First simultaneous extraction of spin PDFs and FFs from a global QCD analysis

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Proton spin from DIS

- Spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + \mathcal{L}$
- Spin PDFs: $\Delta q^+ = \Delta q + \Delta\bar{q}$

→ Quark contribution: $\Delta\Sigma(Q^2) = \int_0^1 dx \left(\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)\right)$

$\approx 0.3_{[10^{-3},1]}$

→ Gluon contribution: $\Delta g(Q^2) = \int_0^1 dx \Delta g(x,Q^2) \approx 0.1_{[0.05,0.2]}$

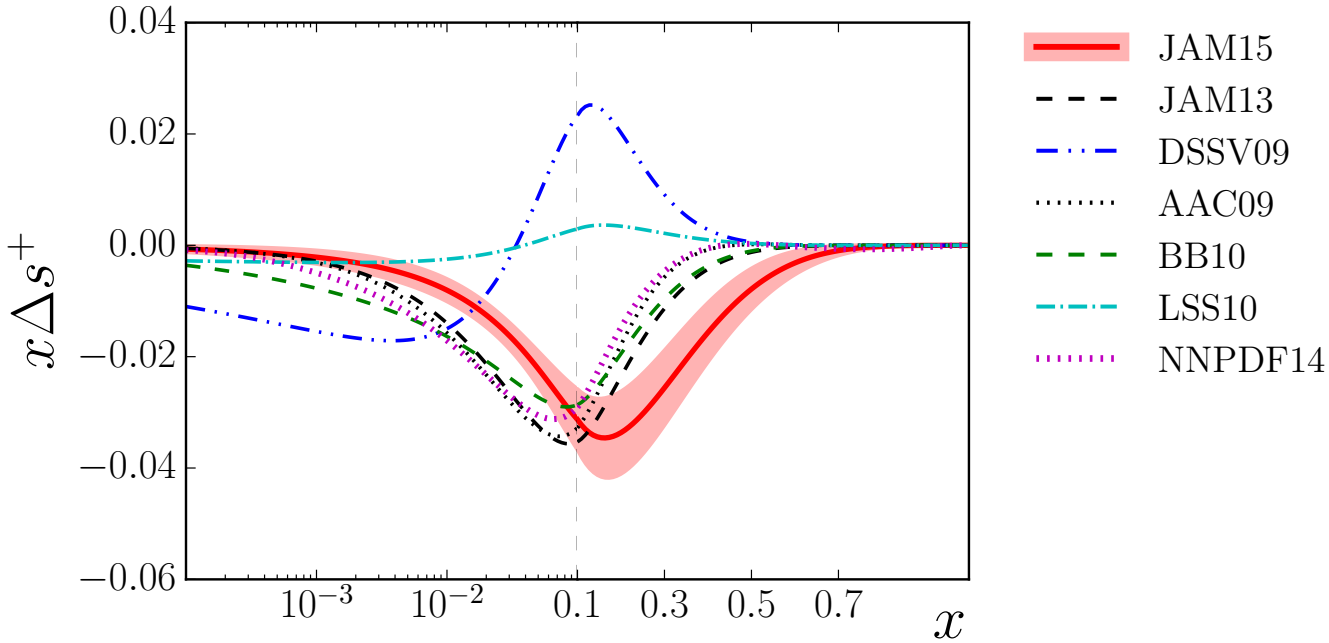
→ Orbital angular momentum: determined from GPDs

- Polarized DIS experiments measure asymmetries to extract information on the polarized structure function

$$g_1^p(x,Q^2) = \frac{2}{9}\Delta u^+ + \frac{1}{18}\Delta d^+ + \frac{1}{18}\Delta s^+ + \mathcal{O}(\alpha_s)$$

- Together with SU(2) and SU(3) constraints from weak baryon and hyperon decays, information on quark and gluon spin contribution can be extracted

Strange polarization



$$\Delta \mathbf{s}^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2)$$

JAM15: $\Delta \mathbf{s}^+ = -0.1 \pm 0.01$ DSSV09: $\Delta \mathbf{s}^+ = -0.11$ $Q^2 = 1\,\mathrm{GeV}^2$

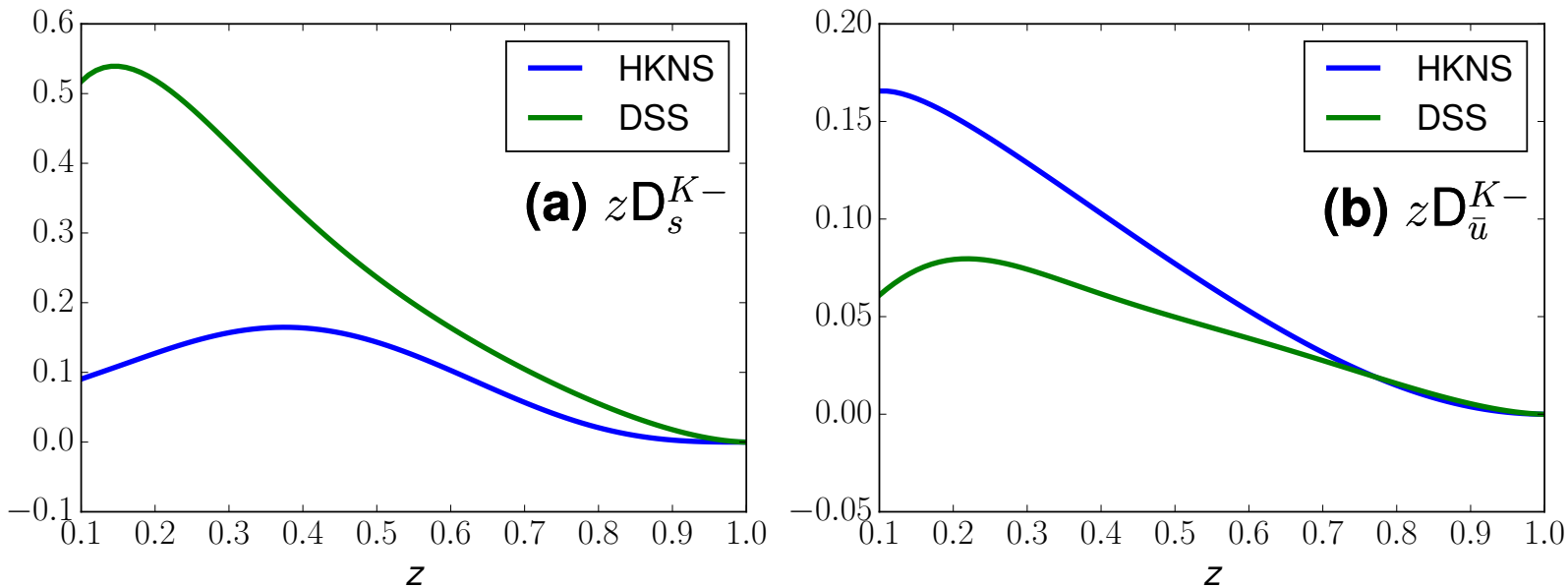
- Discrepancy in shape of strange polarization (DSSV, LSS)

  → Claim: analysis of SIDIS data changes strange polarization shape

Fragmentation Functions

- SIDIS observables require information on FFs → contains information about quark to hadron fragmentation.

  → Choice of kaon FF parameterization influences shape of strange polarization density in SIDIS analysis (Leader, et al)



→ Recent JAM analysis extracted FFs from single-inclusive annihilation using the iterative Monte Carlo technique (arXiv:1609:00899)

JAM17 Combined Analysis

- We perform the first ever combined Monte Carlo analysis of polarized DIS, polarized SIDIS, and SIA data at NLO

$$d\sigma^{DIS} = \sum_f \int d\xi\, \Delta f(\xi) d\hat{\sigma} \qquad d\sigma^{SIA} = \sum_f \int d\zeta\, D_f(\zeta) d\hat{\sigma}$$

$$d\sigma^{SIDIS} = \sum_f \int d\xi d\zeta\, \Delta f(\xi) D_f(\zeta) d\hat{\sigma}$$

- Spin PDFs and FFs are fitted simultaneously
- SU(2) and SU(3) constraints used in DIS only analyses are released

$$a_3 = \int_0^1 dx(\Delta u^+ - \Delta d^+)$$

$$a_8 = \int_0^1 dx(\Delta u^+ + \Delta d^+ - 2\Delta s^+)$$

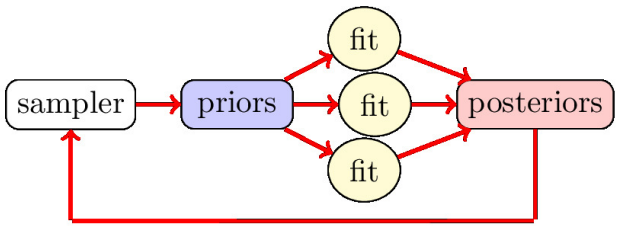
Traditional Fitting Method

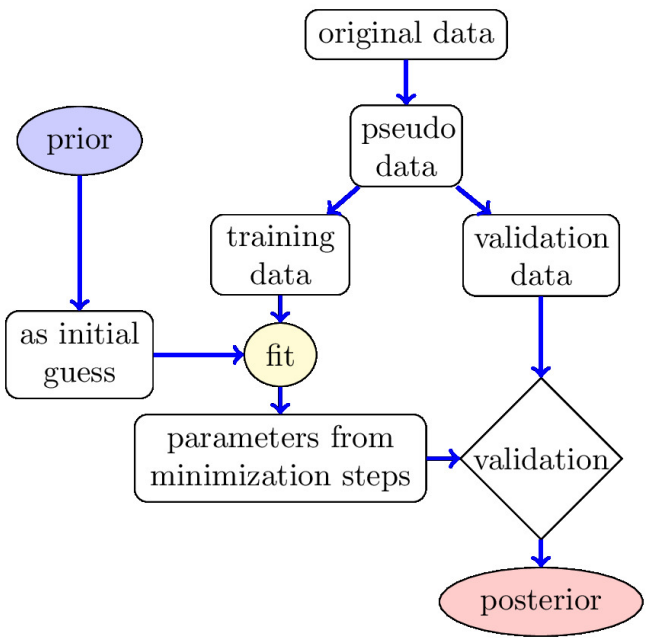
- Functional form for PDF and FF, e.g.

$$xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$$

- Single χ^2 fit of parameters
  - → Typically fix parameters that are difficult to constrain
- Uncertainties determined by Hessian or Lagrange multiplier methods
  - → Introduces tolerance criteria
- Since χ^2 is a highly non-linear function of the fit parameters, there can be various local minima
- We can improve on this using an iterative Monte Carlo procedure

Iterative Monte Carlo (IMC) Fitting Methodology





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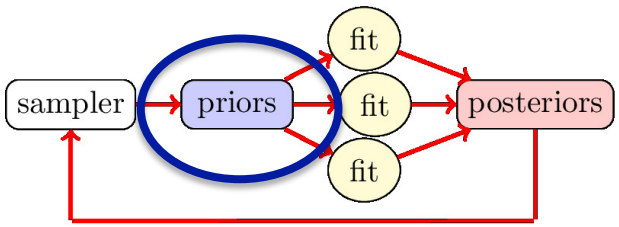
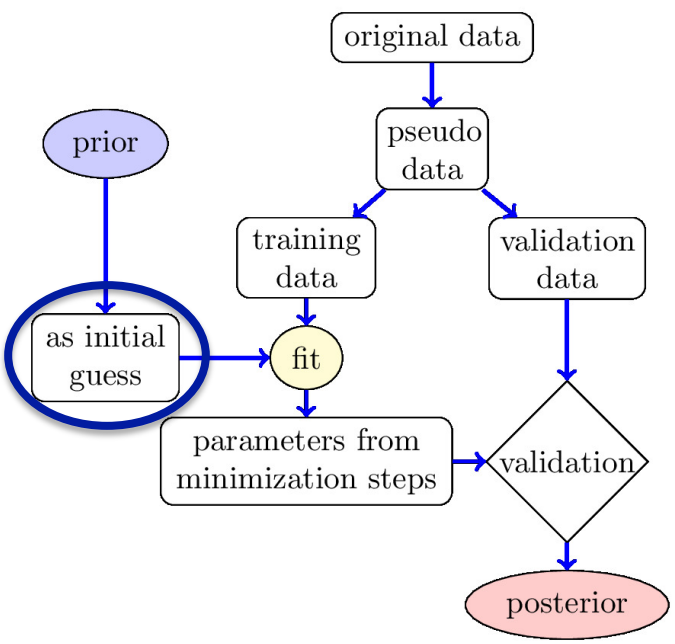
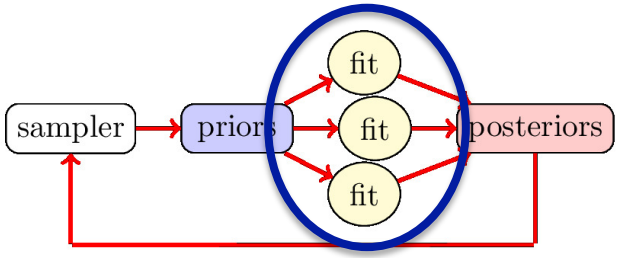




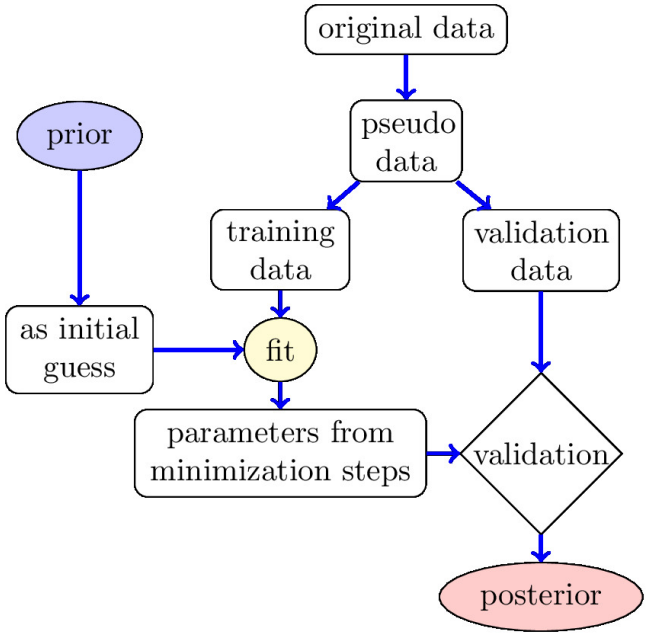
Initial iteration: flat sample priors

→ Set of parameters used as initial guess for least-squares fits

Iterative Monte Carlo (IMC) Fitting Methodology



Perform thousands of fits



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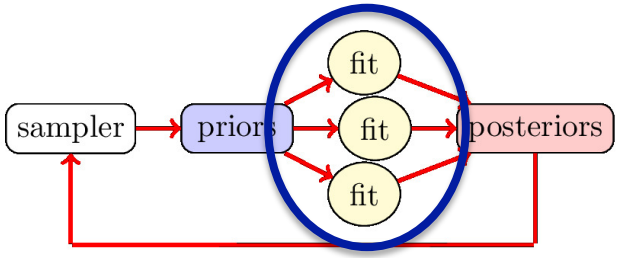
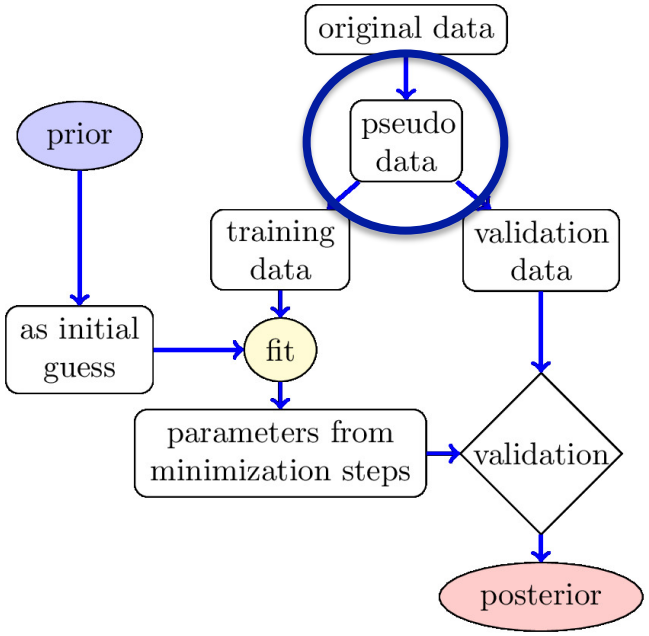
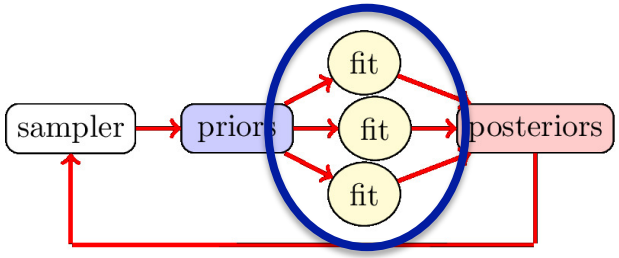


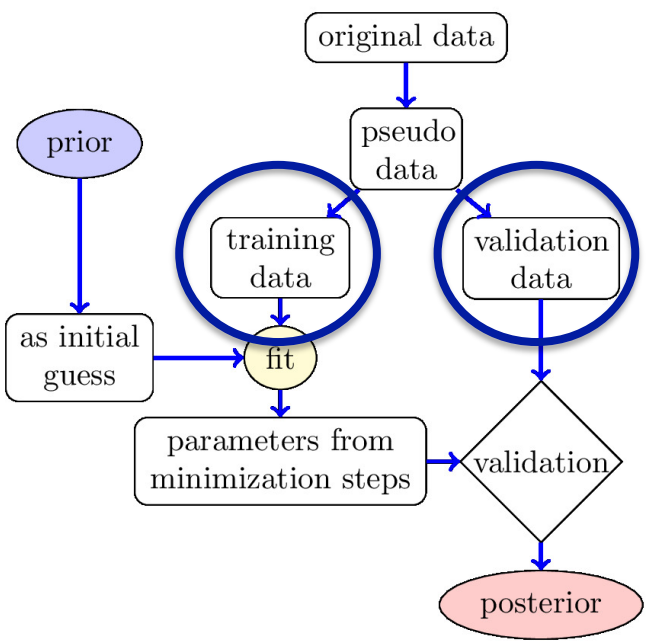


Perform thousands of fits

→ Pseudo-data constructed by bootstrap method

Iterative Monte Carlo (IMC) Fitting Methodology



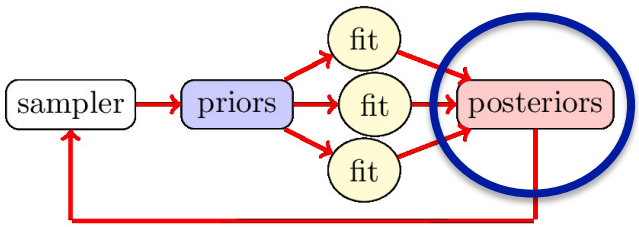


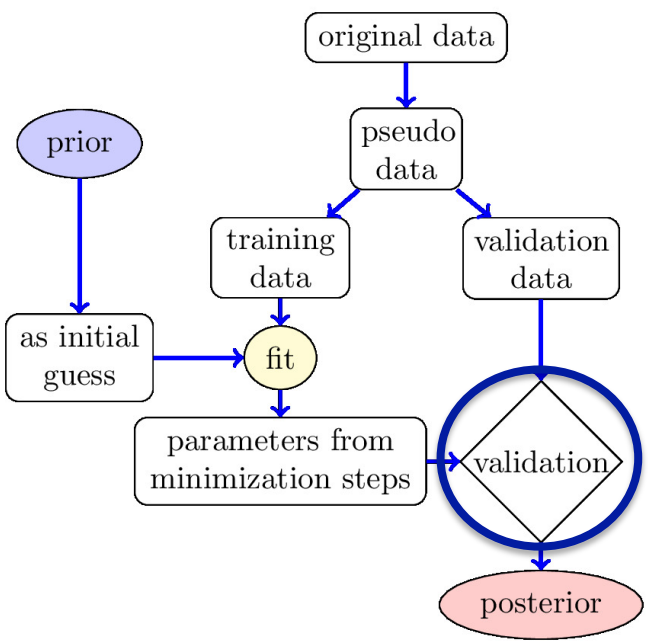
Perform thousands of fits

→ Pseudo-data constructed by bootstrap method

→ Data is partitioned for cross-validation – training set is fitted via chi-square minimization

Iterative Monte Carlo (IMC) Fitting Methodology

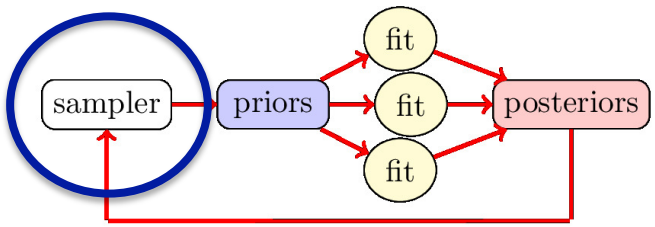


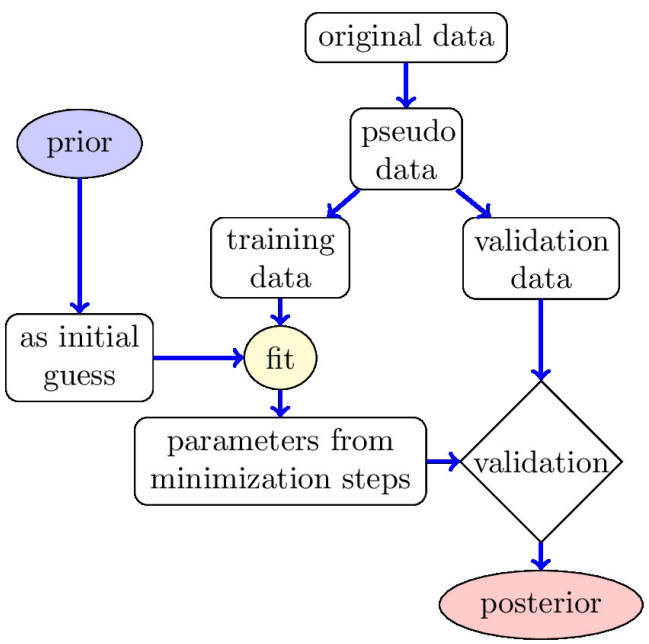


Obtain a set of posteriors

→ Set of parameters that minimize validation chi-square are chosen as posteriors

Iterative Monte Carlo (IMC) Fitting Methodology





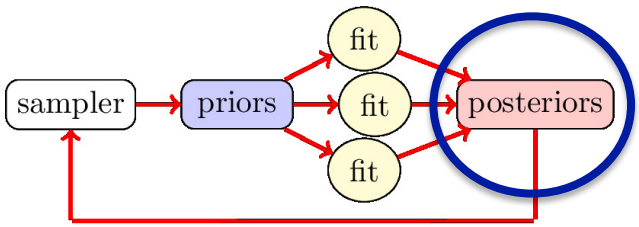
Posteriors are sent through a sampler

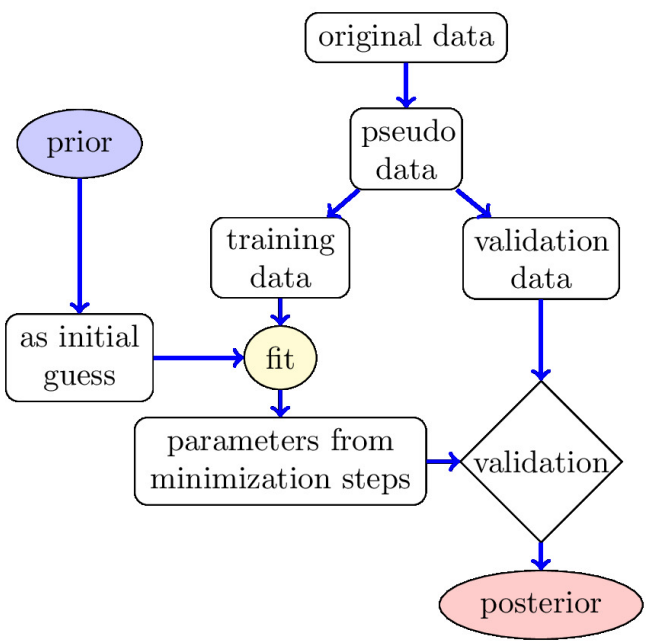
→ Kernel density estimation (KDE): estimates the multi-dimensional probability density function of the parameters

→ A sample of parameters is chosen from the KDE and used as starting priors for the next iteration

→ Iterated until distributions are converged

Iterative Monte Carlo (IMC) Fitting Methodology





Obtain final set of parameters

→ Compute mean and standard deviation of observables

$$\mathrm{E}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n}\mathcal{O}(\boldsymbol{a}_k)$$

$$\mathrm{V}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n}(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

Parameterizations and Chi-square

- PDFs: $\Delta q^+, \Delta g = M \dfrac{x^a (1-x)^b (1 + c\sqrt{x})}{\beta(a+1, b+1) + c\beta(a+1.5, b+1)}$

- FFs: $\mathrm{T}(z; \boldsymbol{a}) = M \dfrac{z^a (1-z)^b}{\beta(a+2, b+1)}$

Favored: $D^h_{q^+} = \mathrm{T}(z; \boldsymbol{a}) + \mathrm{T}(z; \boldsymbol{a}')$

Unfavored: $D^h_{q^+,g} = \mathrm{T}(z; \boldsymbol{a})$

Pions:

$$D^{\pi^+}_{\bar{u}} = D^{\pi^+}_{d} = \mathrm{T}(z; \boldsymbol{a})$$

$$D^{\pi^+}_{s} = D^{\pi^+}_{\bar{s}} = \frac{1}{2} D^{\pi^+}_{s^+}$$

Kaons:

$$D^{K^+}_{\bar{u}} = D^{K^+}_{d} = \frac{1}{2} D^{K^+}_{d^+}$$

$$D^{K^+}_{s} = \mathrm{T}(z; \boldsymbol{a})$$

- Chi-squared definition:

$$\chi^2(\boldsymbol{a}) = \sum_e \left[\sum_i \left(\frac{\mathcal{D}^{(e)}_i N^{(e)}_i - T^{(e)}_i(\boldsymbol{a})}{\alpha^{(e)}_i N^{(e)}_i} \right)^2 + \sum_k \left(r^{(e)}_k \right)^2 \right] + \sum_\ell \left(\frac{a^{(\ell)} - \mu^{(\ell)}}{\sigma^{(\ell)}} \right)^2$$

Parameterizations and Chi-square

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Penalty for fitting normalizations

Parameterizations and Chi-square

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Modified likelihood to include prior information

Simultaneous Fit Details

- Procedure:

→IMC fit of q+ and glue PDF distributions to DIS data (with $W^2 > 10$ GeV^2)

→ Sample above DIS parameters and JAM16 FFs through KDE sampler

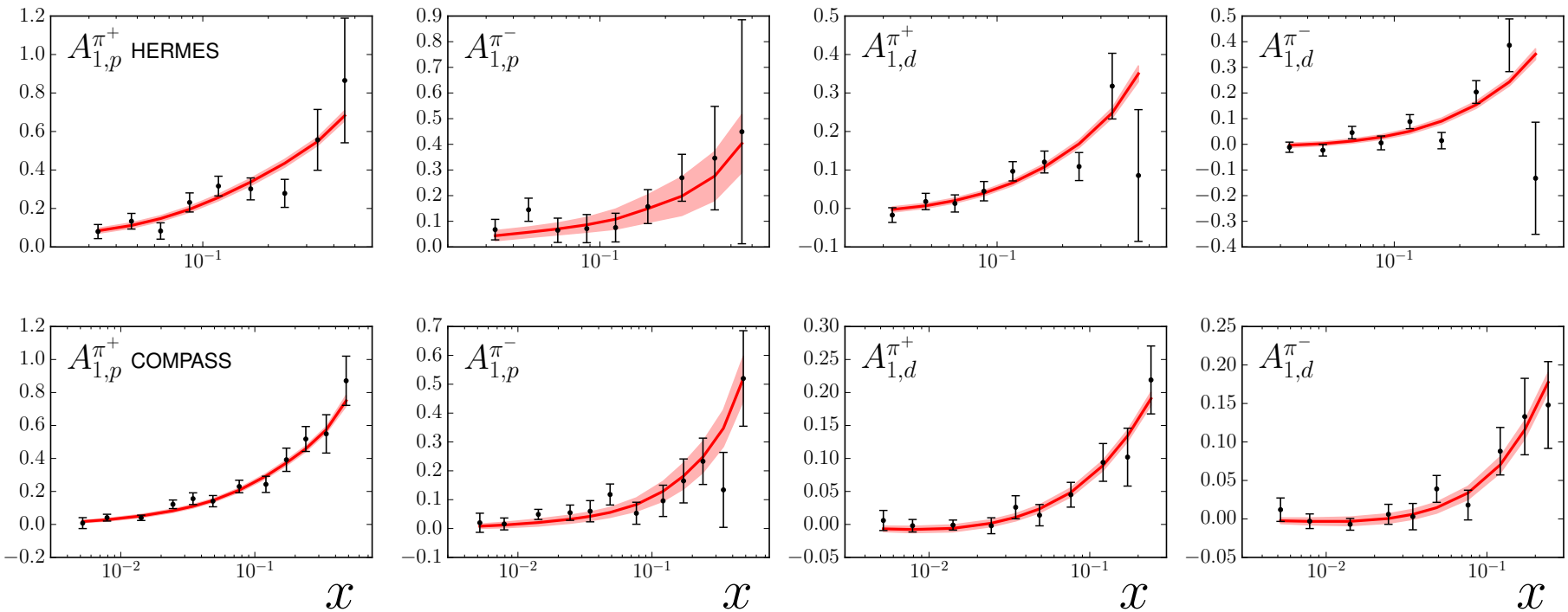
→ Generate priors (flat sampling) for unknown sea quark PDFs and unfavored FFs

→ Fit DIS, SIDIS, and SIA data simultaneously (pion and kaon observables only)

→ Fix r values to DIS/SIA only fits

Data vs Theory – SIDIS pion PRELIMINARY

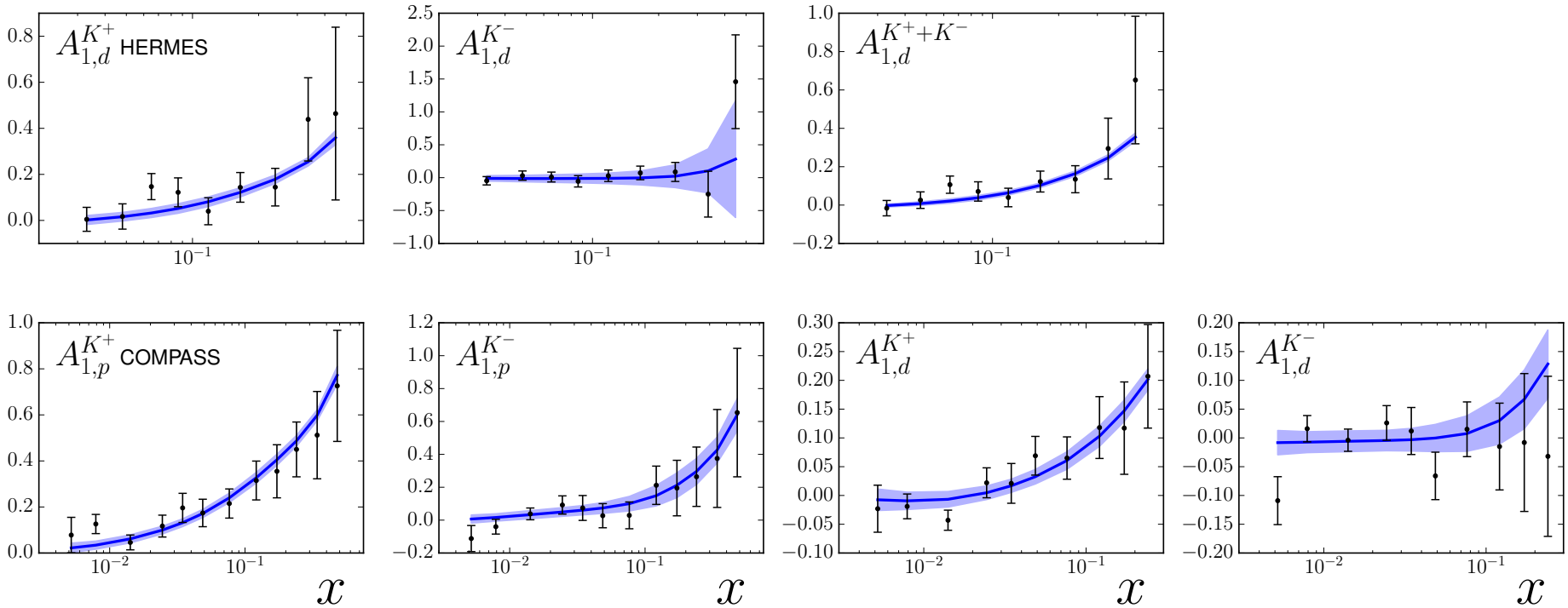
$$\chi^2/N_{pts} = 0.87$$



- Other chi-square information: SIA pion $\chi^2/N_{pts} = 1.30$

Data vs Theory – SIDIS kaon PRELIMINARY

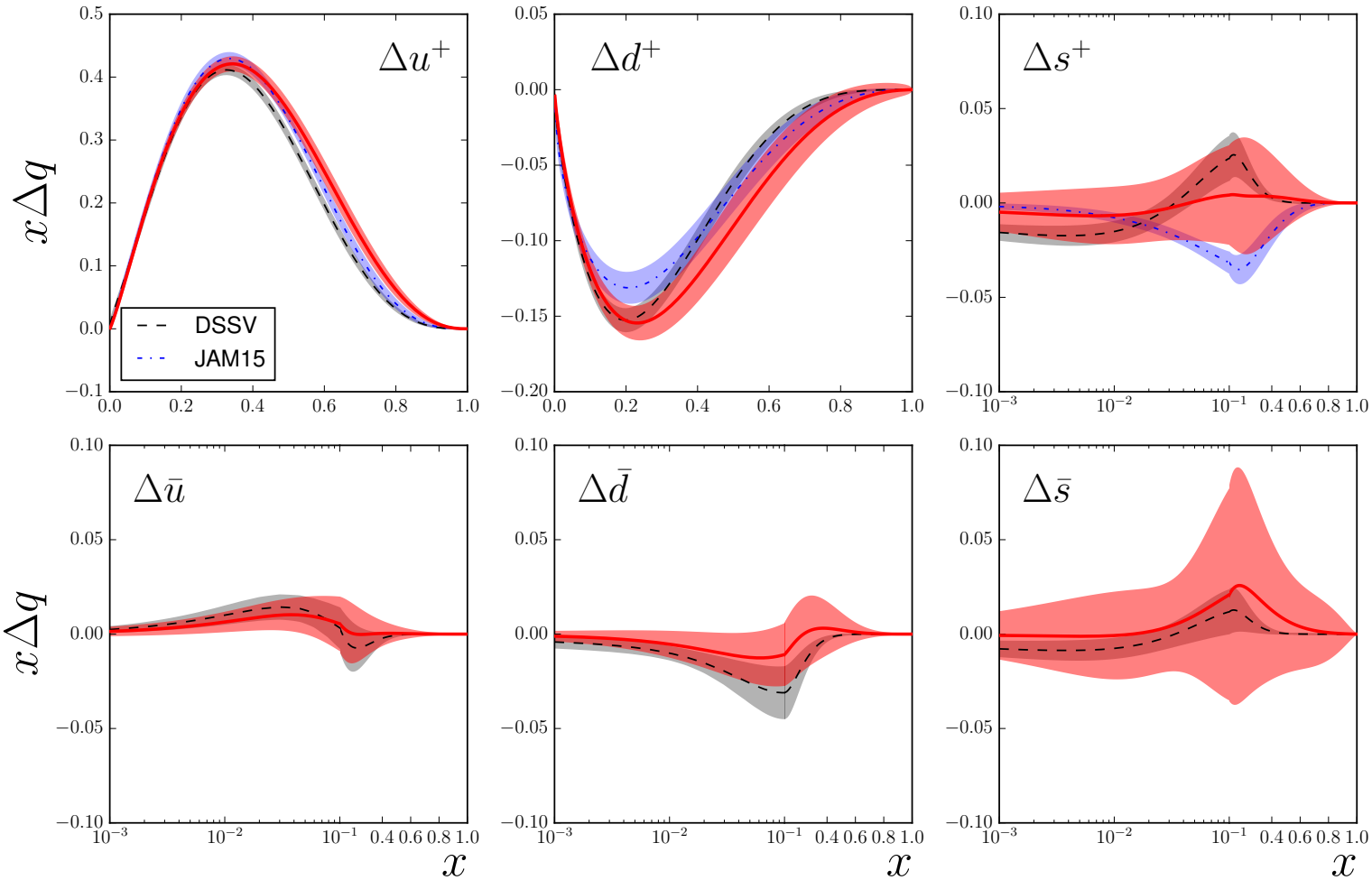
$$\chi^2/N_{pts} = 0.73$$



- Other chi-square information: SIA kaon $\chi^2/N_{pts} = 1.01$

  DIS $\chi^2/N_{pts} = 1.00$

Polarized PDF Distributions PRELIMINARY

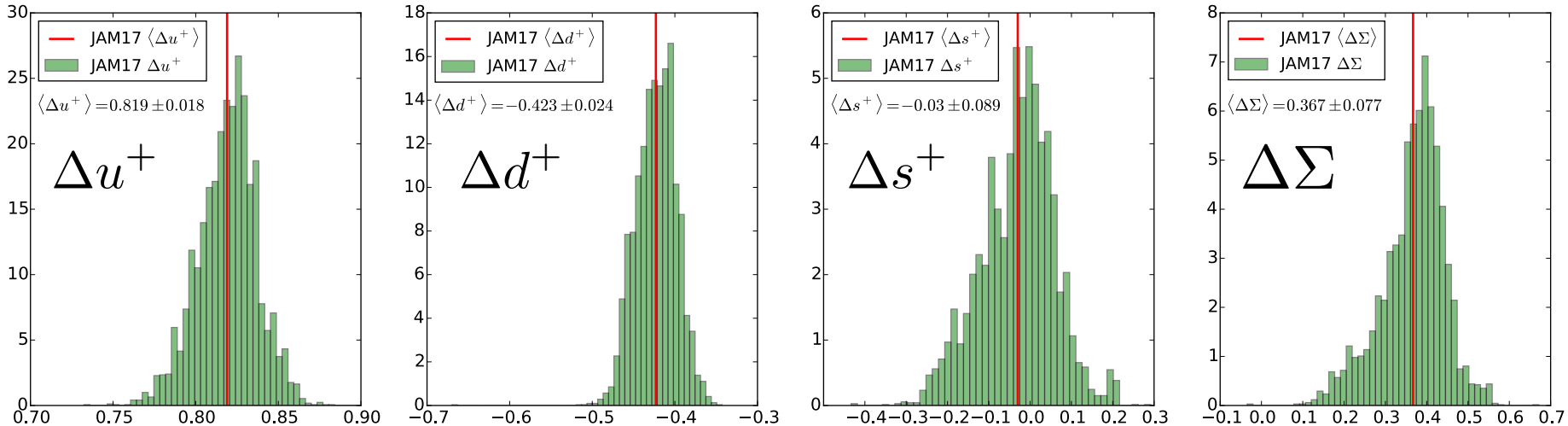


- Δs+ distribution consistent with zero!

$$Q^2 = 1\,\mathrm{GeV}^2$$

Moments

PRELIMINARY



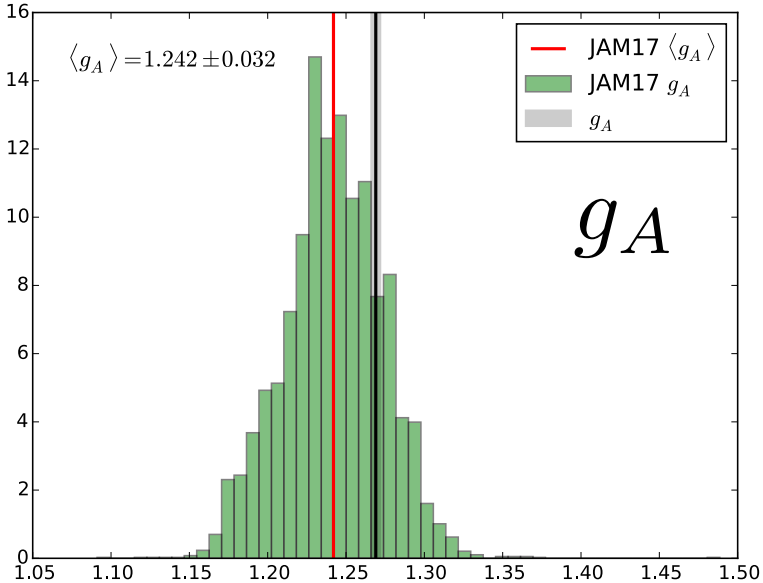
- Δu+ , Δd+ moments are well determined by DIS+SIDIS data

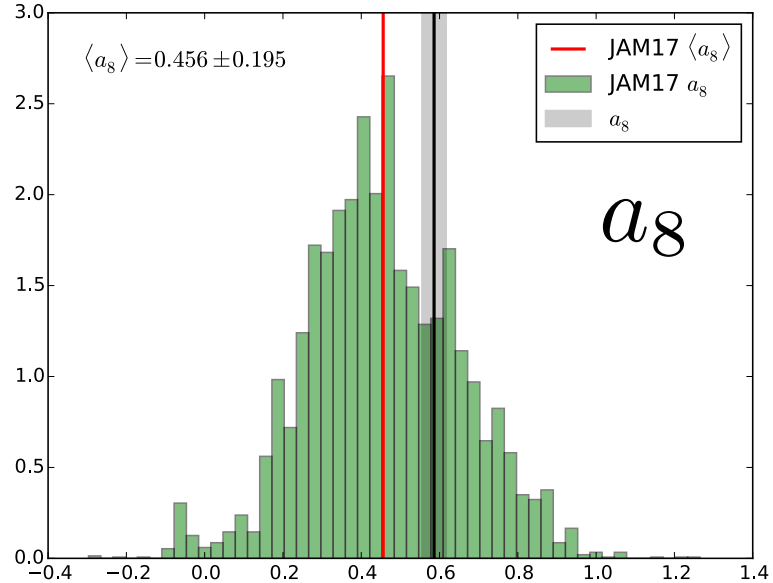
$$\Delta \mathbf{s}^+ = -0.03 \pm 0.089$$ Around zero, but with large uncertainty

$$\Delta \Sigma = 0.367 \pm 0.077$$ Consistent with previous analyses

Moments

PRELIMINARY





$$g_A = 1.242 \pm 0.032$$ SU(2) symmetry confirmed

- Flat sampled octet axial charge in range (-0.2,1.2)

$$a_8 = 0.456 \pm 0.195$$ Not well constrained by pion and kaon SIDIS data

→ SU(3) value within uncertainty range

- Need better determination of Δs+ moment to reduce a_8 uncertainty

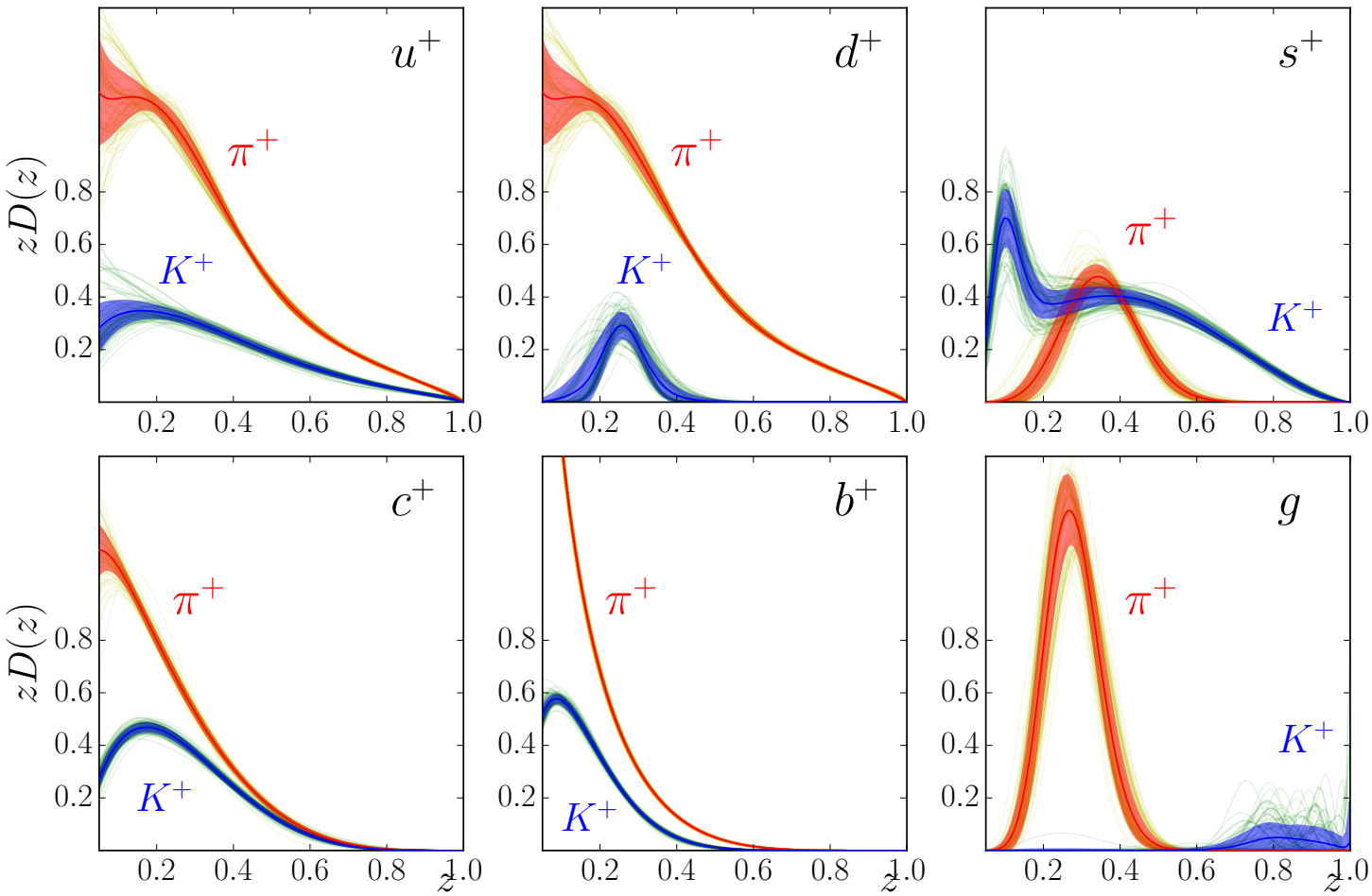
Summary and Outlook

- Δs+ distribution consistent with zero with large uncertainties
  - → Need higher precision polarized SIDIS kaon data
- Difficult to determine a_8 with DIS+SIDIS, but results confirm SU(2) symmetry
- Need to release all r parameters for all data sets
- Would like to lower W cut to include JLab data
- QCD observables yet to be implemented:
  - *W* asymmetries for spin PDFs (up and down sea constraints)
  - Unpolarized SIDIS and single-inclusive *pp* collision for FFs
- Working towards a universal fit of quark helicity distributions
  - Global analyses of combined unpolarized and polarized data

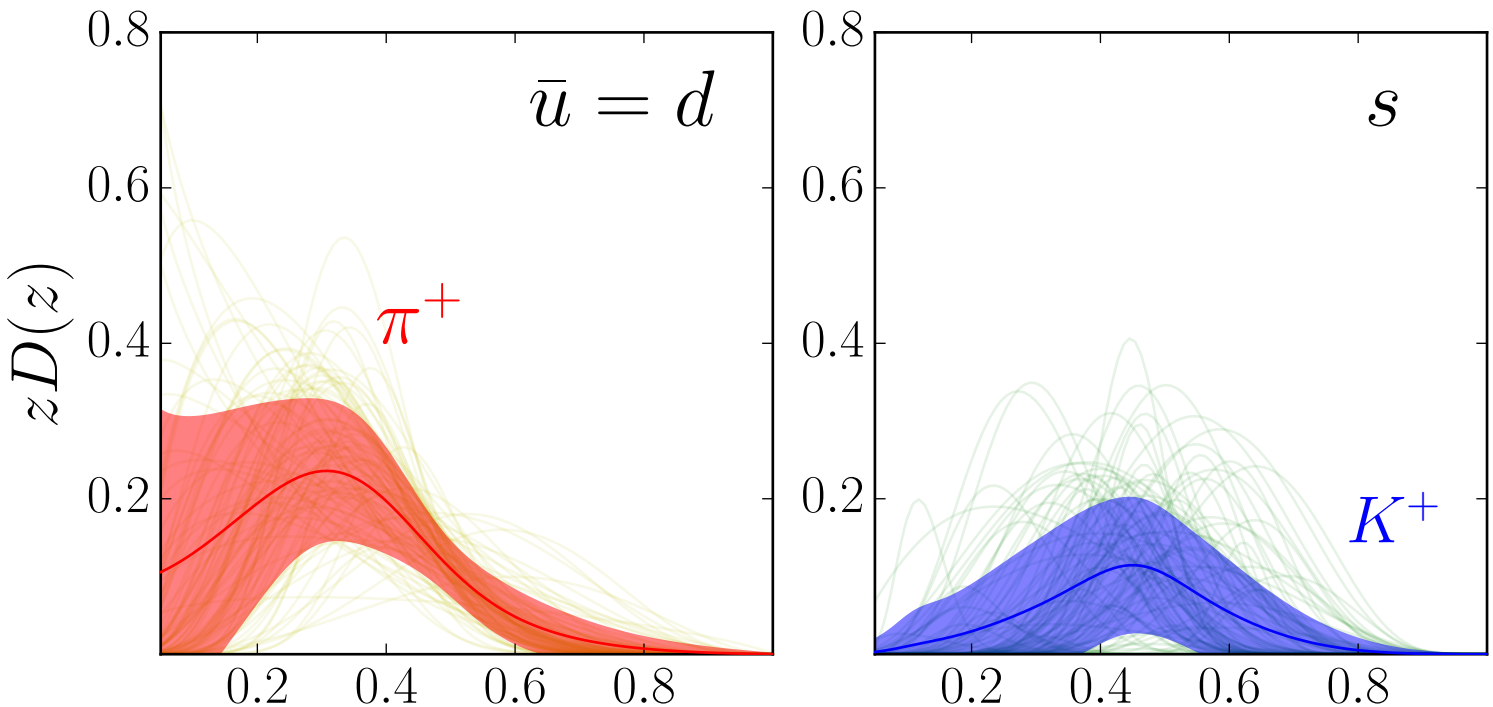
Thank You

BACKUP SLIDES

Fragmentation Functions



Fragmentation Functions



Polarized DIS

- We fit measurements of parallel and perpendicular spin asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\left(A_1 + \eta A_2\right)$$

$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\left(A_2 + \zeta A_1\right)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

- Define our polarized structure functions:

$$g_{1,2}(x, Q^2) = g_{1,2}^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, ...)$$

Leading Twist Structure Functions

- Leading twist structure function defined in collinear factorization as

$$g_1^{(\tau 2)}(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \left[(\Delta C_q \otimes \Delta q^+)(x,Q^2) + \ (\Delta C_g \otimes \Delta g)(x,Q^2)\right]$$

- Leading twist + target mass corrections $\xi = \frac{2x}{1+\rho}, \qquad \rho^2 = 1+\gamma^2$

J. Blümlein and A. Tkabladze
Nucl. Phys. B553, 427 (1999)

$$g_2^{(\tau 2+\mathrm{TMC})}(x,Q^2) = -\frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{1}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{x}{\xi} - (\rho^2-1) + \frac{3(\rho^2-1)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

$$g_1^{(\tau 2+\mathrm{TMC})}(x,Q^2) = \frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{(\rho^2-1)}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{(x+\xi)}{\xi} - \frac{(3-\rho^2)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

- In Bjorken limit $\left(Q^2 \to \infty\right)$:

$$g_1^{(\tau 2+\mathrm{TMC})} = g_1^{(\tau 2)} \qquad g_2^{(\tau 2)}(x,Q^2) = -g_1^{(\tau 2)}(x,Q^2) + \int_x^1 \frac{dz}{z} g_1^{(\tau 2)}(z,Q^2)$$

Semi-inclusive DIS

- In SIDIS, we fit the photon-nucleon spin asymmetries (assuming Bjorken limit)

$$A_1^h\left(x,z,Q^2\right) = \frac{g_1^h(x,z,Q^2)}{F_1(x,z,Q^2)}$$

- The (un)polarized structure functions are defined

$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q e_q^2 \Bigg\{ \Delta q(x,\mu_F) D_q^h(z,\mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi} \times \Big(\Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \Big) \Bigg\}$$

$$F_1^h(x,z,Q^2) = \frac{1}{2}\sum_q e_q^2 \Bigg\{ q(x,\mu_F) D_q^h(z,\mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi} \times \Big(q \otimes C_{qq} \otimes D_q^h + q \otimes C_{gq} \otimes D_g^h + g \otimes C_{qg} \otimes D_q^h \Big) \Bigg\}$$

Single-inclusive Annihilation (SIA)

- Cross sections for $e^+e^- \to hX$

$$\frac{d\sigma^h}{dz} = \frac{d\sigma_L^h}{dz} + \frac{d\sigma_T^h}{dz} \qquad z = \frac{2E_h}{\sqrt{s}}$$

- Transverse and longitudinal cross sections

$$\frac{d\sigma_T^h}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi}\left(C_i^T \otimes D_i\right)(z,Q^2)\right] + \sigma_0 \frac{\alpha_s}{2\pi}\left(C_g^T \otimes D_g\right)(z,Q^2)$$

$$\frac{d\sigma_L^h}{dz} = \sum_i \sigma_i \frac{\alpha_s}{2\pi}\left(C_i^L \otimes D_i\right)(z,Q^2) + \sigma_0 \frac{\alpha_s}{2\pi}\left(C_g^L \otimes D_g\right)(z,Q^2)$$

Single-inclusive Annihilation (SIA)

- Cross sections for $e^+e^- \to hX$

$$\frac{d\sigma^h}{dz} = \frac{d\sigma_L^h}{dz} + \frac{d\sigma_T^h}{dz} \qquad z = \frac{2E_h}{\sqrt{s}}$$

- Transverse and longitudinal cross sections

$$\frac{d\sigma_T^h}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi} \left(C_i^T \otimes D_i \right)(z,Q^2) \right] + \sigma_0 \frac{\alpha_s}{2\pi} \left(C_g^T \otimes D_g \right)(z,Q^2)$$

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- Electroweak cross section $e^+e^- \to \gamma, Z \to q\bar{q}$:

$$\sigma_i(s) = \sigma_0 \left[e_i^2 + 2e_i\, g_V^i g_V^e\, \rho_1(s) + \left(g_A^{e\,2} + g_V^{e\,2} \right) \left(g_A^{i\,2} + g_V^{i\,2} \right) \rho_2(s) \right]$$

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- Typically, observables are normalized by total hadronic cross section

$$\sigma_{\rm tot}(s) = \sum_i \sigma_i \left(1 + \frac{\alpha_s}{\pi} \right)$$