Interpretation of the top-quark mass results

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2. Top mass definitions
3. Standard measurements and Monte Carlo mass
4. Ongoing work on understanding the MC mass
5. Alternative methods for mass measurements and interpretations
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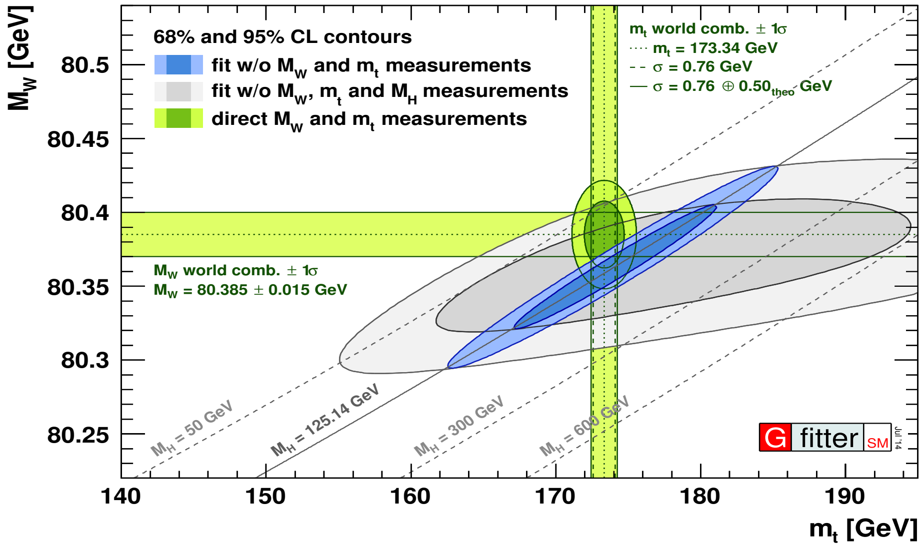
Frascati workshop on 'Top mass: challenges in definition and determination',
https://agenda.infn.it/conferenceDisplay.py?confId=9202

TOP 2015 and 2016 workshops: http://top2015.infn.it/, https://indico.cern.ch/event/486433/

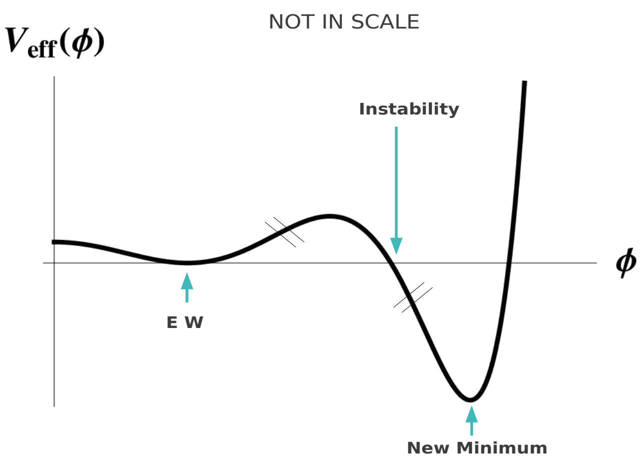
G.C., PoS TOP2015 (2016) 037, arXiv:1511.08429

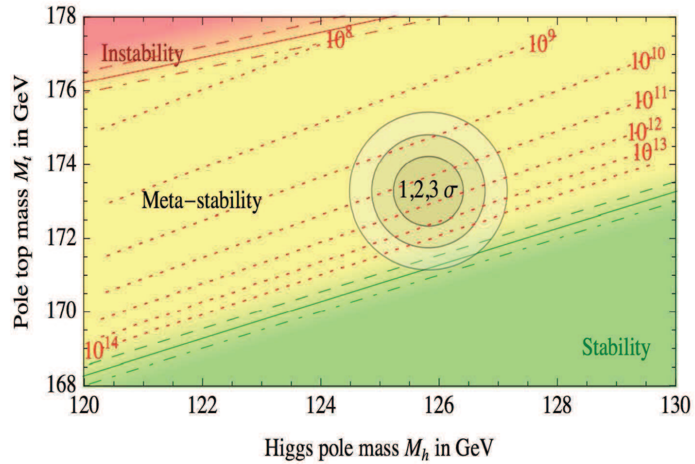
The top mass plays a crucial role in EW symmetry breaking (Gfitter at NNLO, '14)



Stability of SM vacuum depends on top and Higgs masses (Degrassi et al, Branchina et al, '12-'15)

$$V_{RG}(\phi) \simeq \frac{1}{2} m^2(\Lambda)\phi^2(\Lambda) + \frac{1}{4}\lambda^4(\Lambda)\phi^4(\Lambda)\ ,\ \phi \sim \Lambda \gg v$$





Stability: $V_{\rm eff}(v) < V_{\rm eff}(v')$; Instability: $V_{\rm eff}(v) > V_{\rm eff}(v')$; Metastability: $\tau > T_U$

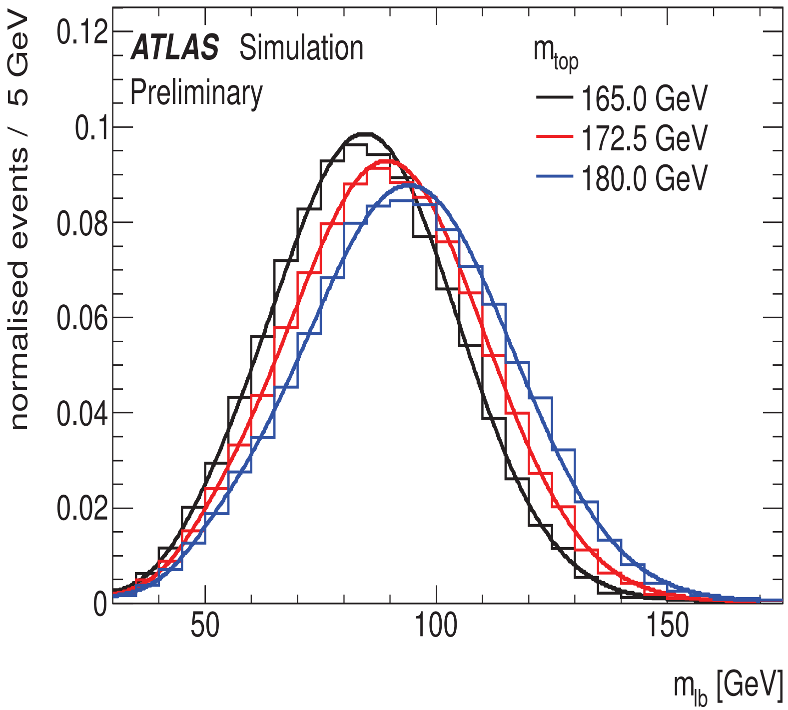
Top mass world average as pole mass in determination of Yukawa coupling

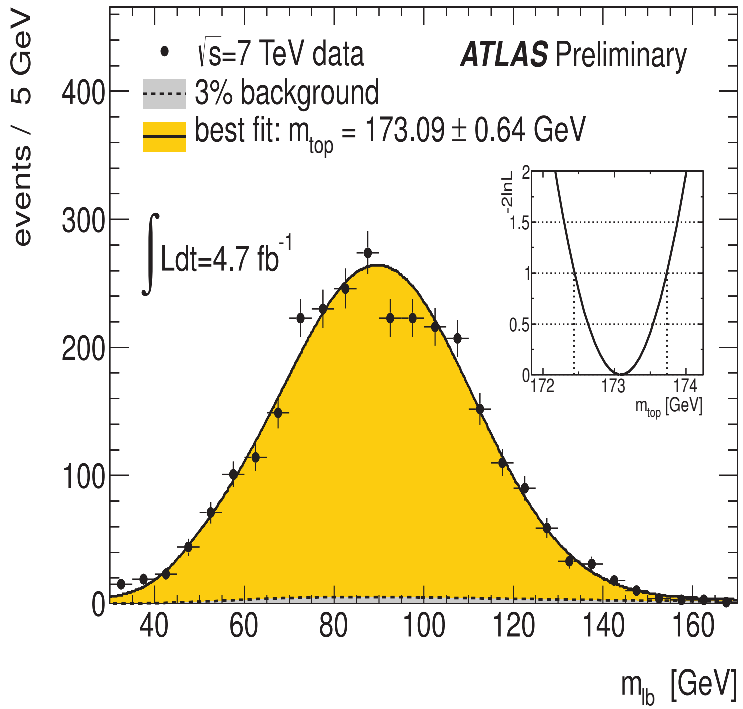
Top-mass measurements compare data with theory: the mass is a parameter in the theoretical prediction

Standard top mass reconstruction relies on top decays: template method

Distribution of observables sensitive to m_t and reconstruction under the assumption that the final state is $WbWb$ and the W mass is fixed

Data confronted with Monte Carlo templates and m_t is the value minimizing the χ^2



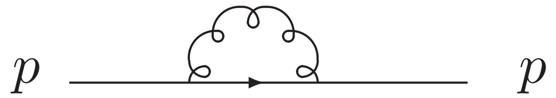


ATLAS: b-jet+lepton invariant mass in dilepton channel

m_t is the parameter in the event generator, often called Monte Carlo mass

Can we relate it to any theoretical mass definition?

Top-quark mass definitions - Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $$S(p) = -\frac{i}{\not p - m_t^0 + \Sigma^R(p, m_t^0, \mu)}$$

$d = 4 - 2\epsilon$

$$\Sigma^R \sim \left[\frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m_t^0, p, \mu)\right] \not p - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi\right) + B(m_t^0, p, \mu)\right] m_t^0 + (Z_2 - 1) \not p - (Z_2 Z_m - 1) m$$

On-shell renormalization (pole mass): $\Sigma^R(p) = 0$, $(\partial \Sigma^R / \partial \not p) = 0$ for $\not p = m$

$\overline{\mathrm{MS}}$ renormalization: counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

$$S^R_{o.s.}(p) \sim \frac{i}{\not p - m_{\mathrm{pole}}} \quad ; \quad S^R_{\overline{\mathrm{MS}}} \sim \frac{i}{\not p - m_{\overline{\mathrm{MS}}} - (A - B) m_{\overline{\mathrm{MS}}}}$$

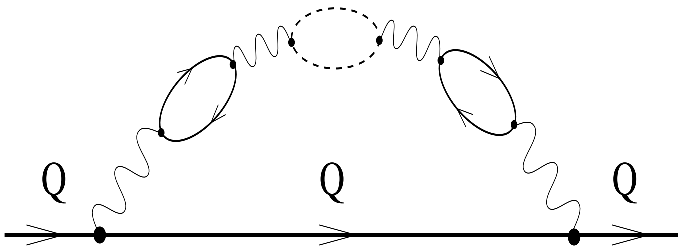
MSR masses in terms of an infrared scale R, e.g. μ_F, $\bar{m}(\mu)$, etc., in SCET (A.Hoang et al)

$$m^{\mathrm{MSR}}(R) = m^{\mathrm{pole}} - \Sigma^{\mathrm{fin}}(R, \mu) \quad \text{for} \quad R < m$$

$m_t^{\mathrm{MSR}}(R) \to m_{\mathrm{pole}}$ for $R \to 0$; $m^{\mathrm{MSR}}(R) \to \bar{m}_t(\bar{m}_t)$ for $R \to \bar{m}_t(\bar{m}_t)$

$$\frac{dm_{\mathrm{pole}}}{d \ln \mu} = 0 \Rightarrow \frac{dm^{\mathrm{MSR}}(R, \mu)}{d \ln \mu} = -R\gamma[\alpha_S(\mu)]$$

Higher-order corrections to the self energy in the infrared limit



$$\Sigma \sim m_{\rm pole} \sum_n \alpha_S^n\, (2\beta_0)^n\, n! \Rightarrow \text{renormalon}$$

$$\delta m_{\rm pole} \approx \mathcal{O}(\Lambda)$$

Relation pole/$\overline{\rm MS}$ mass at 4 loops $[\bar m = \bar m(\bar m)]$ (Marquard et al, PRL'15)

For top quarks:

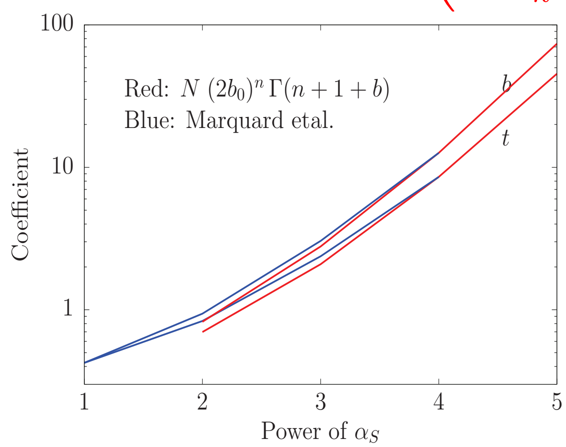
$$m_{\rm pole} = \bar m[1+0.42\;\alpha_S+0.83\;\alpha_S^2+2.37\;\alpha_S^3+(8.49\pm0.25)\;\alpha_S^4]\;;\;\Delta m_{{\rm pole},\overline{\rm MS}} \simeq 195\;{\rm MeV}$$

Renormalon calculation - large-n expansion: (M.Beneke'94)

$$m_{\rm pole} = \bar m \times \left(1+\sum_{n=0}^{\infty} r_n \alpha_S^{n+1}\right)\;;\; r_n \to N(2\beta_0)^n \Gamma\left(n+1+\frac{\beta_1}{\beta_0^2}\right)\left(1+\sum_{k=1}^{\infty}\frac{s_k}{n^k}\right)$$

Fitting N: $N \simeq 0.726$

Can be used to predict higher orders:

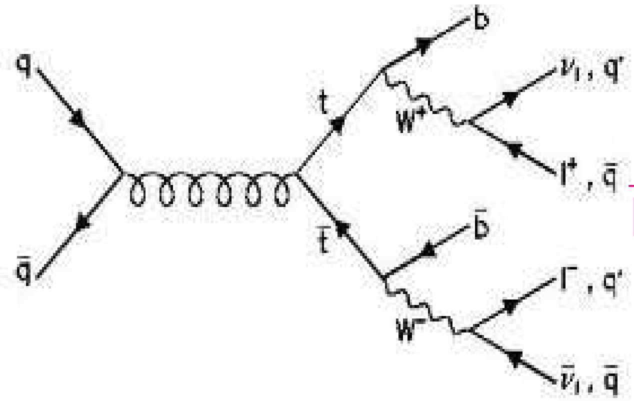


$$m_{\rm pole} = (163.63 + 7.56 + 1.62 + 0.50 + 0.19 + 0.10 + \dots)\;{\rm GeV}$$

Minimum at $n \sim$ 8-9: $\Delta m \approx |r_8\alpha_S^8 - r_9\alpha_S^9| \approx 70$ MeV - below LHC uncertainty

Check with method based on Borel summation (Beneke, Marquard, Nason, Steinhauser, '16)

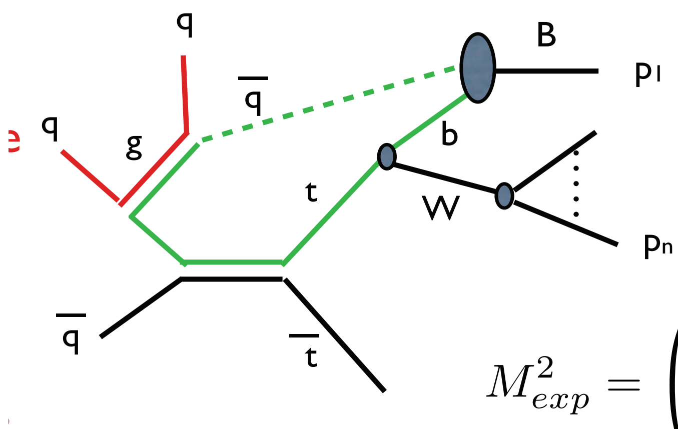
Measured mass must be close to $m_{\rm pole}$: top-decay kinematics is driven by $m_{\rm pole}$



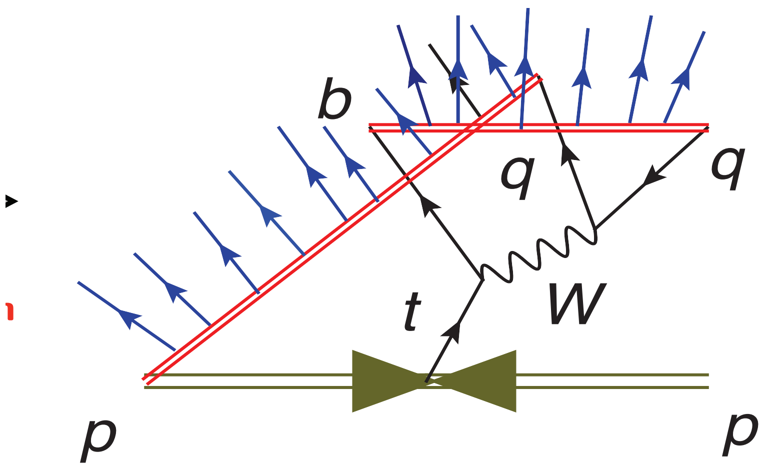
$$\frac{1}{[(p_W+p_b)^2-m_t^2]^2+m_t^2\Gamma_t^2} \sim \frac{1}{\pi}\,\delta[(p_W+p_b)^2-m_t^2] \text{ for } \Gamma_t \ll m_t$$

Reconstructed mass $p^2=(p_{b-\rm jet}+p_\nu+p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the pole mass, up to widths and higher-order corrections

Colour-reconnection effects can spoil this picture



$$M^2_{exp} = \left(\sum_{i=1,\ldots,n} p_i\right)^2$$

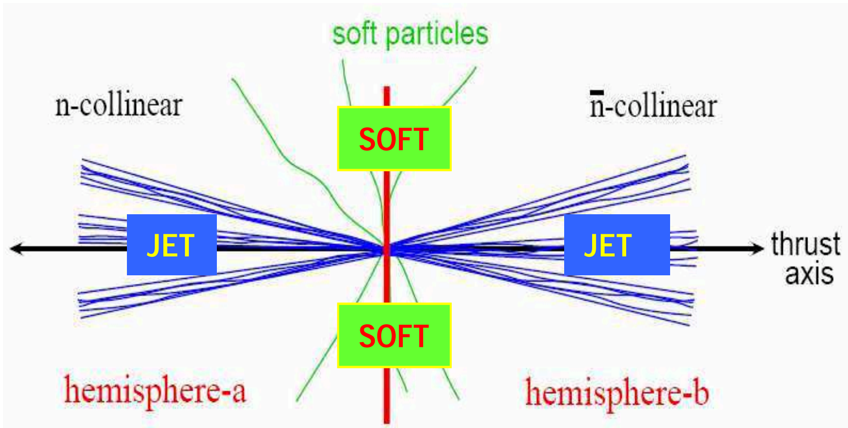


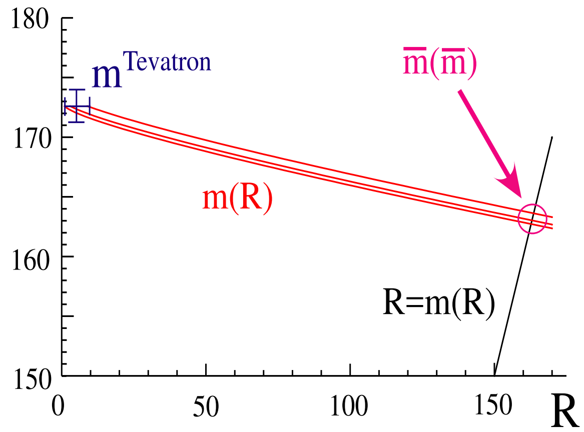
Left: M.Mangano, TOP'13 (HERWIG cluster model), Right: S.Argyropoulos, LNF'15 workshop (PYTHIA string model)

Leptonic observables without reconstructing top decay products minimize such effects

Calibrating the MC mass: measure an observable ξ without any assumption on $m_t^{\rm MC}$, compare it with Monte Carlo simulations and fit ξ and $m_t^{\rm MC}$ (Talk by J.Kieseler)

Attempt to address the MC mass using the SCET formalism $Q \gg m_t \gg \Gamma_t \gg \Lambda_{\rm QCD}$





Factorization theorem $(e^+e^- \to t\bar{t})$: A.H. Hoang and I.Stewart '08

$$\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2} \sim H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_+\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S(\ell^+, \ell)$$

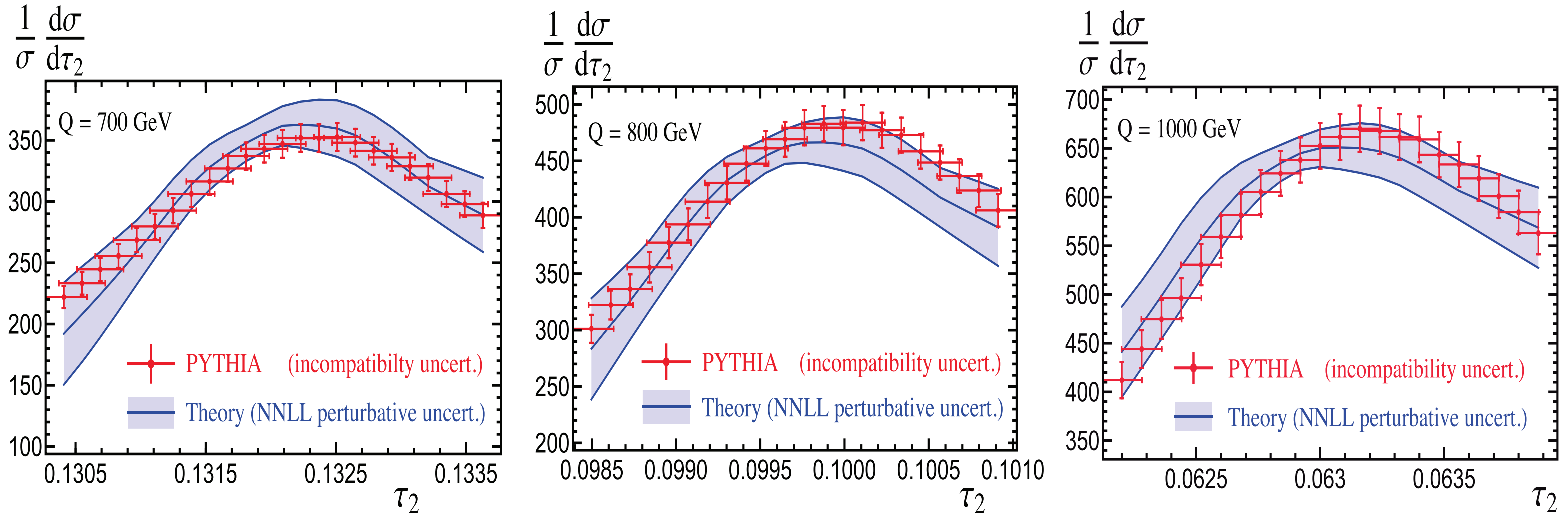
H_Q, H_m: hard scattering; $B_\pm$: jet function; S: soft function

Jet mass as MSR mass with $R \sim \Gamma_t$ and $\mu \sim Q_0 \sim \mathcal{O}(1\ \mathrm{GeV}) \Rightarrow$ MC mass

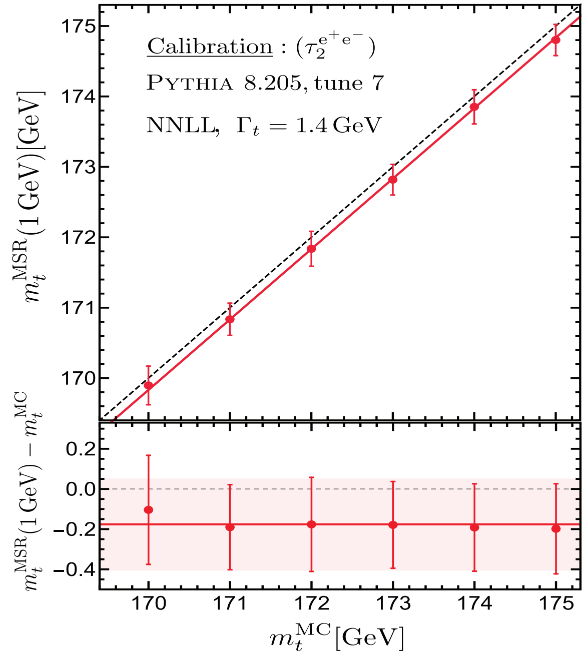
$$m_J(\mu) \sim \left[\frac{d \ln \tilde{B}(y,\mu)}{dy}\right]_{y=-ie^{-\gamma_E}/R} \quad \Rightarrow \quad m_{\rm pole} = m_J(\mu) + e^{\gamma_E}\Gamma_t \frac{\alpha_S(\mu) C_F}{\pi}\left(\ln\frac{\mu}{\Gamma_t} + \frac{1}{2}\right) + \mathcal{O}(\alpha_S^2)$$

$Q_0 \sim$ 1-2 GeV $\Rightarrow m_{\rm pole} - m_J(Q_0) \simeq$ 150-200 MeV, $\bar{m}_t(\bar{m}_t) = 163.0 \pm 1.3 \pm^{+0.6}_{-0.3}$ GeV

Calibrating PYTHIA mass vs resummation using $m_{\rm MSR}$ for $e^+e^- \to t\bar{t}$ Butenschoen et al,'16



$m_t^{\rm MC} = 173$ GeV $(\tau_2^{e^+e^-})$					
Mass	Order	Central	Perturb.	Incompatibility	Total
$m_{t,1\,\rm GeV}^{\rm MSR}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1\,\rm GeV}^{\rm MSR}$	NNLL	172.82	0.19	0.11	0.22
$m_t^{\rm pole}$	NLL	172.10	0.34	0.16	0.38
$m_t^{\rm pole}$	NNLL	172.43	0.18	0.22	0.28



PYTHIA mass compatible with $m_t^{\rm MSR}(1\ {\rm GeV})$

Discrepancy between $m_t^{\rm pole}$ and $m_t^{\rm MC}$ at the level of 600-900 MeV (central values)

e^+e^- collisions and an observable $\left(\tau_2 = 1 - \max\frac{\sum|\vec{n}_t\cdot\vec{p}_i|}{Q}\right)$ not investigated at LHC

Other attempts: run HERWIG with top-hadron states (G.C., TOP'15 proceedings):

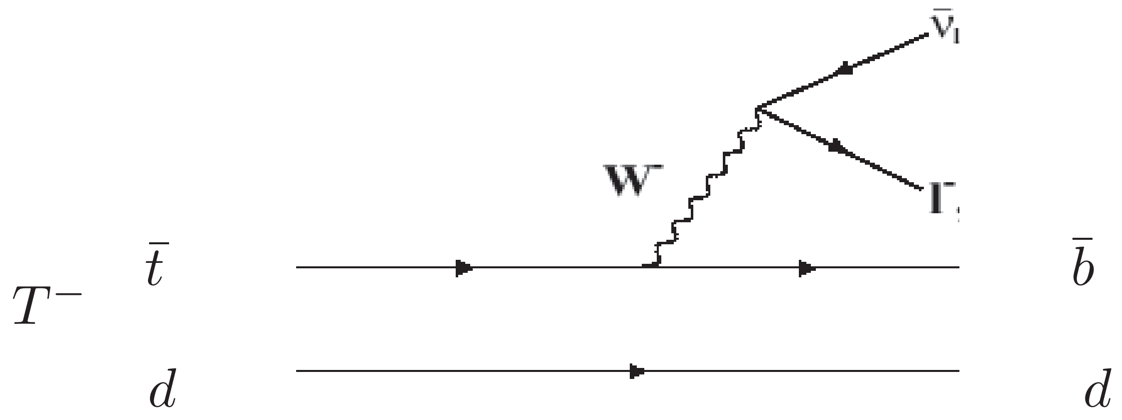
Pretend that top quarks hadronize into $T^{\pm,0}$ mesons and decay via the spectator model

From a given observable R extract the Monte Carlo mass $m_{t,T}^{\rm MC}$

Study the same observable R with standard top samples, get $m_t^{\rm MC}$ and compare the extracted masses : $m_{t,T}^{\rm MC} = m_t^{\rm MC} + \Delta m$

In the hadronized samples, the T-meson mass M_T can be related to pole or $\overline{\rm MS}$ top-quark masses by using lattice, potential models, NRQCD, etc.: $M_T = m_t + m_q + \Delta m_{t,T}$

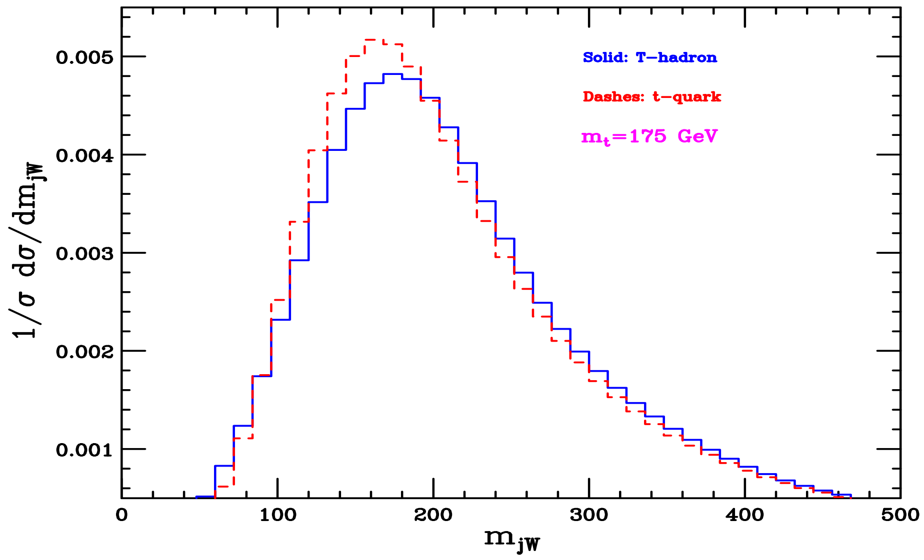
Spectator model decays: $T^- \to (\bar b d)\ell^-\bar\nu_\ell + X \ldots \quad p_T^2 = (p_{\bar b} + p_W + p_q + p_X)^2$

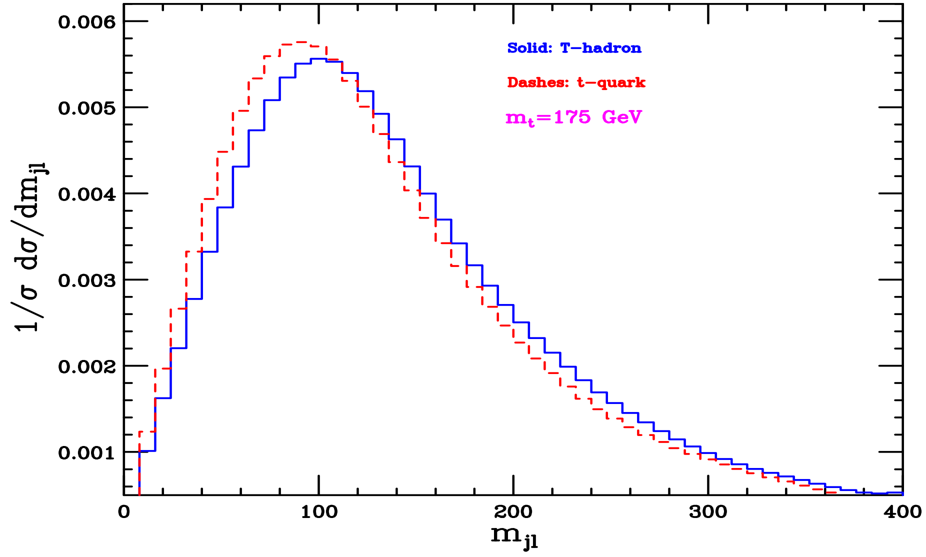


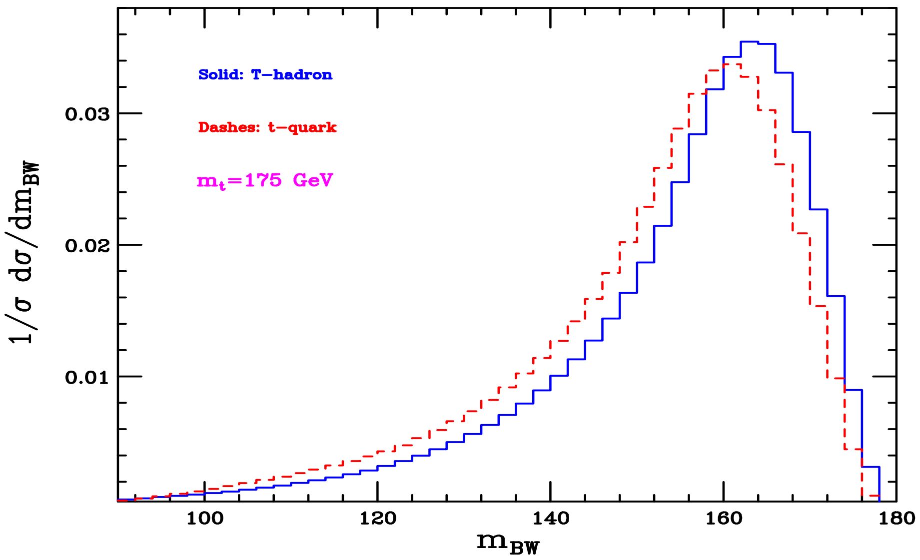
Spectator does not radiate, few events where the b quark does not emit

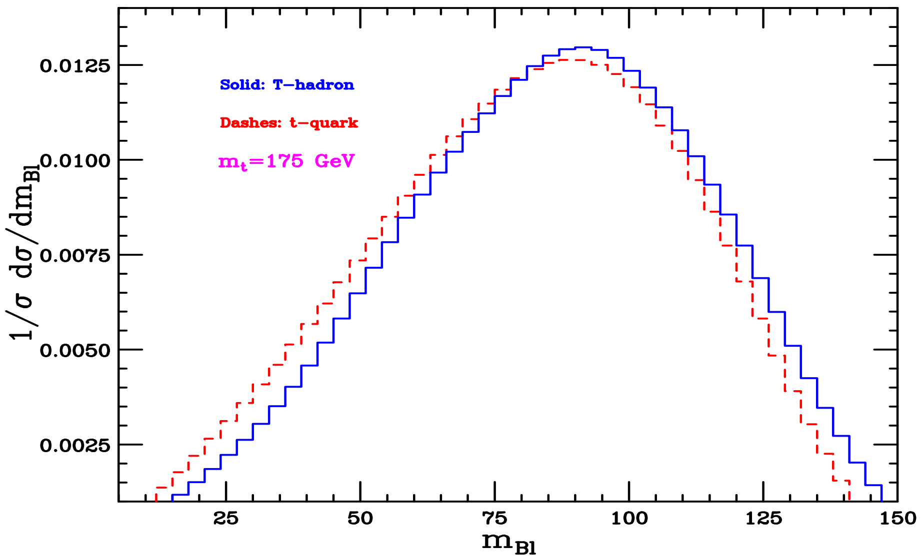
$\bar b$ tends to form clusters with spectator quarks with invariant mass closer to $(p_T - p_W)^2$ with respect to standard top decays

pp collisions at $\sqrt{s} = 8$ TeV, dilepton channel, k_T algorithm, $R = 0.7$, $p_{T,j} > 30$ GeV, $p_{T,\ell} > 20$ GeV, MET> 20 GeV, $|\eta_{j,\ell,\nu}| < 2.5$ (HERWIG 6.510, preliminary)









In progress: relate $\langle \Delta m_{jl,Bl} \rangle$ with $\Delta m_{t,T}$ and using PYTHIA

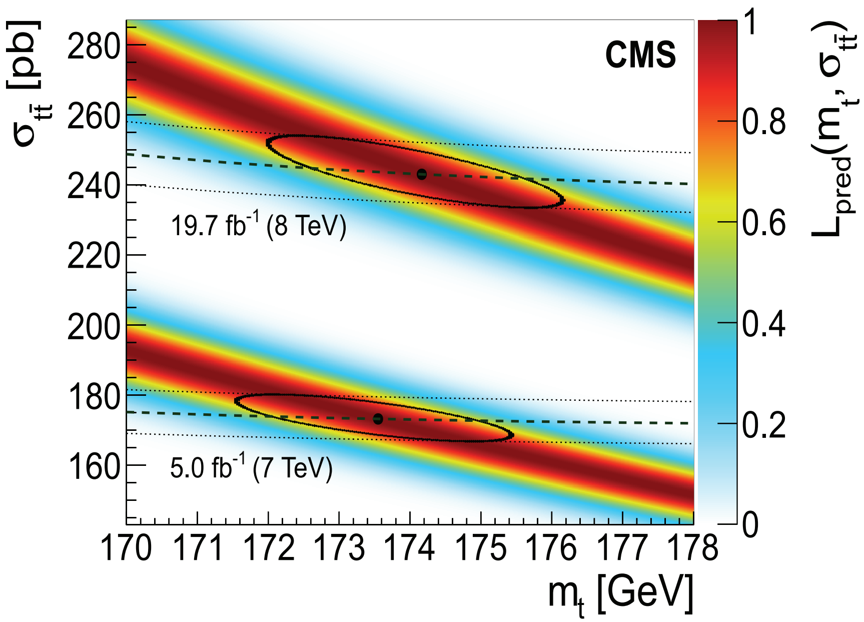
Alternative methods: m_t from NNLO+NNLL $\sigma(pp \to t\bar{t})$ (Czakon, Fielder and Mitov, '13):

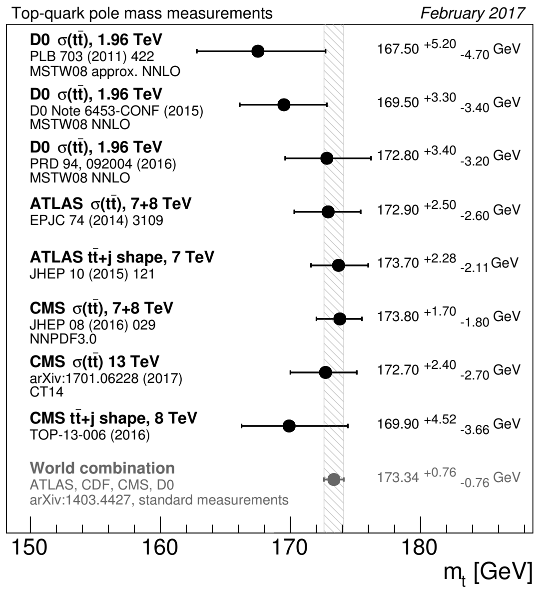
$$\sigma_{\rm tot} = \sum_{i,j} \int d\beta \; \Phi_{ij}(\beta, \mu_F^2)\; \hat{\sigma}_{ij} \quad , \quad \beta = \sqrt{1 - 4m^2/\hat{s}} \quad , \quad \Phi_{ij} = \frac{2\beta}{1-\beta^2}\, x\,(f_i \otimes f_j)$$

At NNLO, $\mu = \mu_F = \mu_R$ and $L = \ln(m_t^2/\mu^2)$, using the pole mass:

$$\hat{\sigma}_{ij} = \frac{\alpha_S^2}{m_t^2}\left\{\sigma^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right]\right\}$$

Threshold logs: $\alpha_S^n[\ln^m(1-z)/(1-z)]_+$, $z = m_t^2/(x_i x_j s) \to 1$, $m \leq 2n-1$





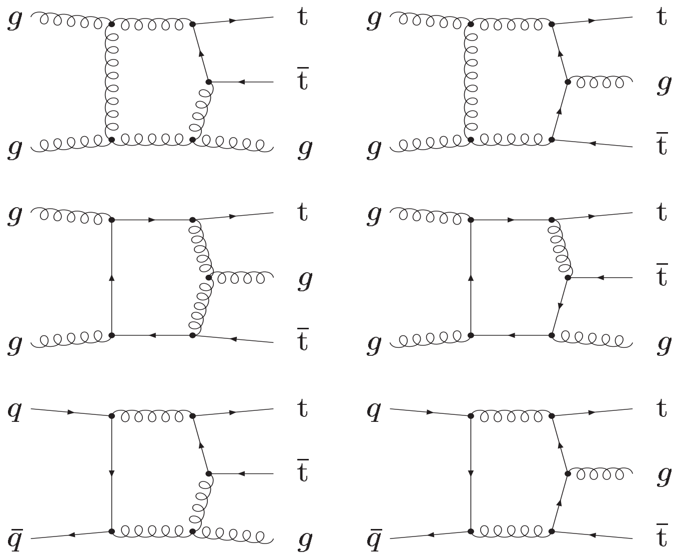
Mild dependence on MC m_t; $\Delta(\alpha_S, {\rm PDF}, \mu) \simeq^{+0.9}_{-1.1}$ GeV, $\Delta({\rm fit}) \simeq^{+2.1}_{-2.0}$ GeV (13 TeV)

Larger errors than standard methods, but larger statistics are expected in Run 2

Late extension to NNLO distributions and prospects to include decays (D.Haymes' talk)

NLO calculation of $t\bar{t}$+jet cross section with the pole mass (S.Dittmaier et al.,'07)

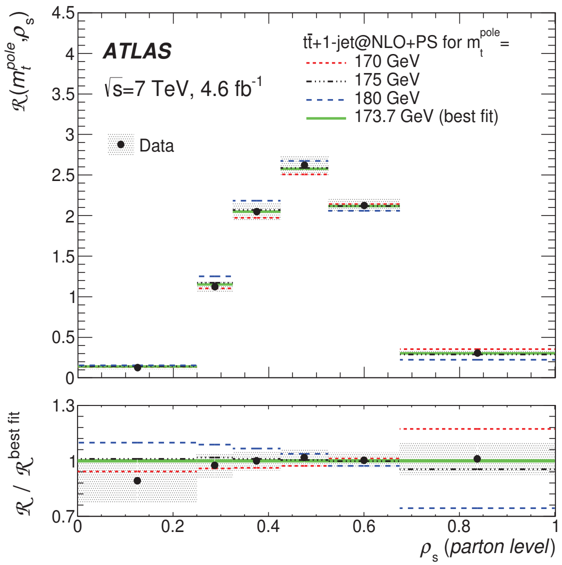
Matching with shower and hadronization through POWHEG+PYTHIA (S.Alioli et al,'13)

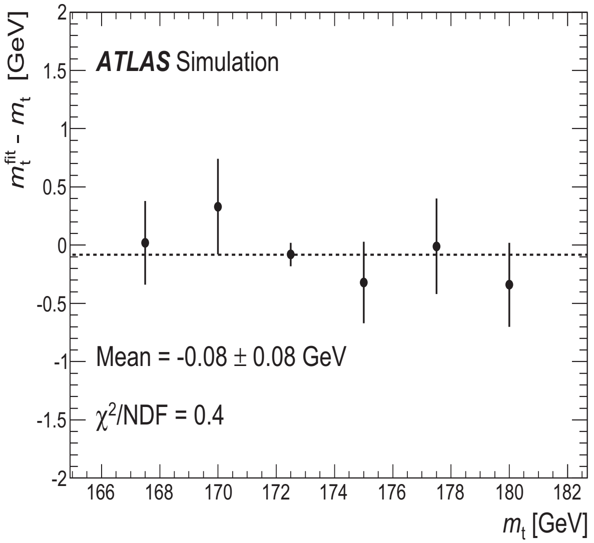


$$\mathcal{R} = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}(m_t^{\rm pole})}{d\rho_S}$$

$$\rho_S = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} \quad , \quad m_0 = 170 \text{ GeV}$$

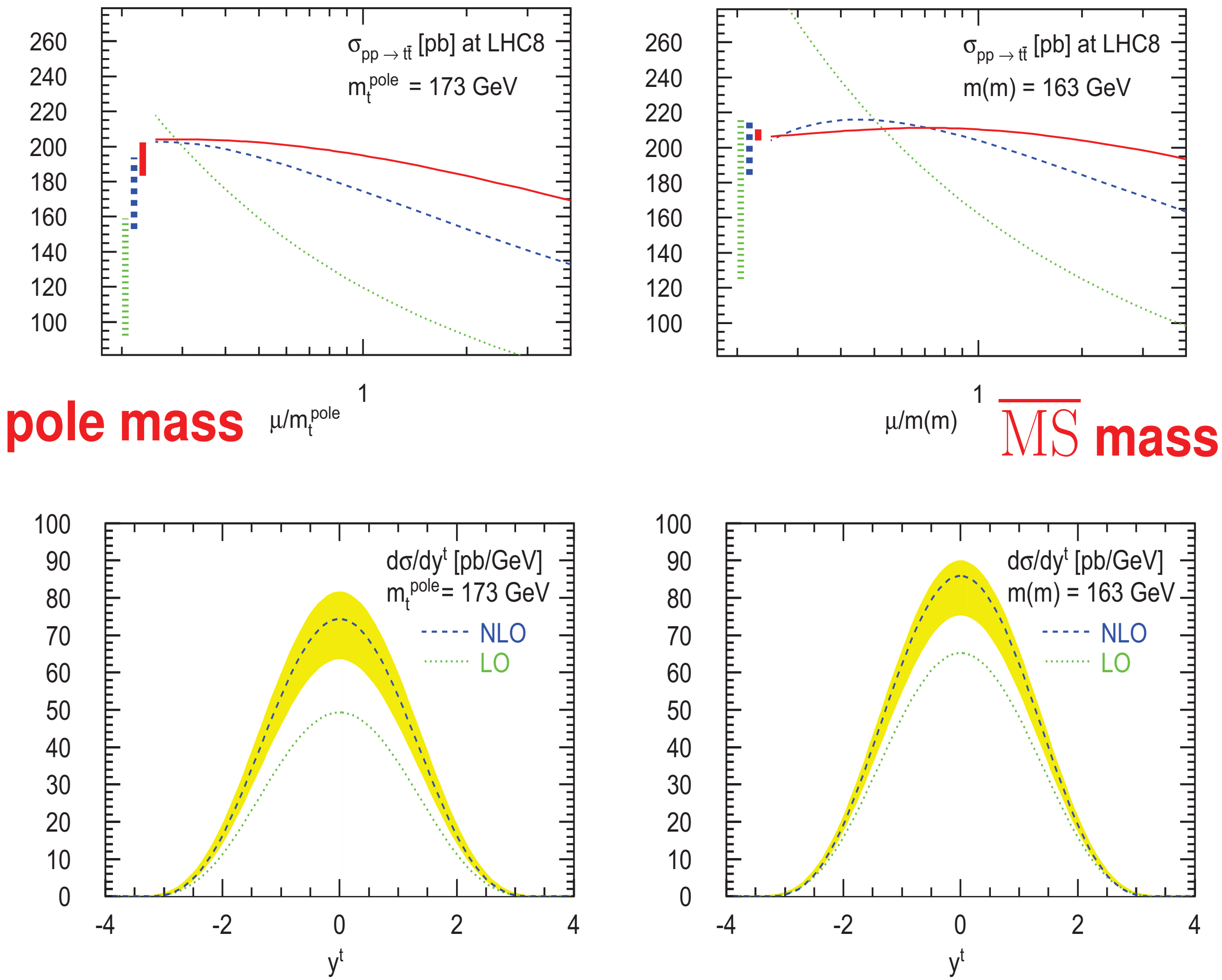
$$t\bar{t} + \text{jet (det.)} \rightarrow t\bar{t} + \text{jet (parton)}$$





$$m_t^{\rm pole} = [173.1 \pm 1.5(\text{stat}) \pm 1.4(\text{syst})^{+1.0}_{-0.5}(\text{theo})] \text{ GeV}$$

Extracting top running mass from NNLO $t\bar{t}$ cross section (M.Dowling and S.Moch'14)

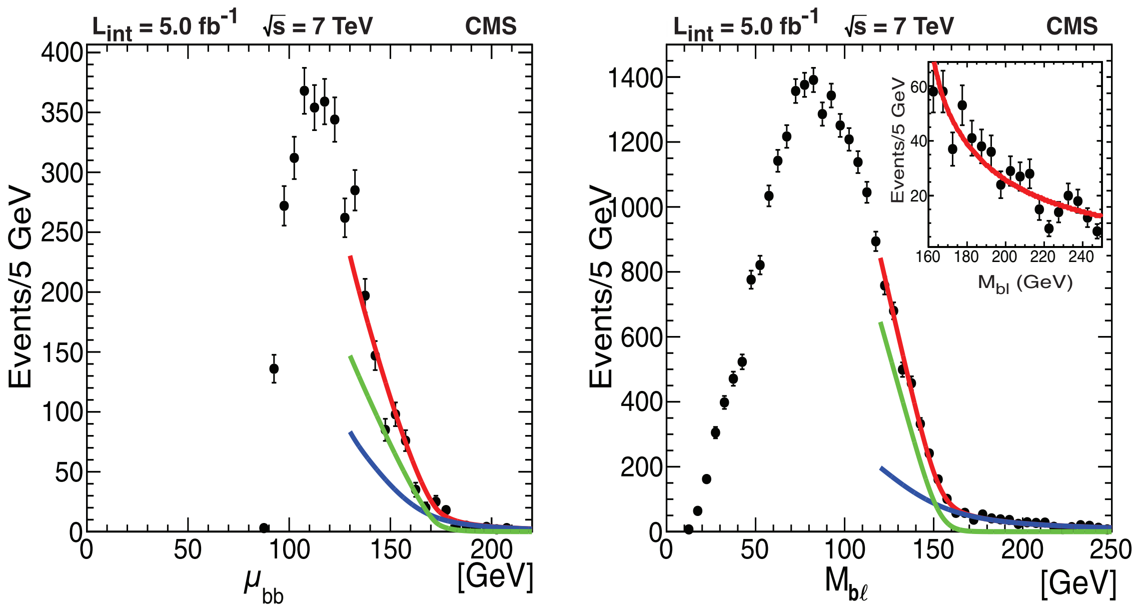


Using the $\overline{\mathrm{MS}}$ mass seems to yield a milder scale dependence

Scale variation: $1/2 < \mu/m_{\mathrm{pole}} < 2$; $1/2 < \mu/m(m) < 2$, $\mu = \mu_F = \mu_R$

ABM fit: $\bar{m}_t(\bar{m}_t) = 162.3 \pm 2.3$ GeV; $m_t^{\mathrm{pole}} = 171.2 \pm 2.4$ GeV

Endpoint method (CMS): in dilepton channels, the endpoint of ℓ+b-jet, '$\ell\ell$' or 'bb' invariant mass distributions are sensitive to m_t



Endpoint relations at LO (no radiative corrections) from energy-momentum conservation; shapes normalized to NLO cross sections

$$\mu_{bb}^{\max} = \frac{m_t}{2}\left(1 - \frac{m_W^2}{m_t^2}\right) + \sqrt{\frac{m_t^2}{4}\left(1 - \frac{m_W^2}{m_t^2}\right) + m_W^2}\ , \ m_{b\ell}^{\max} = \sqrt{m_b^2 + \left(1 - \frac{m_\nu^2}{m_W^2}\right)(E_W^* + p^*)(E_b + p^*)}$$

Minimizes generator impact since b-jet calibration uses data; theory error mostly due to colour reconnection effects

As it is based on kinematic constraints, the measured m_t must be close to the pole mass

Interesting to compare with POWHEG with NLO top decays and approximate Γ_t effects

Conclusions

Top-quark mass is a fundamental SM parameter

Standard reconstruction methods at LHC have reached 0.5% precision

Measured mass close to the pole mass, with renormalon ambiguity even below 100 MeV, after 4-loop calculation of pole vs. $\overline{\rm MS}$ mass relation

Ongoing work using SCET formalism with MSR mass and simulating fictitious T-hadrons may help to shed light on deviations from the pole mass

Higher statistics at Run 2 will reduce the uncertainty on pole mass from $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+j}$ to about 1 GeV

Theoretical uncertainties on pole vs. MC mass relation expected to go down thanks to novel higher order calculations and width effects implementation in NLO+shower codes

Top-quark phenomenology on the road to become precision physics

Small and understood theory errors and a clear relation between MC and pole masses should be reachable in the near future