Probing the gluon Wigner distribution in diffractive dijet production

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YH, Xiao, Yuan, PRL 116 (2016) 202301
Hagiwara, YH, Ueda, PRD 94 (2016) 094036
YH, Nakagawa, Yuan, Zhao, 1612.02445
Hagiwara, YH, Pasechnik, Tasevsky , Teryaev, work in progress

Nucleon tomography

Partons are characterized by not only the longitudinal momentum fraction x,
but also by the transverse momentum $f(x, \vec{k}_\perp)$ (TMD)
and impact parameter $f(x, \vec{b}_\perp)$ (GPD)

➡ 3D tomography

5D tomography:
Wigner distribution— the "mother distribution"

Belitsky, Ji, Yuan (2003);

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \frac{\Delta}{2}|\bar{q}(-z/2)\gamma^+ q(z/2)|P + \frac{\Delta}{2}\rangle$$

$$\begin{tikzcd}
\text{TMD} \quad f(x,\vec{k}_\perp) \arrow[dr, "\int d\vec{k}_\perp"'] & & \text{GPD} \quad f(x,\vec{b}_\perp) \arrow[dl, "\int d\vec{b}_\perp"] \arrow[dr, "\int dx"] & \\
& f(x) \arrow[dr, "\int dx"'] & & F(\vec{b}_\perp) \quad \text{Form factor} \arrow[dl, "\int d\vec{b}_\perp"] \\
& & Q & \\
& & \text{charge} &
\end{tikzcd}$$

$$\int d\vec{b}_\perp \qquad \int d\vec{k}_\perp$$

PDF

5D tomography: GTMD and Husimi

GTMD Meissner, Metz, Schlegel (2009)

Husimi Hagiwara, YH (2015)

$$W(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

$$H(x, \vec{k}_\perp, \vec{b}_\perp)$$

$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$

Gaussian smearing in k, b

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$

$\int d\vec{b}_\perp$ $\int d\vec{k}_\perp$

TMD $f(x, \vec{k}_\perp)$

GPD $f(x, \vec{b}_\perp)$

$\int d\vec{k}_\perp$ $\int d\vec{b}_\perp$ $\int dx$

$f(x)$

PDF

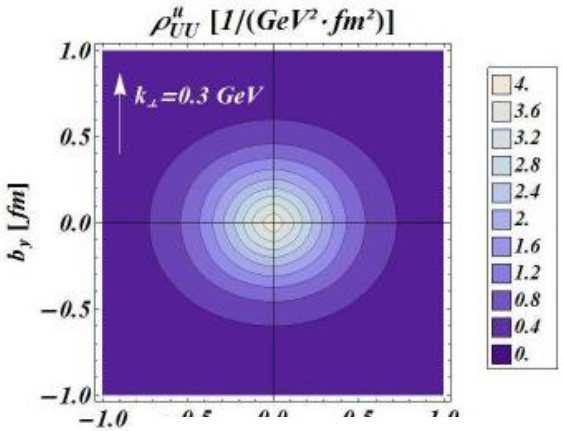
$F(\vec{b}_\perp)$ Form factor

$\int dx$ $\int d\vec{b}_\perp$

Q

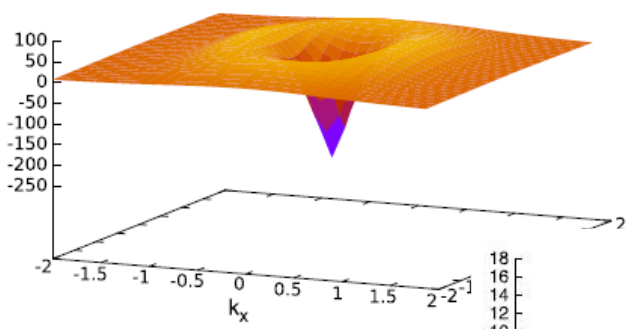
charge

Wigner/Husimi/GTMD
model calculations, 1-loop calculations, evolution...

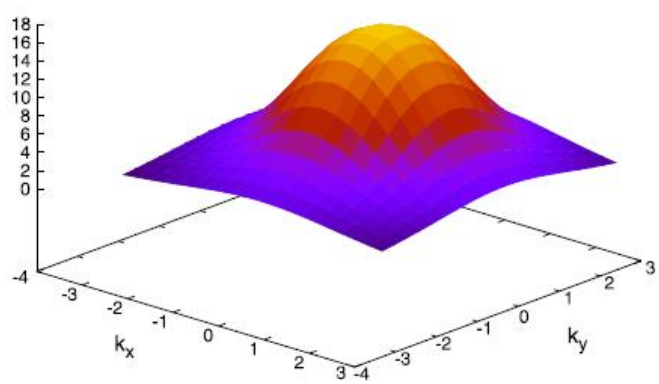


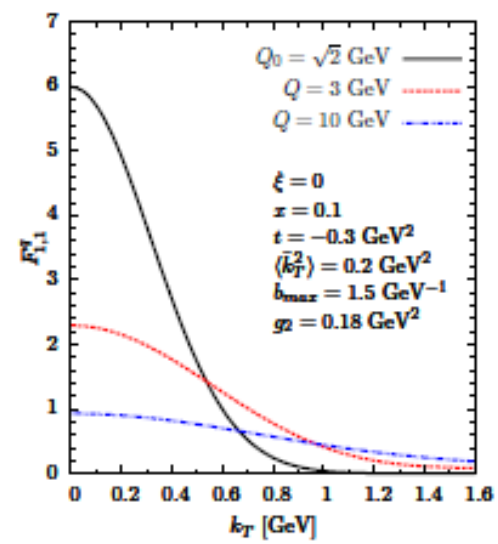
Wigner

Lorce, Pasquini
Liu, Ma
Mukherjee, Nair, Ojha
Courtoy, Goldstein, Hernandez, Liuti, Rajan
Hagiwara, YH、
Echevarria, Idilbi, Kanazawa, Lorce,
Metz, Pasquini, Schlegel
Gutsche, Lyubovitskij, Schmidt
.....



Husimi





GTMD

Parton orbital angular momentum

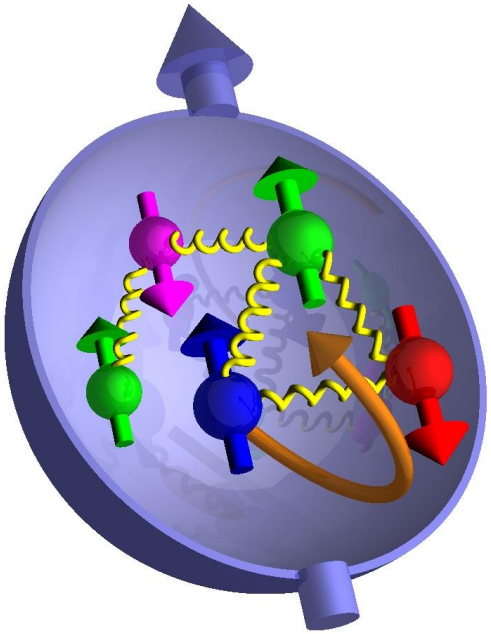
Nucleon spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Canonical Orbital angular momentum



$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \times \begin{cases} W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \\ H^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \end{cases}$$

Lorce, Pasquini, (2011);
YH (2011)

Wigner distribution: Is it measurable?

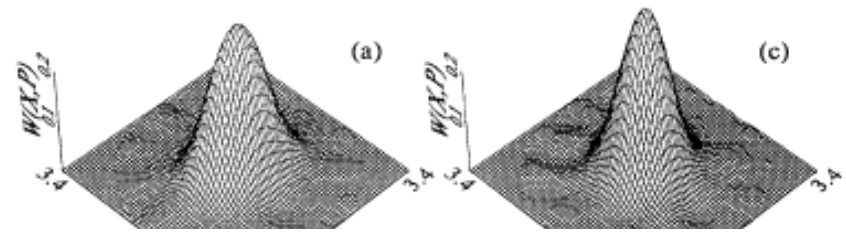
In quantum optics, yes!

VOLUME 70, NUMBER 9 PHYSICAL REVIEW LETTERS 1 MARCH 1993

Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

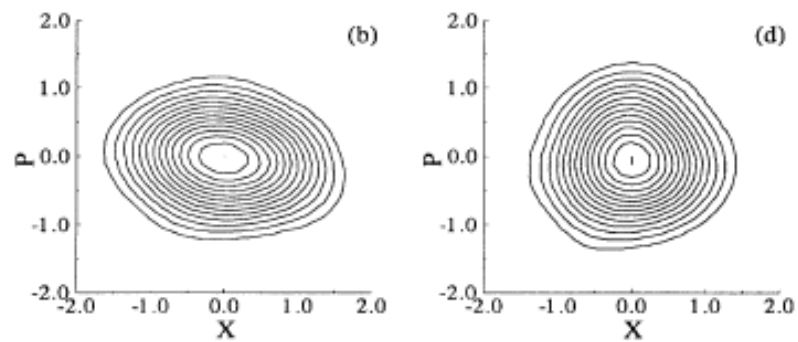
D. T. Smithey, M. Beck, and M. G. Raymer
Department of Physics and Chemical Physics Institute, U

A. Faridani
Department of Mathematics, Oregon State Uni
(Received 16 Novembe



What about in QCD? Go to small-x!

FIG. 1. Measured Wigner distributions for (a),(b) a squeezed state and (c),(d) a vacuum state, viewed in 3D and as contour plots, with equal numbers of constant-height contours. Squeezing of the noise distribution is clearly seen in (b).



Dipole gluon Wigner distribution at small-x

YH, Xiao, Yuan (2016)

$$xW(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int_{\Delta_\perp, z^-, z_\perp} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \frac{\Delta}{2}| \mathrm{Tr} F^{+i}(-z/2) U^{[+]} F^+_i(z/2) U^{[-]} | P + \frac{\Delta}{2}\rangle$$

Approximate $e^{ixP^+z^-} \approx 1$

$$\boxed{xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp,\vec{r}_\perp)}$$

``Dipole S-matrix" $S_x(\vec{b}_\perp,\vec{r}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$

Find a process sensitive to both $\vec{b}_\perp$ and $\vec{r}_\perp$.

Diffractive vector meson production

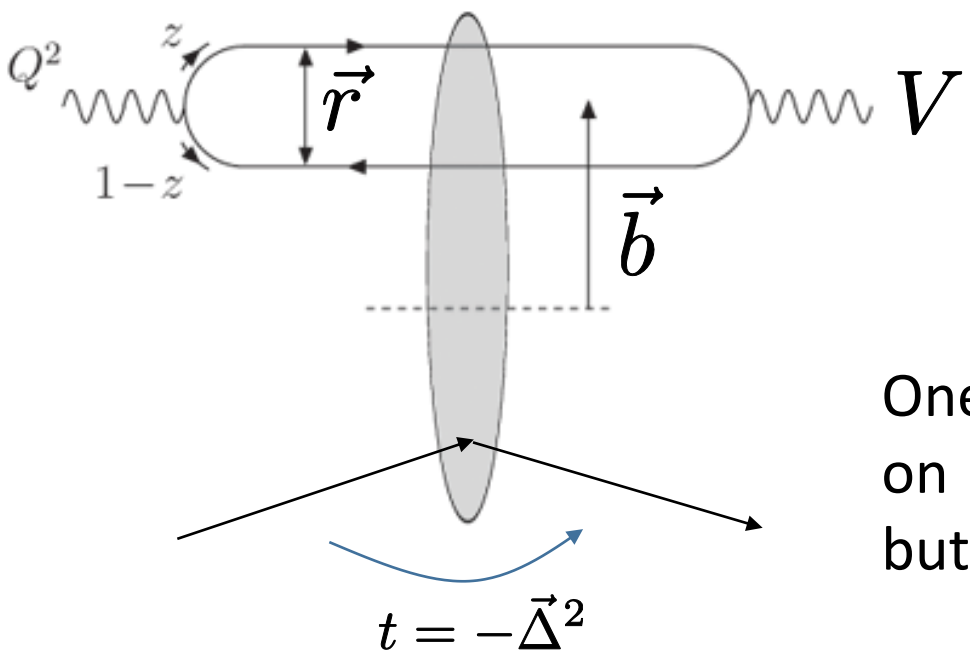
$$\frac{d\sigma}{dt} = \frac{1}{4\pi}\left|\int d^2b\, e^{-i\vec{\Delta}\cdot\vec{b}} A(\vec{b})\right|^2$$

$$A(\vec{b}) = i\int d^2\vec{r}\int dz \Psi^{\gamma^*}(Q,z,\vec{r})(1 - S(\vec{r},\vec{b}))\Psi^V(\vec{r},z)$$

$$e^{-i\left(\frac{1}{2}-z\right)\Delta_\perp\cdot r_\perp}$$

A phase factor needed

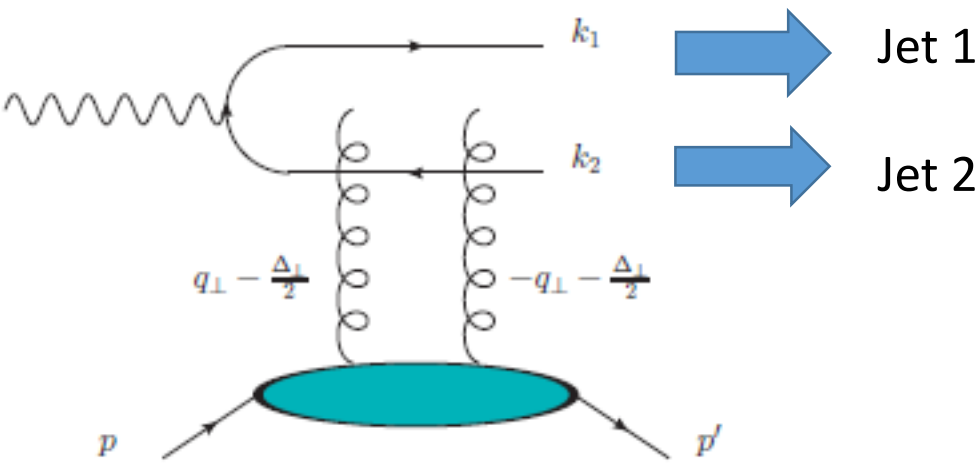
YH, Xiao, Yuan 1703.02085



One can study the dependence on $\vec{b}\ (\leftrightarrow \vec{\Delta})$ (Munier, Stasto, Mueller, 2001) but not on $\vec{r}\ (\leftrightarrow \vec{k})$

Diffractive dijet production

Altinoluk, Armesto, Beuf, Rezaeian (2015)
YH, Xiao, Yuan (2016)



$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$

Proton recoil momentum

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

Dijet relative momentum

Fourier transform of $S(\vec{r}_\perp, \vec{b}_\perp)$

$$\frac{d\sigma^{\gamma_T^* A \to q\bar{q}X}}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} \propto z(1-z)[z^2 + (1-z)^2] \int d^2q_\perp d^2q'_\perp S(q_\perp, \Delta_\perp) S(q'_\perp, \Delta_\perp)$$

$$\times \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2 + \epsilon^2} \right] \cdot \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon^2} \right]$$

$$\sim \left(S(\vec{P}_\perp, \vec{\Delta}_\perp) \right)^2$$

dominated by $q_\perp \sim P_\perp$ for small-Q^2

`Elliptic' Wigner distribution

$\cos 2\phi$ correlation between $\vec{P}_\perp$ and $\vec{\Delta}_\perp$ expected at small-x

$$W(x, \vec{k}_\perp, \vec{b}_\perp) = W_0(x, k_\perp, b_\perp) + 2\cos 2(\phi_k - \phi_b) W_1(x, k_\perp, b_\perp) + \cdots$$

$$\frac{d\sigma}{d^2 k_{1\perp} d^2 k_{2\perp}} \sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta) d\tilde{\sigma}$$

Small in magnitude (a few percent effect)
Distinct functional dependence on $x, k_\perp$

Hagiwara, YH, Ueda (2016)

Elliptic Wigner also relevant to:

`elliptic flow' v_2 in pA and pp collisions Hagiwara, YH, Xiao, Yuan (2017)
gluon transversity GPD and $\cos 2\phi$ correlation in DVCS YH, Xiao, Yuan (2017)
$\cos 2\phi$ correlation in quasielastic scattering $\gamma_T^* p \to p' X$ Zhou (2016)

Computing Wigner from BK

Hagiwara, YH, Ueda (2016)

Solve Balitsky-Kovchegov with b-dependence.

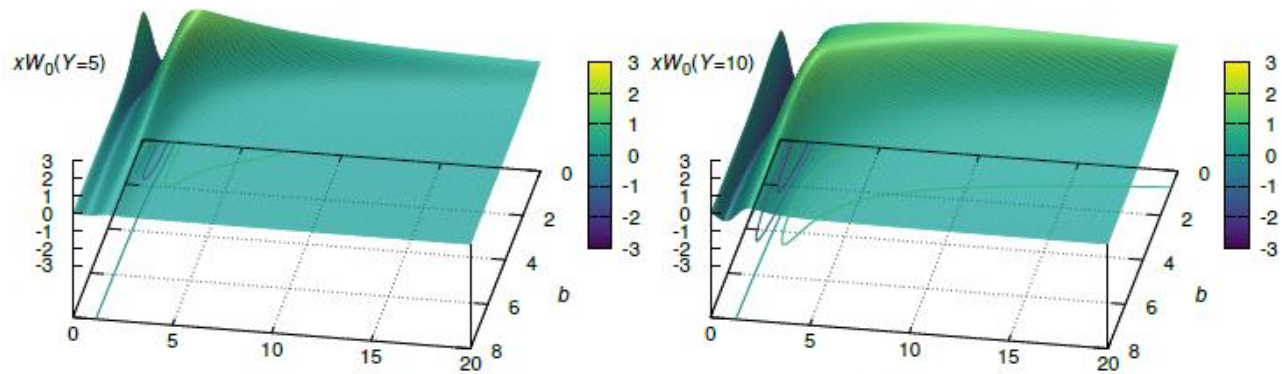
Assume that a SO(3) subgroup of conformal group survives

Gubser (2011)

$$S(\vec{x},\vec{y}) = S(d^2(\vec{x},\vec{y}))$$

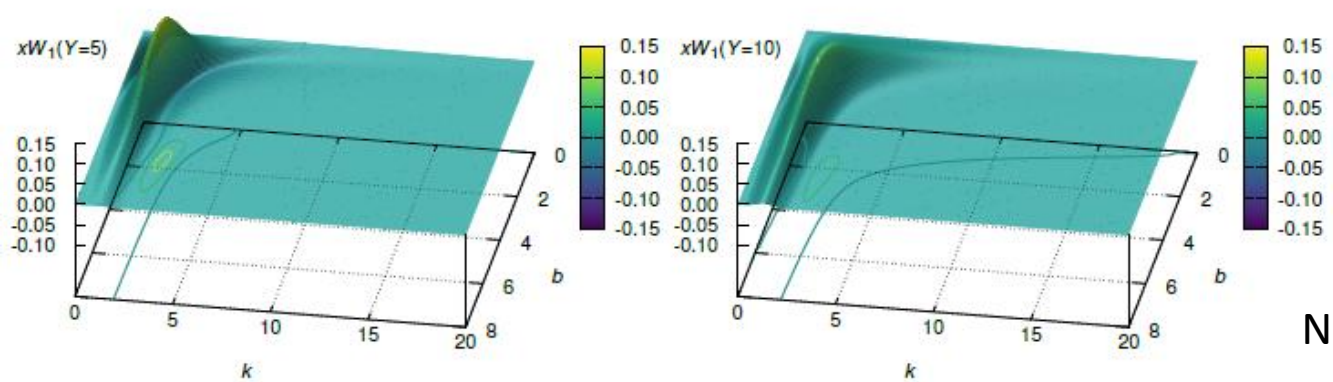
$$d^2(\vec{x},\vec{y}) = \frac{R^2(\vec{x}-\vec{y})^2}{(R^2+x^2)(R^2+y^2)}$$

W_0



Traveling wave (geometric scaling)

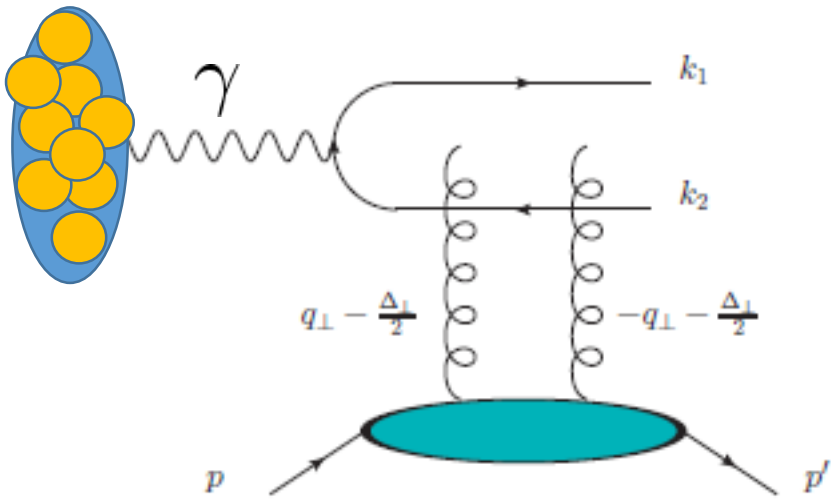
W_1



No traveling wave!

Measuring Wigner in ultra-peripheral pA collisions

Hagiwara, YH, Pasechnik, Tasevsky, Teryaev, in progress



Q^2 preferably small YH, Xiao, Yuan (2016)

Use the Weiszacker-Williams photons with $Q^2 \approx 0$ in UPC!

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega \frac{dN}{d\omega} \frac{N_c \alpha_{em} (2\pi)^4}{P_\perp^2} \sum_f e_f^2 2z(1-z)(z^2 + (1-z)^2) \left(A^2 + 2\cos 2(\phi_P - \phi_\Delta) AB\right)$$

photon flux $\propto Z^2$

$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp} \frac{\partial}{\partial P_\perp} A(P_\perp, \Delta_\perp).$$

$$S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2} \int^{P_\perp^2} \frac{dP_\perp'^2}{P_\perp'^2} B(P_\perp', \Delta_\perp)$$

Gluon orbital angular momentum

Challenge: Can we measure OAM?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

OAM parton distribution

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$L^{q,g}(x) = 2\int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Look at the component $W_g(x) = i\frac{S^+}{P^+}\epsilon^{ij} q_\perp^i \Delta_\perp^j f(x, q_\perp) + \cdots$

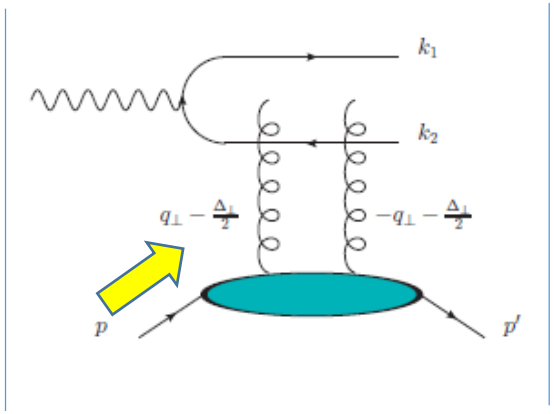
Then $L_g(x) = 2\int d^2q_\perp q_\perp^2 f(x, q_\perp)$

Find a process sensitive to $f(x, q_\perp)$

Longitudinal single spin asymmetry in diffractive dijet production

YH, Nakagawa, Yuan, Zhao (2016)
Yong Zhao, talk on Thursday

$d\Delta\sigma \sim$



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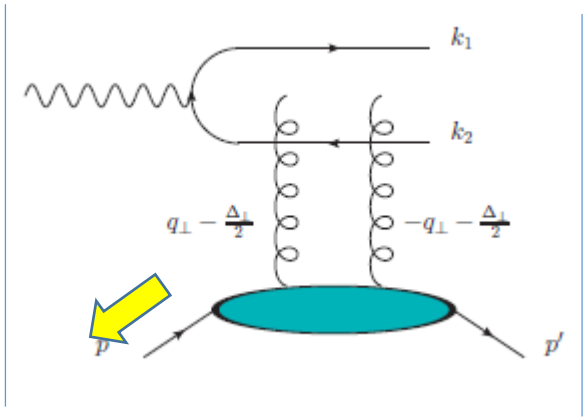

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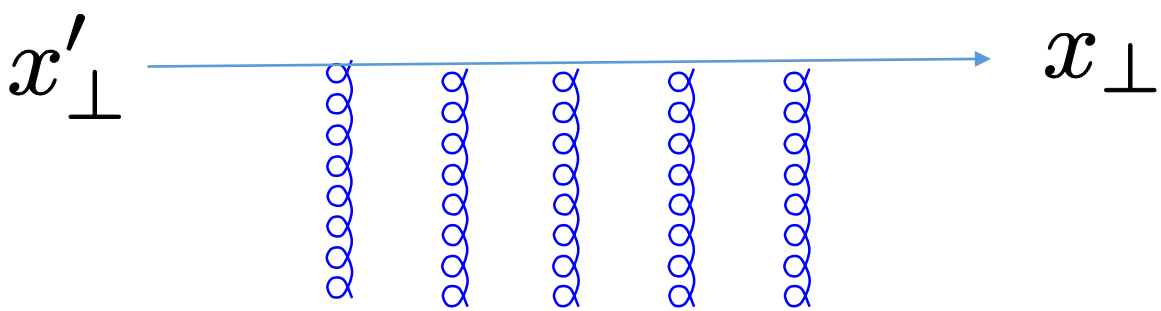
Vanishes in the eikonal approximation. Go to next-to-eikonal.

cf. Altinoluk, Armeso, Beuf, Martinez, Salgado (2014)



The asymmetry

$$\frac{d\Delta\sigma}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} \propto \frac{x(1-2z)\sin\phi_{P\Delta} P_\perp \Delta_\perp}{z(1-z)Q^2(P_\perp^4 \text{ or } Q^4)} \times \int d^2q'_\perp q'^2_\perp O(q'_\perp) \overbrace{\underbrace{\int d^2q_\perp q_\perp^2 (f+g)}_{\frac{1}{2}L_g(x)}}^{-\Delta G(x)}$$

Odderon!

Conjecture : $L_g(x) \approx -2\Delta G(x)$ at small-x

Conclusions

- Gluon Wigner distribution in QCD measurable in diffractive dijet production in DIS and UPC
- Elliptic Wigner responsible for $\cos 2\phi$ correlations in dijet, v_2, DVCS,…
- OAM at small-x also accessible.

  Unexpected connection between $L_g(x)$ and $\Delta G(x)$
- What about quark Wigner? Bhattacharya, Metz, Zhou 1702.04387