Interpretation of the top-quark mass results

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Frascati workshop on ‘Top mass: challenges in definition and determination’,
https://agenda.infn.it/conferenceDisplay.py?confId=9202


The top mass plays a crucial role in EW symmetry breaking (Gfitter at NNLO, ’14)

![Graph showing EW symmetry breaking](image)

Stability of SM vacuum depends on top and Higgs masses (Degrassi et al, Branchina et al, ’12-'15)

\[ V_{RG}(\phi) \approx \frac{1}{2} m^2(\Lambda)\phi^2(\Lambda) + \frac{1}{4} \lambda^4(\Lambda)\phi^4(\Lambda), \quad \phi \sim \Lambda \gg v \]

Stability: \( V_{\text{eff}}(v) < V_{\text{eff}}(v') \); Instability: \( V_{\text{eff}}(v) > V_{\text{eff}}(v') \); Metastability: \( \tau > T_U \)

Top mass world average as pole mass in determination of Yukawa coupling
Top-mass measurements compare data with theory: the mass is a parameter in the theoretical prediction

Standard top mass reconstruction relies on top decays: template method

Distribution of observables sensitive to $m_t$ and reconstruction under the assumption that the final state is $WbWb$ and the $W$ mass is fixed

Data confronted with Monte Carlo templates and $m_t$ is the value minimizing the $\chi^2$

**ATLAS: $b$-jet+lepton invariant mass in dilepton channel**

$m_t$ is the parameter in the event generator, often called Monte Carlo mass

Can we relate it to any theoretical mass definition?
Top-quark mass definitions - Subtraction of the UV divergences in the self energy $\Sigma(p)$

Renormalized propagator:

$$S(p) = - \frac{i}{\not{p} - m^0_t + \Sigma^R(p, m^0_t, \mu)}$$

$$\Sigma^R \sim \left[ \frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m^0_t, p, \mu) \right] \not{p} - \left[ 4 \left( \frac{1}{\epsilon} - \gamma + \ln 4\pi \right) + B(m^0_t, p, \mu) \right] m^0_t + (Z_2 - 1) \not{p} - (Z_2 Z_m - 1)m$$

On-shell renormalization (pole mass): $\Sigma^R(p) = 0$, $(\partial \Sigma^R / \partial \not{p}) = 0$ for $\not{p} = m$

$\overline{\text{MS}}$ renormalization: counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

$$S^{R}_{o.s.}(p) \sim \frac{i}{\not{p} - m_{\text{pole}}}; \quad S^{R}_{\overline{\text{MS}}} \sim \frac{i}{\not{p} - m_{\overline{\text{MS}}} - (A - B)m_{\overline{\text{MS}}}}$$

MSR masses in terms of an infrared scale $R$, e.g. $\mu_F$, $\bar{m}(\mu)$, etc., in SCET (A.Hoang et al)

$$m^{\text{MSR}}(R) = m^{\text{pole}} - \Sigma^{\text{fin}}(R, \mu) \quad \text{for} \quad R < m$$

$$m^{\text{MSR}}_t(R) \to m_{\text{pole}} \quad \text{for} \quad R \to 0; \quad m^{\text{MSR}}(R) \to \bar{m}_t(\bar{m}_t) \quad \text{for} \quad R \to \bar{m}_t(\bar{m}_t)$$

$$\frac{dm_{\text{pole}}}{d \ln \mu} = 0 \Rightarrow \frac{dm^{\text{MSR}}(R, \mu)}{d \ln \mu} = -R \gamma[A_S(\mu)]$$
Higher-order corrections to the self energy in the infrared limit

\[ \Sigma \sim m_{\text{pole}} \sum_n \alpha_S^n (2\beta_0)^n n! \Rightarrow \text{renormalon} \]

\[ \delta m_{\text{pole}} \approx \mathcal{O}(\Lambda) \]

Relation pole/\overline{MS} mass at 4 loops \( [\bar{m} = \bar{m}(\bar{m})] \) (Marquard et al, PRL’15)

For top quarks:
\[ m_{\text{pole}} = \bar{m}[1 + 0.42 \alpha_S + 0.83 \alpha_S^2 + 2.37 \alpha_S^3 + (8.49 \pm 0.25) \alpha_S^4] ; \quad \Delta m_{\text{pole},\overline{\text{MS}}} \approx 195 \text{ MeV} \]

Renormalon calculation - large-\( n \) expansion: (M.Beneke’94)

\[ m_{\text{pole}} = \bar{m} \times \left( 1 + \sum_{n=0}^{\infty} r_n \alpha_S^{n+1} \right) ; \quad r_n \to N(2\beta_0)^n \Gamma \left( n + 1 + \frac{\beta_1}{\beta_0} \right) \left( 1 + \sum_{k=1}^{\infty} \frac{s_k}{n^k} \right) \]

Fitting \( N \): \( N \approx 0.726 \)

Can be used to predict higher orders:
\[ m_{\text{pole}} = (163.63 + 7.56 + 1.62 + 0.50 + 0.19 + 0.10 + \ldots) \text{ GeV} \]

Minimum at \( n \approx 8-9 \): \( \Delta m \approx |r_8 \alpha_S^8 - r_9 \alpha_S^9| \approx 70 \text{ MeV} \) - below LHC uncertainty

Check with method based on Borel summation (Beneke, Marquard, Nason, Steinhauser, ’16)
Measured mass must be close to $m_{\text{pole}}$: top-decay kinematics is driven by $m_{\text{pole}}$

\[
\frac{1}{[(p_W + p_b)^2 - m_t^2]^2 + m_t^2 \Gamma_t^2} \sim \frac{1}{\pi} \delta[(p_W + p_b)^2 - m_t^2] \text{ for } \Gamma_t \ll m_t
\]

Reconstructed mass $p^2 = (p_{b-\text{jet}} + p_\nu + p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the pole mass, up to widths and higher-order corrections

Colour-reconnection effects can spoil this picture

\[
M_{\exp}^2 = \left( \sum_{i=1,\ldots,n} p_i \right)^2
\]

Left: M.Mangano, TOP’13 (HERWIG cluster model), Right: S.Argyropoulos, LNF’15 workshop (PYTHIA string model)

Leptonic observables without reconstructing top decay products minimize such effects
Calibrating the MC mass: measure an observable $\xi$ without any assumption on $m_t^{MC}$, compare it with Monte Carlo simulations and fit $\xi$ and $m_t^{MC}$ (Talk by J.Kieseler)

Attempt to address the MC mass using the SCET formalism $Q \gg m_t \gg \Gamma_t \gg \Lambda_{QCD}$

Factorization theorem ($e^+e^- \rightarrow t\bar{t}$): A.H. Hoang and I.Stewart '08

$$\frac{d\sigma}{dM_t^2dM_{\bar{t}}^2} \sim H_Q(Q, \mu_m)H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell^+d\ell^-B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right)B_+\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right)S(\ell^+, \ell^-)$$

$H_Q, H_m$: hard scattering; $B_\pm$: jet function; $S$: soft function

Jet mass as MSR mass with $R \sim \Gamma_t$ and $\mu \sim Q_0 \sim \mathcal{O}(1 \text{ GeV}) \Rightarrow \text{MC mass}$

$$m_J(\mu) \sim \left[ \frac{d\ln \tilde{B}(y, \mu)}{dy} \right]_{y=-ie^{-\gamma_E/R}} \Rightarrow m_{pole} = m_J(\mu) + e^{\gamma_E\Gamma_t} \frac{\alpha_S(\mu)C_F}{\pi} \left( \ln \frac{\mu}{\Gamma_t} + \frac{1}{2} \right) + \mathcal{O}(\alpha_S^2)$$

$Q_0 \sim 1-2 \text{ GeV} \Rightarrow m_{pole} - m_J(Q_0) \sim 150-200 \text{ MeV}$, $\bar{m}_t(\bar{m}_t) = 163.0 \pm 1.3^{+0.6}_{-0.3} \text{ GeV}$
Calibrating PYTHIA mass vs resummation using $m_{\text{MSR}}$ for $e^+e^- \rightarrow t\bar{t}$ Butenschoen et al.,'16

PYTHIA mass compatible with $m_{t}^{\text{MSR}}(1 \text{ GeV})$

Discrepancy between $m_{t}^{\text{pole}}$ and $m_{t}^{\text{MC}}$ at the level of 600-900 MeV (central values) $e^+e^-$ collisions and an observable $\left(\tau_2 = 1 - \max \sum_{i} |\vec{n}_i \cdot \vec{p}_i|\right)$ not investigated at LHC
Other attempts: run HERWIG with top-hadron states (G.C., TOP’15 proceedings):

Pretend that top quarks hadronize into $T^{\pm,0}$ mesons and decay via the spectator model.

From a given observable $R$ extract the Monte Carlo mass $m_{t,T}^{\text{MC}}$:

Study the same observable $R$ with standard top samples, get $m_{t}^{\text{MC}}$ and compare the extracted masses: $m_{t,T}^{\text{MC}} = m_{t}^{\text{MC}} + \Delta m$.

In the hadronized samples, the $T$-meson mass $M_T$ can be related to pole or $\overline{\text{MS}}$ top-quark masses by using lattice, potential models, NRQCD, etc.: $M_T = m_t + m_q + \Delta m_{t,T}$

Spectator model decays: $T^- \to (\overline{b}d)\ell^-\overline{\nu}_\ell + X \ldots$  \[ p_T^2 = (p_{\overline{b}} + p_{W} + p_q + p_X)^2 \]

Spectator does not radiate, few events where the $b$ quark does not emit.

$\overline{b}$ tends to form clusters with spectator quarks with invariant mass closer to $(p_T - p_W)^2$ with respect to standard top decays.
In progress: relate $\langle \Delta m_{j,l,Bl} \rangle$ with $\Delta m_{t,T}$ and using PYTHIA
Alternative methods: $m_t$ from NNLO+NNLL $\sigma(pp \to t\bar{t})$ (Czakon, Fielder and Mitov, '13):

$$\sigma_{tot} = \sum_{i,j} \int d\beta \, \Phi_{ij}(\beta, \mu_F^2) \hat{\sigma}_{ij}, \quad \beta = \sqrt{1 - 4m^2/\hat{s}}, \quad \Phi_{ij} = \frac{2\beta}{1 - \beta^2} x (f_i \otimes f_j)$$

At NNLO, $\mu = \mu_F = \mu_R$ and $L = \ln(m_t^2/\mu^2)$, using the pole mass:

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_t^2} \left\{ \sigma^{(0)} + \alpha_S \left[ \sigma^{(1)}_{ij} + L\sigma^{(1,1)}_{ij} \right] + \alpha_S^2 \left[ \sigma^{(2)}_{ij} + L\sigma^{(2,1)}_{ij} + L^2\sigma^{(2,2)}_{ij} \right] \right\}$$

Threshold logs: $\alpha_s^n \left[ \ln m (1 - z)/(1 - z) \right]^+, \quad z = m_t^2/(x_i x_j s) \to 1, \quad m \leq 2n - 1$

Mild dependence on MC $m_t$; $\Delta(\alpha_S, PDF, \mu) \sim^{+0.9}_{-1.1}$ GeV, $\Delta$(fit) $\sim^{+2.1}_{-2.0}$ GeV (13 TeV)

Larger errors than standard methods, but larger statistics are expected in Run 2

Late extension to NNLO distributions and prospects to include decays (D.Haymes’ talk)
NLO calculation of $t\bar{t}$+jet cross section with the pole mass (S.Dittmaier et al.,’07)

Matching with shower and hadronization through POWHEG+PYTHIA (S.Alioli et al,’13)

\[
\mathcal{R} = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}(m_t^{\text{pole}})}{d\rho_S}
\]

\[
\rho_S = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} , \quad m_0 = 170 \text{ GeV}
\]

$t\bar{t}$ + jet (det.) → $t\bar{t}$ + jet (parton)

\[m_t^{\text{pole}} = [173.1 \pm 1.5 \text{(stat)} \pm 1.4 \text{(syst)}^{+1.0}_{-0.8} \text{(theo)}] \text{ GeV}\]
Extracting top running mass from NNLO $t\bar{t}$ cross section (M.Dowling and S.Moch'14)

Using the $\overline{\text{MS}}$ mass seems to yield a milder scale dependence
Scale variation: $1/2 < \mu/m_{\text{pole}} < 2$; $1/2 < \mu/m(m) < 2$, $\mu = \mu_F = \mu_R$
ABM fit: $\bar{m}_t(\bar{m}_t) = 162.3 \pm 2.3$ GeV; $m_t^{\text{pole}} = 171.2 \pm 2.4$ GeV
Endpoint method (CMS): in dilepton channels, the endpoint of $\ell+b$-jet, ‘$\ell\ell$’ or ‘$bb$’ invariant mass distributions are sensitive to $m_t$

Endpoint relations at LO (no radiative corrections) from energy-momentum conservation; shapes normalized to NLO cross sections

$$
\mu_{bb}^{\text{max}} = \frac{m_t}{2} \left(1 - \frac{m_W^2}{m_t^2}\right) + \sqrt{\frac{m_t^2}{4} \left(1 - \frac{m_W^2}{m_t^2}\right) + m_W^2}, \quad m_{b\ell}^{\text{max}} = \sqrt{m_b^2 + \left(1 - \frac{m_{\nu}^2}{m_W^2}\right)(E_W^* + p^*)(E_b + p^*)}
$$

Minimizes generator impact since $b$-jet calibration uses data; theory error mostly due to colour reconnection effects

As it is based on kinematic constraints, the measured $m_t$ must be close to the pole mass

Interesting to compare with POWHEG with NLO top decays and approximate $\Gamma_t$ effects
Conclusions

Top-quark mass is a fundamental SM parameter

Standard reconstruction methods at LHC have reached 0.5% precision

Measured mass close to the pole mass, with renormalon ambiguity even below 100 MeV, after 4-loop calculation of pole vs. $\overline{\text{MS}}$ mass relation

Ongoing work using SCET formalism with MSR mass and simulating fictitious $T$-hadrons may help to shed light on deviations from the pole mass

Higher statistics at Run 2 will reduce the uncertainty on pole mass from $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+j}$ to about 1 GeV

Theoretical uncertainties on pole vs. MC mass relation expected to go down thanks to novel higher order calculations and width effects implementation in NLO+shower codes

Top-quark phenomenology on the road to become precision physics

Small and understood theory errors and a clear relation between MC and pole masses should be reachable in the near future