New Corrections to the r/cMSSM Higgs Masses and Mixings

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1. Introduction

2. Corrections of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

3. Corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in the cMSSM

4. Conclusions

1. Introduction

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - (m_{12}^2 \epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$ (no $\mathcal{CP}$V at tree-level)

2 $\mathcal{CP}$-violating phases: ξ, $\arg(m_{12})$ $\Rightarrow$ can compensate each other

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A or $M_{H^\pm}$

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Contrary to the SM:

m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of m_h, Higgs couplings $\Rightarrow$ test of the theory

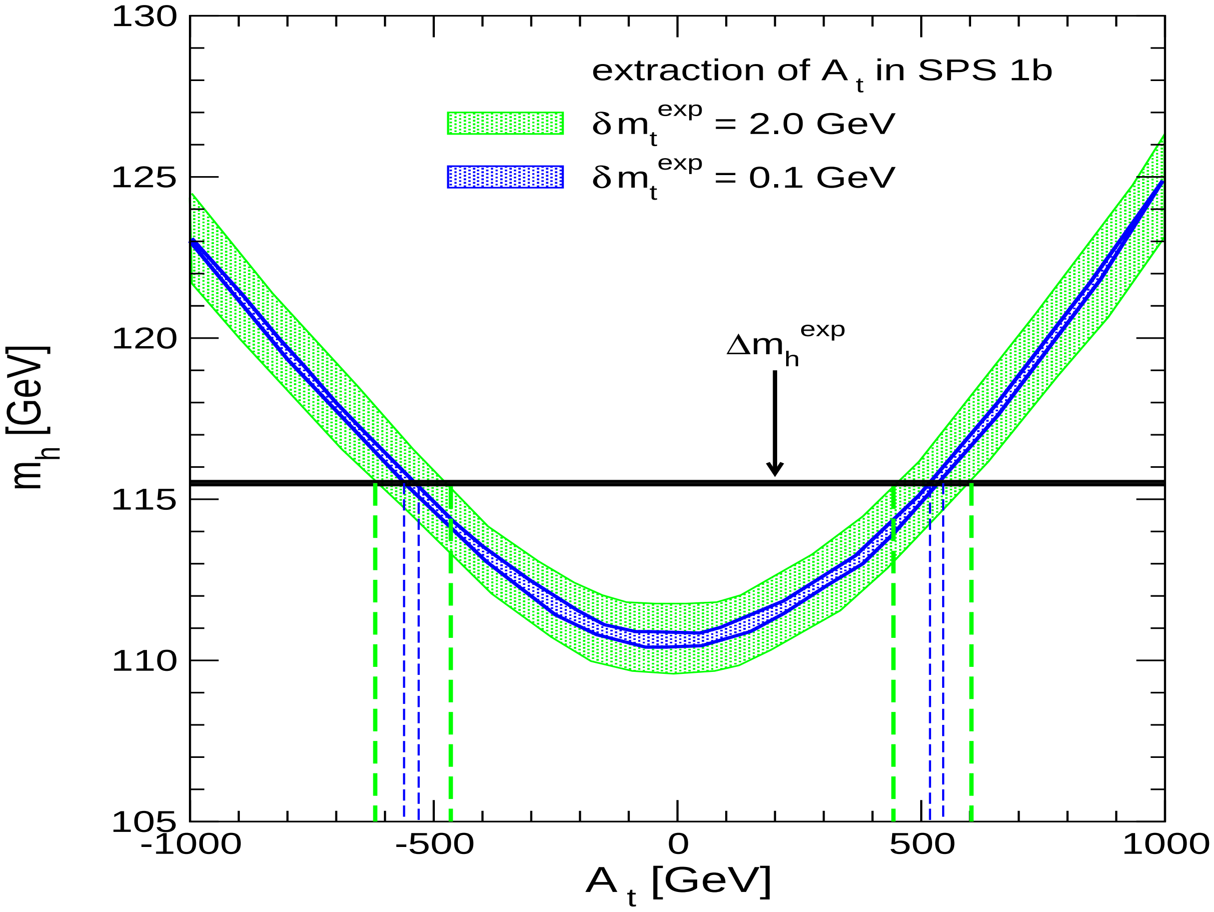
LC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow$ aim for theoretical precision!

($\Rightarrow m_h$ will be (the best?) electroweak precision observable)

Example of application: m_h prediction as a function of A_t

[S.H., S. Kraml, W. Porod, G. Weiglein '02]



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan\beta, M_A$ known,

realistic parametric errors assumed (from SUSY exp. errors)

$\Rightarrow$ extraction of A_t possible
Theory error neglected

$\Rightarrow$ m_h is crucial input for SUSY fit programs (Fittino, Sfitter)

2. Correction of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

Evaluation of Higgs boson masses in the MSSM with real parameters:

Two-point vertex function:

$$\Gamma(q^2) = \begin{pmatrix} q^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

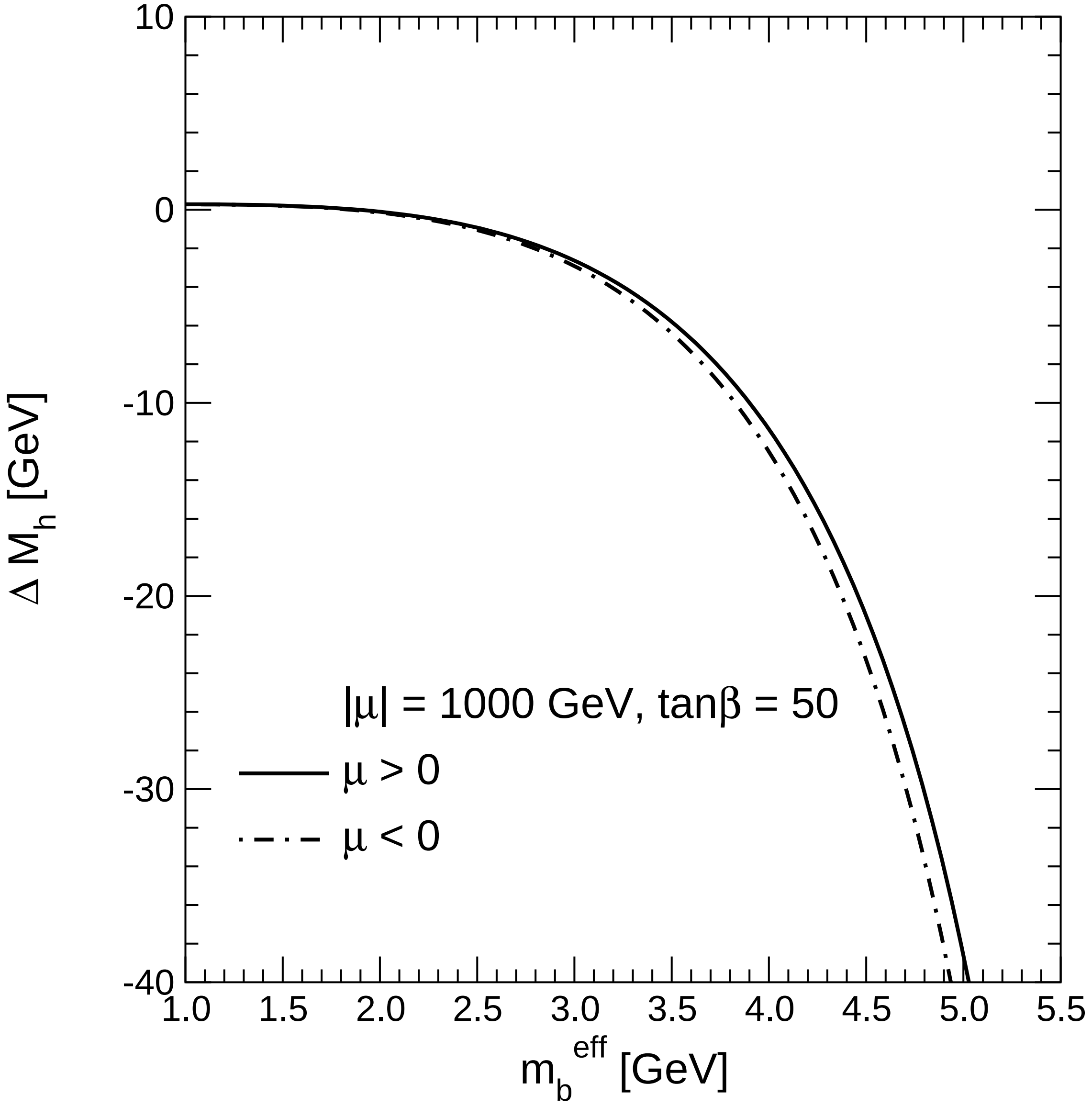
determination of $\det(\Gamma(q^2)) = 0 \Rightarrow M_h, M_H, \alpha_{\text{eff}}, \ldots$

Main task: calculation of $\hat{\Sigma}(q^2)$, including renormalization

Here:

- evaluation of 2-loop corrections of $\mathcal{O}(\alpha_b\alpha_s)$
- comparison of 4 different renormalization schemes

Motivation: Why 2-loop corrections in the $b/\tilde{b}$ sector?



1-loop corrections $\mathcal{O}(\alpha_b)$ to M_h can be sizable

Precise M_h prediction
$\Rightarrow$ 2-loop corrections necessary

The Higgs self-energy at 2-loop:

$\rightarrow \alpha_s$ correction to the leading 1-loop term $\sim m_b^4$

Approximations:

- only m_b^2 ($\sim y_b^2$) terms
- gauge couplings are set to 0
- external momentum is set to 0

$$\Rightarrow \hat{\Sigma}^{(2)}_{22}(q^2) \approx \Sigma^{(2)}_{22}(0) + \cos^2\beta\, \delta M_A^{2\,(2)} - \frac{e}{2\, M_W s_W}\left(\sin^2\beta \cos\beta\, \delta t_1^{(2)} - \sin\beta\,(1+\cos^2\beta\,)\delta t_2^{(2)}\right)$$

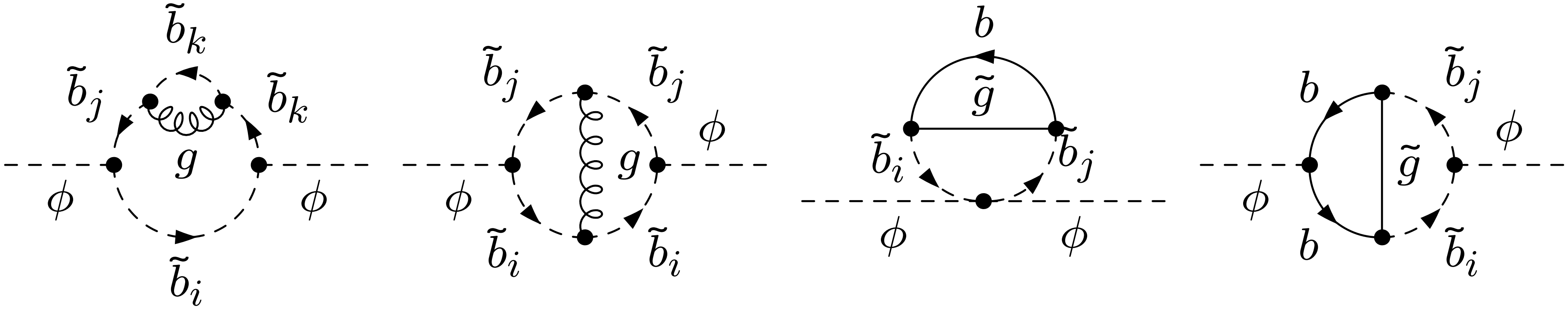
in the $\phi_1\phi_2$ basis with

$\Sigma^{(2)}_{22}(0)$: unrenormalized 2-2 self-energy

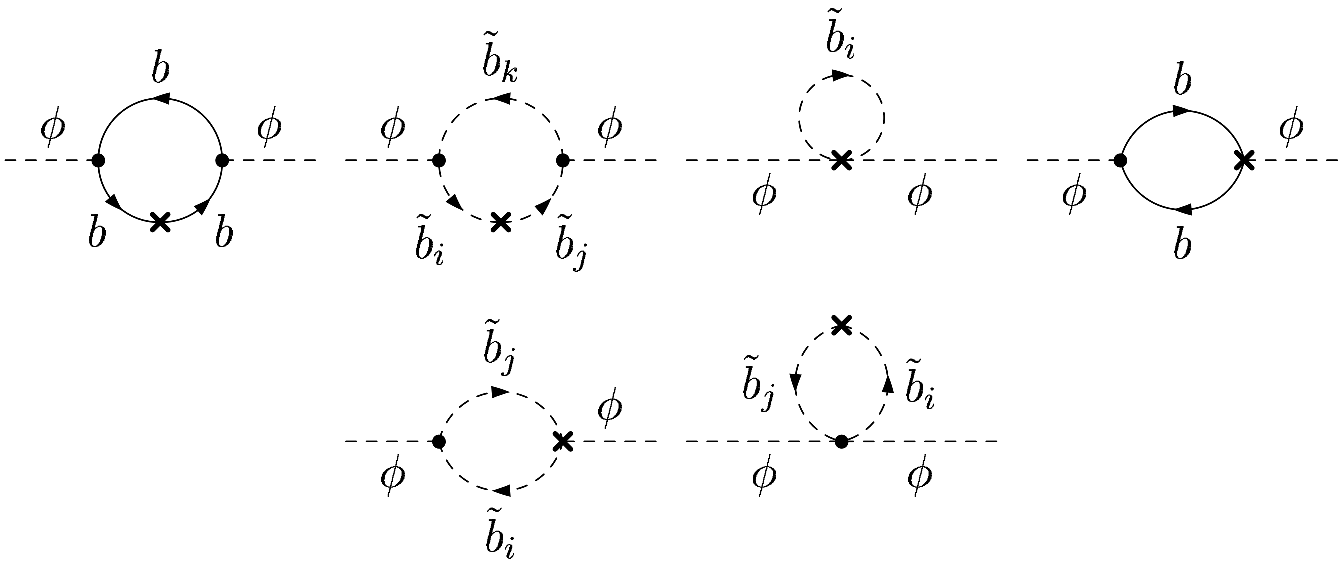
$\delta M_A^{2\,(2)} = \Sigma_A^{(2)}(0)$: A mass counter term

$\delta t_i^{(2)} = -T_i^{(2)}$: ϕ_i tad-pole

2-loop self-energy diagrams:



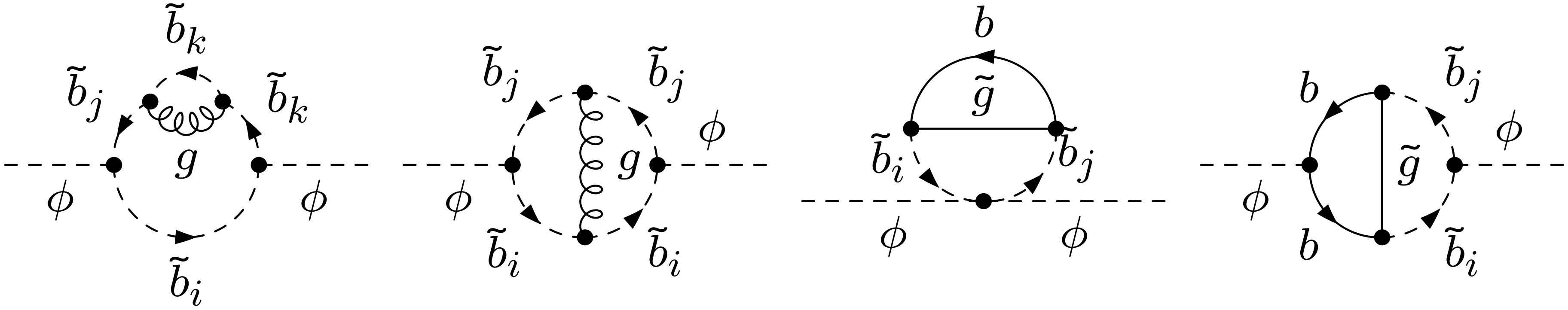
diagrams with counter term insertion:



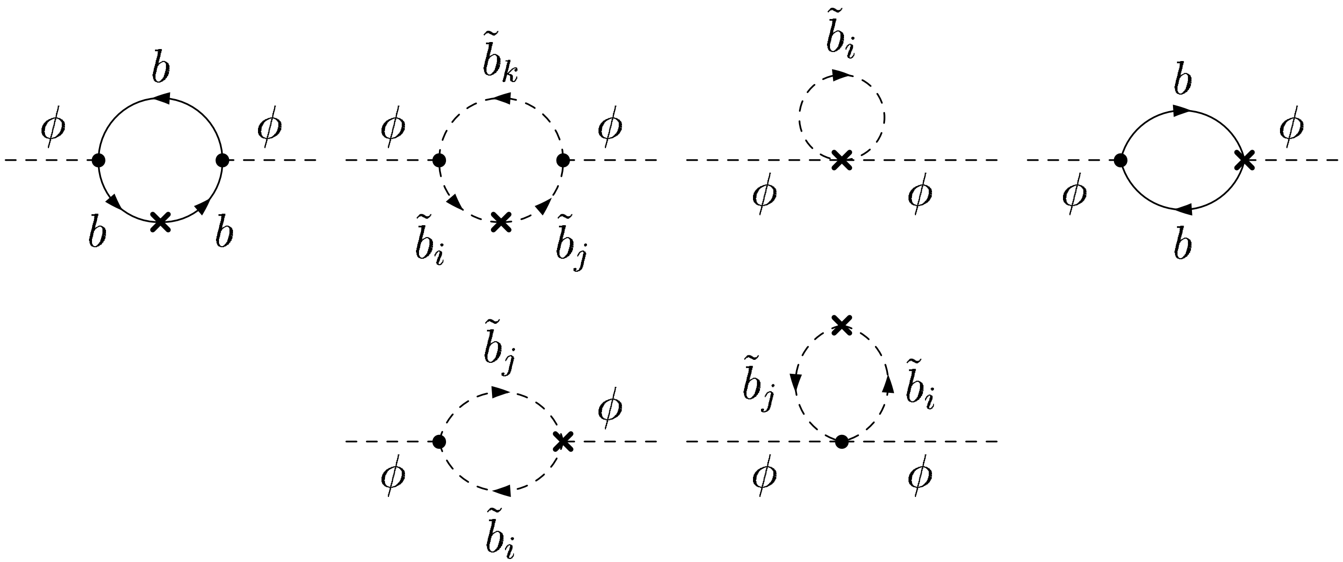
$\rightarrow$ different renormalization schemes enter

Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



diagrams with counter term insertion:



→ different renormalization schemes enter ⇒ **the real problem!**

Renormalization:

Calculation of two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$ and $\mathcal{O}(\alpha_b \alpha_s)$

$\Rightarrow$ parameters of the $t/\tilde{t}$ and $b/\tilde{b}$ are defined at the 1-loop level

$\Rightarrow$ different choices of renormalization possible

$t/\tilde{t}$ sector:

one renormalization scheme:
4 independent parameters:
$m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, m_t on-shell
$\rightarrow A_t$ given in terms of the others

$b/\tilde{b}$ sector: four schemes analyzed

Investigation of scheme dependence:

$\Rightarrow$ information about size of missing higher order corrections

$\Rightarrow$ estimate of theory uncertainty

Renormalization schemes in the $b/\tilde{b}$ sector:

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$ out of $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\theta_{\tilde{b}}$, A_b, m_b only 3 are independent

$\Rightarrow$ two parameters (incl. CTs) are given in terms of the others

In all four schemes: $m_{\tilde{b}_1}$ dep. ($SU(2)$ relation), $m_{\tilde{b}_2}$ OS

scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	$\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.
A_b, $\theta_{\tilde{b}}$ OS	dep.	OS	OS
A_b, $\theta_{\tilde{b}}$ $\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.	$\overline{\text{DR}}$
m_b OS	OS	dep.	OS

$\Rightarrow$ scheme m_b OS: analogous to the $t/\tilde{t}$ sector

$\rightarrow$ obvious choice ?

Resummed bottom quark mass:

$\rightarrow$ absorb the leading corrections in a resummed form in the bottom quark mass at the 1-loop level

$$m_b^{\overline{\mathrm{DR}}} = \frac{\tilde{m}_b^{\mathrm{pole}} + \Sigma_b^{\tan\beta\mathrm{non-enh.}}{}_{|\mathrm{fin}}}{1 + \Delta m_b}$$

with

$$\Delta m_b = \frac{2\,\alpha_s}{3\,\pi} \tan\beta\,\mu\, m_{\tilde{g}}\, I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2)$$

$$\Sigma_b^{\tan\beta\mathrm{non-enh.}}{}_{|\mathrm{fin}} = \tan\beta\,\text{non-enhanced terms in } \Sigma_{b,s}$$

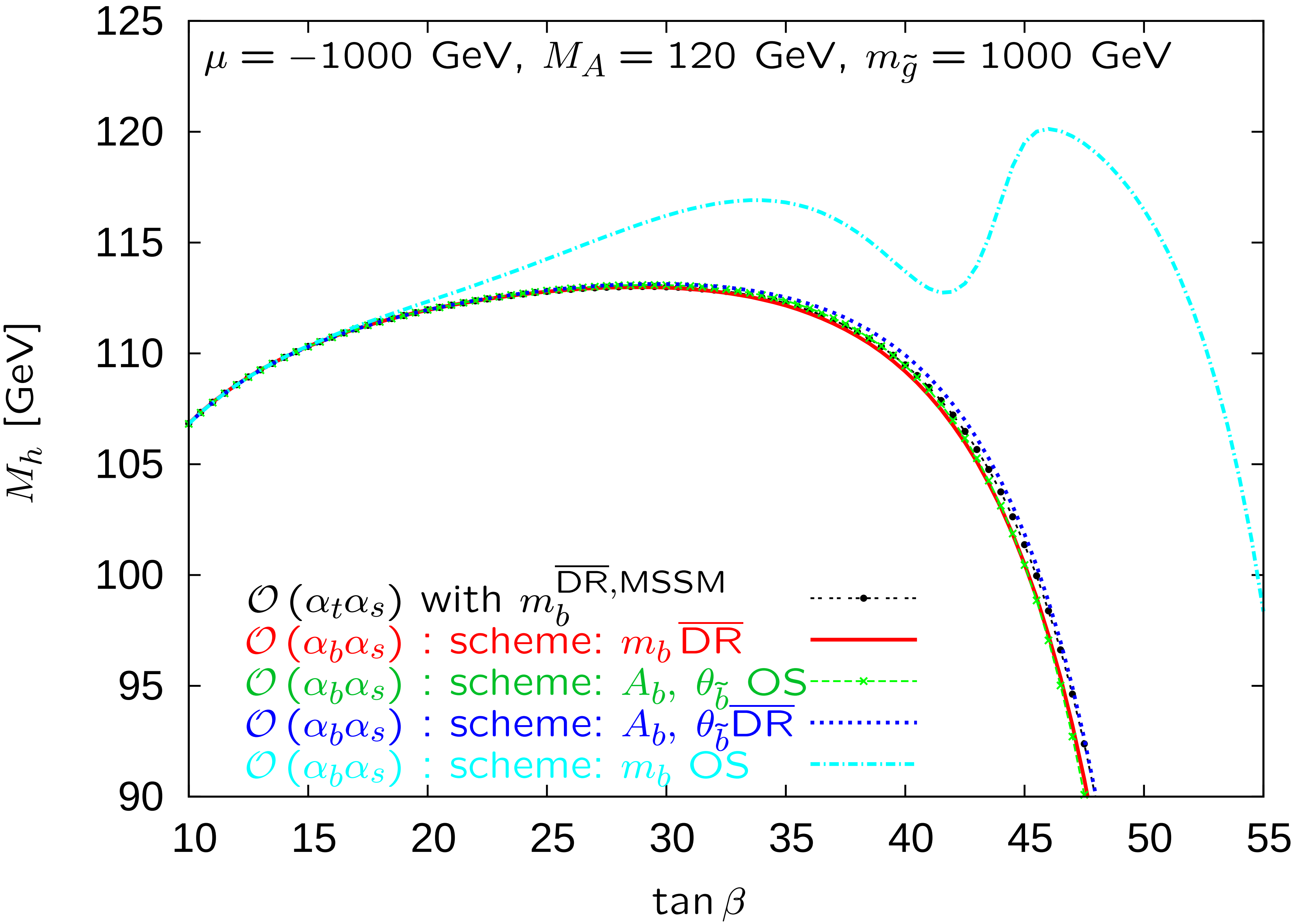
$$\tilde{m}_b^{\mathrm{pole}} = m_b^{\overline{\mathrm{MS}}}(M_Z) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \log\frac{(m_b^{\overline{\mathrm{MS}}})^2}{M_Z^2}\right)\right]$$

"formal" pole mass obtained from the $\overline{\mathrm{MS}}$ mass

$\Rightarrow$ large higher-order corrections included at the 1-loop level

$\rightarrow$ other renormalization schemes by finite shift

M_h as a function of $\tan\beta$, $\mu < 0$:



Observations:

- Scheme m_b OS gives very large corrections

  Reason: A_b is a dependent quantity $\Rightarrow$ large corrections via δA_b

$$\delta A_b = \frac{1}{m_b}\left[-\frac{\delta m_b}{2\,m_b}(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)\sin 2\theta_{\tilde{b}} + \ldots\right]$$
$$= \frac{1}{m_b}\left[-\delta m_b(A_b - \mu \tan\beta) + \ldots\right]$$

$$\hat{\Sigma}_{HH} \sim (\cos\alpha\, A_b)^2, \quad \hat{\Sigma}_{hh} \sim (\sin\alpha\, A_b)^2$$

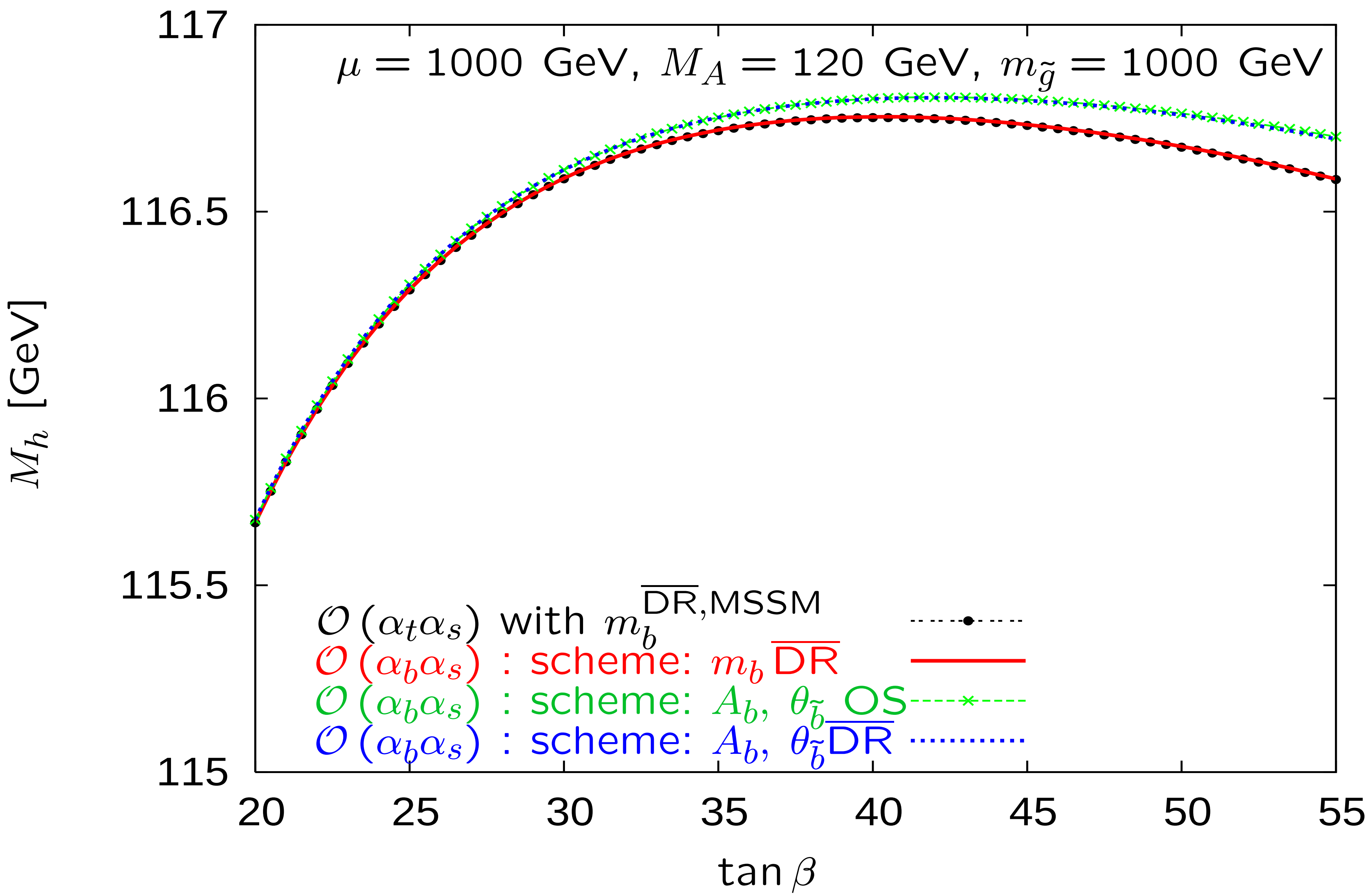
  $\Rightarrow$ effect more pronounced for M_H

  $\Rightarrow$ Scheme m_b OS is discarded as a useful renormalization scheme

- Other schemes: differences of $\mathcal{O}$ (1 GeV) for large $\tan\beta$

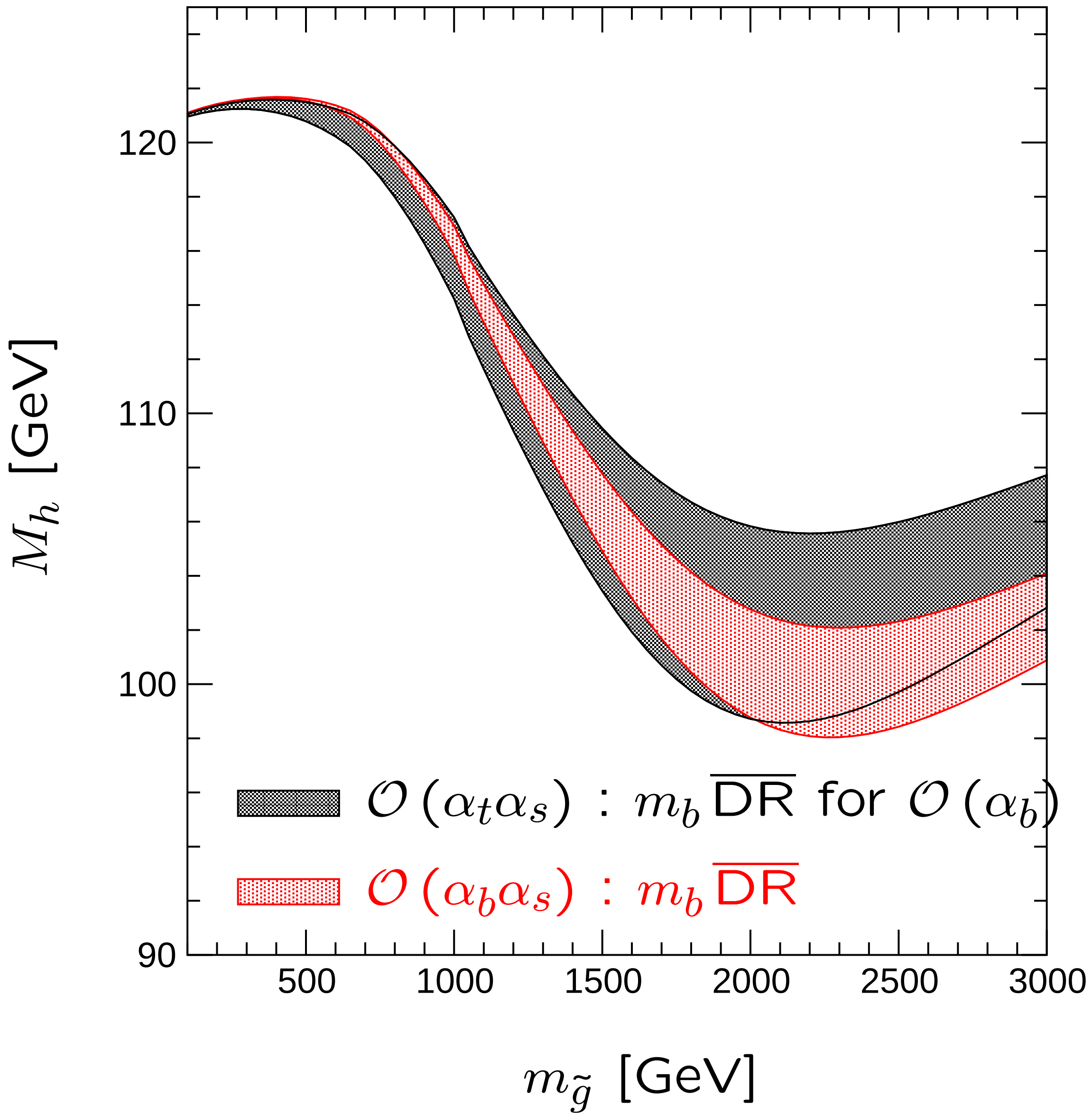
  $\Rightarrow$ non-negligible

M_h as a function of $\tan\beta$, $\mu > 0$:



$\Rightarrow$ small corrections, scheme m_b $\overline{\text{DR}}$: "no" correction

Dependence on renormalization scale $\mu^{\overline{\rm DR}}$:



$M_A = 700$ GeV
$\mu = -1000$ GeV
$\tan\beta = 50$
$m_t/2 \leq \mu^{\overline{\rm DR}} \leq 2\,m_t$

$\Rightarrow$ scale dependence
$\mathcal{O}(\pm 2$ GeV) for large $m_{\tilde{g}}$

3. Corrections of $\mathcal{O}(\alpha_t \alpha_s)$ in the cMSSM

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– M_3 : gluino mass parameter

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

Inclusion of higher-order corrections:

($\to$ Feynman-diagrammatic approach)

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \Rightarrow \mathcal{CPV}$, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Our result for $\hat{\Sigma}_{ij}$:

– full 1-loop evaluation: dependence on all possible phases included

– New: $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach

rMSSM: difference between FD and RGiEP approach $\mathcal{O}$ (few GeV)

Differences to the real case:

- use $M_{H^\pm}$ as on-shell mass, since A mixes with h, H in higher orders
  $\Rightarrow$ $\tilde{b}$ sector enters via $\Sigma_{H^\pm}$
  $\Rightarrow$ renormalization of the $\tilde{b}$ sector

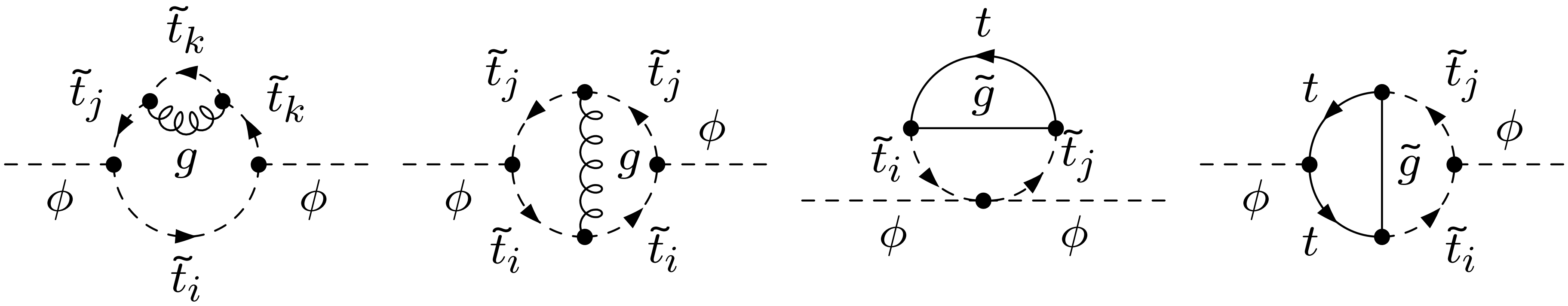
- A_t complex $\Rightarrow$ renormalization of $|A_t|$ and ϕ_{A_t}
  (no renormalization of μ, no $\mathcal{O}(\alpha_s)$ corrections)

- M_3 complex, but $m_{\tilde{g}}$ is real (and positive)
  $\Rightarrow$ phase of M_3 enters gluino vertices

- $T_A \neq 0$ $\Rightarrow$ renormalized to zero
  $\Rightarrow$ δt_A enters renormalized self-energies $\hat{\Sigma}_{hA}$, $\hat{\Sigma}_{HA}$

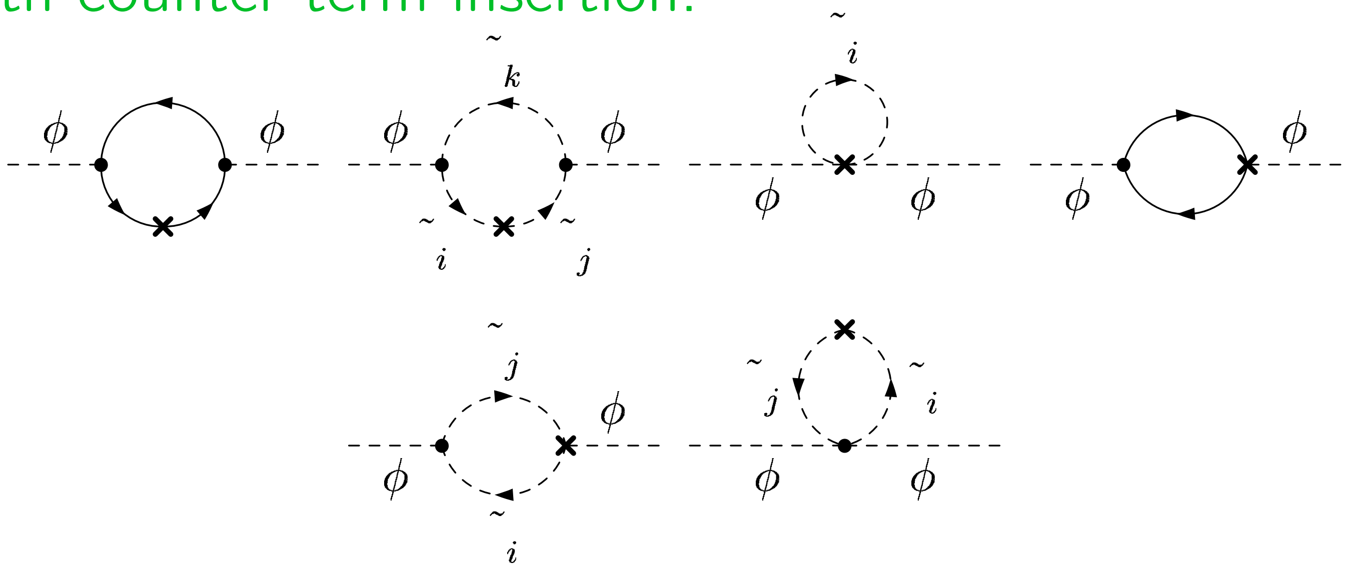
Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



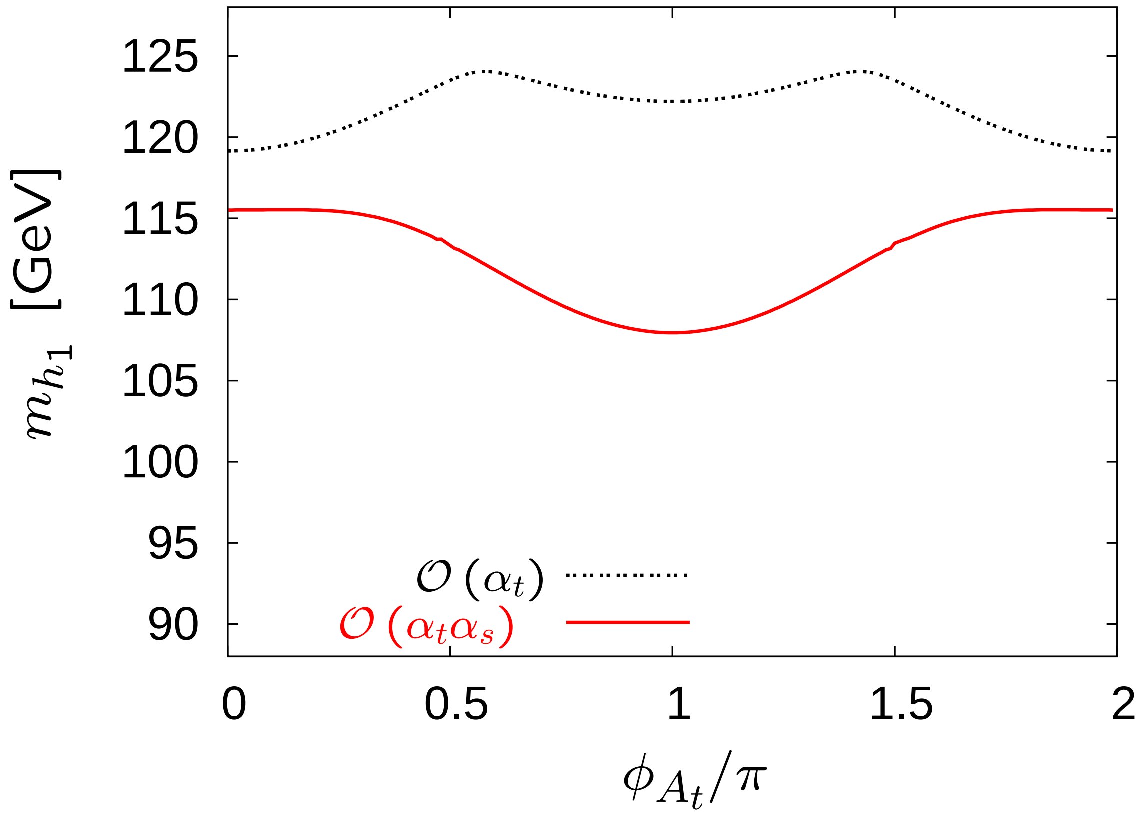
→ complex phases enter ⇒ **something new**

diagrams with counter term insertion:



→ renormalization os complex parameters enter ⇒ **something new**

m_{h_1} as a function of ϕ_{A_t}:

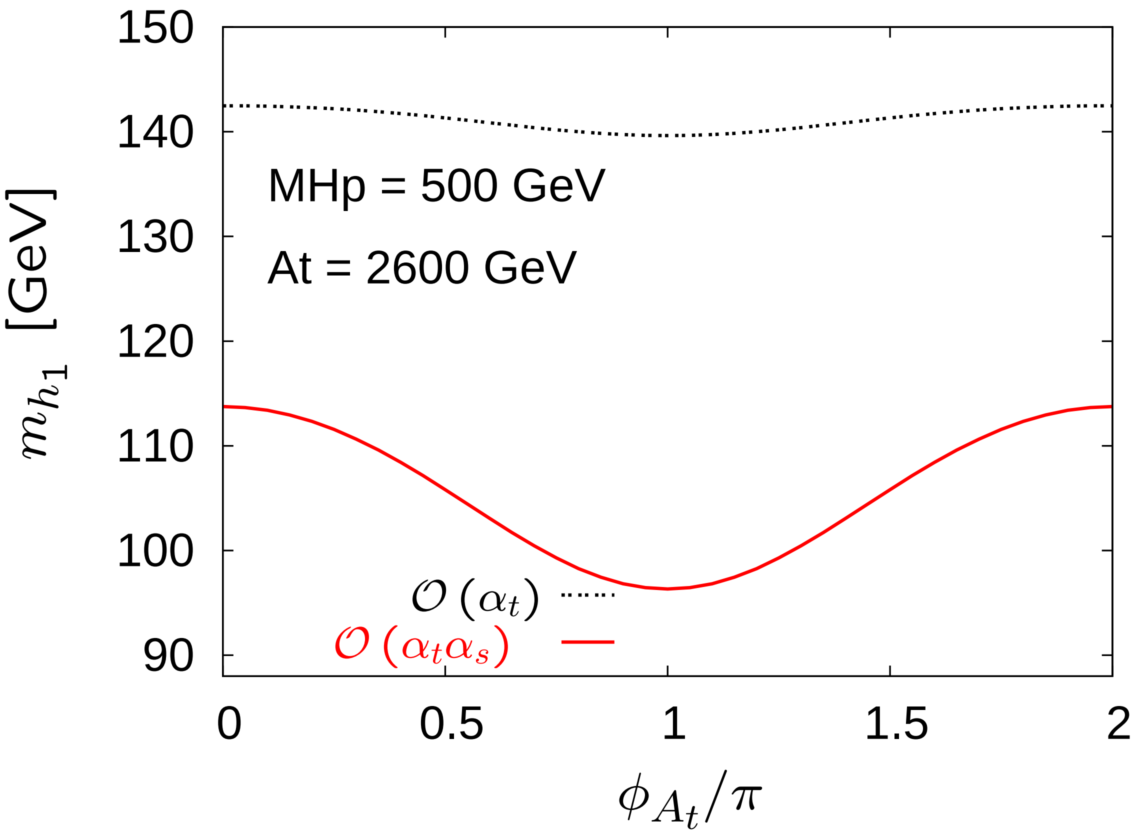


$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence
on ϕ_{A_t} at the 2-loop level

m_{h_1} as a function of ϕ_{A_t}:

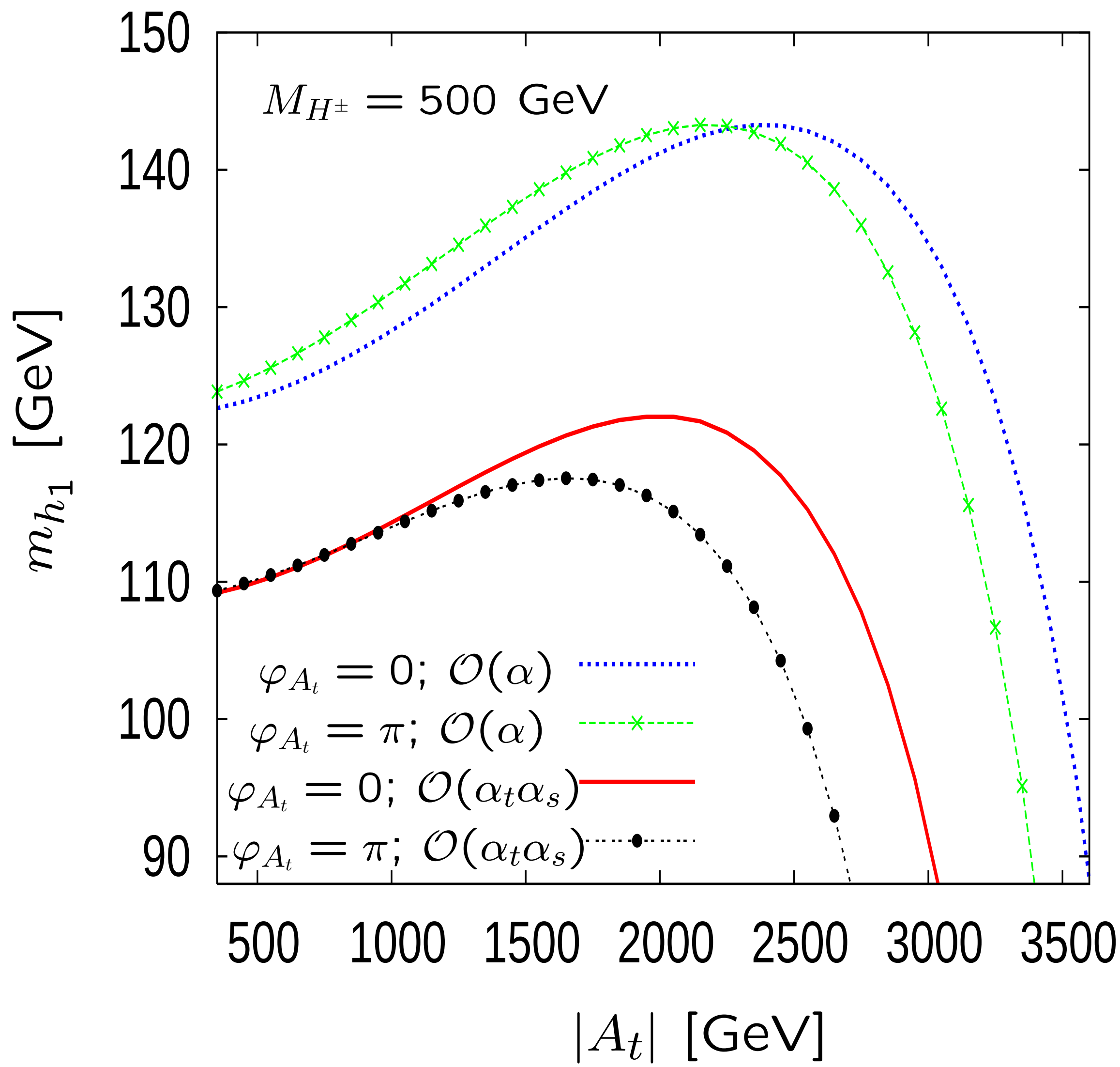


$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ modified dependence on ϕ_{A_t} at the 2-loop level

m_{h_1} as a function of A_t:



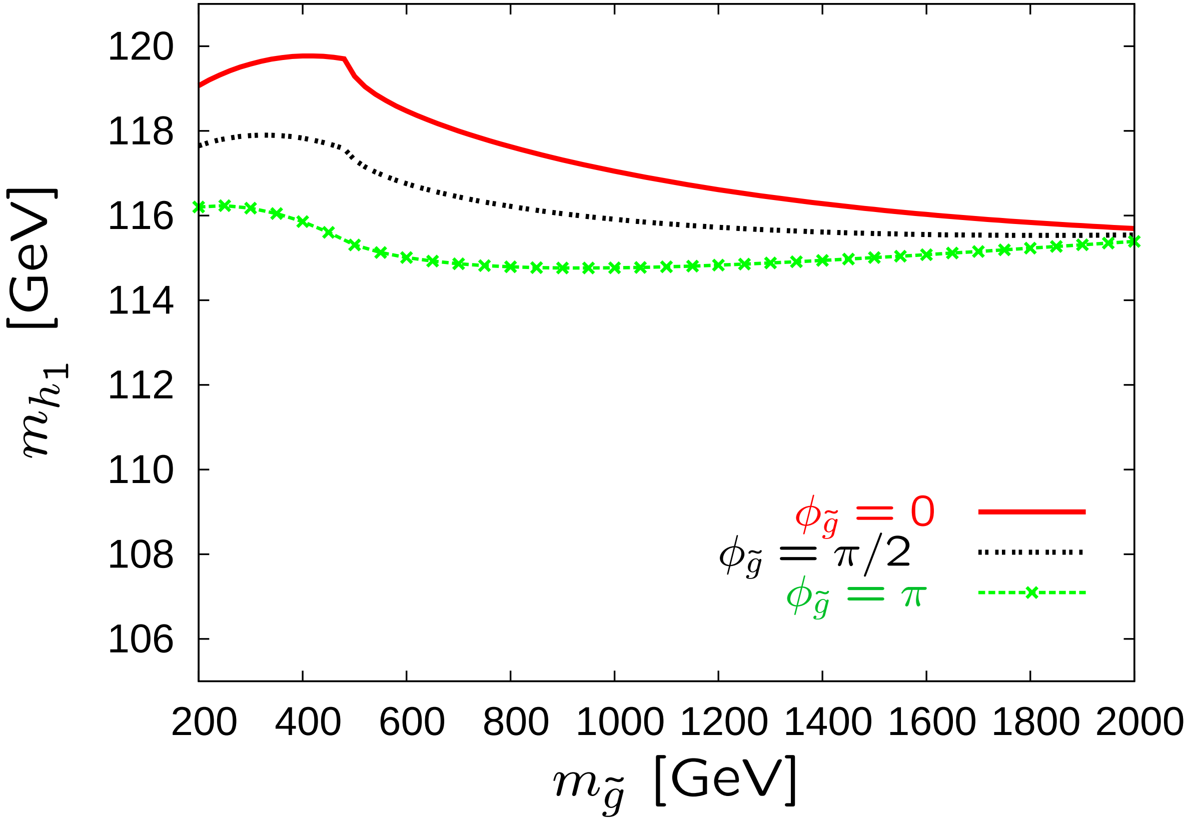
$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

1L: shift in $|A_t|$,
no change in m_{h_1}

2L: m_{h_1} depends on ϕ_{A_t}

m_{h_1} as a function of $\phi_{\tilde{g}}$:



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account

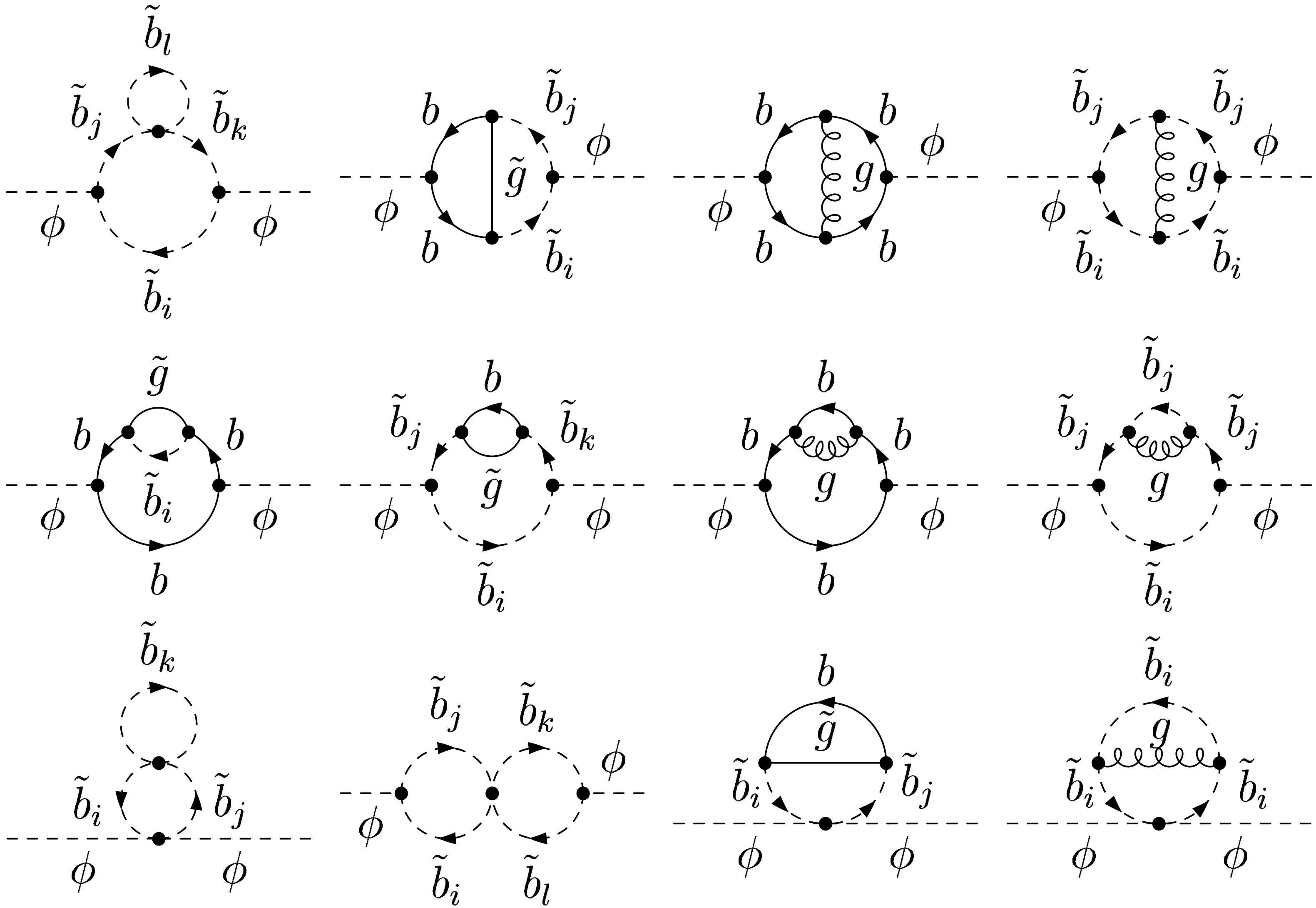
4. Conclusinos

- The LC will provide high precision results for a light r/cMSSM Higgs
- MSSM Higgs masses and couplings is connected via radiative corrections to all other sectors
- Evaluation of $\mathcal{O}(\alpha_b\alpha_s)$ corrections in the rMSSM:
  - real problematic part: renormalization
  - investigation of different renormalization schemes
    $\Rightarrow$ error estimate from scheme and scale dependence
  - $\mu > 0$: corrections $\mathcal{O}(100$ MeV$)$ $\Rightarrow$ under control
  - $\mu < 0$: corrections $\mathcal{O}(2 - 3$ GeV$)$
    error estimate $\mathcal{O}(2$ GeV$)$ $\Rightarrow$ not under control
- Evaluation of $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the cMSSM:
  - new: renormalization for complex parameters
  - $\tilde{b}$ sector enters
  - $|A_t|$ and ϕ_{A_t} dependence modified
  - $\phi_{\tilde{g}}$ dependence $\mathcal{O}(2$ GeV$)$ possible
- Results will are currently implemented into *FeynHiggs* (www.feynhiggs.de)

Backup slides

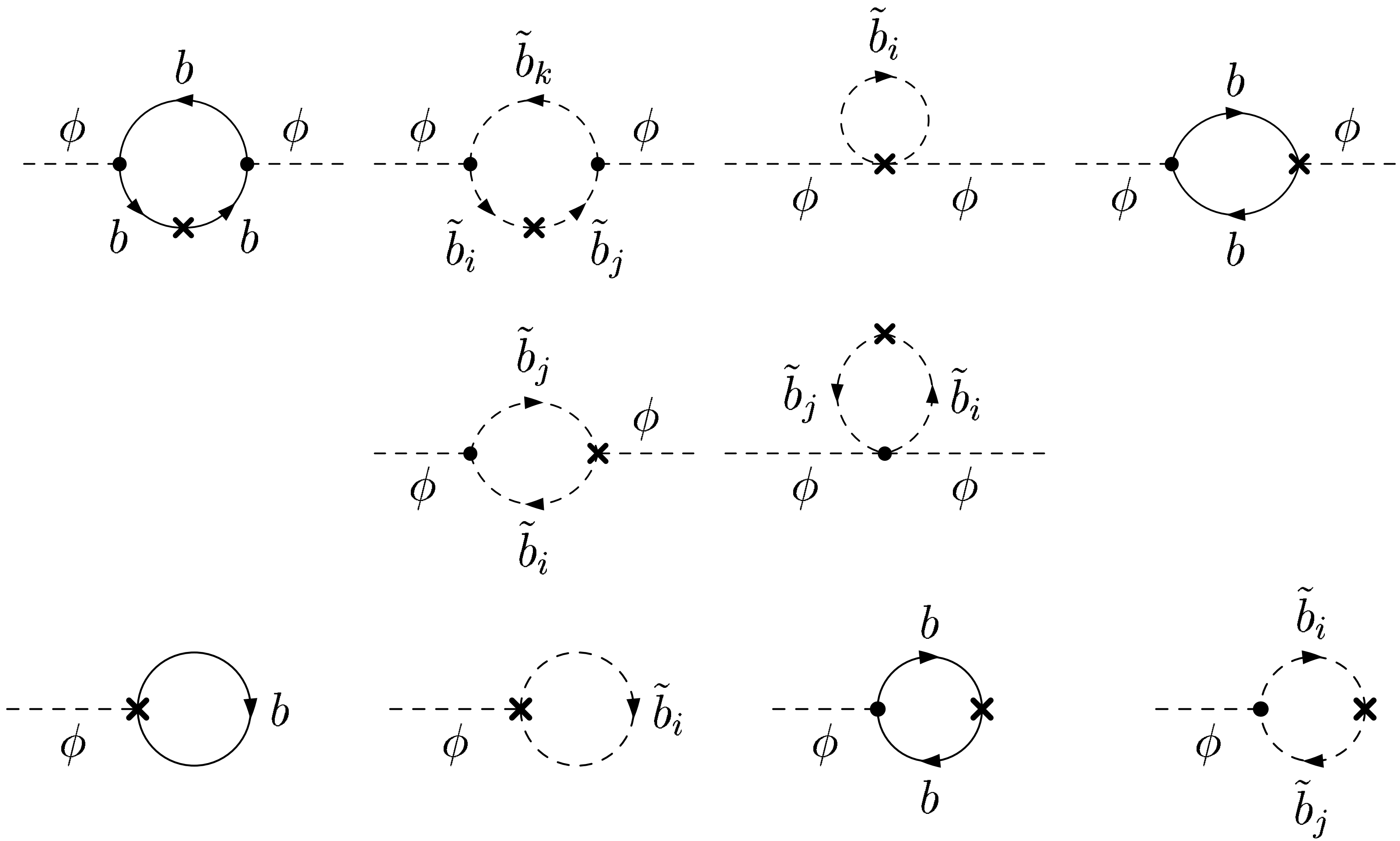
Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



Contributions to the 2-loop self-energy:

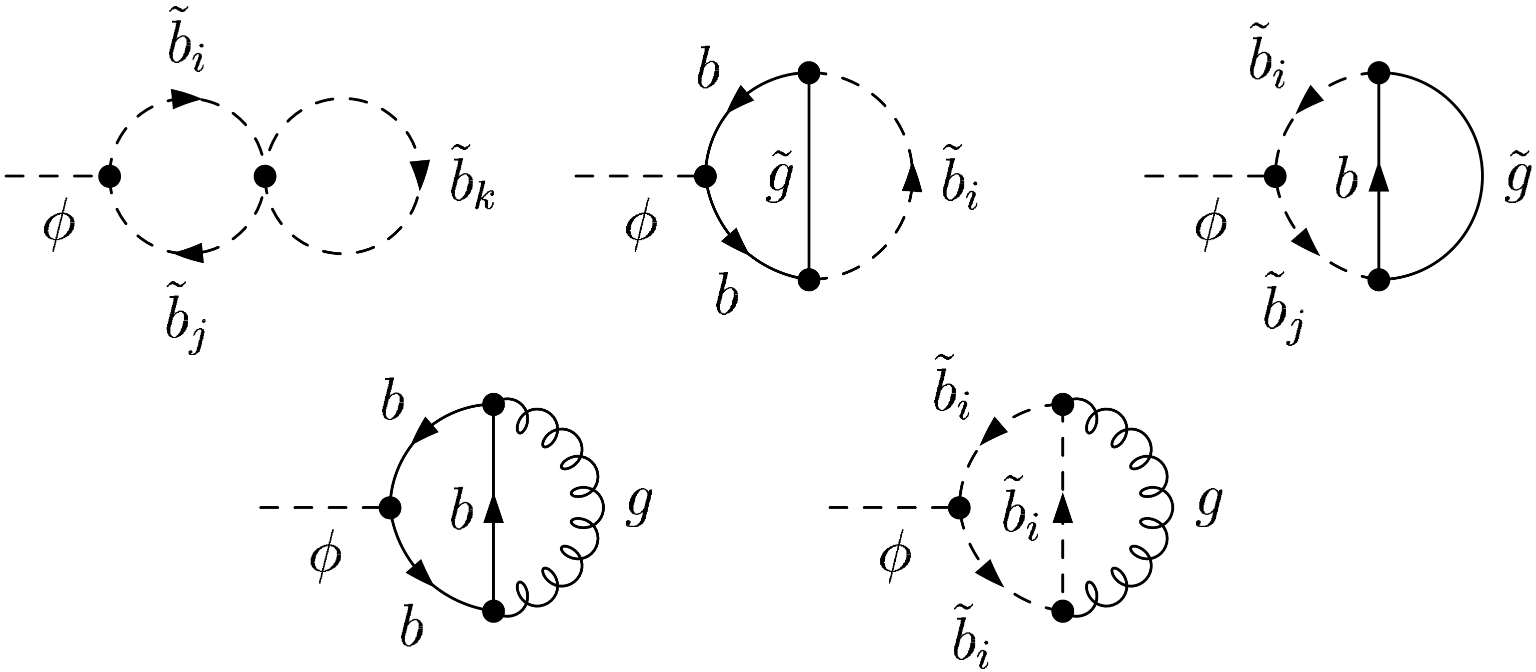
diagrams with counter term insertion:



→ different renormalization schemes enter

Contributions to the 2-loop self-energy:

2-loop tad-pole diagrams:



Evaluation of 2-loop diagrams:

1. Generation of diagrams and amplitudes with FeynArts
   [*Küblbeck, Böhm, Denner '90*] [*Hahn '00 - '05*]

2. Algebraic evaluation and tensor integral reduction to scalar integrals: TwoCalc

   (works for two-loop self-energies)

   [*G. Weiglein '92*] [*G. Weiglein, R. Scharf, M. Böhm '94*]

3. Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
   $\rightarrow$ algebraical check: cancellation of divergencies

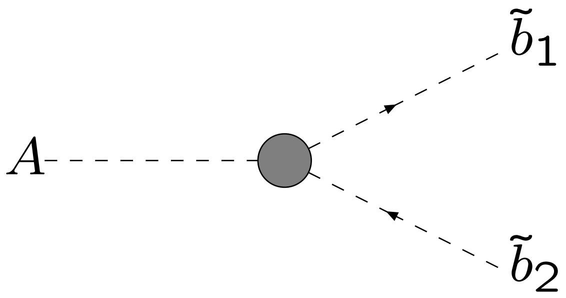
4. Result:

   – algebraic Mathematica code
   – Fortran code (planned: implementation into *FeynHiggs*)

Some more details:

– scheme m_b OS: analogous to the $t/\tilde{t}$ sector
→ obvious choice ?

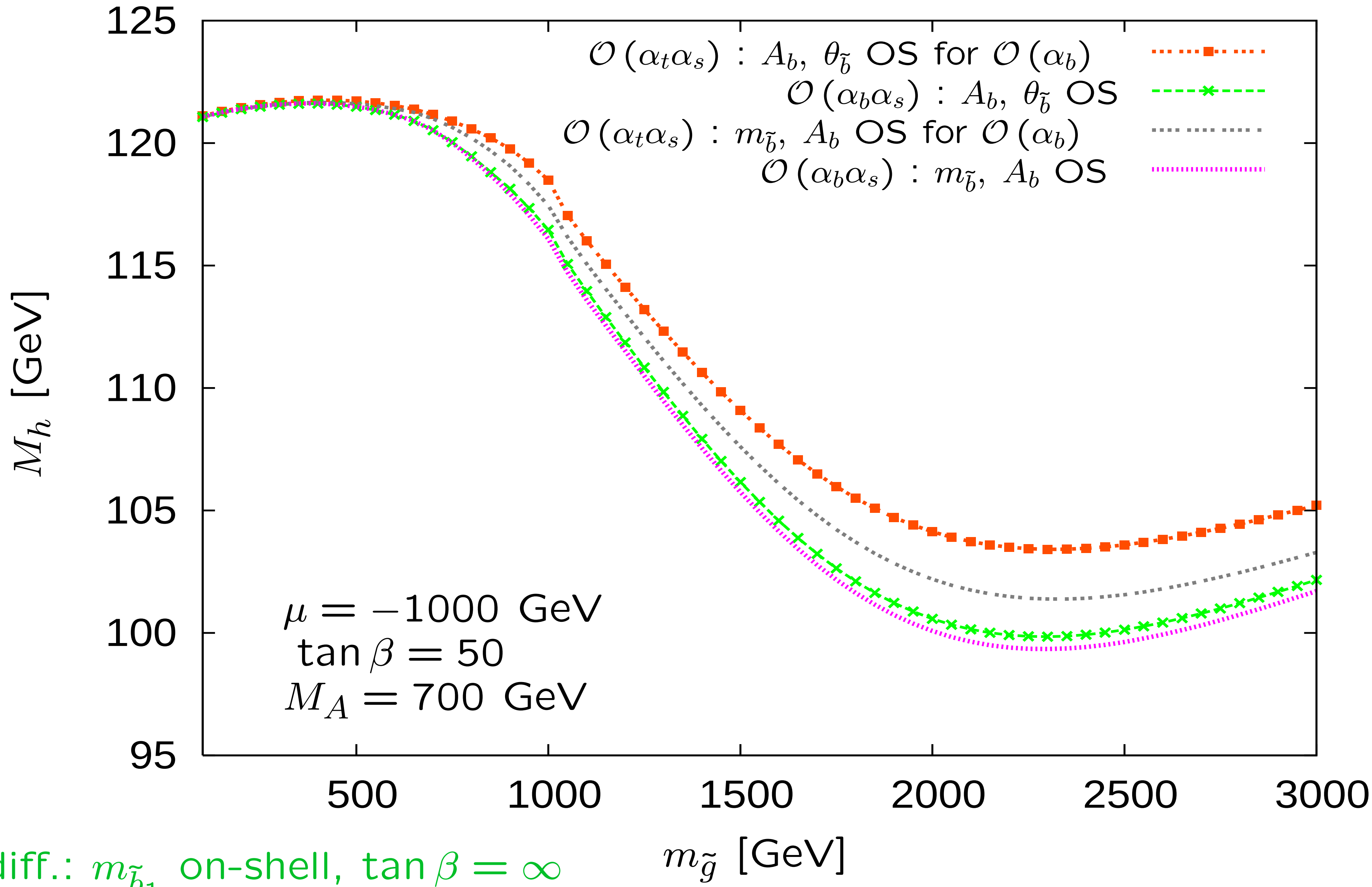
– A_b OS: determined via



analogous to [A. Brignole, G. Degrassi, P. Slavich and F. Zwirner '02]

– $\theta_{\tilde{t}}$ OS: $$\delta\theta_{\tilde{b}} = \frac{\mathrm{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \mathrm{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2}}$$

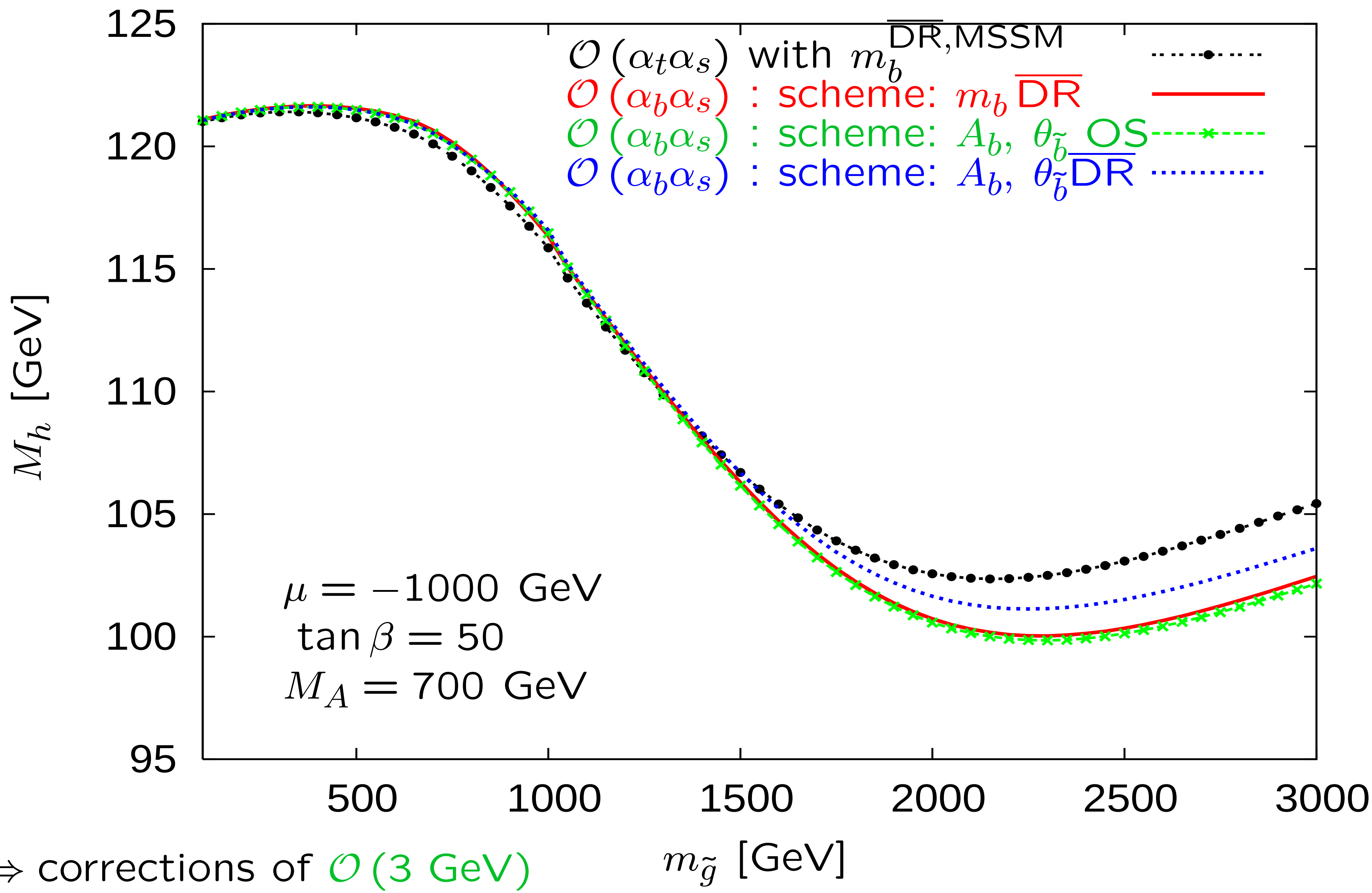
Comparison with existing calculation: [A. Brignole et al. '02]



diff.: $m_{\tilde{b}_1}$ on-shell, $\tan\beta = \infty$

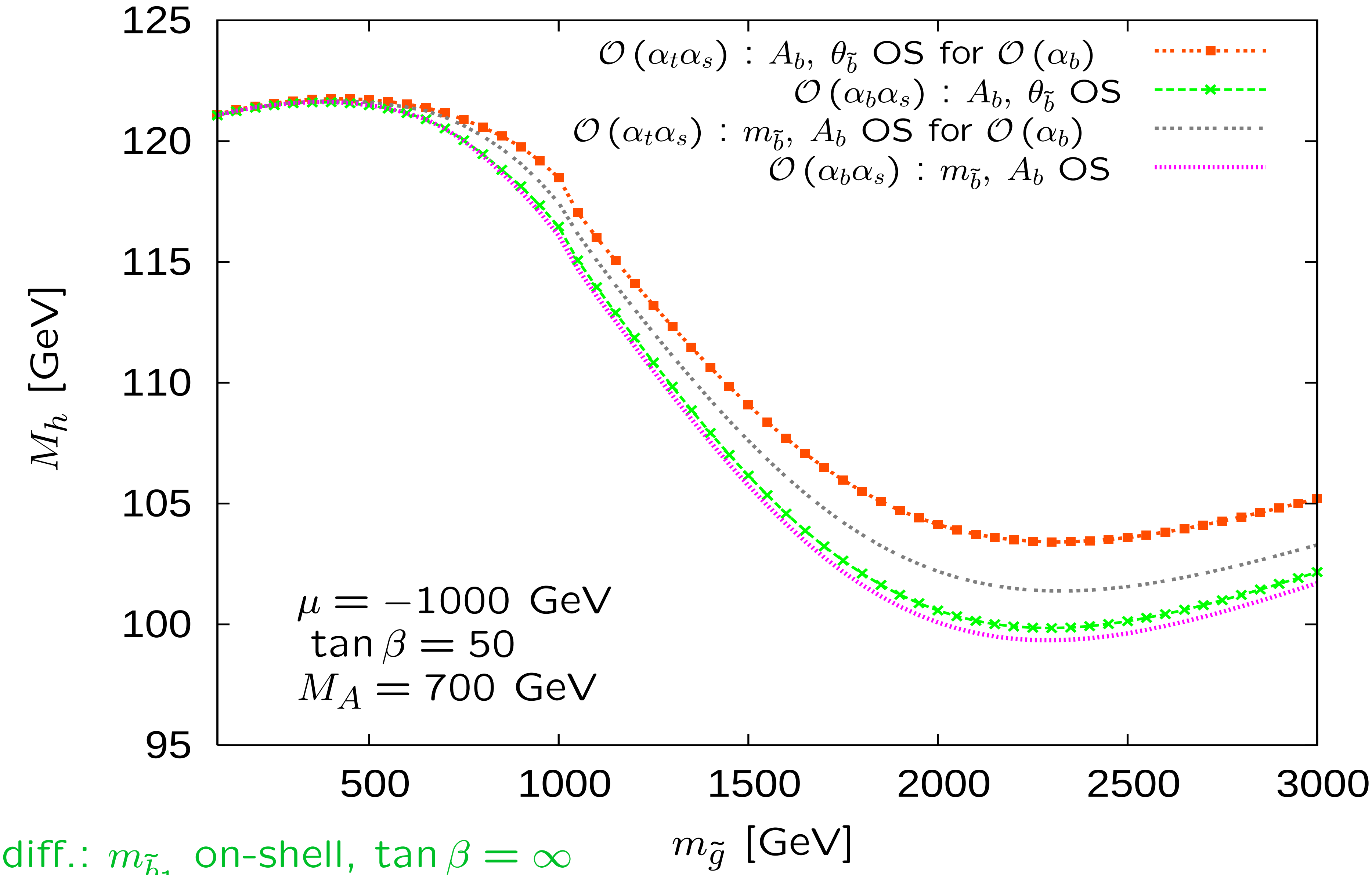
$\Rightarrow$ 2-loop: differences $\mathcal{O}(0.5$ GeV$)$

M_h as a function of $m_{\tilde{g}}$, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}$(3 GeV)
scheme difference $\mathcal{O}$(2 GeV)

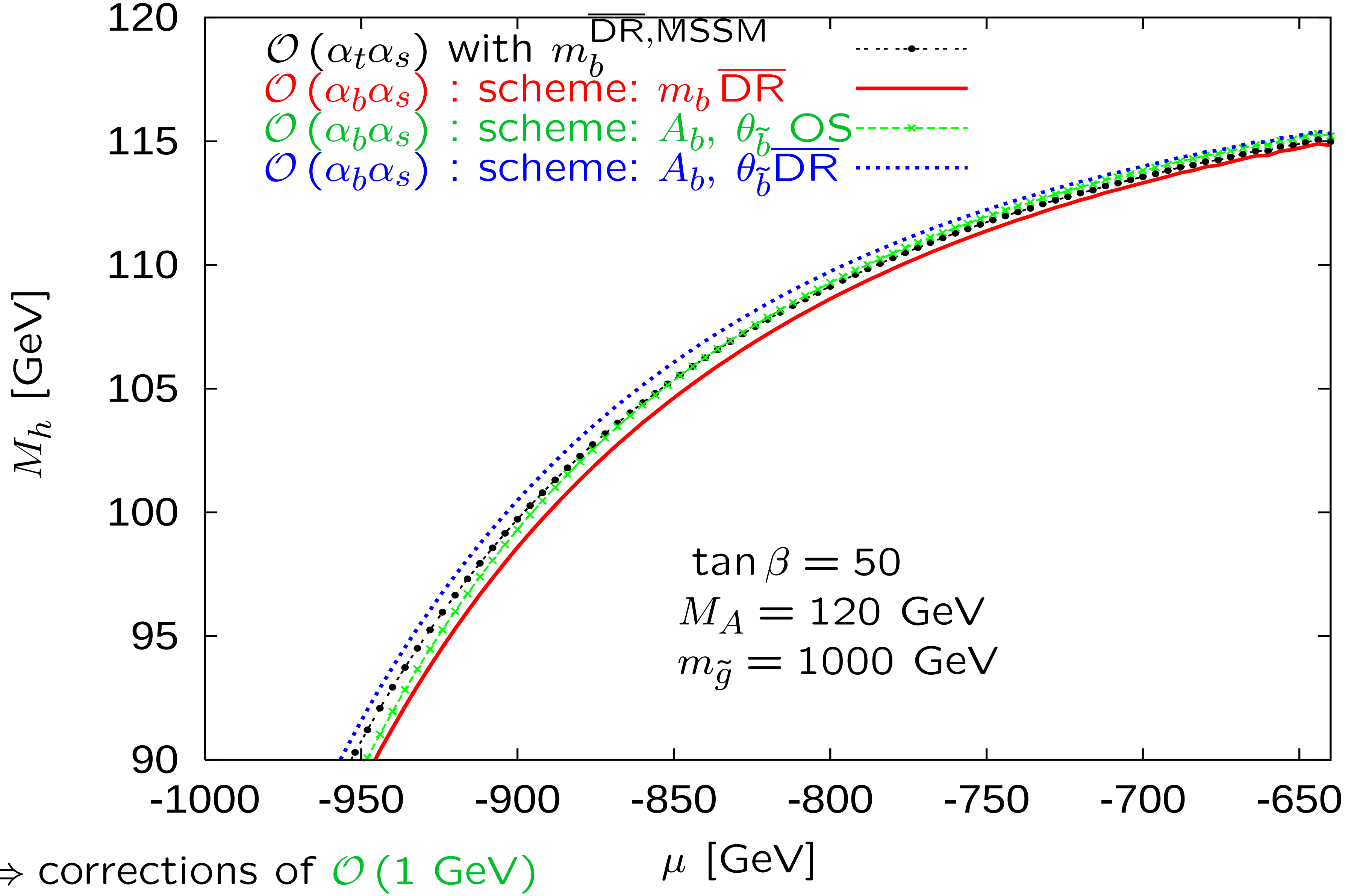
Comparison with existing calculation: [A. Brignole et al. '02]



diff.: $m_{\tilde{b}_1}$ on-shell, $\tan\beta = \infty$

$\Rightarrow$ 2-loop: differences $\mathcal{O}(0.5$ GeV$)$

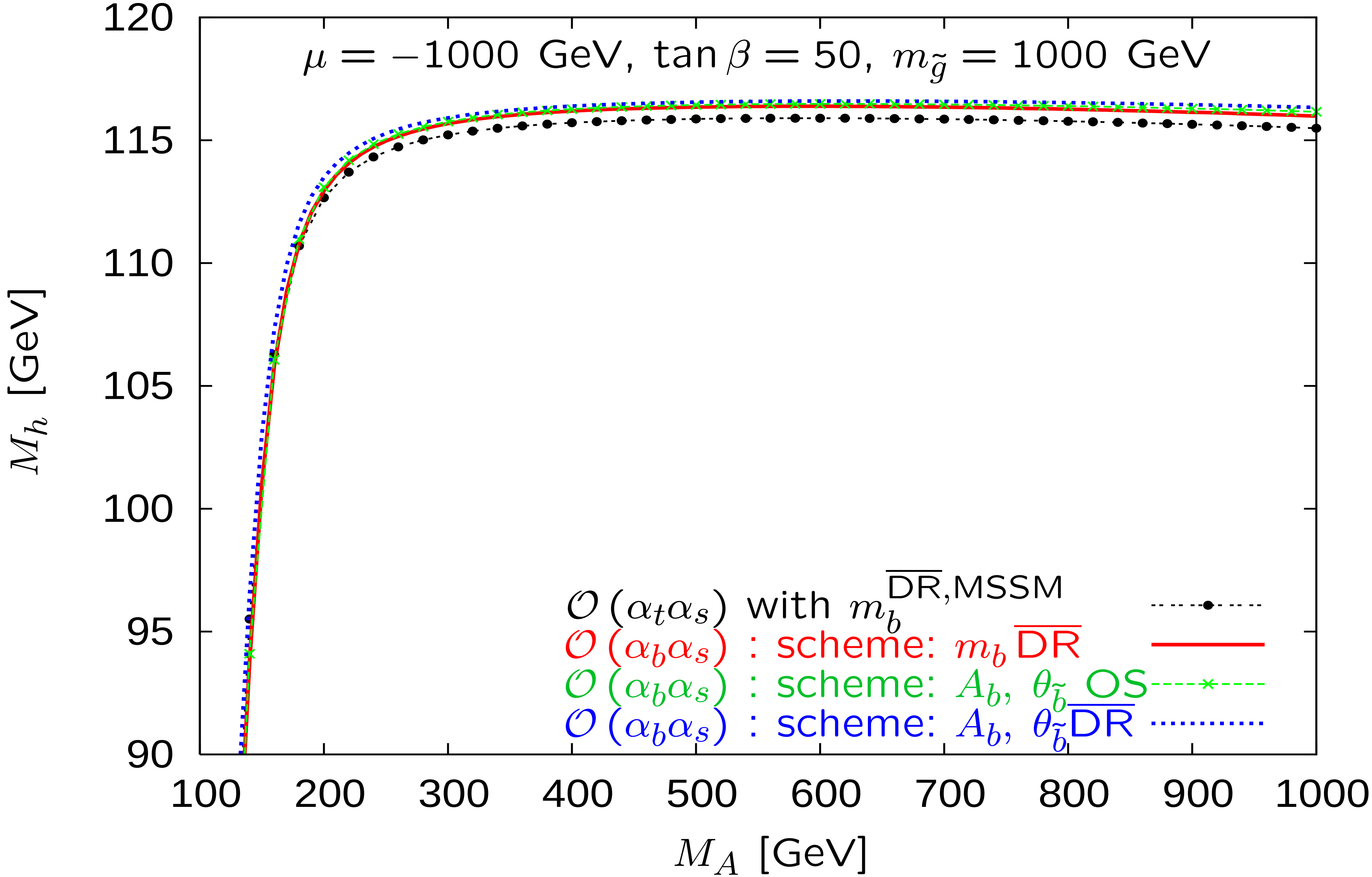
M_h as a function of μ, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}(1$ GeV$)$

scheme difference similar

M_h as a function of M_A, $\mu < 0$:



⇒ subleading corrections of $\mathcal{O}(1$ GeV$)$