

# *Jet Mass Dependent Fragmentation*

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# Message

- *It might be worthy  
not to neglect  
parton virtualities?*
- Suggestion  
*It might be more suitable to  
characterise JETs  
with their MASS  
*instead of thier  $P$  or  $E$**

# Conclusion

- Suggestion

Parametrise fragmentation functions as

$$D \left[ x = \frac{2 P_\mu^{jet} p_h^\mu}{M_{jet}^2}, Q^2 = M_{jet}^2 \right]$$

*Energy fraction the hadron takes away in the frame co-moving with the jet*

*Fragmentation scale: jet mass*

# *Outline*

- Fragmentation of *off-shell partons* into  
*high-mass jets* (new parametrisation of FFs)

$$D \left[ x = \frac{2 P_{\mu}^{jet} p_h^{\mu}}{M_{jet}^2}, Q^2 = M_{jet}^2 \right]$$

- *Statistical fragmentation model* (as non-perturbative input)  
*at starting scale*
- *Fits: mass dependence* of fragmentation functions
- *Off-shell scale evolution in Fat jets*

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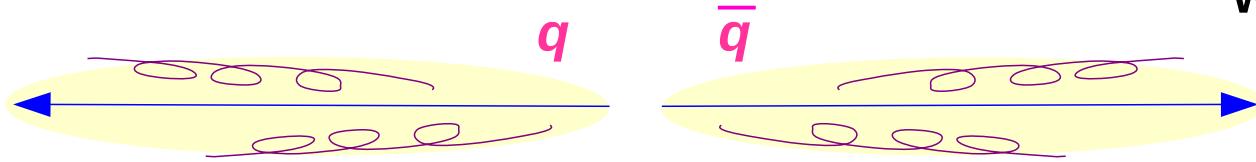
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- *Statistical fragmentation model* (as non-perturbative input)  
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**Ideal world:**

**e<sup>+</sup>e<sup>-</sup> annihilations in the factorized picture**

2 identical jets:



width:

$$\updownarrow \sim \sqrt{p_q^2} \approx 0 \ll M_{jet}$$

$$p_\mu^{q,\bar{q}} = (\sqrt{s}/2, 0, 0, \pm\sqrt{s}/2)$$

**Problem:**  $P^2 \sim 0$  quark produces a **heavy jet** of mass  $M \sim [0.1 - 0.5] \sqrt{s}$

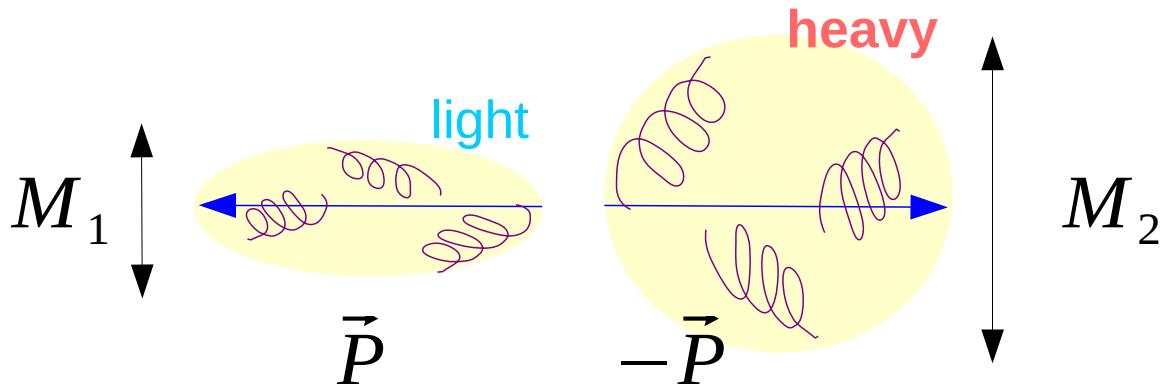
- **energy fraction** of the hadron takes away from the energy of the jet:
- **fragmentation scale:**

$$x = \frac{p_h^0}{\sqrt{s}/2}$$

$$Q \sim \sqrt{s}$$

**Real world:**

the 2 jets are *not identical*



**Energy-momentum conservation:**

$$P_1^u = (P^0, 0, 0, |\mathbf{P}|)$$

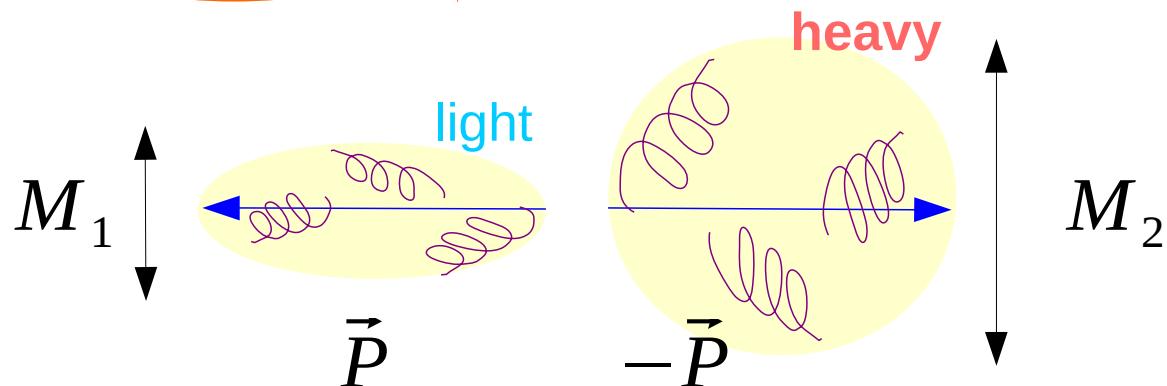
$$P_2^u = (\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|)$$

### Problems:

- **the energy of a jet**  $P^0 \neq (\sqrt{s}/2)$ , so  $x = \frac{p_h^0}{\sqrt{s}/2}$  is **no longer the energy fraction**, the hadron takes away from the energy of the jet.
- **fragmentation scale** is **no longer**  $\sqrt{s}/2$

**Real world:**

the 2 jets are not identical



**Energy-momentum conservation:**

$$P_1^\mu = (P^0, 0, 0, |\mathbf{P}|)$$

$$P_2^\mu = (\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|)$$

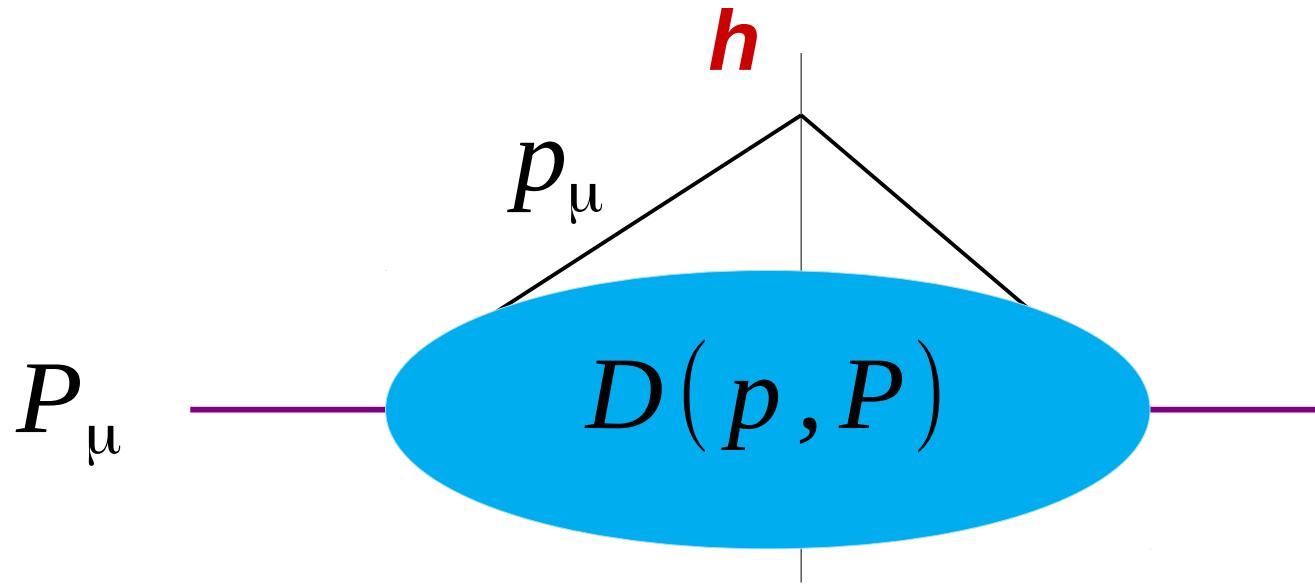
**We propose to use:**

- the **real energy fraction** the hadron takes away from the energy of the jet in the **frame co-moving** with jet:
- the **jet mass as fragmentation scale**:

$$\chi = \frac{2 p_h^\mu P_\mu^{\text{jet}}}{M_{\text{jet}}^2}$$

$$Q \sim M_{\text{jet}}$$

## Natural variables?



What invariants can we make out from  $P_\mu$  and  $p_\mu$  ?

- $p^2 \approx 0$
- $P^2 = M_{jet}^2$
- $(P - p)^2 = M_{jet}^2 - 2P_\mu p^\mu = M_{jet}^2 \left( 1 - \frac{2P_\mu p^\mu}{M_{jet}^2} \right) = M_{jet}^2 (1 - x)$

# *Outline*

- Fragmentation of *off-shell partons* into  
*high-mass jets* (new parametrisation of FFs)
- ***Statistical fragmentation model*** (as non-perturbative input)  
***at starting scale***
- *Fits: mass dependence* of fragmentation functions
- *Off-shell scale evolution in Fat jets*

## The non-perturbative input

To make the ***evolution*** of ***fragmentation functions***, we need a ***non-perturbative*** input: the form of ***FF*** at starting scale  $t_0$

$$D(x, t) = \int_x^1 \frac{dz}{z} f(z, t) D_0(x/z)$$

*non-perturbative*                    *non-perturbative*

For  $D_0(x)$ , let us a ***statistical fragmentation model***:

$$D_0(x) = \left(1 + \frac{q_0 - 1}{\tau_0} x\right)^{-1/(q_0 - 1)}$$

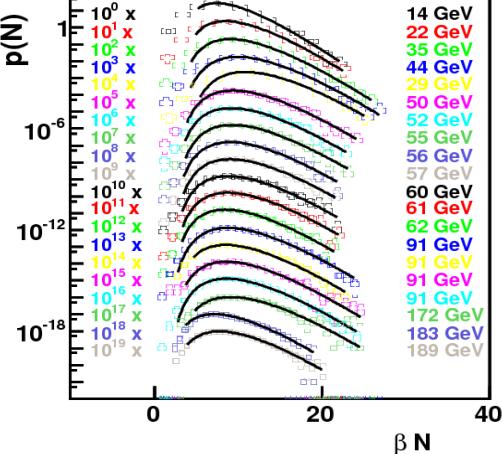
**Idea of our *statistical model* is to *combine***

***Negative Binomial hadron multiplicity distribution***

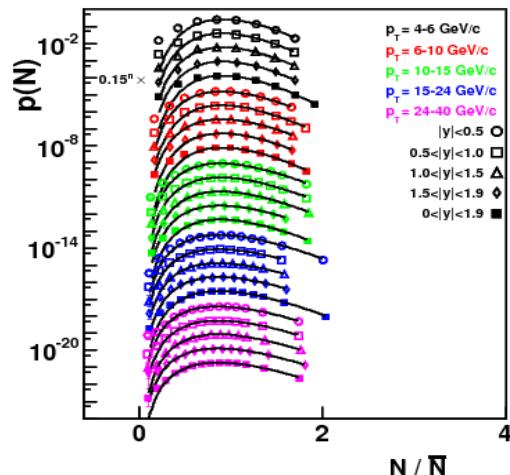
***Power-law hadron spectra***

# Particle Multiplicity fluctuates according to the Negative-binomial distribution

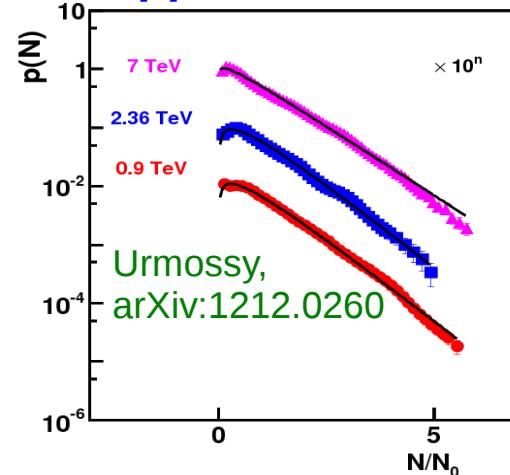
$e^-e^+ \rightarrow h^\pm$



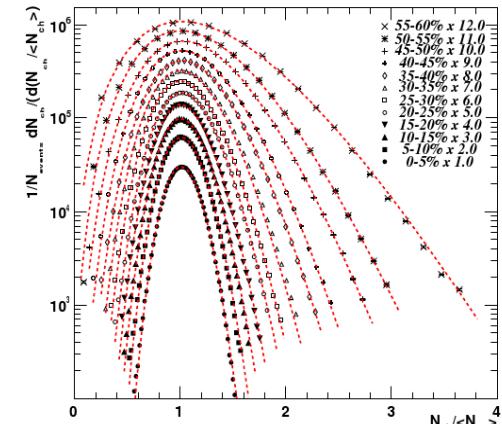
$pp \rightarrow \text{jets} @ 7 \text{ TeV}$



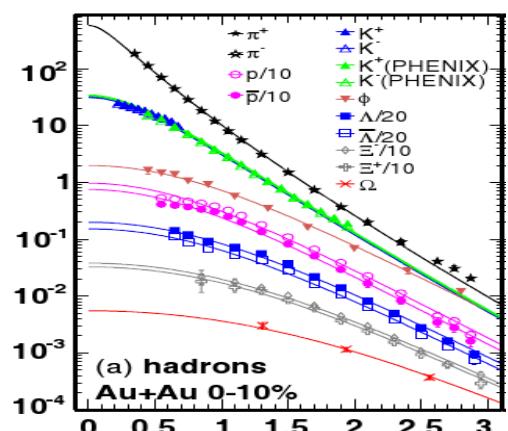
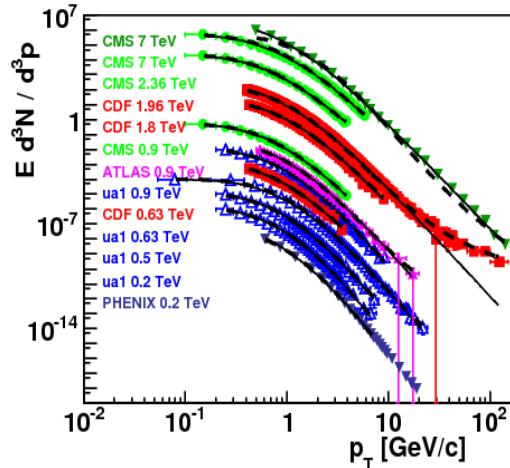
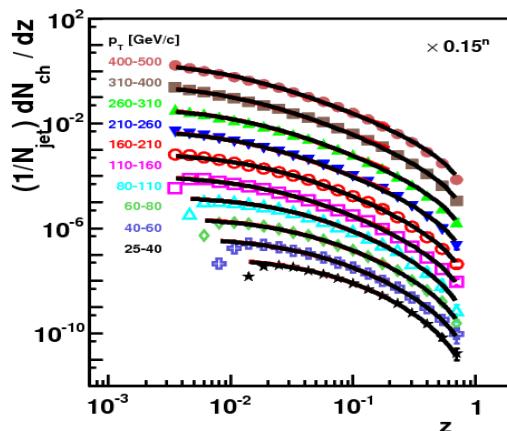
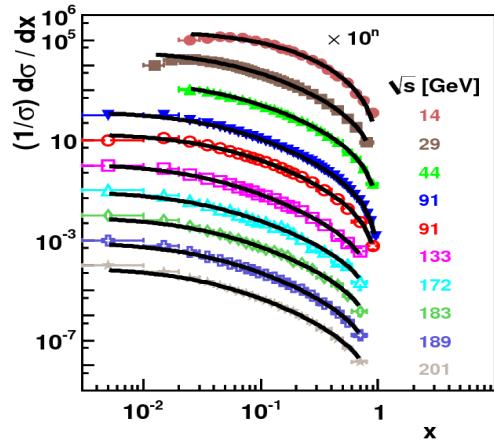
$pp \rightarrow h^\pm @ \text{LHC}$



$AuAu \rightarrow h^\pm @ \text{RHIC}$



## Power-law hadron spectra



Urmossy et.al., PLB,  
**701**: 111-116 (2011)

Urmossy et. al., PLB,  
**718**, 125-129, (2012)

Barnaföldi et.al, J. Phys.: Conf. Ser., **270**, 012008 (2011 )

J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010),

## Statistical jet-fragmentation

The cross-section of the creation of hadrons  $h_1, \dots, h_N$  in a jet of N hadrons

$$d\sigma^{h_1, \dots, h_N} = |M|^2 \delta^{(4)} \left( \sum_i p_{h_i}^\mu - P_{tot}^\mu \right) d\Omega_{h_1, \dots, h_N}$$

If  $|M| \approx \text{constans}$ , we arrive at a *microcanonical ensemble*:

$$d\sigma^{h_1, \dots, h_n} \sim \delta \left( \sum_i p_{h_i}^\mu - P_{tot}^\mu \right) d\Omega_{h_1, \dots, h_n} \propto (P_\mu P^\mu)^{n-2} = M^{2n-4}$$

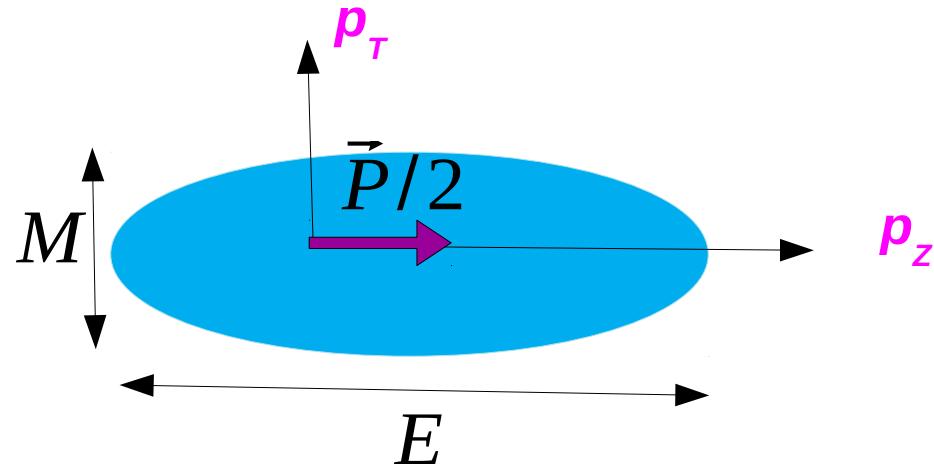
Thus, the hadron distribution in a jet of  $n$  hadron is

$$p^0 \frac{d\sigma}{d^3 p} \stackrel{n=fix}{\propto} \frac{\Omega_{n-1} (P_\mu - p_\mu)}{\Omega_n (P_\mu)} \propto (1-x)^{n-3}, \quad x = \frac{P_\mu p^\mu}{M^2/2}$$

Energy of the hadron  
in the co-moving frame

# Statistical ~ we **only** focus on the *phasespace*

The haron distribution in a jet of  $n$  hadron with total momentum  $\vec{P}$



$$p^0 \frac{d\sigma}{d^3 p} {}^{n=fix} \propto (1-x)^{n-3}, \quad x = \frac{P_\mu p^\mu}{M^2/2}$$

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

## Problems

- Averaging over *multiplicity fluctuations*

$$p^0 \frac{d\sigma}{d^3 p} = A \left[ 1 + \frac{q-1}{\tau} x \right]^{-1/(q-1)}$$

### Refs.:

Urmossy et.al., *PLB*, **701**: 111-116 (2011)

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

## Interpretation of $q$ and $\tau$

$q$  measures '*deviation*' from the *exponential* distribution

$$\left[1 + \frac{q-1}{\tau} x\right]^{-1/(q-1)} \rightarrow \exp\{-x/\tau\}$$

## Equipartition

$$\langle p^0 \rangle = d\tau \frac{M/2}{1 - (d+2)(q-1)} \rightarrow d\tau(M/2) \quad (\text{if } q \rightarrow 1, )$$

$d=2$  is the effective dimension of  $\frac{d^3 p}{p^0}$

***Scale evolution***

## The $\Phi^3$ theory case

The evolved fragmentation function doesn't preserve its shape

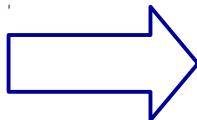
$$D(x, t) = \int_x^1 \frac{dz}{z} f(z, t) \left(1 + \frac{q_0 - 1}{\tau_0} \frac{x}{z}\right)^{1/(q_0 - 1)} \neq \left(1 + \frac{q(t) - 1}{\tau(t)} x\right)^{1/(q(t) - 1)}$$

Let it be approx equal!

Let us prescribe the approximations:

$$\begin{aligned} \int D_{apx}(x, t) &= \int D(x, t) \\ \int x D_{apx}(x, t) &= \int x D(x, t) = 1 \\ \int x^2 D_{apx}(x, t) &= \int x^2 D(x, t) \end{aligned}$$

(by definition)

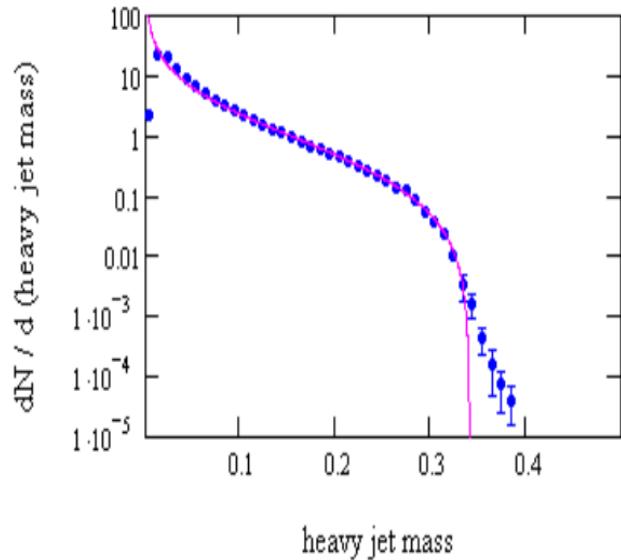


$$\begin{aligned} q(t) &= \frac{\alpha_1(t/t_0)^{a1} - \alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1} - \alpha_4(t/t_0)^{-a2}} \\ \tau(t) &= \frac{\tau_0}{\alpha_4(t/t_0)^{-a2} - \alpha_3(t/t_0)^{a1}} \\ a_1 &= \tilde{P}(1)/\beta_0, \quad a_2 = \tilde{P}(3)/\beta_0 \end{aligned}$$

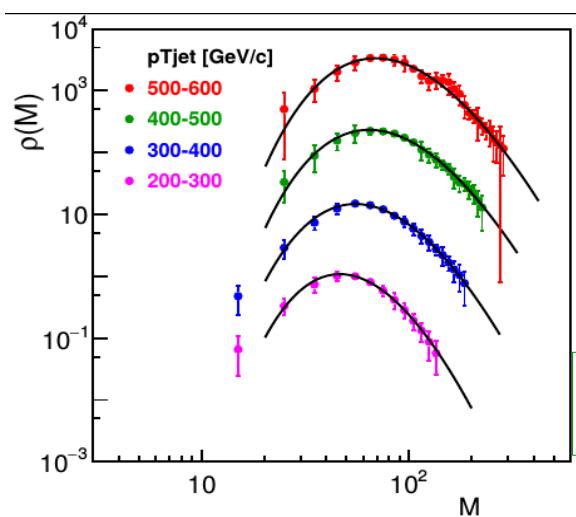
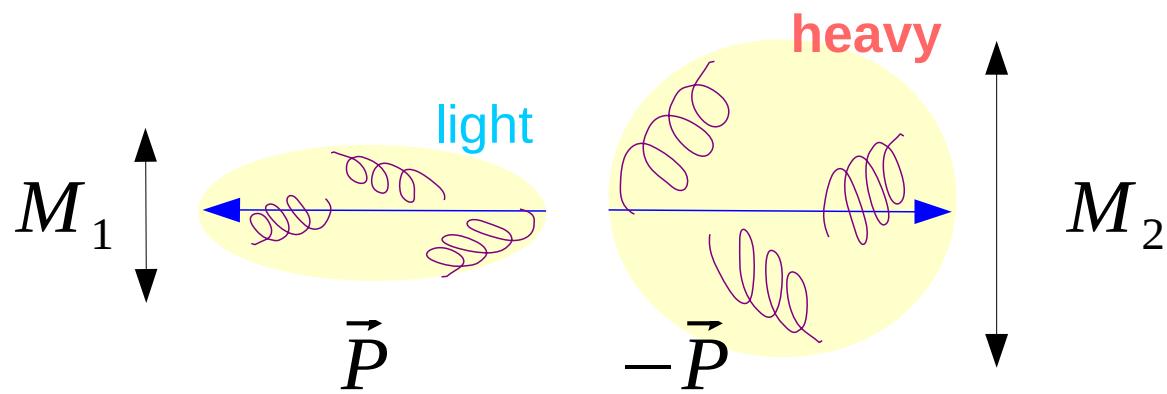
# *Fits*

- *mass dependence* of fragmentation functions

## Jet mass fluctuations

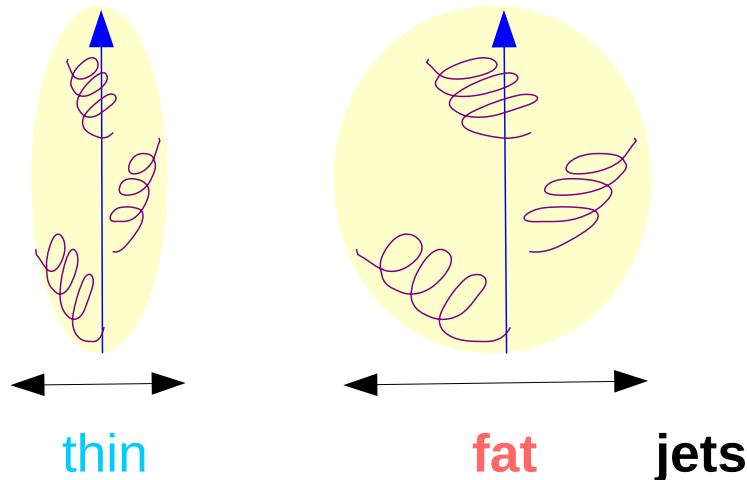


$e^+e^- \rightarrow 2 \text{ jet}$ : both  $E$  and  $\vec{P}$  of the jets fluctuate

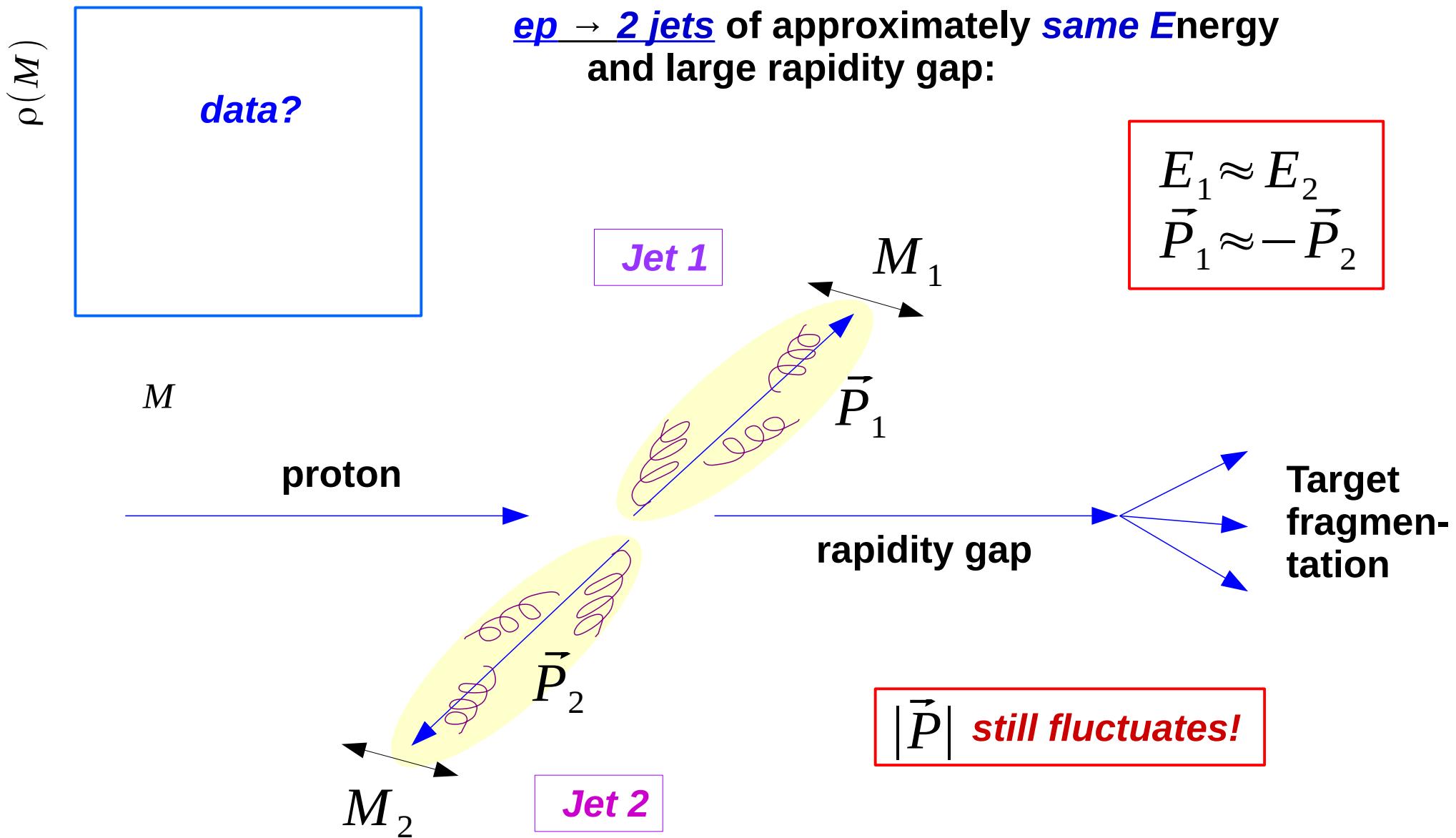


$pp$  collisions: jet  $\vec{P}$  is measured,  $E, M$  fluctuates

K.U, Z. Xu,  
arXiv:1605.06876



## Problems



## What dataset to analyse?

We have a haron distribution, which depends on  $x = \frac{P_\mu}{M^2} p^\mu / 2$

**but,** in case of available data, the *jet E* or *P fluctuate*:

- pp collisions:  $\vec{P}$  is measured,  $E$  fluctuates
- $e^+e^- \rightarrow 2\text{ jet}$ : both  $E$  and  $\vec{P}$  of the jets fluctuate
- $e^+p \rightarrow 2\text{ jet}$ :  $\vec{P}$  of the jets fluctuate

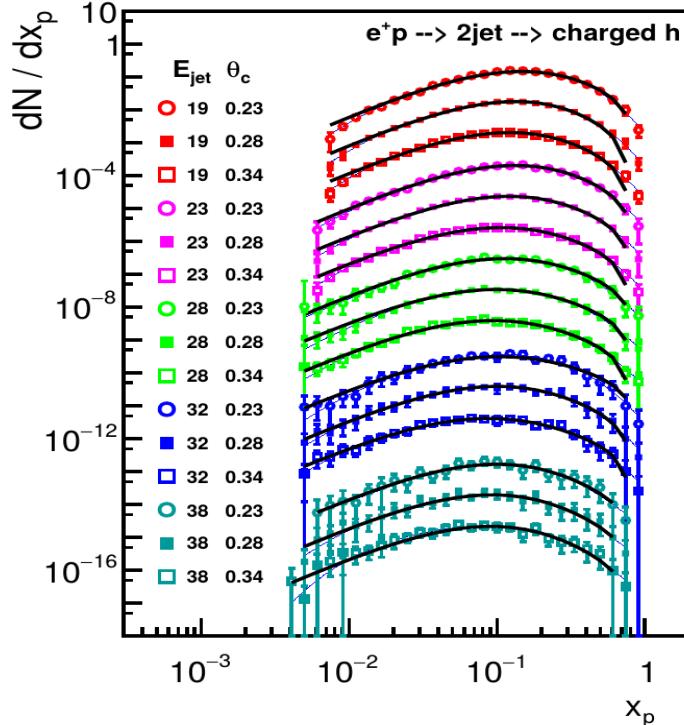
**So,** we *fit* a *characteristic/average jet mass* and extract the scale dependence of the parameters of the model

-  $p p$  collisions:  $\vec{P}$  is measured,  $E$  fluctuates

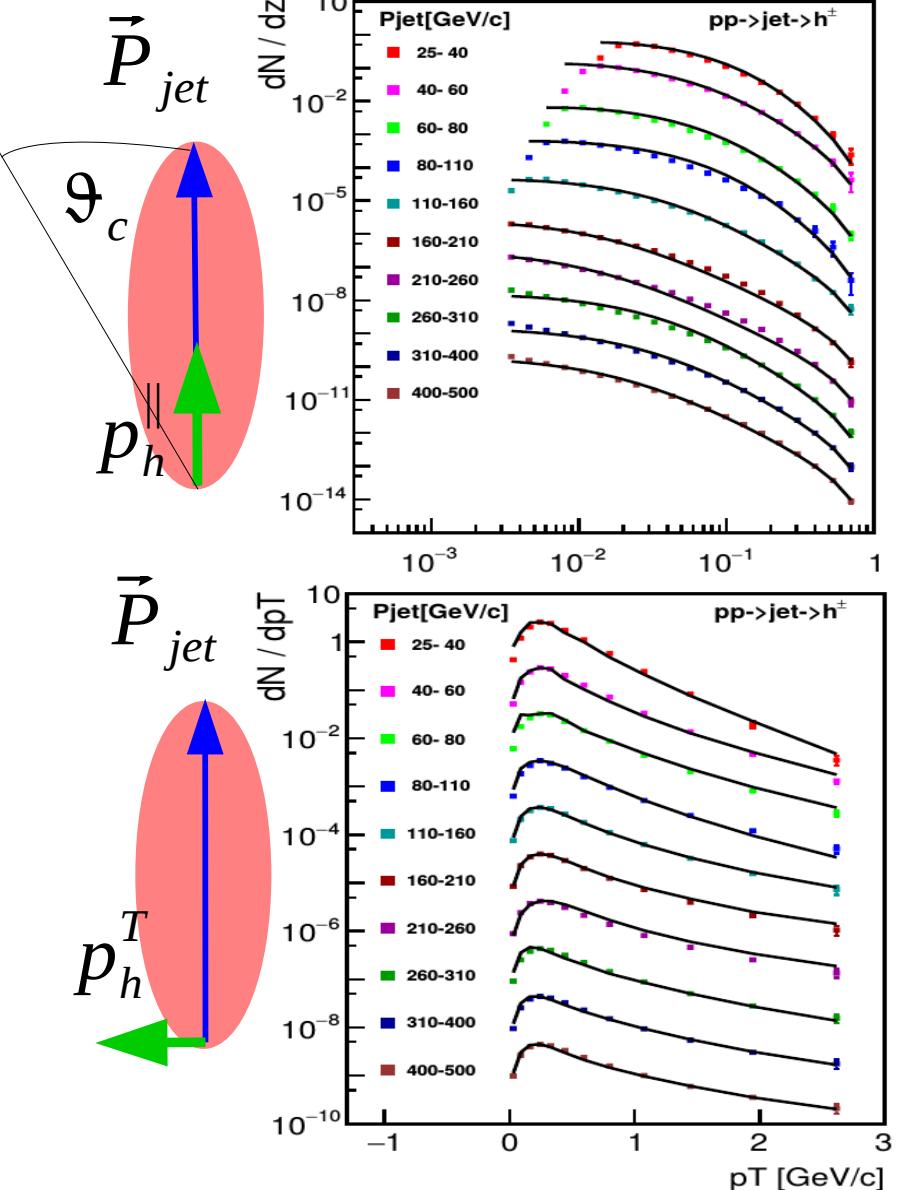
-  $e^+ p \rightarrow 2 \text{ jet}$ :  $\vec{P}$  of the jets fluctuate

**So,** we *fit* a **characteristic/average jet mass** and extract the scale dependence of the parameters of the model

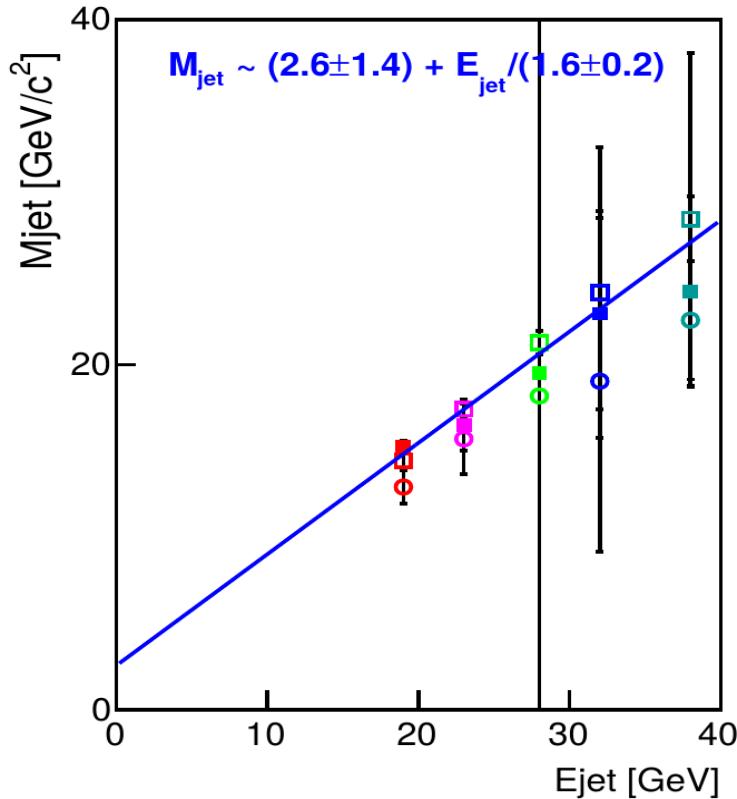
$e^+P \rightarrow 2 \text{ jets}$



$PP \rightarrow \text{jets}$

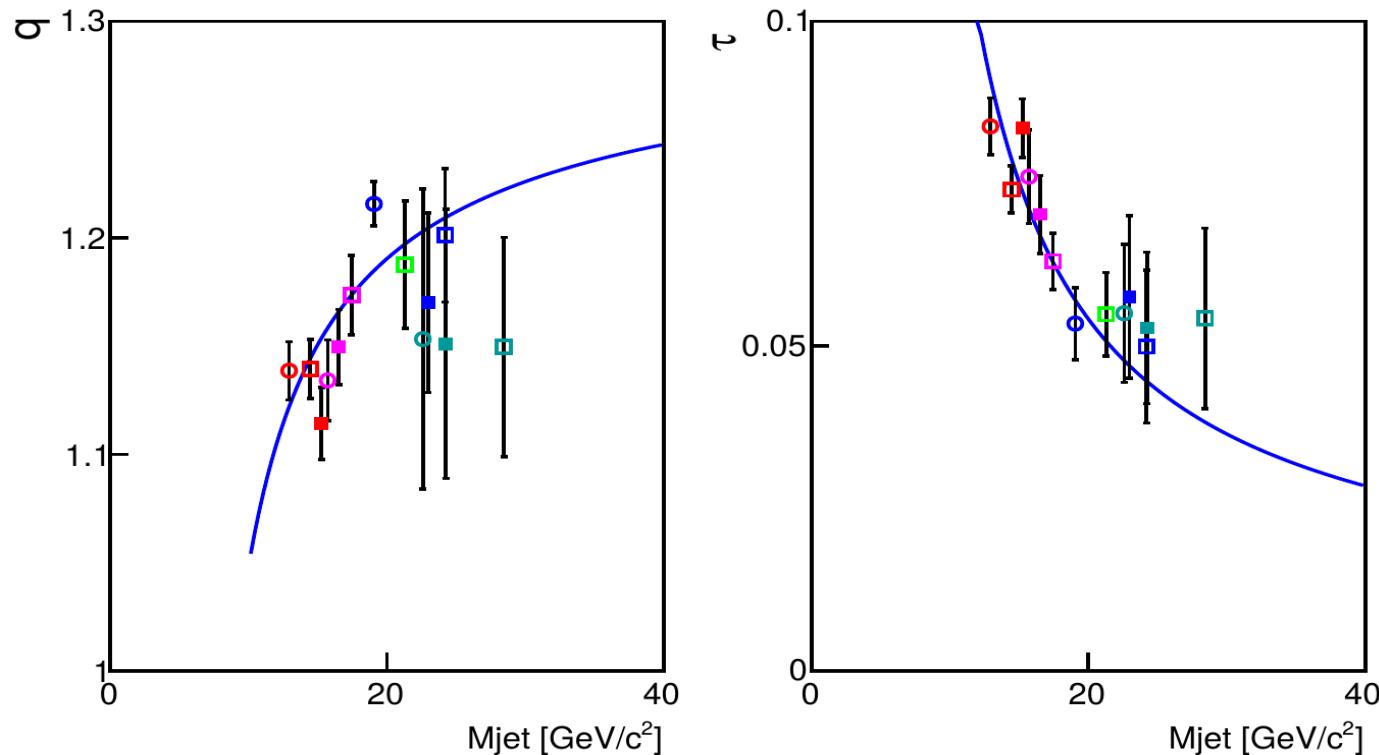


## Fitted average characteristic jet mass



$$\text{fitted } \langle M_{JET} \rangle = M_0 + E_{JET}/E_0$$

## Scale evolution of the fit parameters



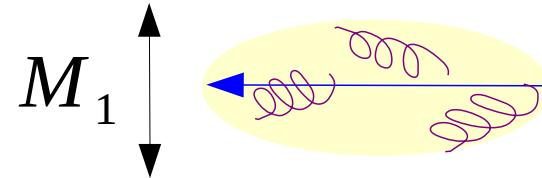
$$q(t) = \frac{\alpha_1(t/t_0)^{a1} - \alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1} - \alpha_4(t/t_0)^{-a2}}$$

$$\tau(t) = \frac{\tau_0}{\alpha_4(t/t_0)^{-a2} - \alpha_3(t/t_0)^{a1}}$$

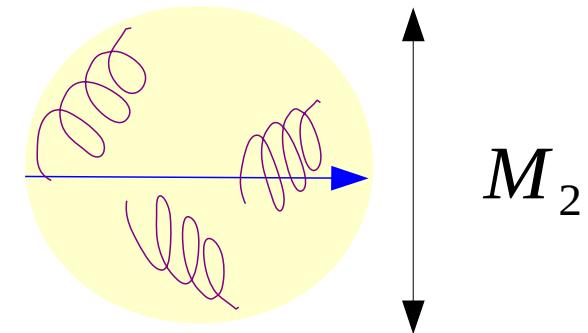
$$t = \ln \left( \frac{M_{jet}^2}{\Lambda^2} \right)$$

## Interpretation of the results

Inside *light jets*



Inside *heavy jets*



The *fragmentation function*:

$$D(x) \approx \exp\{-x/\tau\}$$

The *multiplicity distribution*:

$$P(n) \approx \frac{(1/\tau)^n}{n!} e^{-1/\tau}$$

$$D(x) \approx \left(1 + \frac{q-1}{\tau} x\right)^{-1/(q-1)}$$

$$P(n) \approx \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

$$\begin{aligned}\tilde{p} &= (q-1)/(\tau+q-1) \\ r &= 1/(q-1)-3\end{aligned}$$

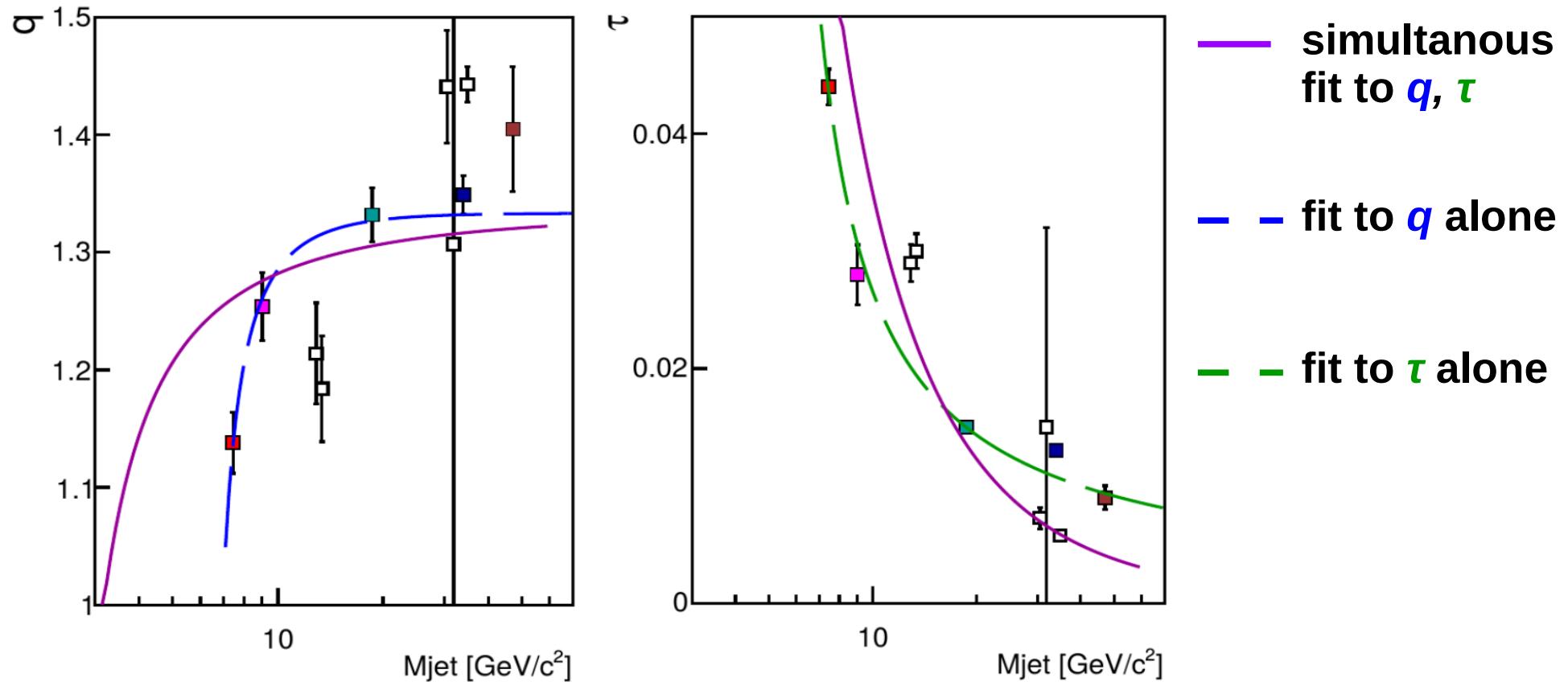
## Interpretation of the results

Evolution of the mean *multiplicity* and its *dispersion*:

$$\langle n \rangle = \frac{4 - 3q_0}{\tau_0} (t/t_0)^{-a_2} \sim \ln^a(M_{jet})$$

$$\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle \left[ \frac{3 - 2q_0}{\tau_0} (t/t_0)^{a_1} + 1 - \langle n \rangle \right]$$

## Scale evolution of the fit parameters



Why does it look so messy? *Jet mass fluctuations spoil things?*

## Mass-averaged fits are better

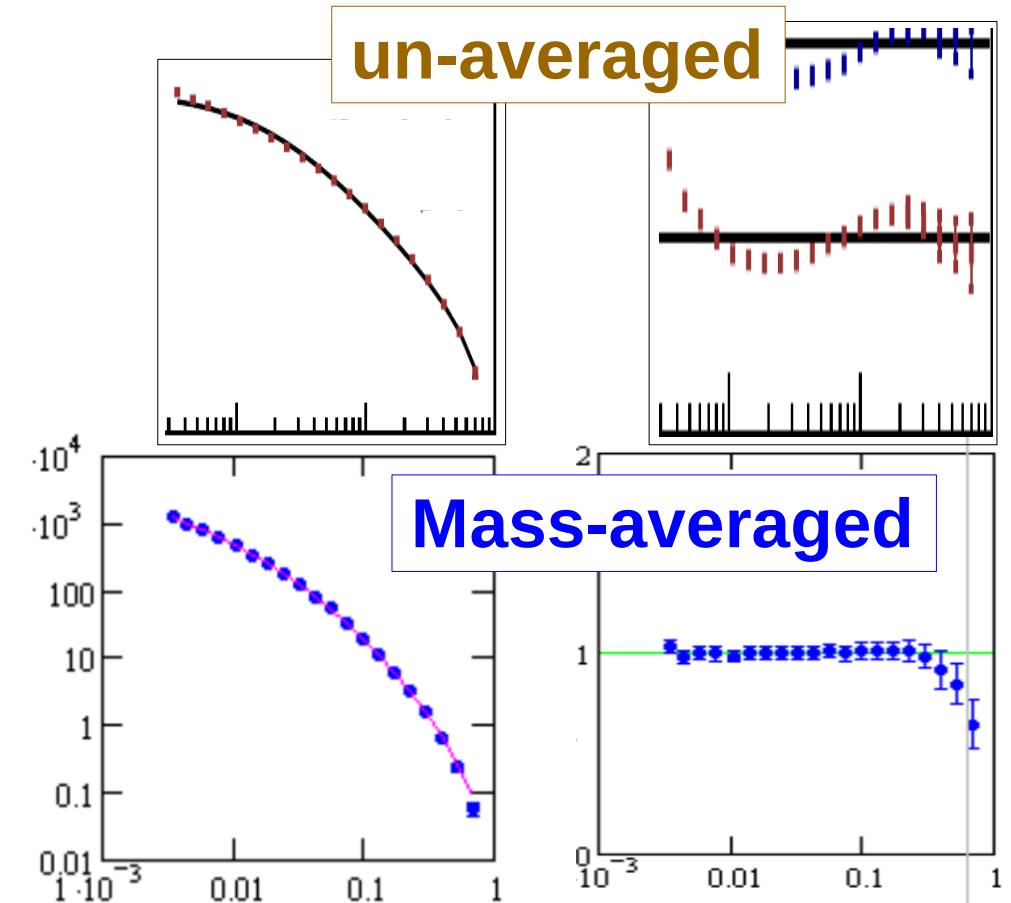
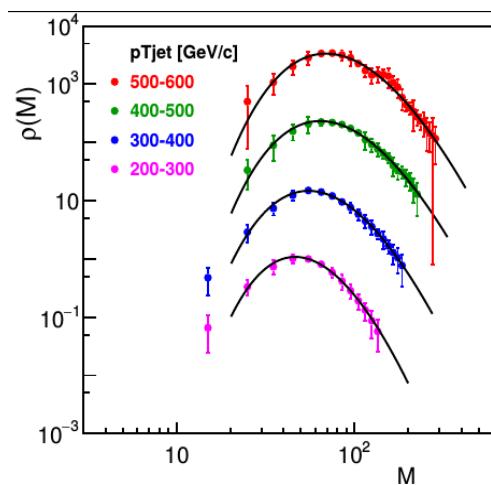
The *fragmentation function* is jet mass dependent

$$D(x, t) = \int_x^1 \frac{dz}{z} f(z, t) D_0(x/z)$$

$t = \ln\left(\frac{M_{jet}^2}{\Lambda^2}\right)$

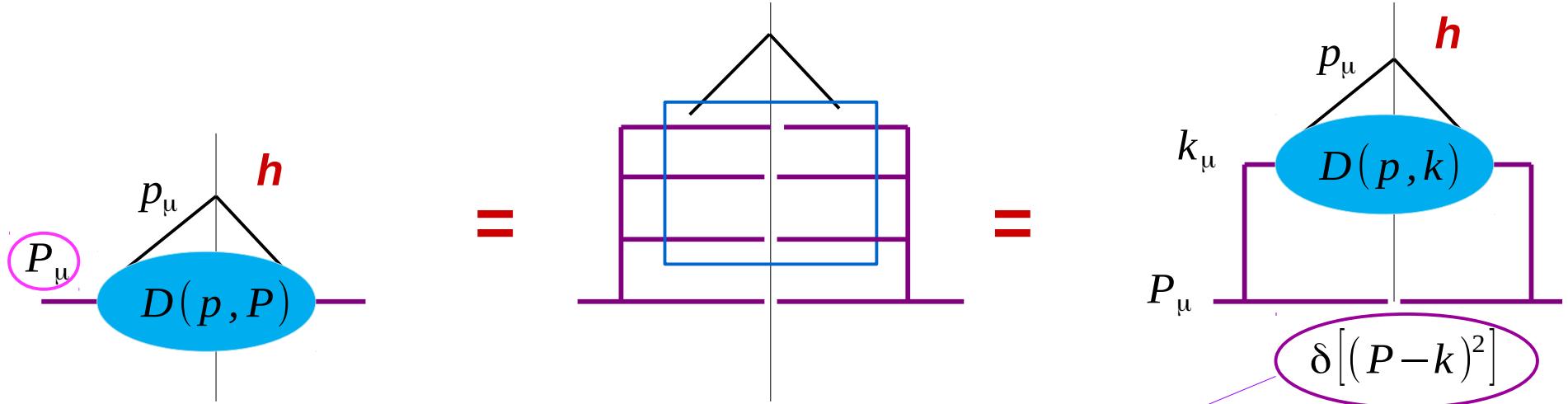
The *jet mass* fluctuates as

$$\rho(M_{jet}) \sim \ln^b(M_{jet}/M_0)/M_{jet}^c$$



*Fat jets*  
*need*  
*Off-shell*  
***scale evolution?***

# Resumming the ladder



Thus, the equation for  $D$  is

$$D(p, P) \sim \int d^D k \frac{g^2(k^2)}{k^4} \delta[(P-k)^2] D(p, k)$$

**Light cone coordinates:**  $k_u = (k_+, k_-, \mathbf{k}_T)$   $k_{\pm} = (k_0 \pm k_z)/\sqrt{2}$

$$P_u = (M/\sqrt{2}, M/\sqrt{2}, \mathbf{0})$$

$$p_u = (\sqrt{2} p, 0, \mathbf{0})$$

with  $x = p_+/P_+$ ,  $z = k_+/P_+$

The equation for  $D$  is

$$D(x, M^2) \sim \int_x^1 \frac{dz}{z} \Pi(z) \int_0^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(zM^2\alpha) D\left(\frac{x}{z} \frac{1-z(1-\alpha)}{\alpha}, zM^2\alpha\right)$$

The transverse momentum dependence is in

$$\alpha = 1 - \frac{\mathbf{k}_T^2/M^2}{z(1-z)}$$

Nearly collinear radiations:

$$\mathbf{k}_T^2 \approx 0, \rightarrow \alpha = 1$$

$$D(x, M^2) \approx \int_x^1 \frac{dz}{z} \Pi(z) \int_0^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(zM^2\alpha) D\left(\frac{x}{z}, zM^2\alpha\right)$$

Factorized form in Mellin space:

$$\tilde{D}(\omega, M^2) \approx \Pi(\omega + \partial/\partial Y) \int_a^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(M^2 \alpha) D(\omega, M^2 \alpha)$$

regularisation

DGALP-like equation:

$$\frac{\partial}{\partial a} \tilde{D}(\omega, M^2) \approx -\Pi(\omega + \partial/\partial Y) \frac{1-a}{a^2} g^2(M^2 a) D(\omega, M^2 a)$$

Solution:

$$\tilde{D}(\omega, M^2) \approx D(\omega, 0) \exp \left\{ \Pi(\omega + \partial/\partial Y) \int_a^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(M^2 \alpha) \right\}$$

usual  
splitting  
function

transverse  
phasespace

If it was  $\int \frac{d\alpha}{\alpha} g^2 \rightarrow \ln \ln M^2$

# Message

- *It might be worthy  
not to neglect  
parton virtualities?*
- Suggestion  
*It might be more suitable to  
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with their MASS  
*instead of thier P or E**

# Conclusion

- Suggestion

Parametrise fragmentation functions as

$$D \left[ x = \frac{2 P_\mu^{jet} p_h^\mu}{M_{jet}^2}, Q^2 = M_{jet}^2 \right]$$

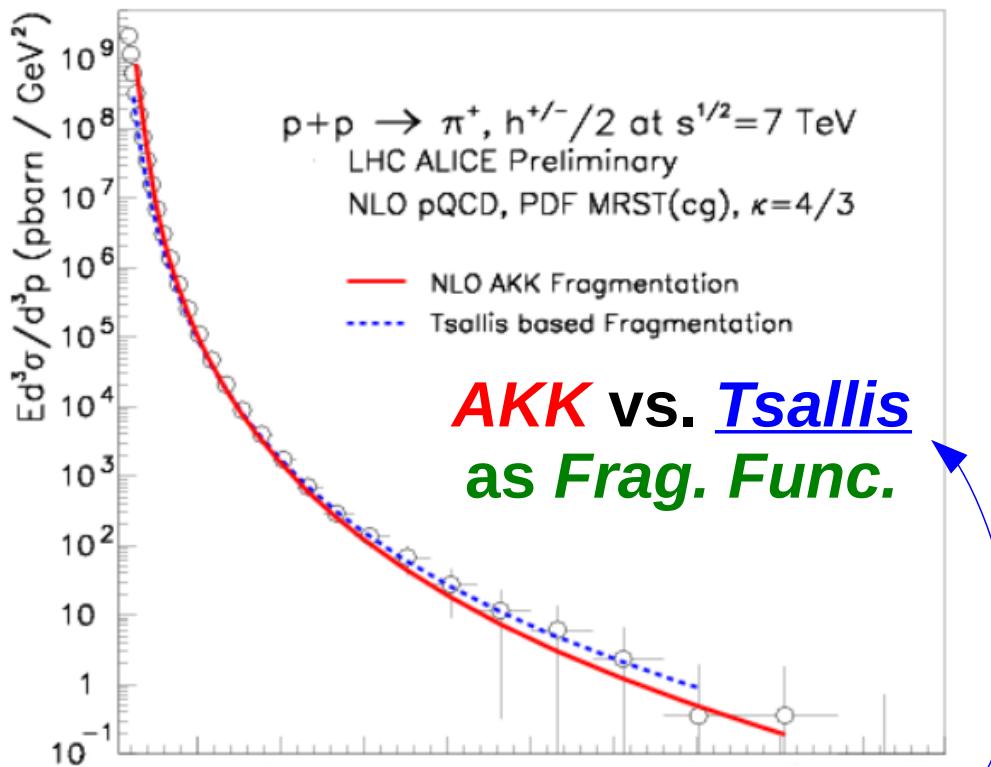
*Energy fraction the hadron takes away in the frame co-moving with the jet*

*Fragmentation scale: jet mass*

# *Back-up*

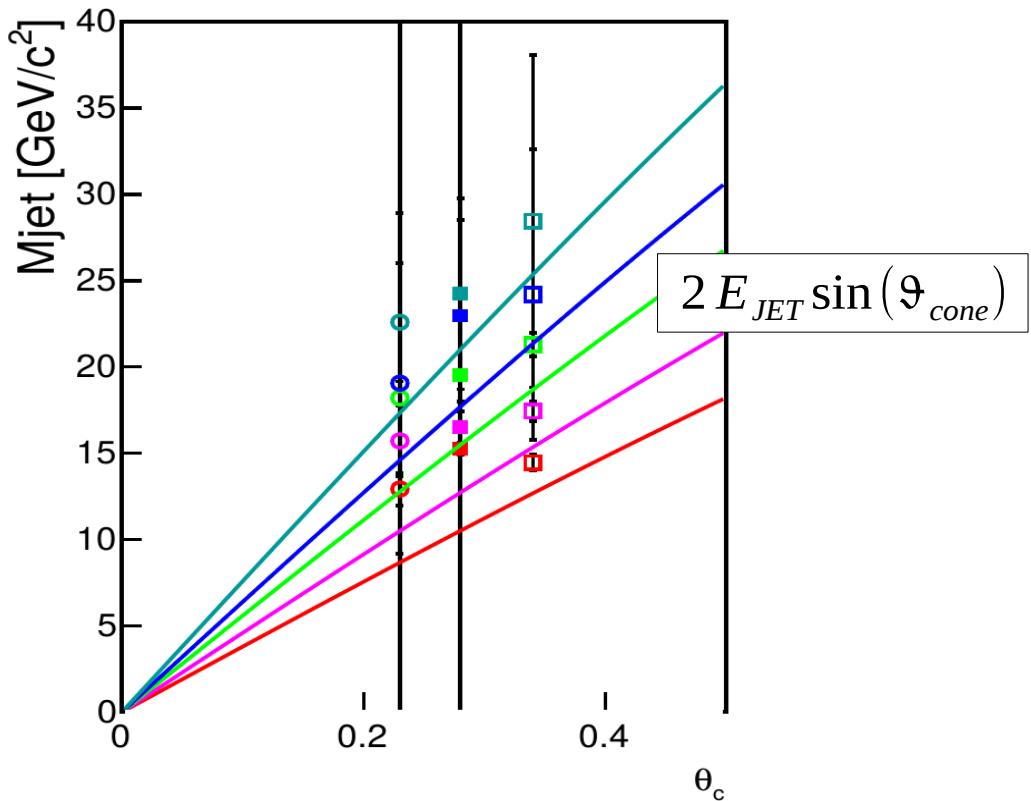
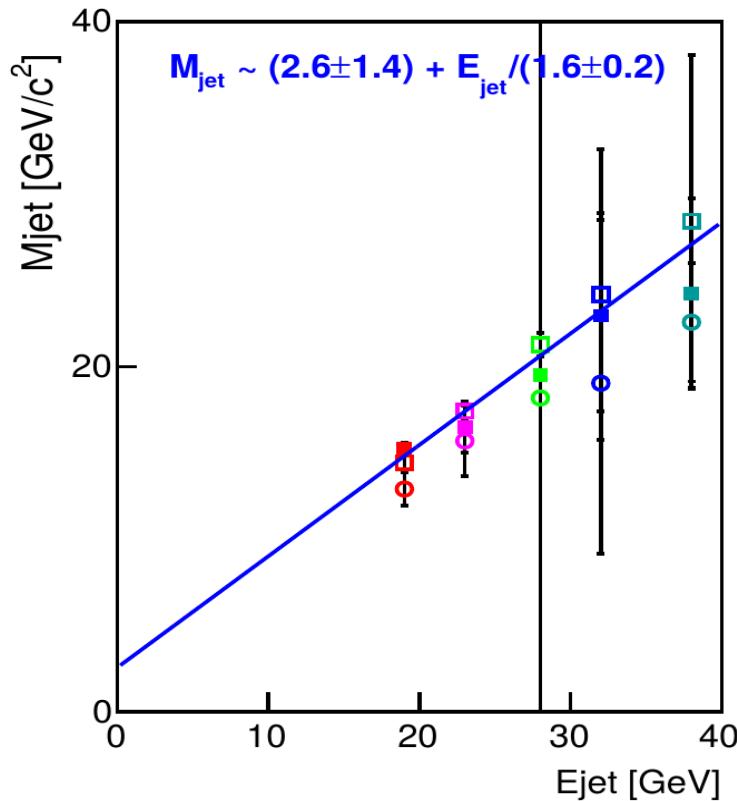
## *Application in a pQCD calculation*

$\pi^+$  spectrum in  $pp \rightarrow \pi^\pm X$  @  $\sqrt{s}=7$  TeV (NLO pQCD)



$$D_{p_i}^{\pi^+}(z) \sim (1 + (q_i - 1)z/T_i)^{-1/(q_i - 1)}$$

## Fitted average characteristic jet mass



fitted  $\langle M_{JET} \rangle = M_0 + E_{JET}/E_0$

Fitted average jet mass is of the order of that used in DGLAP calcs.

$$\langle M_{JET} \rangle \sim 2 E_{JET} \sin(\theta_{cone})$$

## The $\Phi^3$ theory case

**Resummation of branchings with DGLAP**

$$\frac{d}{dt} D(x,t) = g^2 \int_x^1 \frac{dz}{z} P(z) D(x/z, t), \quad t = \ln(Q^2/\Lambda^2), \quad g^2 = 1/(\beta_0 t)$$

with **LO splitting function**:  $P(z) = z(1-z) - \frac{1}{12}\delta(1-z)$

Let the non-perturbative input at starting scale  $Q_0$  be:  $D_0(x) = \left(1 + \frac{q_0-1}{\tau_0}x\right)^{-1/(q_0-1)}$

The full solution is

$$D(x,t) = \int_x^1 \frac{dz}{z} f(z,t) D_0(x/z)$$

with  $f(x) \sim \delta(1-x) + \sum_{k=1}^{\infty} \frac{b^k}{k!(k-1)!} \sum_{j=0}^{k-1} \frac{(k-1+j)!}{j!(k-1-j)!} x \ln^{k-1-j} \left[ \frac{1}{x} \right] [(-1)^j + (-1)^k x]$

**D does not preserve its shape:**

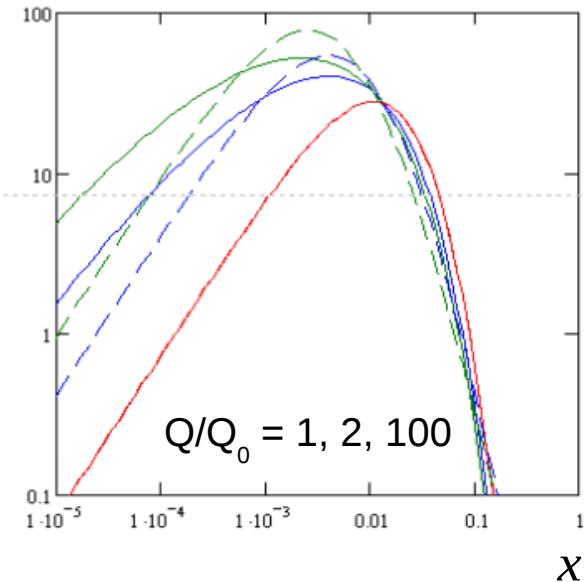
$$b = \beta_0^{-1} \ln(t/t_0)$$

$$\int_x^1 \frac{dz}{z} f(z,t) \left(1 + \frac{q_0-1}{\tau_0} \frac{x}{z}\right)^{1/(q_0-1)} \neq \left(1 + \frac{q(t)-1}{\tau(t)} x\right)^{1/(q(t)-1)}$$

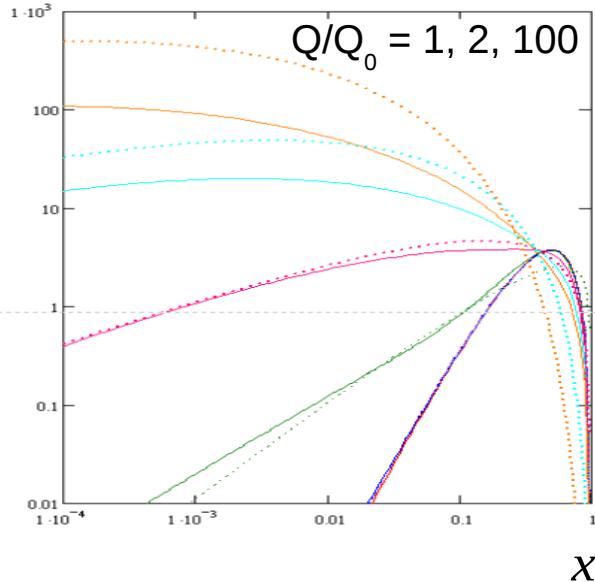
**It is only an approximation!**

## How good is the approximation?

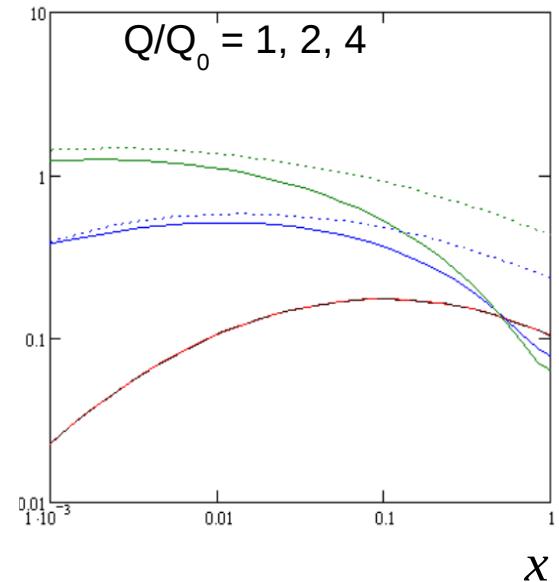
$$x \left(1 + \frac{q-1}{\tau} x\right)^{-1/(q-1)}$$



$$x^b \ln^a(1/x)$$



**Dist. Gauss in  $\ln(1/x)$**



Requirement:  
moments:  $\langle x^j \rangle$

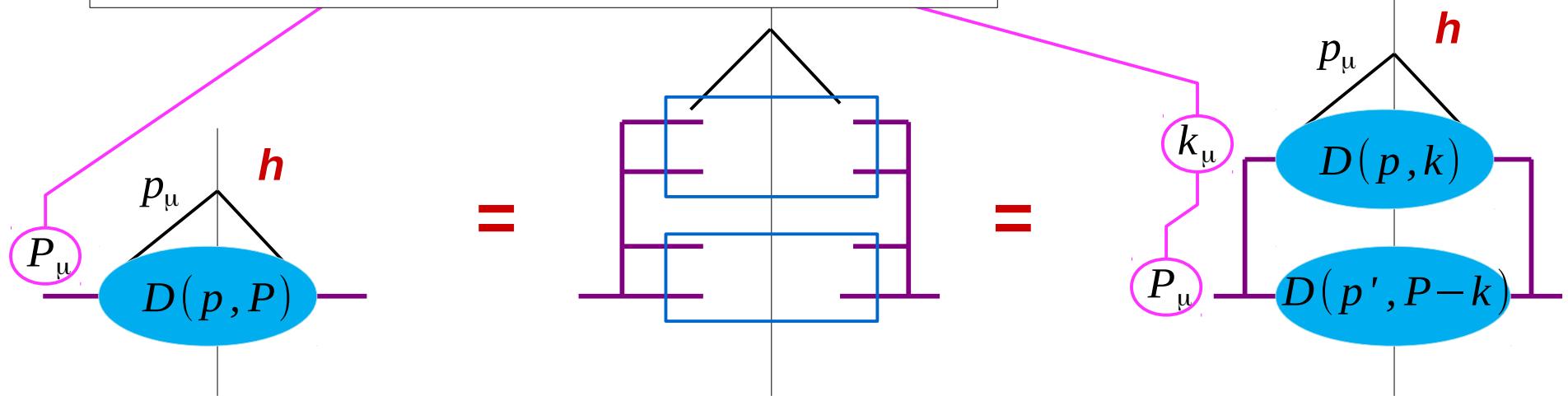
$$\langle \ln^j(1/x) \rangle$$

$$\langle \ln^j(1/x) \rangle$$

be equal in case of the shape preserving, approximate solution and the exact solution

## Off-shell scale evolution

Resumming the splittings in the  $\Phi^3$  theory



Thus, the equation for  $D$  is

$$D(p, P) \sim \int d^D k \frac{g^2(k^2) D(p, k)}{k^4 (P - k)^4} \int d^D p' D(p', P - k)$$

Let us parametrise  $D$  as

$$D(p, P) \sim P^4 \boxed{\rho(P^2)} \boxed{f(p, P)}$$

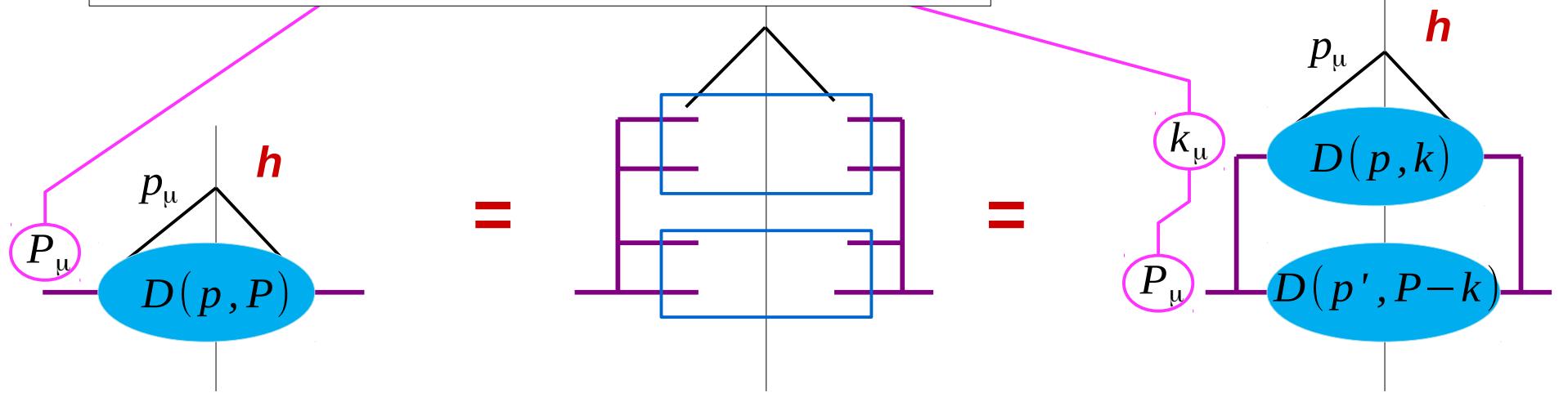
jet mass distribution

conditional probability of a **hadron** with  $p$  in a **jet** with  $P$

$$\int d^D p f(p, P) = 1$$

## Off-shell scale evolution

Resumming the splittings in the  $\Phi^3$  theory



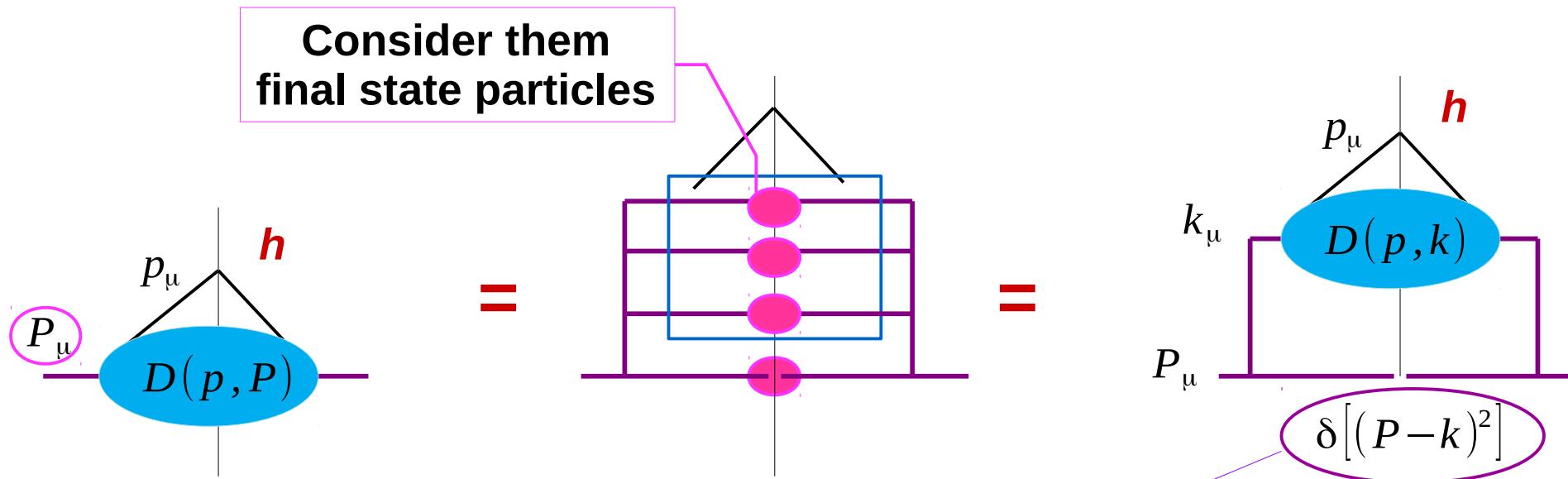
Thus, we obtain 2 equations for  $f$  and  $\rho$

$$D(p, P) \sim \int d^D k \frac{g^2(k^2) D(p, k)}{k^4} \rho[(P - k)^2]$$

$$\rho(P^2) \sim \int d^D k g^2(k^2) \rho(k^2) \rho[(P - k)^2]$$

**Non-linear :**

## Simplified version



Thus, the equation for  $D$  is

$$D(p, P) \sim \int d^D k \frac{g^2(k^2)}{k^4} \delta[(P-k)^2] D(p, k)$$

**Nearly collinear radiations:**  $k_T^2 \approx 0, \rightarrow \alpha = 1$

$$D(x, M^2) \approx \int_x^1 \frac{dz}{z} \Pi(z) \int_0^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(zM^2\alpha) D\left(\frac{x}{z}, zM^2\alpha\right)$$

We get rid of  $z$  dependence in  $g$  and  $D$  by a trick:

$$\xi = \ln(1/z), \quad Y = \ln M^2, \quad y = \ln \alpha$$

$$g(zM^2\alpha) = g(e^{-\xi+Y+y}) = e^{-\xi \partial/\partial Y} g(e^{Y+y}) = z^{\partial/\partial Y} g(M^2\alpha)$$

Now we have

$$D(x, M^2) \approx \int_x^1 \frac{dz}{z} \Pi(z) z^{\partial/\partial Y} \int_0^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(M^2\alpha) D\left(\frac{x}{z}, M^2\alpha\right)$$

This has a form of

$$D(x) \approx \int_x^1 \frac{dz}{z} f(z) D\left(\frac{x}{z}\right) \rightarrow \tilde{D}(\omega) \approx \tilde{f}(z) \tilde{D}(\omega), \quad \tilde{f}(\omega) = \int_0^1 x^{\omega-1} f(x)$$

**Solution:**

$$\tilde{D}(\omega, M^2) \approx D(\omega, 0) \exp \left\{ \Pi(\omega + \partial/\partial Y) \int_a^1 d\alpha \frac{1-\alpha}{\alpha^2} g^2(M^2 \alpha) \right\}$$

**Handling differential operators in the denominator:**

$$\frac{1}{\omega + 1 + \partial_Y} b(Y) = \int_0^\infty ds e^{-s(\omega+1+\partial_Y)} b(Y) = \int_0^\infty ds e^{-s(\omega+1)} b(Y-s)$$