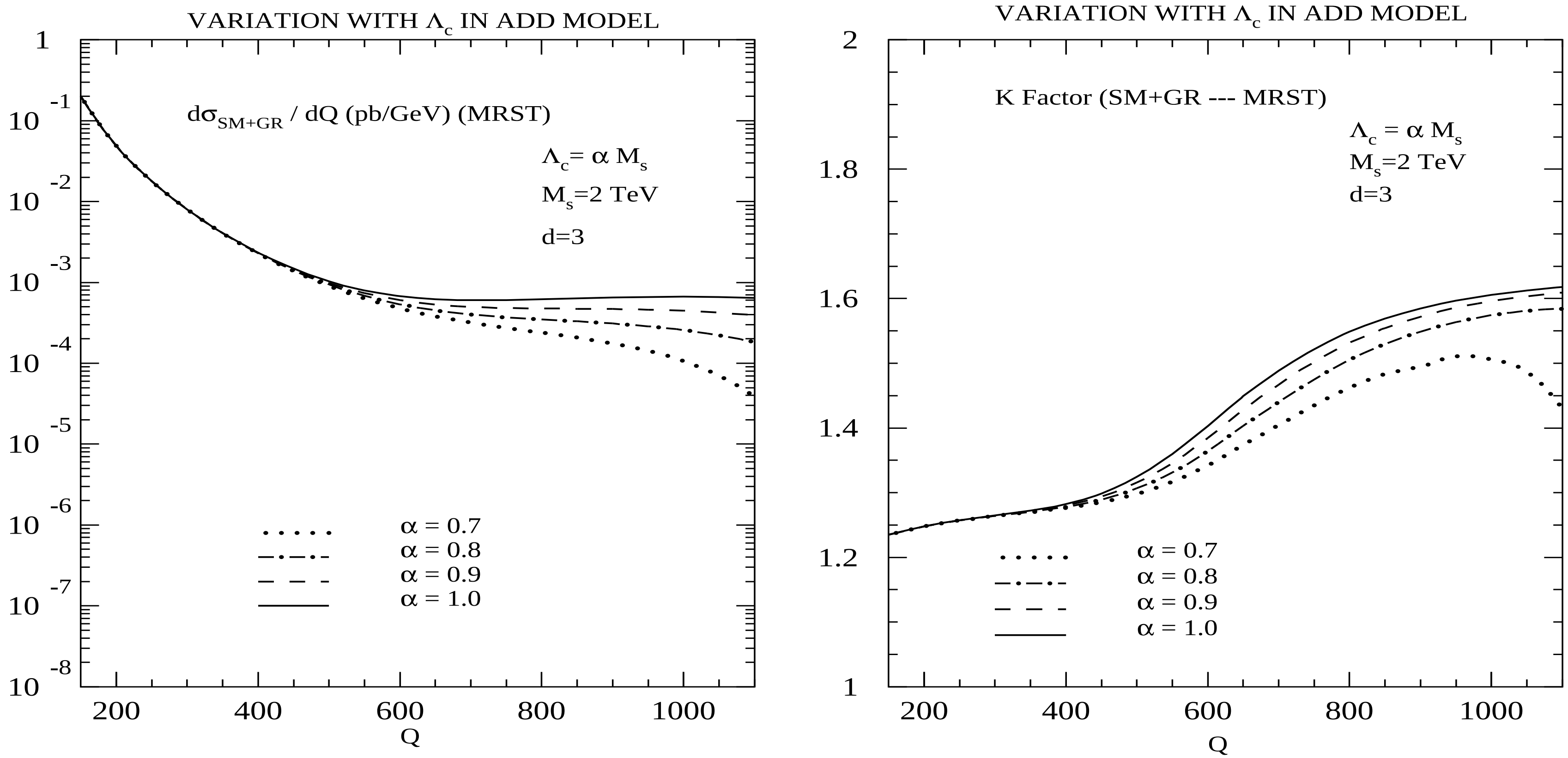
Factorisation scale dependence of angular distribution:



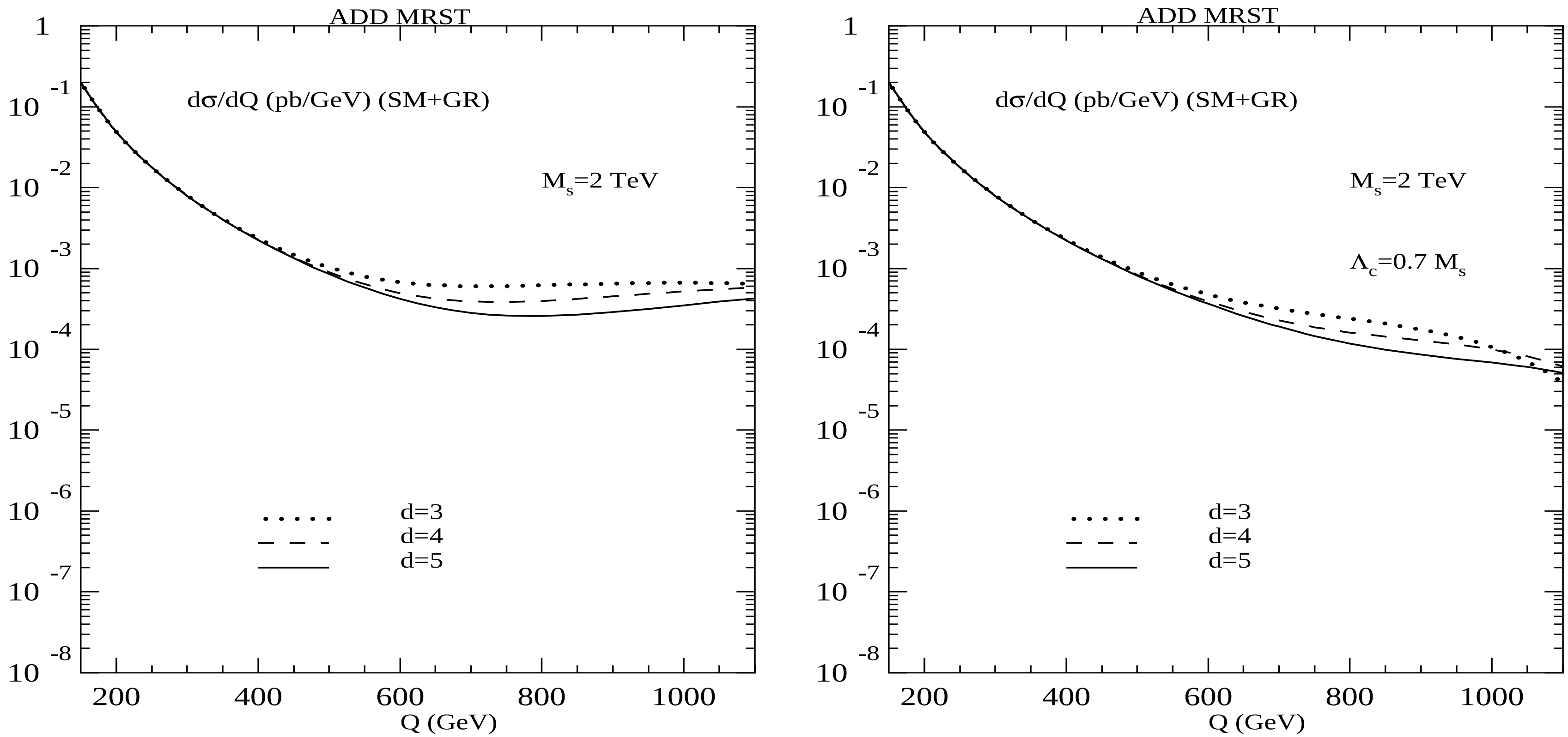
Summary

- Next to Leading Order coefficient functions for DY process in models of TeV-scale gravity are now available
- Various distributions $viz.$ Q, Y, $\cos\theta^*$ to NLO have been studied for ADD & RS models
- Theoretical uncertainties get significantly reduced at NLO level
- Dependence of the results on PDFs have also been studied
- Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders investigated

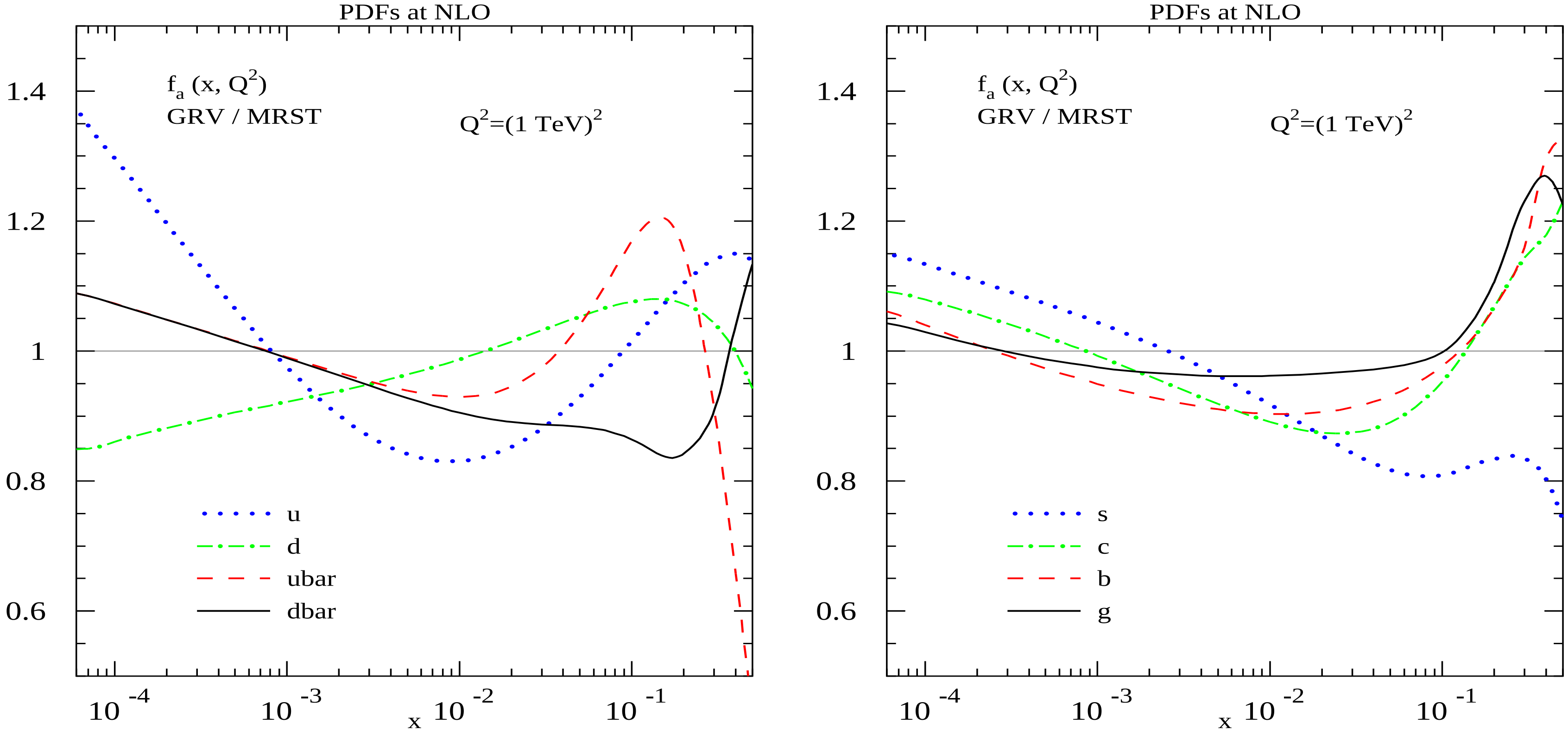
Variation with Λ_c



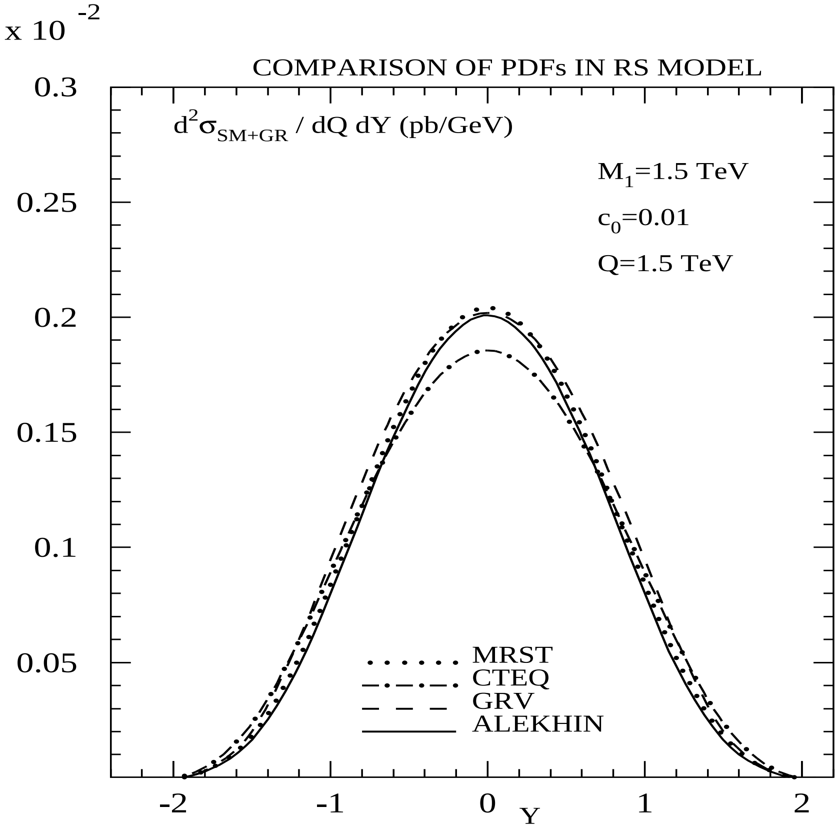
Variation with d



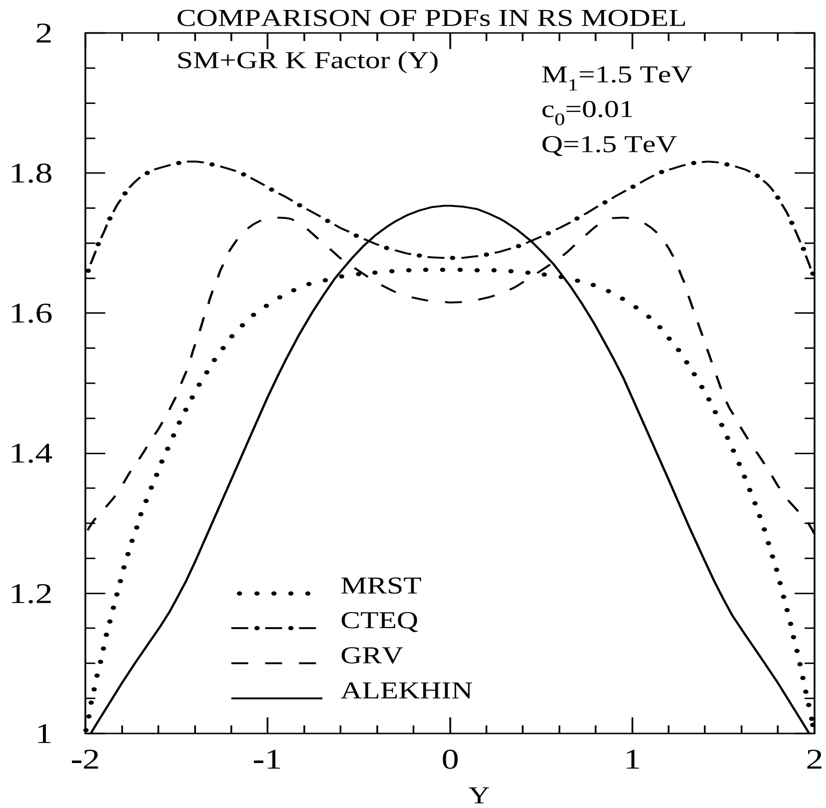
GRV/MRST


PDFs at NLO
$f_a(x, Q^2)$
GRV / MRST
$Q^2=(1\ TeV)^2$
1.4
1.2
1
0.8
0.6
u
d
ubar
dbar
10^{-4}
10^{-3}
x 10^{-2}
10^{-1}
PDFs at NLO
$f_a(x, Q^2)$
GRV / MRST
$Q^2=(1\ TeV)^2$
1.4
1.2
1
0.8
0.6
s
c
b
g
10^{-4}
10^{-3}
x 10^{-2}
10^{-1}

RS Rapidity



x 10 -2
COMPARISON OF PDFs IN RS MODEL
d2σSM+GR / dQ dY (pb/GeV)
M1=1.5 TeV
c0=0.01
Q=1.5 TeV
MRST
CTEQ
GRV
ALEKHIN
0.3
0.25
0.2
0.15
0.1
0.05
-2
-1
0
1
2
Y



COMPARISON OF PDFs IN RS MODEL
SM+GR K Factor (Y)
M1=1.5 TeV
c0=0.01
Q=1.5 TeV
MRST
CTEQ
GRV
ALEKHIN
2
1.8
1.6
1.4
1.2
1
-2
-1
0
1
2
Y

PDF ERROR ESTIMATION (MRST)