TMD splitting functions and the corresponding evolution equation

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[PRD 94, 114013 (2016), 1607.01507]

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Introduction

- Parton distribution functions (PDFs) together with parton level matrix elements allow for a very accurate description of 'hard' events in hadron-hadron and hadron-electron collisions.

- The bulk of such analysis is carried out within the framework of collinear factorization.

- However, there exist classes of multi-scale processes where the use of more general schemes is of advantage
  - E.g. high-energy or low x limit of hard processes $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$ where $x = M^2/s$.
  - In such a scenario it is necessary to resum terms enhanced by logarithms $\ln 1/x$ to all orders in the α_s, which is achieved by BFKL evolution equation.
  - The resulting formalism called high energy (or k_T) factorization provides a factorization of such cross-sections into a TMD coefficient or 'impact factor' and an 'unintegrated' gluon density.

Introduction

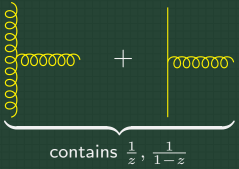
Limitations of high-energy factorization framework:

- valid only in low $x \lesssim 10^{-2}$ region
  - problems for observables involving fragmentation functions which involve integrals over the full x range of initial state PDFs
  - limited to exclusive observables which allow to fix x of both gluons
- limited to gluon-to-gluon splittings in the low x evolution, with quarks being absent.
  - omits a resummation of collinear logarithms associated with quark splittings which can provide sizable contributions at intermediate and large x
  - For hard processes initiated by quarks the appropriate unintegrated parton density functions are needed

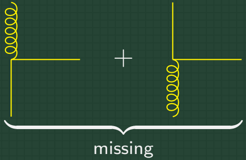
What can we do?

- Partial solution: use CCFM evolution equation instead of BFKL

  ✓ based on QCD coherence
  → includes also resummation
  of soft logarithms



  ✗ still limited to low x
  ✗ evolution equation for gluons only
  ✗ missing collinear logarithms



What do we want?

- Resum low x logarithms.
- Smooth continuation to the large x region.
- Reproduce the correct collinear limit (DGLAP).
- Ultimately: a coupled system of evolution equations for unintegrated PDFs
  - need: k_T-dependent splitting functions

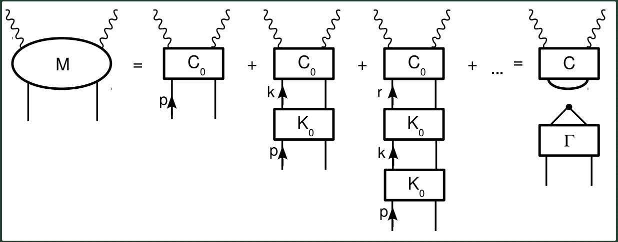
We will try to achieve this goal by extending Curci-Furmanski-Petronzio (CFP) and Catani-Hautmann (CH) formalisms.

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.
2. Generalization to the high-energy case and kernel calculation.
3. First attempt to formulate evolution equation including quark contributions.

Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

- Factorization based on generalized ladder expansion (in terms of 2PI kernels) [Ellis et al. NPB152 (1979), 285]

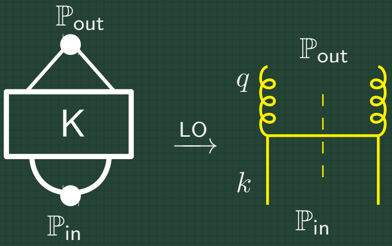


- Axial gauge instrumental
  - integration over outgoing legs leads to collinear singularities
  - incoming propagators amputated
- 2PI kernels connected only by convolution in x
  - this is achieved by introducing appropriate projector operators.

Curci-Furmanski-Petronzio (CFP) methodology

[Nucl. Phys. B175 (1980) 2792]

- CFP projector operators:



$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{out}} = -g^{\mu\nu} \\ \mathbb{P}_{q,\,\text{out}}(q) = \frac{\not{n}}{2q\cdot n} \end{cases}$$

$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{in}}(k) = \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n}\right) \\ \mathbb{P}_{q,\,\text{in}} = \frac{\not{k}}{2} \\ \text{incoming legs put on-shell } k^2 = 0 \end{cases}$$

- performs integration over $dq^2 d^{m-2}\mathbf{q}$

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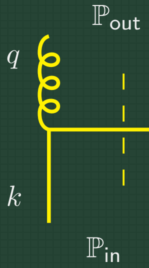
Generalization to high energy kinematics

- High energy kinematics



$$k^\mu = y p^\mu + k_\perp^\mu$$

$$q^\mu = x p^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2xp \cdot n} n^\mu$$

We will define/constrain splitting functions by requiring:

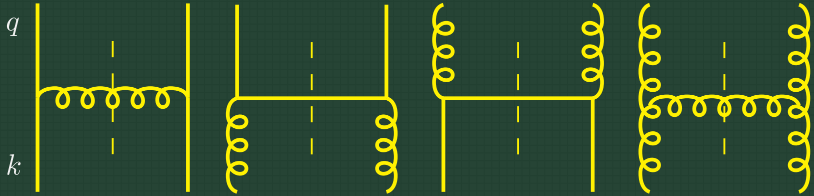
- gauge invariance/current conservation of vertices
- correct collinear limit
- correct high energy limit

Generalization to high energy kinematics

[JHEP 01 (2016) 181, 1511.08439]

- Partly obtained by Catani and Hautmann for the case of P_{qg}
  [Catani, Hautmann NPB427 (1994) 475524, hep-ph/9405388]

- We want to extend it to general case including all splittings



P_{qq} P_{qg} P_{gq} P_{gg}

To achieve this goal we need to provide:

- Appropriate projector operators
- Generalize QCD vertices

Generalization of projector operators

Since the incoming momentum is no longer collinear the corresponding projector operators need to be modified.

- Gluon case [Catani, Hautmann NPB427 (1994) 475524]:

$$\mathbb{P}_{g,\,\mathrm{in}}^{\mu\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2}$$

- Quark case [JHEP 01 (2016) 181]:

$$\mathbb{P}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}$$

✓ Both operators reduce to the CFP projectors in the collinear limit

$$< \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2} >_\phi \overset{k_\perp \to 0}{=} \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n}\right)$$

$$k \overset{k_\perp \to 0}{=} yp$$

Generalization of QCD vertices

We need to ensure gauge invariance of vertices in the presence of off-shell k, q momenta.

- This can be obtained using the reggeized quark formalism (Lipatov high-energy action) [Lipatov, Vyazovsky, NPB 597 (2001) 399]
- Alternatively the spin helicity formalism can be also used to construct appropriate gauge invariant amplitudes and extracted vertices from them [Kutak, van Hameren, Serino, JHEP 02, 009 (2017)]

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = igt^a\left(\gamma^\mu - \frac{n^\mu}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = igt^a\left(\gamma^\mu - \frac{p^\mu}{p\cdot q}\not{k}\right)$$

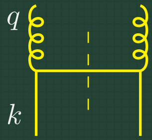
$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = igt^a\left(\gamma^\mu - \frac{p^\mu}{p\cdot p'}\not{k} + \frac{n^\mu}{n\cdot p'}\not{q}\right)$$

TMD splitting function definition

- Angular-dependent splitting function

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon\right) = z \int \frac{d^{2+2\epsilon}\mathbf{q}}{2(2\pi)^{4+2\epsilon}} \underbrace{\int dq^2\, \mathbb{P}_{j,\,\text{in}} \otimes \hat{K}^{(0)}_{ij}(q,k) \otimes \mathbb{P}_{i,\,\text{out}}}_{\tilde{P}^{(0)}_{ij}(z,\mathbf{k},\tilde{\mathbf{q}},\epsilon)}\, \Theta(\mu_F^2 + q^2)$$

$$\hat{K}^{(0)}_{ij}(q,k) =$$



$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2+\mathbf{q}^2}{2xp\cdot n} n^\mu$$

$$k^\mu = yp^\mu + k_\perp^\mu$$

$$\tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}, \qquad z = x/y$$

- Angular-average splitting function

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \epsilon\right) = \frac{\alpha_s}{2\pi}\, z \int_0^{(1-z)(\mu_F^2 - z\mathbf{k}^2)} \frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^{\epsilon} \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P^{(0)}_{ij}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right)$$

Results for splitting functions

So far we have calculated the real contributions to qq, qg and gq splitting functions at LO.

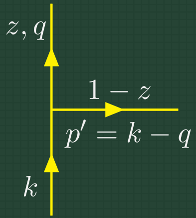
$$P_{qg}^{(0)} = T_R \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\,\mathbf{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right]$$

$$P_{gq}^{(0)} = C_F \left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} - \frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z) + \mathbf{k}^2 z(1-z^2))}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} + \frac{\epsilon z\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2 + (1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} \right]$$

$$P_{qq}^{(0)} = C_F \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2} \right) \left[\frac{\tilde{\mathbf{q}}^2 + (1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} + \frac{z^2\tilde{\mathbf{q}}^2 - z(1-z)(1-3z+z^2)\mathbf{k}^2 + (1-z)^2\epsilon(\tilde{\mathbf{q}}^2 + z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)} \right]$$

✓ In the collinear limit, $\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \to 0$, standard DGLAP results are reproduced.

Splitting functions: singularity structure



- $z \to 1$ limit:

$$\lim_{z\to 1} P_{qq}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right) = C_F \frac{2}{1-z}$$

  - This singularity is also present in the collinear case and it is regularized by the corresponding virtual correction.
  - We expect similar mechanism to apply here (TO BE DONE).

- $\mathbf{p'^2} \to 0$ limit (or equivalently $|\tilde{\mathbf{q}}| \to (1-z)|\mathbf{k}|$):
  - both P_{qq} and P_{gq} are singular

$$\lim_{\mathbf{p'^2}\to 0} \frac{P_{qq}}{\mathbf{p}'^2} = \frac{C_F}{\mathbf{p}'^2}\frac{2}{1-z} \qquad \lim_{\mathbf{p'^2}\to 0} \frac{P_{gq}}{\mathbf{p}'^2} = \frac{C_F}{\mathbf{p}'^2}\frac{2}{z}$$

  - and this singularity overlaps with the high energy ($z \to 0$) limit of P_{gq} and soft limit for P_{qq}.

P_{gq} problem

$$\lim_{\mathbf{p}'^2 \to 0} \frac{P_{gq}}{\mathbf{p}'^2} = \frac{C_F}{\mathbf{p}'^2} \frac{2}{z}$$

- There is no virtual correction that could regularize the $\mathbf{p}'^2 \to 0$ singularity.
- We will try to formulate evolution equation including P_{gq} that will (at least partly) help to overcome this issue.

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.

2. Generalization to the high-energy case and kernel calculation.

3. First attempt to formulate evolution equation including quark contributions.

Evolution equation first try [PRD 94, 114013 (2016), 1607.01507]

Our starting point is the LO BFKL equation:

$$\mathcal{F}(x,\mathbf{q}^2) = \mathcal{F}^0(x,\mathbf{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{p}'}{\pi \mathbf{p}'^2} \left[\mathcal{F}(x/z, |\mathbf{q}+\mathbf{p}'|^2) - \theta(\mathbf{q}^2 - \mathbf{p}'^2)\mathcal{F}(x/z,\mathbf{q}^2)\right]$$

We supplement it by a contribution from quarks:

$$\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\int \frac{d^2\mathbf{p}'}{\pi\mathbf{p}'^2} P_{gq}(z,\mathbf{p}',\mathbf{q})\mathcal{Q}(x/z,|\mathbf{q}+\mathbf{p}'|^2)$$

where $\mathcal{Q}(x/z,|\mathbf{q}+\mathbf{p}'|^2)$ is distribution of quarks.

Evolution equation first try

1. We introduce resolution scale μ for gluons
   - $\mathbf{p}'^2 > \mu^2$ – resolved real emissions
   - $\mathbf{p}'^2 < \mu^2$ – unresolved real emissions to be combined with virtual ($|\mathbf{q}+\mathbf{p}'|^2 \approx \mathbf{q}^2$)

$$\mathcal{F}(x,\mathbf{q}^2) = \mathcal{F}^0(x,\mathbf{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z}\left[\int_{\mu^2} \frac{d^2\mathbf{p}'}{\pi\mathbf{p}'^2}\mathcal{F}(x/z,|\mathbf{q}+\mathbf{p}'|^2) - \ln\frac{\mathbf{q}^2}{\mu^2}\mathcal{F}(x/z,\mathbf{q}^2)\right]$$

2. Integral over $\mathbf{p}'$ in the quark part is divergent and needs to be regulated $\rightarrow$ cut-off $\mu_q \equiv \mu \rightarrow \Theta(\mathbf{p}'^2 - \mu^2)$

$$\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\int_{\mu^2}\frac{d^2\mathbf{p}'}{\pi\mathbf{p}'^2}P_{gq}(z,\mathbf{p}',\mathbf{q})\mathcal{Q}(x/z,|\mathbf{q}+\mathbf{p}'|^2)$$

Evolution equation first try [PRD 94, 114013 (2016), 1607.01507]

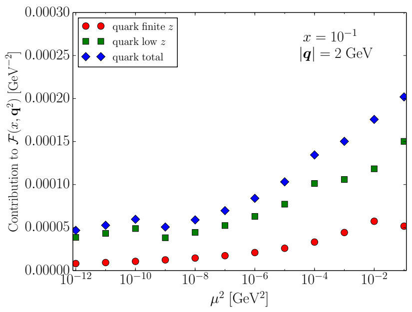
3. We resum virtual and unresolved emissions by going to Mellin space ($x \to \omega \to x$).

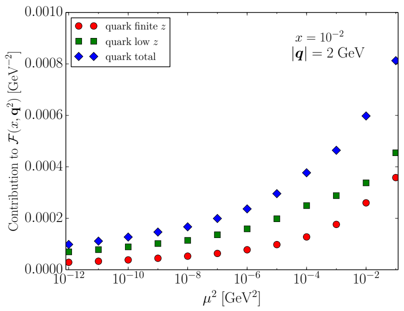
$$\mathcal{F}(x,\mathbf{q}^2) = \tilde{\mathcal{F}}^0(x,\mathbf{q}^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\int_{\mu^2}\frac{d^2\mathbf{p}'}{\pi\mathbf{p}'^2}\Bigg[\Delta_R(z,\mathbf{q}^2,\mu^2)\left(2C_A\mathcal{F}\left(\frac{x}{z},|\mathbf{q}+\mathbf{p}'|^2\right) + C_F\mathcal{Q}\left(\frac{x}{z},|\mathbf{q}+\mathbf{p}'|^2\right)\right) - \int_z^1 \frac{dz_1}{z_1}\Delta_R(z_1,\mathbf{q}^2,\mu^2)\left[\tilde{P}'_{gq}\left(\frac{z}{z_1},\mathbf{p},\mathbf{q}\right)\frac{z}{z_1}\right]\mathcal{Q}\left(\frac{x}{z},|\mathbf{q}+\mathbf{p}'|^2\right)\Bigg]$$

- $\frac{1}{\mathbf{p}'^2}$ singularity is regularized by Regge formfactor $\Delta_R(z,\mathbf{q}^2,\mu^2) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z}\ln\frac{\mathbf{q}^2}{\mu^2}\right)$

Numerical stability with respect to cut-off μ

- We need to assume some form of the quark distribution $\mathcal{Q}$
  - DLC 2016 set of parton densities [Kutak et al. JHEP 04 (2016) 175]





- As $\mu \to 0$ the cutoff dependence gets weaker

Summary and Outlook

- We have calculated real emission $k_\perp$-dependent P_{qq}, P_{gq} and P_{qg} splitting functions.
- We used them to construct evolution equation for gluons, receiving contribution from quarks.
- We found that resummation of gluon virtual contribution partially helps with the treatment of $\mathbf{p}'^2 \to 0$ singularity of P_{gq} splitting.
- P_{gg} splitting function and virtual corrections to P_{qq} should be computed using the same formalism.
- We need to better understand $\mathbf{p}'^2 \to 0$ limit.

BACKUP SLIDES

Splitting functions: collinear limit

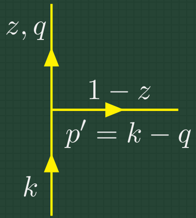
Collinear limit is obtained when $\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \to 0$

$$P_{qg}^{(0)}(z,0,\epsilon) = T_R\left(z^2 + (1-z)^2\right)$$

$$P_{gq}^{(0)}(z,0,\epsilon) = C_F\frac{1 + (1-z)^2 + \epsilon z^2}{z}$$

$$P_{qq}^{(0)}(z,0,\epsilon) = C_F\frac{1 + z^2 + \epsilon(1-z)^2}{1-z}$$

Angular-dependent splitting functions: $\mathrm{p}'^2 \to 0$ limit



- $\mathbf{p'^2} \to 0$ limit:

$$\hat{K}_{ij}\left(z, \frac{\mathbf{q}^2}{\mu_F^2}\right) = \frac{\alpha_s}{2\pi} z \int \frac{d^2\mathbf{p}'}{\pi}\, \Theta\left(\mu_F^2 - \frac{z\,\mathbf{p}'^2 + (1-z)\,\mathbf{q}^2}{1-z}\right) \frac{P_{ij}(z, \mathbf{p}', \mathbf{q})}{\mathbf{p}'^2}$$

$$\lim_{\mathbf{p'^2}\to 0} \frac{P_{qq}}{\mathbf{p}'^2} = \frac{C_F}{\mathbf{p}'^2}\frac{2}{1-z} \qquad \lim_{\mathbf{p'^2}\to 0} \frac{P_{gq}}{\mathbf{p}'^2} = \frac{C_F}{\mathbf{p}'^2}\frac{2}{z}$$

Mellin transforms

Mellin transform is defined as

$$\begin{aligned}
\overline{\mathcal{F}}(\omega, \mathbf{q}^2) &= \int_0^1 dx x^{\omega-1} \mathcal{F}(x, \mathbf{q}^2) \\
\overline{\mathcal{Q}}(\omega, \mathbf{q}^2) &= \int_0^1 dx x^{\omega-1} \mathcal{Q}(x, \mathbf{q}^2) \\
\overline{P}_{gq}(\omega, \mathbf{p}, \mathbf{q}) &= \int_0^1 dz P_{gq}(z, \mathbf{p}, \mathbf{q}) z^{\omega}
\end{aligned}$$

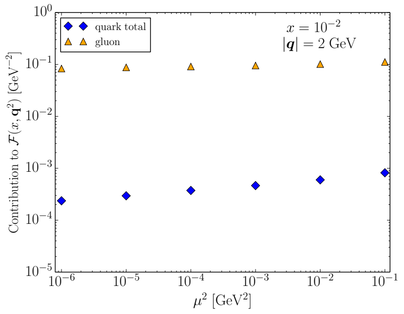
while the inverse transform reads

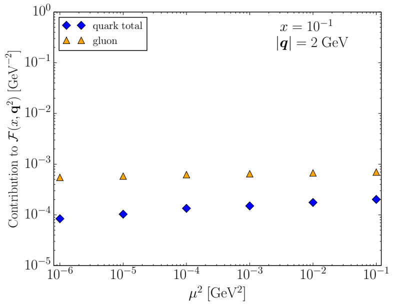
$$\begin{aligned}
\mathcal{F}(x, \mathbf{q}^2) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega} \overline{\mathcal{F}}(\omega, \mathbf{q}^2) \\
\mathcal{Q}(x, \mathbf{q}^2) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega} \overline{\mathcal{Q}}(\omega, \mathbf{q}^2)
\end{aligned}$$

Additionally

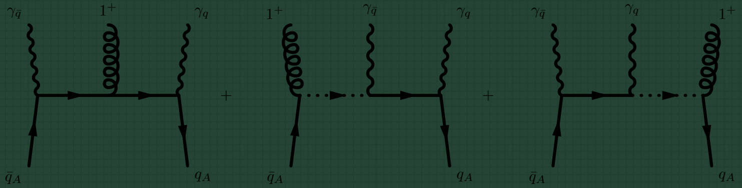
$$\tilde{P}'_{gq}(z, \mathbf{p}, \mathbf{q}) \equiv \frac{d}{dz}\left(\frac{1}{z} P_{gq}(z, \mathbf{p}, \mathbf{q})\right) = -C_F \frac{2(1-z)\mathbf{p}^2\mathbf{q}^4}{\left(z\mathbf{p}^2 + (1-z)\mathbf{q}^2\right)^3}.$$

Importance of the quark contribution





$\Gamma^{\mu}_{q^*,q^*,g}$ from $\mathcal{A}(g^+, \bar{q}^{*+}, q^{*-})$



$$\mathcal{A}(1^+, \bar{q}^{*+}, q^{*-}) \to \langle \bar{q}| \frac{\not{k}_{\bar{q}}}{k_{\bar{q}}^2} \left\{ \gamma^\mu + \frac{p_{\bar{q}}^\mu}{p_{\bar{q}} \cdot k_q} \not{k}_{\bar{q}} + \frac{p_q^\mu}{p_q \cdot k_{\bar{q}}} \not{k}_q \right\} \frac{\not{k}_q}{k_q^2} |q]$$