Proton structure in a light-front quark diquark model: Single Spin Asymmetry

Tanmay Maji
Indian Institute of Technology Kanpur, India

April 5, 2017

Collaborator:
Dipankar Chakrabarti
Indian Institute of Technology Kanpur, India

25th International Workshops on Deep-inelastic scattering and related topics
University of Birmingham, Birmingham, UK

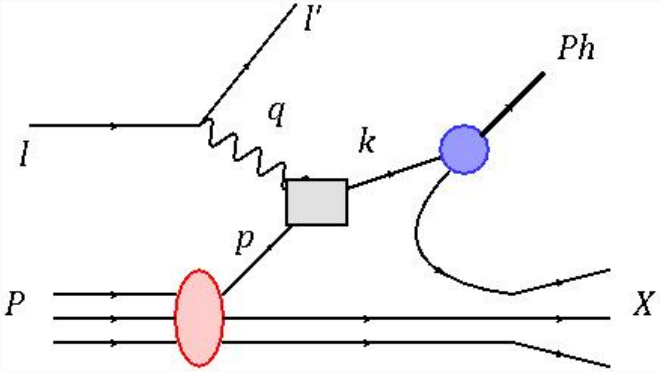
Contents

Introduction

- In past few decades several experiment collaborations, e.g., HERMES, COMPASS, JLAB etc., have came up with one of the most exciting feature of the nucleons: Single spin asymmetry(SSA).
- These asymmetries indicate existence of non-vanishing transverse momentum of interior partons and collinear picture is no longer sufficient to describe the transverse structure of nucleons.
- The information of asymmetries are encoded into the transverse momentum dependent distributions(TMDs) and fragmentation functions(FFs).

Single spin asymmetries in SIDIS

$$\ell(\ell) + N(P) \to \ell(\ell') + h(P_h) + X$$



A_{UU}	$\sim f_1 \otimes D_1$
$A_{UT}^{\sin(\phi_h - \phi_S)}$	$\sim f_{1T}^{\perp} \otimes D_1$
$A_{UT}^{\sin(\phi_h + \phi_S)}$	$\sim h_1 \otimes H_1^{\perp}$
$A_{UT}^{\sin(3\phi_h - \phi_S)}$	$\sim h_{1T}^{\perp} \otimes H_1^{\perp}$
$A_{UL}^{\sin\phi_h}$	$\sim h_{1L}^{\perp} \otimes H_1^{\perp}$
	$\ldots$

- At small $\mathbf{P}_{h\perp}$ and large Q region: $P_{h\perp}^2 \simeq \Lambda_{QCD}^2 \ll Q^2$

$$d\sigma^{\ell N \to \ell' h X} = \sum_{\nu} \hat{f}_{\nu/P}(x, \mathbf{p}_{\perp}^2; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q} \otimes \hat{D}_{h/\nu}(z, \mathbf{k}_{\perp}^2; Q^2);$$

$$A_{S_\ell S_N} = \frac{d\sigma^{\ell P^{\uparrow} \to \ell' h X} - d\sigma^{\ell P^{\downarrow} \to \ell' h X}}{d\sigma^{\ell P^{\uparrow} \to \ell' h X} + d\sigma^{\ell P^{\downarrow} \to \ell' h X}} \neq 0$$

$$A_{S_\ell S_N}^{\mathcal{W}}(x, z, \mathbf{P}_{h\perp}, y) = 2\frac{\int d\phi_h d\phi_S [d\sigma^{\ell P^{\uparrow} \to \ell' h X} - d\sigma^{\ell P^{\downarrow} \to \ell' h X}]\mathcal{W}}{\int d\phi_h d\phi_S [d\sigma^{\ell P^{\uparrow} \to \ell' h X} + d\sigma^{\ell P^{\downarrow} \to \ell' h X}]}$$

- **Phenomenological extractions:** TMDs and FFs by fitting the experimental data of SSAs considering the Gaussian ansatz— $h_1(x, \mathbf{p}_\perp)$ and $H_1^{\perp\nu}$ by Anselmino $et.\ al.$ [PRD75, PRD87] $h_{1T}^{\perp}(x, \mathbf{p}_\perp)$ by Prokudin $et.\ al.$ [PRD91] etc.

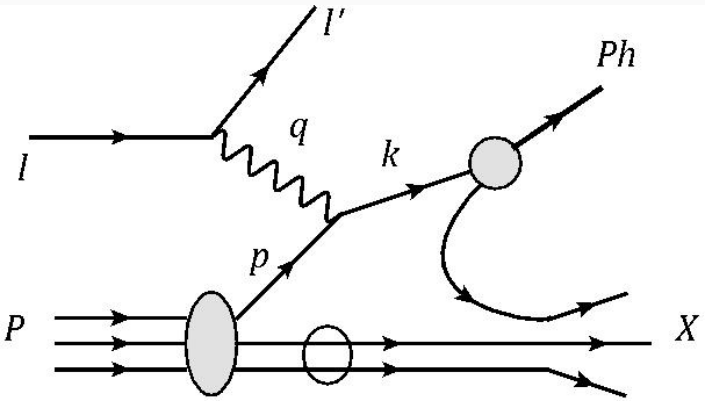
$$
\begin{aligned}
f_1^\nu(x, \mathbf{p}_\perp^2) &= f_1^\nu(x) \frac{e^{-\mathbf{p}_\perp^2/\langle p^2{}_\perp\rangle}}{\pi\langle p^2{}_\perp\rangle} \\
D_1^{h/\nu}(z, \mathbf{k}_\perp^2) &= D_1^{h/\nu}(z) \frac{e^{-\mathbf{k}_\perp^2/\langle k^2{}_\perp\rangle}}{\pi\langle k^2{}_\perp\rangle}
\end{aligned}
$$

- **Model calculation:** prediction to the data of different asymmetries— in Bag model(H. Avakian $et.\ al.$[PRD78]), in light-cone constituent quark model(S.Boffi, $et.\ al.$ PRD79), in Spectator model(A. Bacchetta $et.\ al.$ [PLB659]) etc.

Motivation

- We calculate the leading twist TMDs in a light-front quark-diquark model(LFQDM) under the framework of soft-wall AdS/QCD and give model predictions to spin asymmetries. Particularly, here we present model result for Collins asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}(x, z, \mathbf{P}_{h\perp}, y)$ of proton in SIDIS($\ell P \to \ell' h X$) for the π^+ and π^- channels.

Light-front quark-diquark Model(LFQDM)



S=0, 1

- In the parton model, the flavor singlet scalar diquark and the flavor triplet axial-vector diquark are combined to a symmetric spin-flavor wave function of $SU(4)$ structure. The proton state is written as[1]

$$|P;\pm\rangle = C_S|u\ S^0\rangle^{\pm} + C_V|u\ A^0\rangle^{\pm} + C_{VV}|d\ A^1\rangle^{\pm}$$

- The two particle Fock-state expansion for $J^z = \pm 1/2$

$$|u\ S\rangle^{\pm} = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_\lambda \psi_\lambda^{\pm(u)}(x,\mathbf{p}_\perp)|\lambda\ \Lambda_S; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_S=0}$$

$$|\nu\ A\rangle^{\pm} = \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_\lambda \sum_{\Lambda_A} \psi_{\lambda\Lambda_A}^{\pm(\nu)}(x,\mathbf{p}_\perp)|\lambda\ \Lambda_A; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_A=1,0,-1}$$

[1] TM, D.Chakrabarti, PRD94,094020(2016); R. Jakob $et.\ al.$ NPA626(1997)937

Wave function

- The light-front wave functions:

$$\psi^{\pm(\nu)}_{\lambda\Lambda}(x,\mathbf{p}_\perp) = \quad N^\nu f(x,\mathbf{p}_\perp,\lambda,\Lambda)\,\varphi^{(\nu)}_i(x,\mathbf{p}_\perp)\Big|_{i=1,2}$$

- Modified soft-wall AdS/QCD wave function[2] for two particle bound state:

$$\varphi^{(\nu)}_i(x,\mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a^\nu_i}\,(1-x)^{b^\nu_i}\,\exp\left[-\delta^\nu\,\frac{\mathbf{p}^2_\perp}{2\kappa^2}\,\frac{\log(1/x)}{(1-x)^2}\right].$$

- We determine the parameters $a^\nu_i,\ b^\nu_i,\ \delta^\nu$ by fitting the experimental data of the Dirac $F_1(Q^2)$ and Pauli $F_2(Q^2)$ form factors.
- In this model, scale evolution of the distributions is modelled by making the parameters in the PDFs scale dependent and fit the DGLAP evolution of unpolarized PDFs.
—-*T.Maji, D.Chakrabarti, PRD94,094020(2016)*

[2] G. F. de Teramond and S. J. Brodsky, arXiv:1203.4025 [hep-ph].
D. Chakrabarti and C. Mondal, Eur. Phys. J. C **73**, 2671 (2013).

TMDs at leading twist

- TMD correlator:

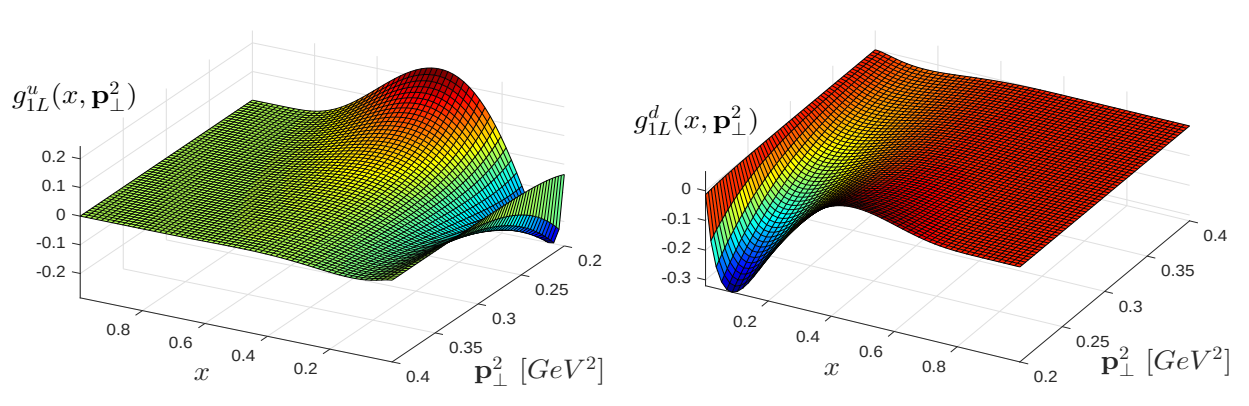
$$\Phi^{\nu[\Gamma]}(x, \mathbf{p}_\perp; S) = \frac{1}{2}\int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip.z} \langle P; S|\overline{\psi}^\nu(0)\Gamma \mathcal{W}_{[0,z]}\psi^\nu(z)|P; S\rangle\Big|_{z^+=0}$$

- Leading twist TMDs are related as:

$$\begin{aligned}
\Phi^{\nu[\gamma^+]}(x, \mathbf{p}_\perp; S) &= f_1^\nu(x, \mathbf{p}_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i S_T^j}{M} f_{1T}^{\perp\nu}(x, \mathbf{p}_\perp^2), \\
\Phi^{\nu[\gamma^+\gamma^5]}(x, \mathbf{p}_\perp; S) &= \lambda g_{1L}^\nu(x, \mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp.\mathbf{S}_T}{M} g_{1T}^\nu(x, \mathbf{p}_\perp^2), \\
\Phi^{\nu[i\sigma^{j+}\gamma^5]}(x, \mathbf{p}_\perp; S) &= S_T^j h_1^\nu(x, \mathbf{p}_\perp^2) + \lambda \frac{p_\perp^j}{M} h_{1L}^{\perp\nu}(x, \mathbf{p}_\perp^2) \\
&\quad + \frac{2p_\perp^j \mathbf{p}_\perp.\mathbf{S}_T - S_T^j \mathbf{p}_\perp^2}{2M^2} h_{1T}^{\perp\nu}(x, \mathbf{p}_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i}{M} h_1^{\perp\nu}(x, \mathbf{p}_\perp^2).
\end{aligned}$$

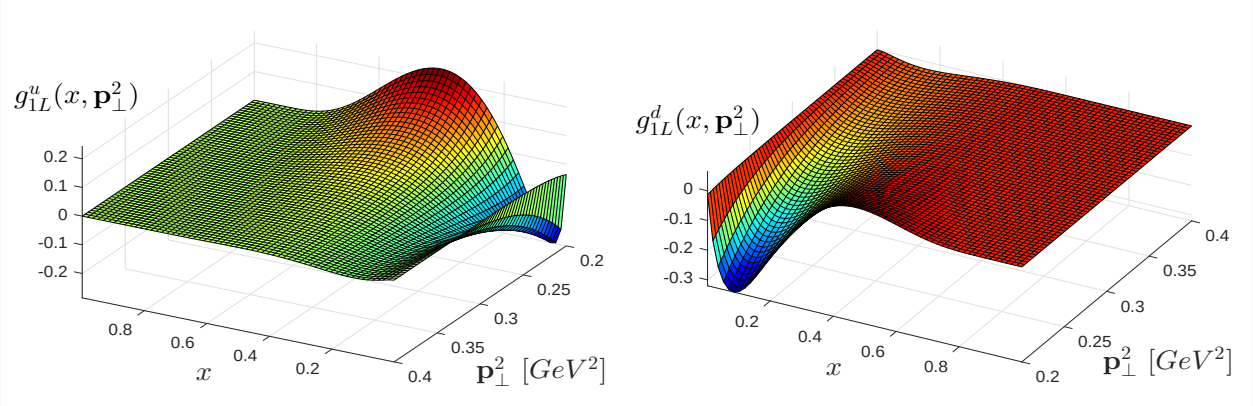
Helicity g_{1L}^{ν} and Axial charge g_A

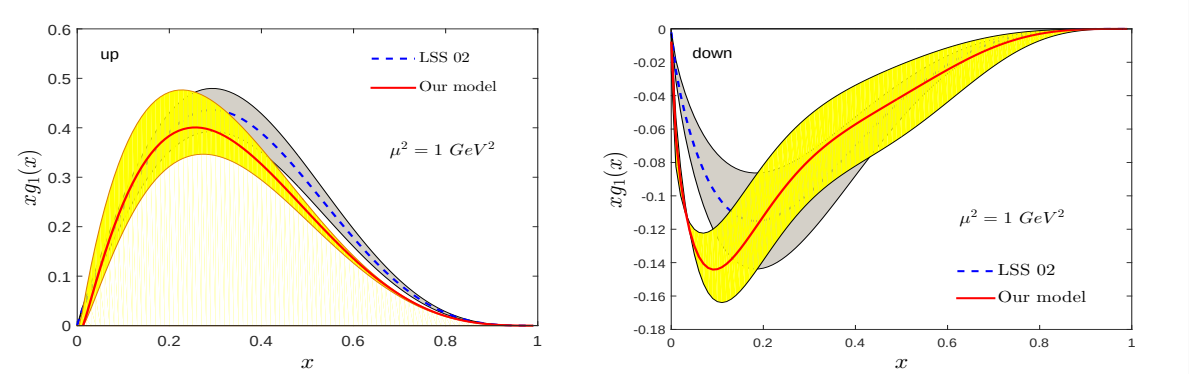
$$g_{1L}^{\nu}(x,\mathbf{p}_\perp^2) = N_{g_{1L}}^{\nu}\,\frac{\ln(1/x)}{\pi\kappa^2}\left[x^{2a_1^\nu}(1-x)^{2b_1^\nu-1} - \frac{\mathbf{p}_\perp^2}{M^2}x^{2a_2^\nu-2}(1-x)^{2b_2^\nu-1}\right]e^{-\mathbf{p}_\perp^2\frac{\delta^\nu\ln(1/x)}{\kappa^2(1-x)^2}}$$



Helicity g_{1L}^{ν} and Axial charge g_A

$$g_{1L}^{\nu}(x,\mathbf{p}_\perp^2) = N_{g_{1L}}^{\nu}\,\frac{\ln(1/x)}{\pi\kappa^2}\left[x^{2a_1^\nu}(1-x)^{2b_1^\nu-1} - \frac{\mathbf{p}_\perp^2}{M^2}x^{2a_2^\nu-2}(1-x)^{2b_2^\nu-1}\right]e^{-\mathbf{p}_\perp^2\frac{\delta^\nu\ln(1/x)}{\kappa^2(1-x)^2}}$$



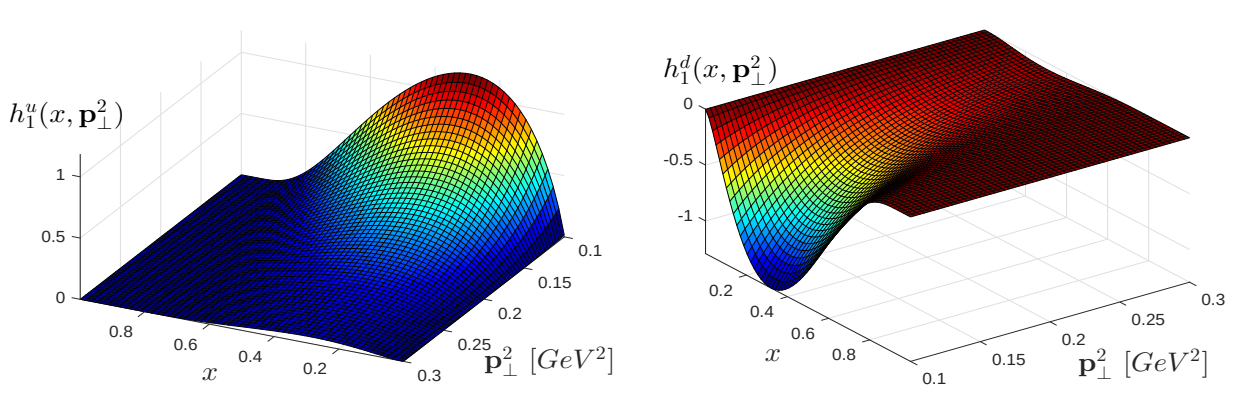


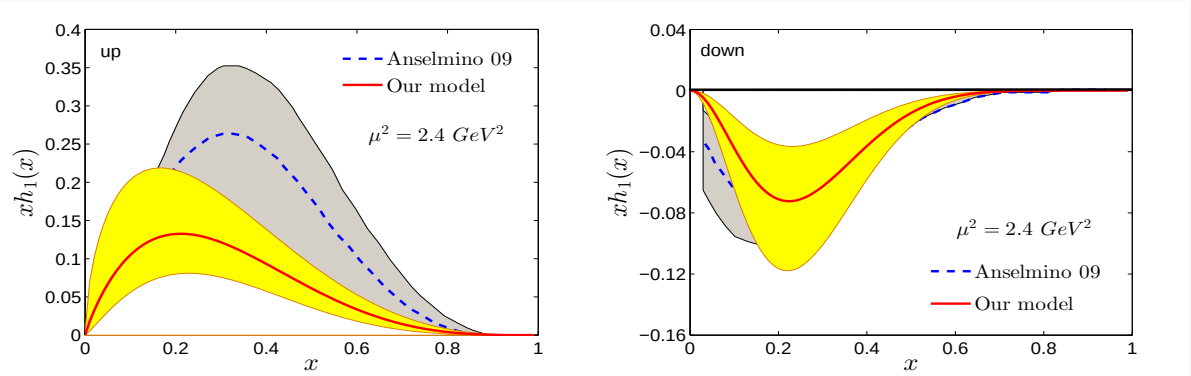
$\mu^2 = 1\ GeV^2$	g_A^u	g_A^d	$g_A = g_A^u - g_A^d$
Our result:	0.71 ± 0.09	$-0.54^{+0.19}_{-0.13}$	$1.25^{+0.28}_{-0.22}$
Measured Data:	0.82 ± 0.07	-0.45 ± 0.07	1.27 ± 0.14

—E. Leader $et.al.$,Phys. Rev. D 67 (2003) 037503

Transversity h_1^ν and Tensor Charge g_T

$$h_1^\nu(x,\mathbf{p}_\perp^2) = \left(C_S^2 N_S^{\nu 2} - C_V^2 \frac{1}{3} N_0^{\nu 2}\right) \frac{\ln(1/x)}{\pi\kappa^2} x^{2a_1^\nu}(1-x)^{2b_1^\nu - 1} e^{-\mathbf{p}_\perp^2 \frac{\delta^\nu \ln(1/x)}{\kappa^2(1-x)^2}}$$





$\mu^2 = 0.8\ GeV^2$	g_T^u	g_T^d	$g_T = g_T^u - g_T^d$
Our result:	$0.37^{+0.06}_{-0.05}$	$-0.14^{+0.05}_{-0.06}$	$0.51^{+0.12}_{-0.11}$
Measured Data:	$0.59^{+0.14}_{-0.13}$	$-0.20^{+0.05}_{-0.07}$	$0.79^{+0.19}_{-0.20}$

—M. Anselmino $et.al.$ Nucl.Phys.Proc.Suppl.191(2009)98

Model predictions to SSAs in SIDIS: Collins Asymmetry

Collins Asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}$

- Collins asymmetry provides correlation between the transversely polarised quark in a transversely polarised nucleon and transverse momentum of the produced hadron.

- Collins asymmetry is defined as

$$A_{UT}^{\sin(\phi_h+\phi_S)}(x,z,\mathbf{P}_{h\perp},y) = \frac{4\pi^2\alpha^2\frac{(1-y)}{sxy^2}\mathcal{C}\left[\frac{P_{h\perp}-z(\hat{\mathbf{P}}_{h\perp}\cdot\mathbf{p}_\perp)}{zM_h}h_1^\nu(x,\mathbf{p}_\perp^2)H_1^{\perp\nu}(z,\mathbf{k}_\perp)\right]}{2\pi^2\alpha^2\frac{1+(1-y)^2}{sxy^2}\mathcal{C}[f_1^\nu(x,\mathbf{p}_\perp^2)D_1^{h/\nu}(z,\mathbf{k}_\perp)]}$$

- The fragmentation functions $H_1^{\perp\nu}(z,\mathbf{k}_\perp)$ and $D_1^{h/\nu}(z,\mathbf{k}_\perp)$ are taken as phenomenological inputs from the parametrization by Anselmino $et.\ al.$(2007,2013) and Kretzer $et.\ al.$(2001) at the scale $\mu^2 = 2.5\ GeV^2$.

Collins Asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}$

- Collins asymmetry provides correlation between the transversely polarised quark in a transversely polarised nucleon and transverse momentum of the produced hadron.

- Collins asymmetry is defined as

$$A_{UT}^{\sin(\phi_h+\phi_S)}(x,z,\mathbf{P}_{h\perp},y) = \frac{4\pi^2\alpha^2\frac{(1-y)}{sxy^2}\mathcal{C}\left[\frac{P_{h\perp}-z(\hat{\mathbf{P}}_{h\perp}\cdot\mathbf{p}_\perp)}{zM_h}h_1^\nu(x,\mathbf{p}_\perp^2)H_1^{\perp\nu}(z,\mathbf{k}_\perp)\right]}{2\pi^2\alpha^2\frac{1+(1-y)^2}{sxy^2}\mathcal{C}[f_1^\nu(x,\mathbf{p}_\perp^2)D_1^{h/\nu}(z,\mathbf{k}_\perp)]}$$

- The fragmentation functions $H_1^{\perp\nu}(z,\mathbf{k}_\perp)$ and $D_1^{h/\nu}(z,\mathbf{k}_\perp)$ are taken as phenomenological inputs from the parametrization by Anselmino $et.\ al.$(2007,2013) and Kretzer $et.\ al.$(2001) at the scale $\mu^2 = 2.5\ GeV^2$.

- In the LFQDM:

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \frac{\frac{2(1-y)}{sxy^2}\frac{P_{h\perp}\sqrt{2e}}{M_h}\sum_\nu e_\nu^2\hat{h}_1^\nu(x)}{\frac{1+(1-y)^2}{sxy^2}\sum_\nu e_\nu^2 N_{f_1}^\nu\frac{\ln(1/x)}{\pi\kappa^2}\left[T_1^\nu(x)-\frac{\langle m_\perp^2\rangle}{M^2}T_2^\nu(x)\right]} \times \frac{\mathcal{N}_\nu^C(z)D_1^{h/\nu}(z)\frac{\langle k^2{}_\perp\rangle_C^2\langle p^2{}_\perp\rangle_x}{\langle k^2{}_\perp\rangle\langle P^2{}_{h\perp}\rangle_C}\frac{e^{-\mathbf{P}_{h\perp}^2/\langle P^2{}_{h\perp}\rangle_C}}{\langle P^2{}_{h\perp}\rangle_C}}{D_1^{h/\nu}(z)\langle p^2{}_\perp\rangle_x\frac{e^{-\mathbf{P}_{h\perp}^2/\langle P^2{}_{h\perp}\rangle}}{\langle P^2{}_{h\perp}\rangle}}.$$

Scale evolution

- A complete QCD evolutions of all the leading twist TMDs are not known.

- However,
  (i) TMDs evolutions from DGLAP evolution of collinear part:

$$f_1^{\nu}(x, \mathbf{p}_\perp^2, Q^2) \quad = \quad f_1^{\nu}(x, Q^2)\Big|_{\mathrm{DGLAP}} \frac{e^{-\mathbf{p}_\perp^2/\langle p^2{}_\perp\rangle}}{\pi\langle p^2{}_\perp\rangle}$$

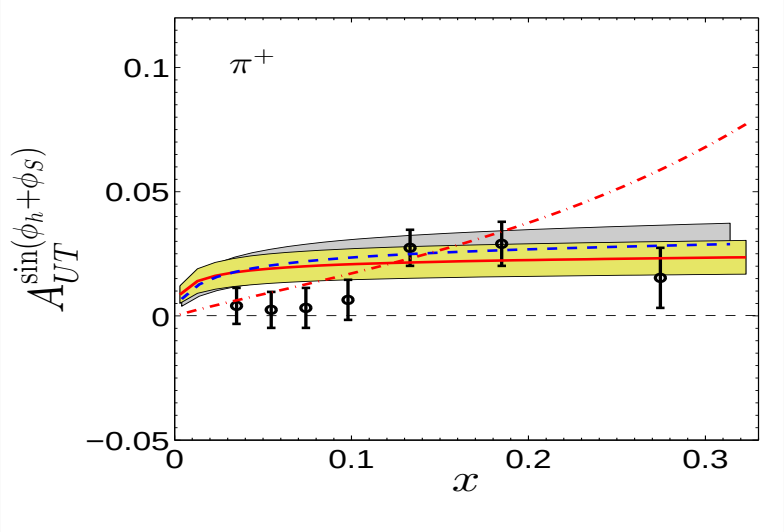
  (ii) QCD evolution for $f_1(x, \mathbf{p}_\perp^2)$ and $D_1^{h/\nu}(z, \mathbf{k}_\perp^2)$:

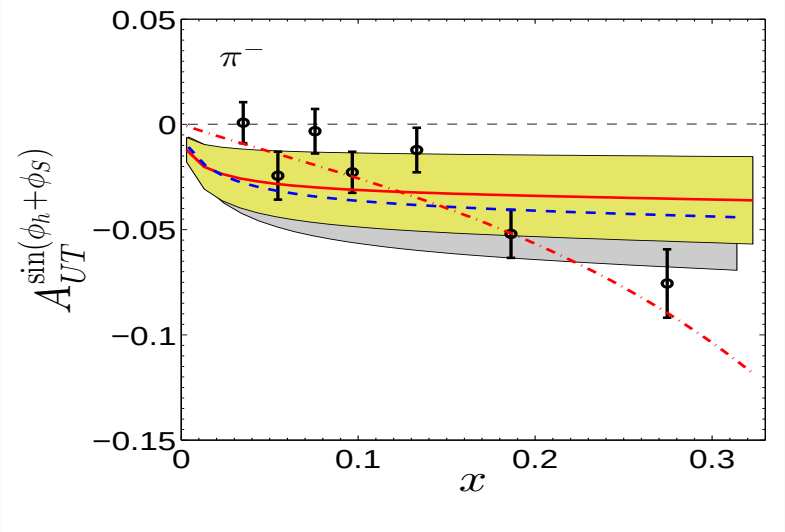
  —Collins(2011,2012), Aybat and Rogers(2011), Anselmino(2012)

- Parameter evolution approach: assuming that the evolution information encoded into the parameters(PRD94,094020(2016)) of this model generates the TMD evolution.

Collins Asymmetry with x

- HERMES data(Proton): —A. Airapetian [HERMES Collaboration], Phys. Lett. B 693, 11(2010)



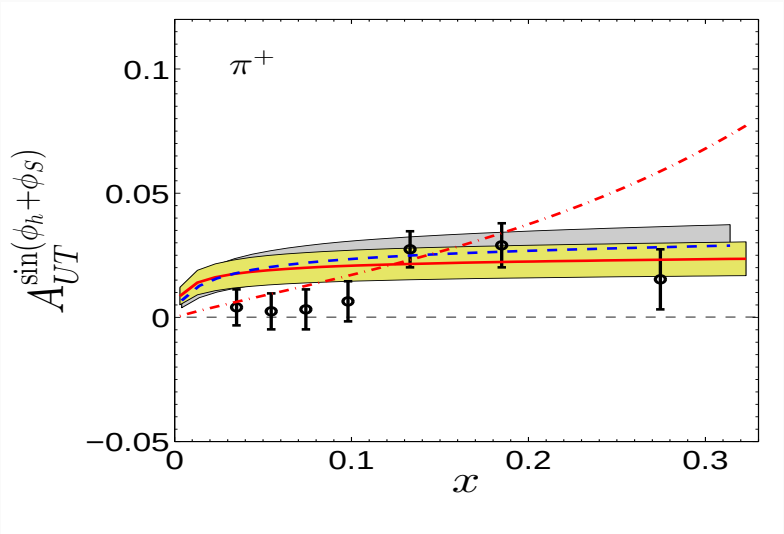


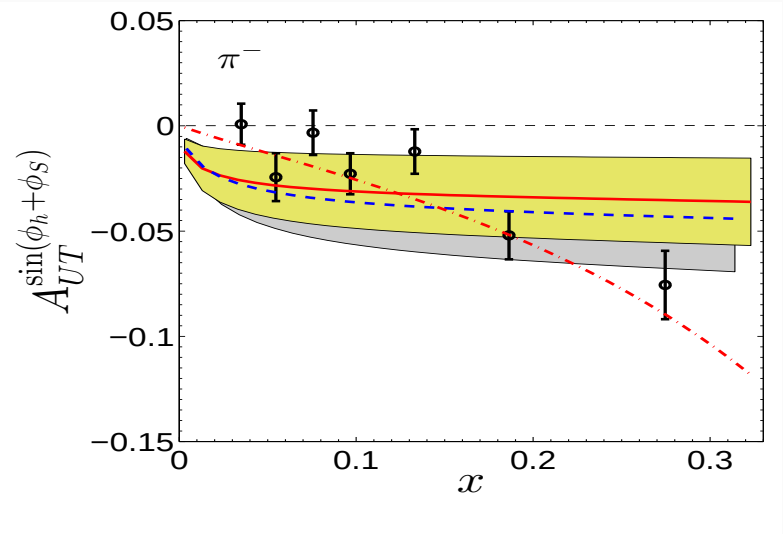
- Red continuous lines(yellow error region): $h_1(x, \mathbf{p}_\perp^2)$ and $f_1(x, \mathbf{p}_\perp^2)$ both are at the scale $\mu^2 = 2.5\ GeV^2$. TMDs are evolved in parameter evolution approach of LFQDM.
- Blue dashed lines(gray error bar): $h_1(x, \mathbf{p}_\perp^2)$ is at initial scale and $f_1(x, \mathbf{p}_\perp^2)$ is at $\mu^2 = 2.5\ GeV^2$. Here $f_1(x, \mathbf{p}_\perp^2)$ is evolved in QCD evolution approach.
- Red dot-dashed lines: Same as blue dashed line but for parameter evolution approach.

- $H_1^\perp$ and $D_1^{h/\nu}$ are taken as phenomenological inputs from the parametrization by Anselmino $et.al.$(2007,2013) and Kretzer $et.\ al.$(2001) at the scale $\mu^2 = 2.5\ GeV^2$.

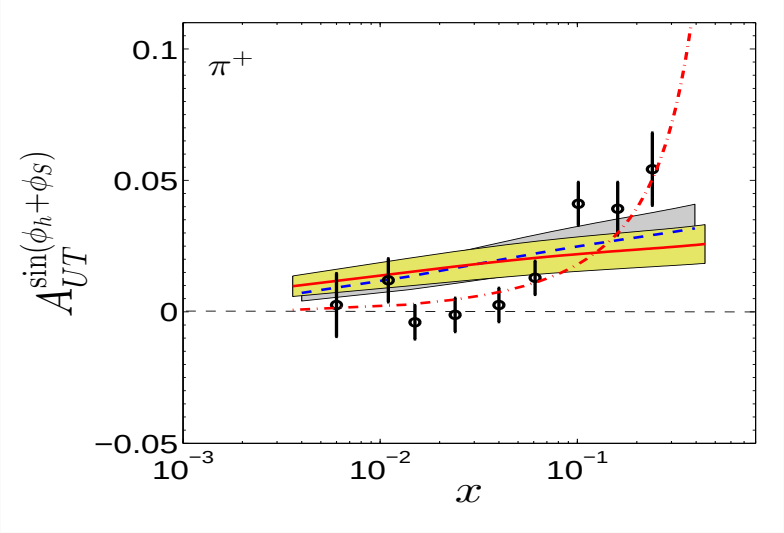
Collins Asymmetry with x

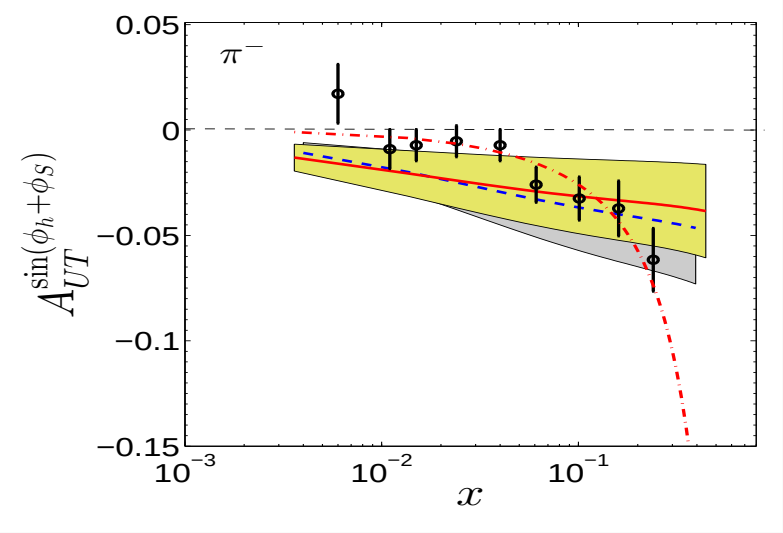
- HERMES data(Proton): —A. Airapetian [HERMES Collaboration], Phys. Lett. B 693, 11(2010)





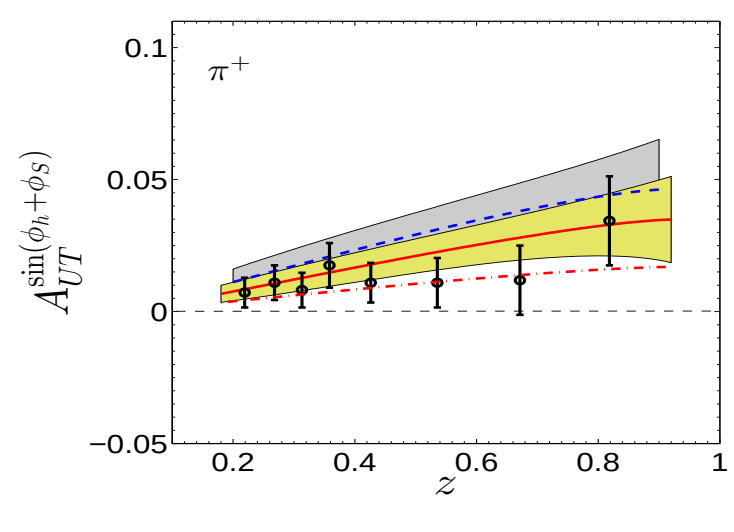
- COMPASS data(Proton): —A. Martin[COMPASS Collaboration],Phys.Part.Nucl.45,141(2014)

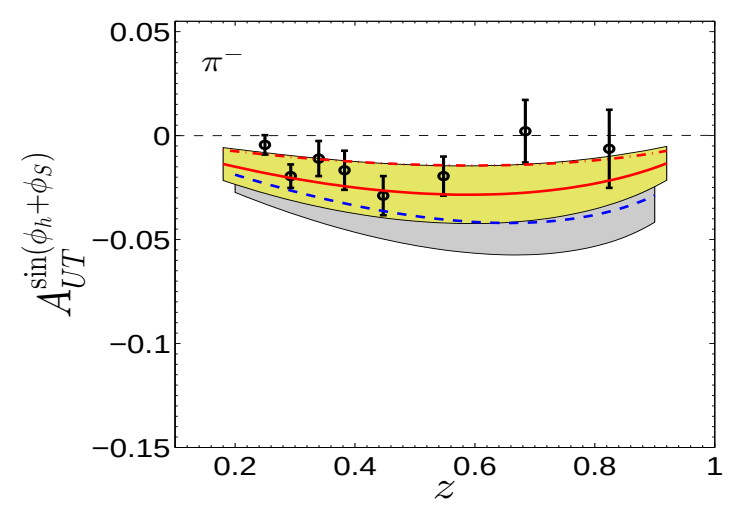




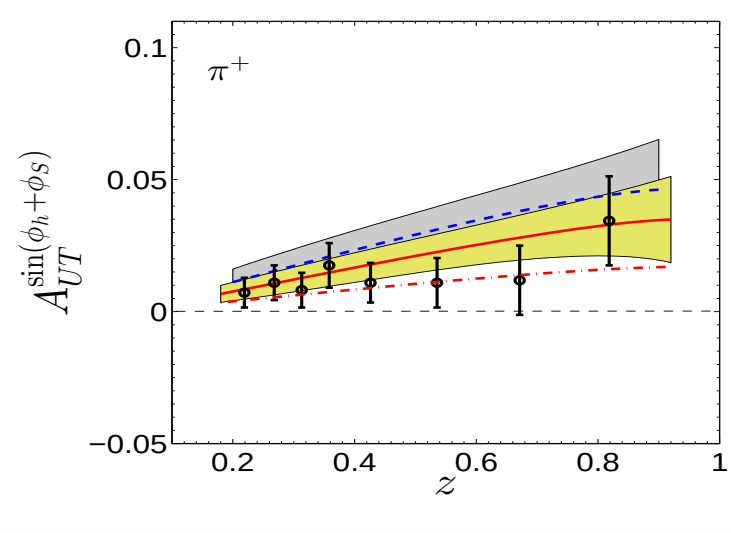
Collins Asymmetry with z

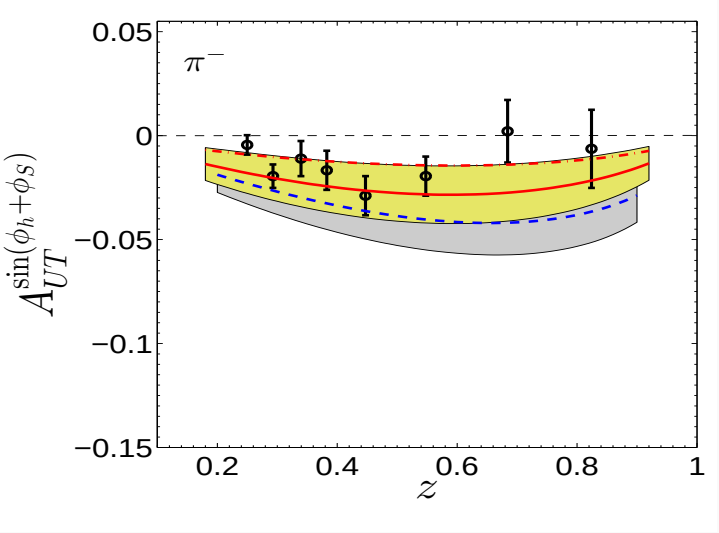
- HERMES data(Proton): —A. Airapetian [HERMES Collaboration], Phys. Lett. B 693, 11(2010)





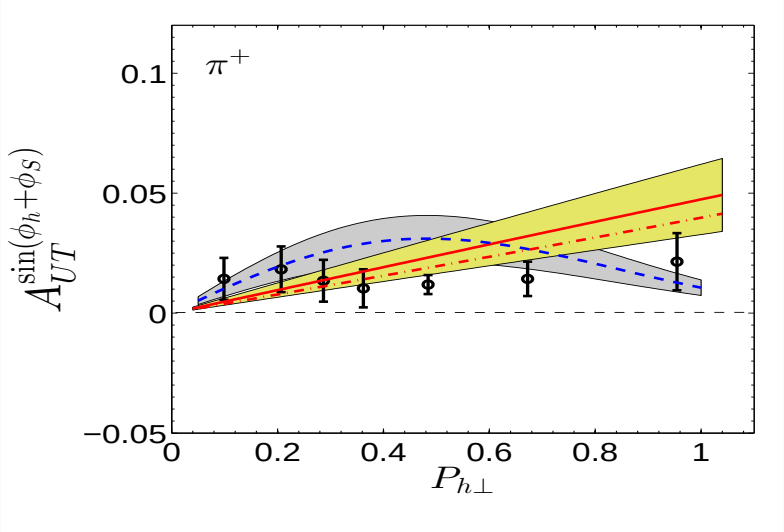
- COMPASS data(Proton): —A. Martin[COMPASS Collaboration],Phys.Part.Nucl.45,141(2014)

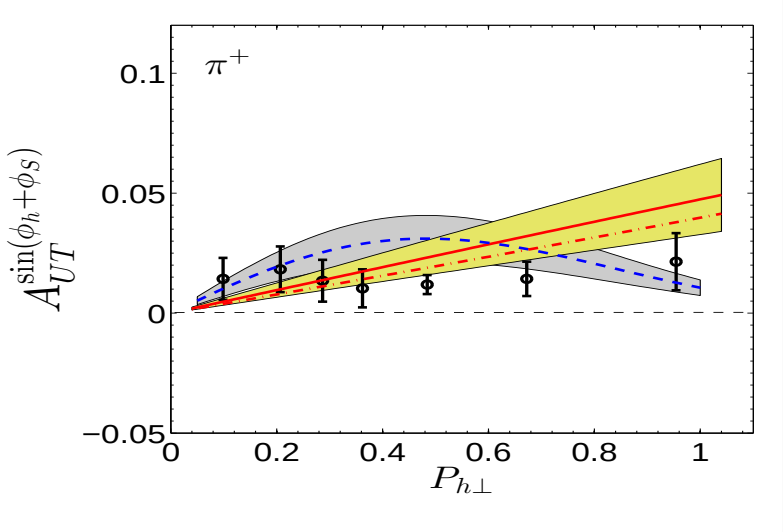




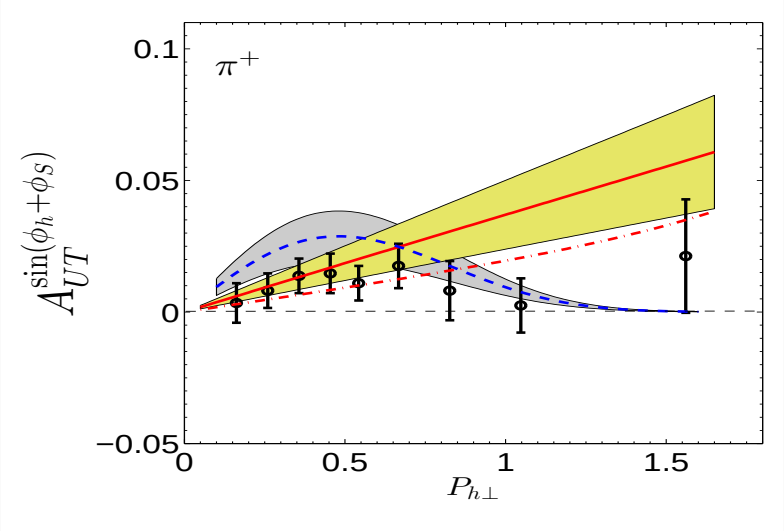
Collins Asymmetry with $P_{h\perp}$

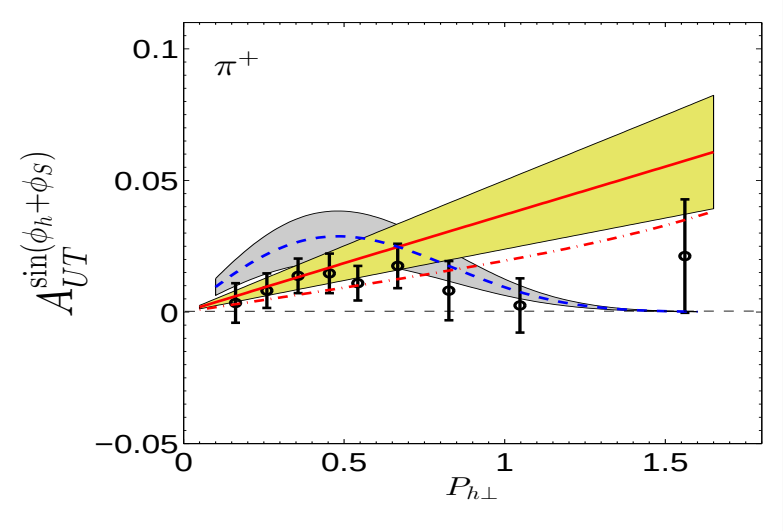
- HERMES data(Proton): —A. Airapetian [HERMES Collaboration], Phys. Lett. B 693, 11(2010)





- COMPASS data(Proton): —A. Martin[COMPASS Collaboration],Phys.Part.Nucl.45,141(2014)

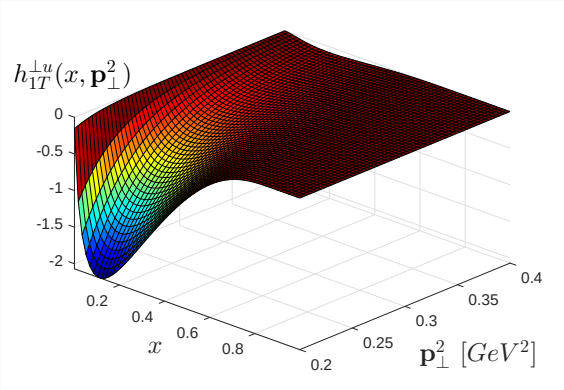


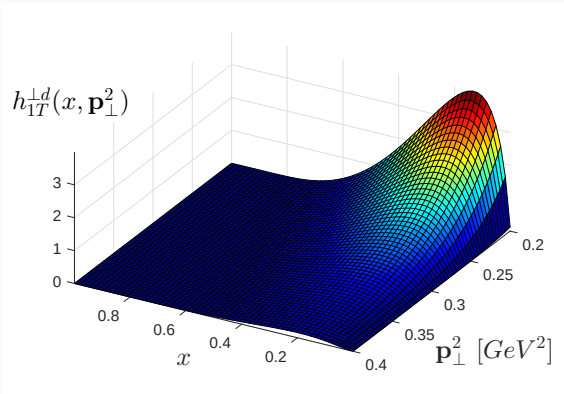


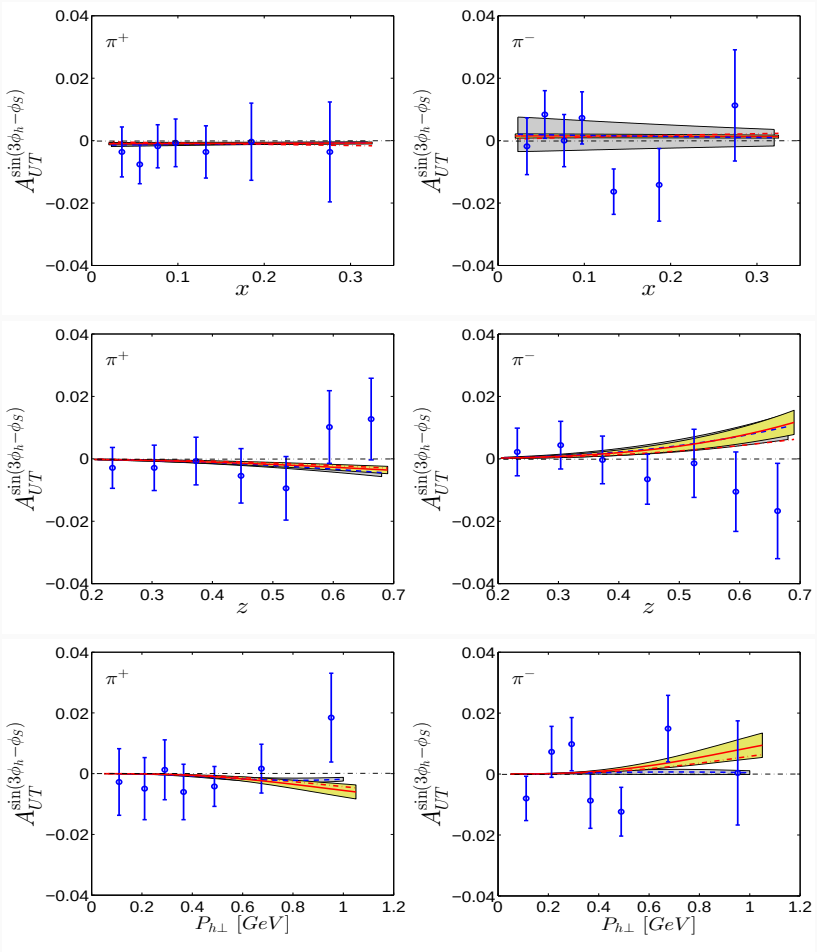
Single spin asymmetry $A_{UT}^{\sin(3\phi_h-\phi_S)}$

$$A_{UT}^{\sin(3\phi_h-\phi_S)}(x,z,\mathbf{P}_{h\perp},y) = \frac{4\pi^2\alpha^2\frac{(1-y)}{sxy^2}\mathcal{C}\left[\tilde{w}\ h_{1T}^{\perp\nu}(x,\mathbf{p}_\perp^2)H_1^{\perp\nu}(z,\mathbf{k}_\perp^2)\right]}{2\pi^2\alpha^2\frac{1+(1-y)^2}{sxy^2}\mathcal{C}[f_1^\nu(x,\mathbf{p}_\perp^2)D_1^{h/\nu}(z,\mathbf{k}_\perp^2)]}.$$

Where, $\tilde{w} = p_\perp^2\left(\frac{-P_{h\perp}+2P_{h\perp}(\hat{\mathbf{P}}_{h\perp}.\mathbf{p}_\perp)^2-zp_\perp[4(\hat{\mathbf{P}}_{h\perp}.\hat{\mathbf{p}}_\perp)^3+3(\hat{\mathbf{P}}_{h\perp}.\hat{\mathbf{p}}_\perp)]}{2zM_hM^2}\right)$



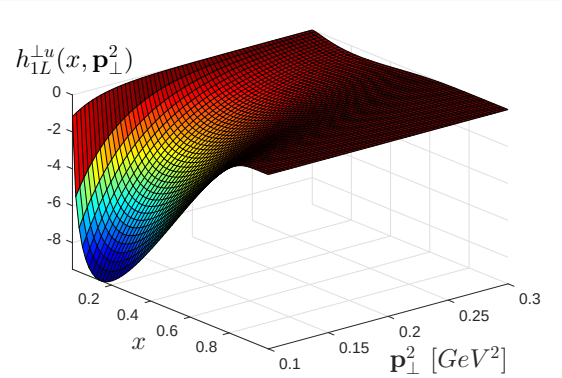


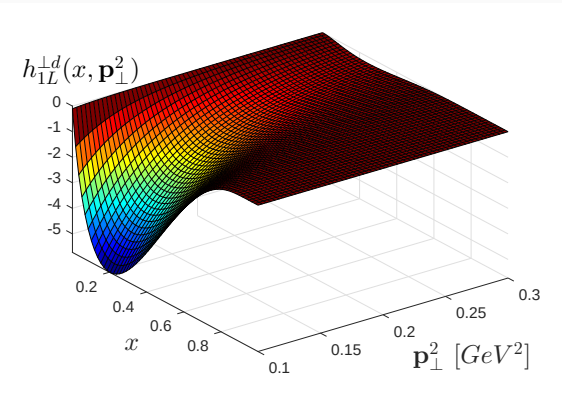


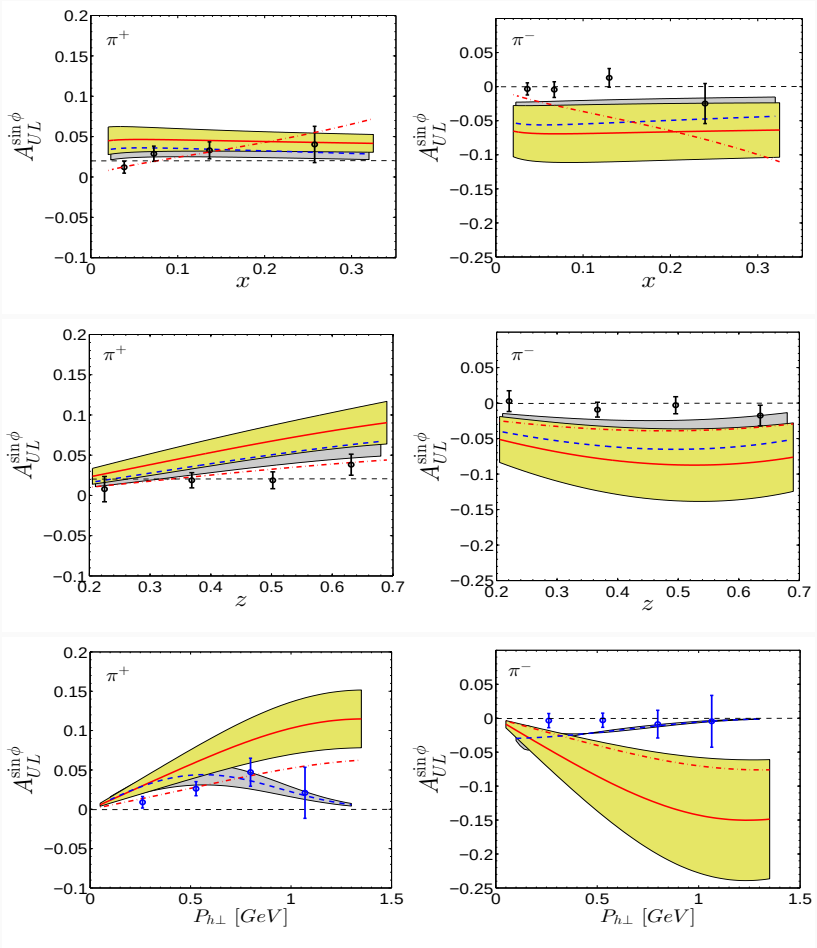
—A. Airapetian et. al.[HERMES],PLB562,182(2003)

Single spin asymmetry $A_{UL}^{\sin\phi_h}$

$$A_{UL}^{\sin\phi_h}(x,z,\mathbf{P}_{h\perp},y) = \frac{4\pi^2\alpha^2\frac{\sqrt{1-y}(2-y)}{sxy^2}\mathcal{C}\left[\frac{p_\perp^2(P_{h\perp}-z\hat{\mathbf{P}}_{h\perp}\cdot\mathbf{p}_\perp)}{zM_hM}h_{1L}^{\perp\nu}(x,\mathbf{p}_\perp^2)H_1^{\perp\nu}(z,\mathbf{k}_\perp^2)\right]}{2\pi^2\alpha^2\frac{1+(1-y)^2}{sxy^2}\mathcal{C}[f_1^\nu(x,\mathbf{p}_\perp^2)D_1^{h/\nu}(z,\mathbf{k}_\perp^2)]}.$$







—A. Airapetian et. al. [HERMES Collaboration], Phys.Lett.B562,182(2003)

Wigner distributions in LFQDM

- We also investigate the Quark Wigner distributions and general transverse momentum dependent distributions(GTMDs) for both u and d quarks in a proton and compare with other models. We present a detailed study of the quark orbital angular momentum and its correlation with the quark spin and the proton spin.

—D. Chakrabarti, T. Maji, C. Mondal and A. Mukherjee, arXiv:1701.08551[hep-ph].

Summary and conclusions

- The light-front quark-diquark model(LFQDM) in the framework of soft-wall AdS/QCD predicts the TMDs whose PDF limits satisfy the phenomenological extractions quite well.
- Model prediction to the Collins asymmetry agrees well with the COMPASS, HERMES data.
- The AdS/QCD prediction to the two particle wave functions is useful to study the novel phenomena in QCD bound states e.g., Collins asymmetry, other single spin asymmetries etc.

Future directions

- Work in progress to predict Sivers asymmetry in this model.
- Double spin asymmetries(DSA) can also be studied in this model and can be compared with experimental data.
- It will be interesting to study other processes like Drell-Yan process, dihadron production etc. in this model and compare with experimental findings.

Thank you!

Parameret evolution approach

- In this model:

$$f_1^\nu(x,\mathbf{p}_\perp^2) = N_{f_1}^\nu \frac{\ln(1/x)}{\pi\kappa^2}\left[x^{2a_1^\nu}(1-x)^{2b_1^\nu-1} + \frac{\mathbf{p}_\perp^2}{M^2}x^{2a_2^\nu-2}(1-x)^{2b_2^\nu-1}\right]e^{-\mathbf{p}_\perp^2\frac{\delta^\nu\ln(1/x)}{\kappa^2(1-x)^2}}$$

$$a_i^\nu(\mu) \quad b_i^\nu(\mu) \quad \delta^\nu(\mu)$$

- Scale variation of the parameters is determined by fitting the DGLAP evolution for unpolarised PDFs. .

$$\begin{aligned} a_i^\nu(\mu) &= a_i^\nu(\mu_0) + A_i^\nu(\mu), \\ b_i^\nu(\mu) &= b_i^\nu(\mu_0) - B_i^\nu(\mu)\frac{4C_F}{\beta_0}\ln\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right), \\ \delta^\nu(\mu) &= \exp\left[\delta_1^\nu\left(\ln(\mu^2/\mu_0^2)\right)^{\delta_2^\nu}\right], \end{aligned}$$

$$P_i^\nu(\mu) = \alpha_{P,i}^\nu\,\mu^{2\beta_{P,i}^\nu}\left[\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]^{\gamma_{P,i}^\nu}\Bigg|_{P=A,B\ at\ i=1,2},$$

TM, D.Chakrabarti, Phys. Rev. D 94,094020(2016)

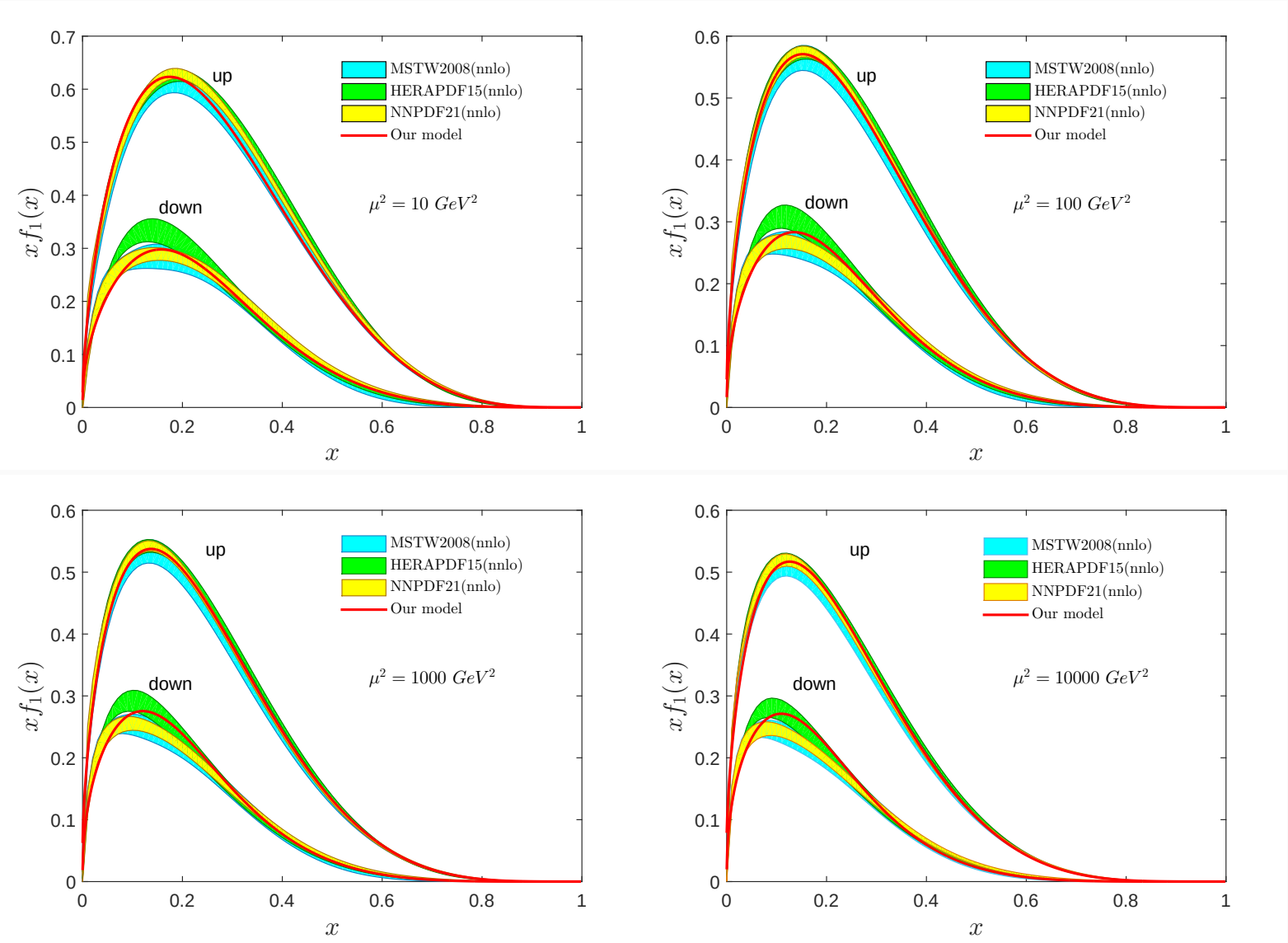
QCD evolution for $f_1(x, \mathbf{p}_\perp^2)$ and $D_1^{h/\nu}(z, \mathbf{k}_\perp^2)$

$$\tilde{F}(x, \mathbf{b}_\perp; \mu) = \tilde{F}(x, \mathbf{b}_\perp; \mu_0) \exp\left(\ln \frac{\mu}{\mu_0} \tilde{K}(b_\perp; \mu) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\mu', \frac{\mu^2}{\mu'^2}\right) \right),$$

where, at $\mathcal{O}(\alpha_s)$

$$\begin{aligned}
\tilde{K}(b_\perp; \mu) &= -\frac{\alpha_s C_F}{\pi} [\ln(b_*^2 \mu_b^2) - \ln(4) + 2\gamma_E] + \left[\int_{\mu}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') \right] - g_K(b_T) \\
b_*(b_T) &= \left. \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{max}^2}}} \right|_{b_{max} = 0.5\ GeV^{-1}} \quad ; \quad \mu_b = \left. \frac{C_1}{b_*(b_T)} \right|_{C_1 = 2e^{-\gamma_E}} \\
& \qquad g_K = \left. \frac{1}{2} g_2 b_T^2 \right|_{b_2 = 0.68\ GeV^2}
\end{aligned}$$

Unpol. PDFs evolution upto $\mu^2 = 10^4\ GeV^2$



- This model evolution predicts the unpolarised PDFs accurately for a wide range of scale.

$x - \mathrm{p}_\perp^2$ factorization in TMDs

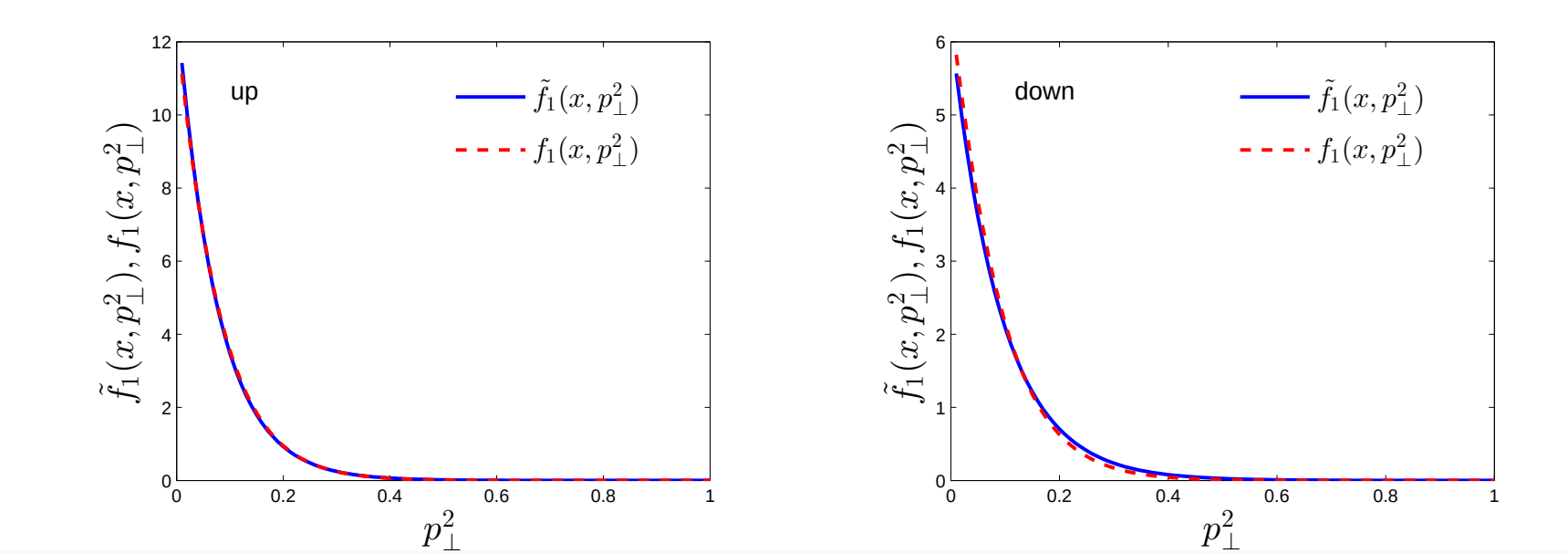
- In the Gaussian ansatz the unpolarized TMDs are written as

$$\tilde{f}_1^\nu(x, \mathbf{p}_\perp^2) = f_1^\nu(x) \frac{e^{-\mathbf{p}_\perp^2/\langle \mathbf{p}_\perp^2(f_1)\rangle^\nu}}{\pi \langle \mathbf{p}_\perp^2(f_1)\rangle^\nu}; \qquad \langle \mathbf{p}_\perp^2(f_1)\rangle^\nu = \frac{\int dx \int d^2p_\perp p_\perp^2 f_1(x, \mathbf{p}_\perp^2)}{\int dx \int d^2p_\perp f_1^\nu(x, \mathbf{p}_\perp^2)}$$

- In this model:

$$f_1^\nu(x, \mathbf{p}_\perp^2) = N_{f_1}^\nu \frac{\ln(1/x)}{\pi\kappa^2} \left[x^{2a_1^\nu}(1-x)^{2b_1^\nu - 1} + \frac{\mathbf{p}_\perp^2}{M^2} x^{2a_2^\nu - 2}(1-x)^{2b_2^\nu - 1} \right] e^{-\mathbf{p}_\perp^2 \frac{\delta^\nu \ln(1/x)}{\kappa^2(1-x)^2}}$$

We calculate $\langle \mathbf{p}_\perp^2(f_1)\rangle^\nu$ and take the PDF $f_1^\nu(x) = \int d^2p_\perp f_1(x, \mathbf{p}_\perp^2)$.



FFs

We use Gaussian anstz for fragmentations functions as discussed by Anselmino $et.al.$ [PRD75].

$$
\begin{aligned}
D_1^{h/\nu}(z,\mathbf{k}_\perp) &= D_1^{h/\nu}(z)\frac{e^{-\mathbf{k}_\perp^2/\langle k^2{}_\perp\rangle}}{\pi\langle k^2{}_\perp\rangle} \\
\frac{2k_\perp}{zM_h}H_1^{\perp\nu}(z,\mathbf{k}_\perp) &= 2\mathcal{N}_\nu^C(z)D_1^{h/\nu}(z)h(k_\perp)\frac{e^{-\mathbf{k}_\perp^2/\langle k^2{}_\perp\rangle}}{\pi\langle k^2{}_\perp\rangle}
\end{aligned}
$$

with

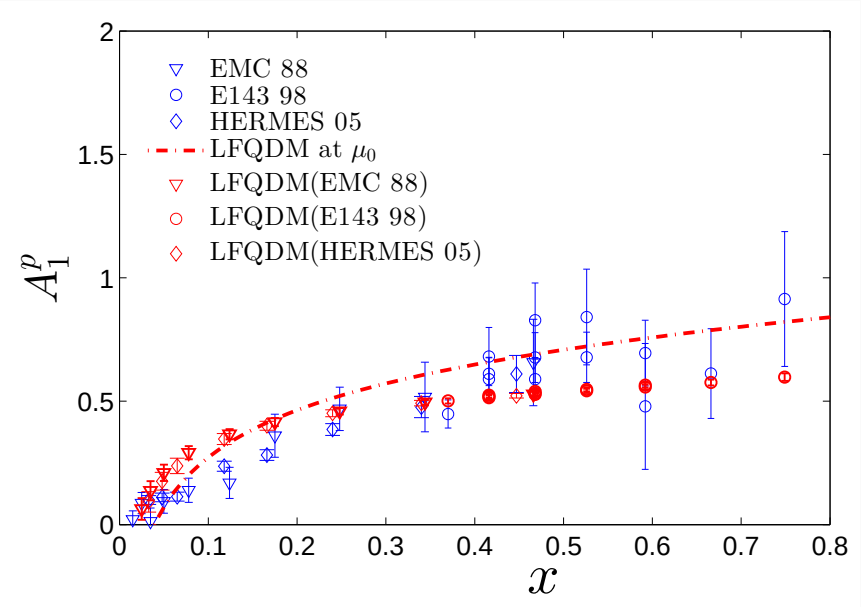
$$
\begin{aligned}
\mathcal{N}_\nu^C(z) &= N_\nu^C z^{\rho_1}(1-z)^{\rho_2}\frac{(\rho_1+\rho_2)^{(\rho_1+\rho_2)}}{\rho_1^{\rho_1}\rho_2^{\rho_2}} \\
h(k_\perp) &= \sqrt{2e}\frac{k_\perp}{M_h}e^{-\mathbf{k}_\perp^2/M_h^2}
\end{aligned}
$$

Where the hadron of momentum $\mathbf{P}_h$ and of energy fraction $z = P_h^-/k^-$ is produced from a fragmanting quark of momentum **k**. The values of the parameters are listed in [PRD87,094019] and $D_1^{h/\nu}(z)$ is taken from the phenomenological extraction [EPJC22,269].

Double spin asymmetry

- When the final state is not observed, the double-spin asymmetry is defined as

$$A_1 = \frac{\sum_\nu e_\nu^2 x g_1^\nu(x)}{\sum_\nu e_\nu^2 x f_1^\nu(x)}.$$



- The SIDIS polarised cross-section at $\mathcal{O}\mathbf{p}_\perp/Q)$ is given as(M.Anselmino $et.al.$ PRD83)

$$
\begin{aligned}
&\frac{d\sigma^{\ell(S_\ell)+P(S)\to\ell' P_h X}}{dx_B dy dz d^2\mathbf{P}_{h\perp} d\phi_S} \\
&= \frac{2\alpha^2}{sxy^2}\Bigg\{\frac{1+(1-y)^2}{2}F_{UU} + (2-y)\sqrt{1-y}\cos\phi_h F_{UU}^{\cos\phi_h} + (1-y)cos2\phi_h F_{UU}^{\cos 2\phi_h} \\
&+S_L\left[(1-y)\sin 2\phi_h F_{UL}^{\sin 2\phi_h} + (2-y)\sqrt{1-y}\sin\phi_h F_{UL}^{\sin\phi_h}\right] \\
&+S_L S_\ell\left[\frac{1-(1-y)^2}{2}F_{LL} + y\sqrt{1-y}\cos\phi_h F_{LL}^{\cos\phi_h}\right] \\
&+S_T\Bigg[\frac{1+(1-y)^2}{2}\sin(\phi_h-\phi_S)F_{UT}^{\sin(\phi_h-\phi_S)} \\
&+(1-y)\Bigg(\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}\Bigg|_{\boxed{h_1,\,H_1^\perp}} + \sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\Bigg|_{h_{1T}^\perp,H_1^\perp}\Bigg) \\
&+(2-y)\sqrt{(1-y)}\Bigg(\sin\phi_S F_{UT}^{\sin\phi_S}\Bigg|_{h_{1L}^\perp,H_1^\perp} + \sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg)\Bigg] \\
&+S_T S_\ell\Bigg[\frac{1-(1-y)^2}{2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} \\
&+y\sqrt{1-y}\Bigg(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg)\Bigg]\Bigg\}
\end{aligned}
$$

$$
\boxed{A_{S_\ell S_N}^{\mathcal{W}}(x,z,\mathbf{P}_{h\perp},y) = 2\frac{\int d\phi_h d\phi_S[d\sigma^{\ell P^\uparrow\to\ell' hX} - d\sigma^{\ell P^\downarrow\to\ell' hX}]\mathcal{W}}{\int d\phi_h d\phi_S[d\sigma^{\ell P^\uparrow\to\ell' hX} + d\sigma^{\ell P^\downarrow\to\ell' hX}]}}
$$