First Monte Carlo Analysis of Fragmentation Functions

Nobuo Sato
University of Connecticut
25th International Workshop on Deep Inelastic Scattering,
Apr 4 2017

Why study fragmentation functions

- Tool to unveil nucleon structure
  - 1D tomography : PDFs, ΔPDFs
  - 3D tomography : TMDPDFs, TMDFFs

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  - 1D tomography : PDFs, ΔPDFs
  - 3D tomography : TMDPDFs, TMDFFs
- Characterization of formation of hadrons
  - Energy distribution of hadrons
  - Relative strength of FFs: favored vs unfavored
  - Interplay with parton shower $\rightarrow$ DGLAP, MLL, threshold resummation

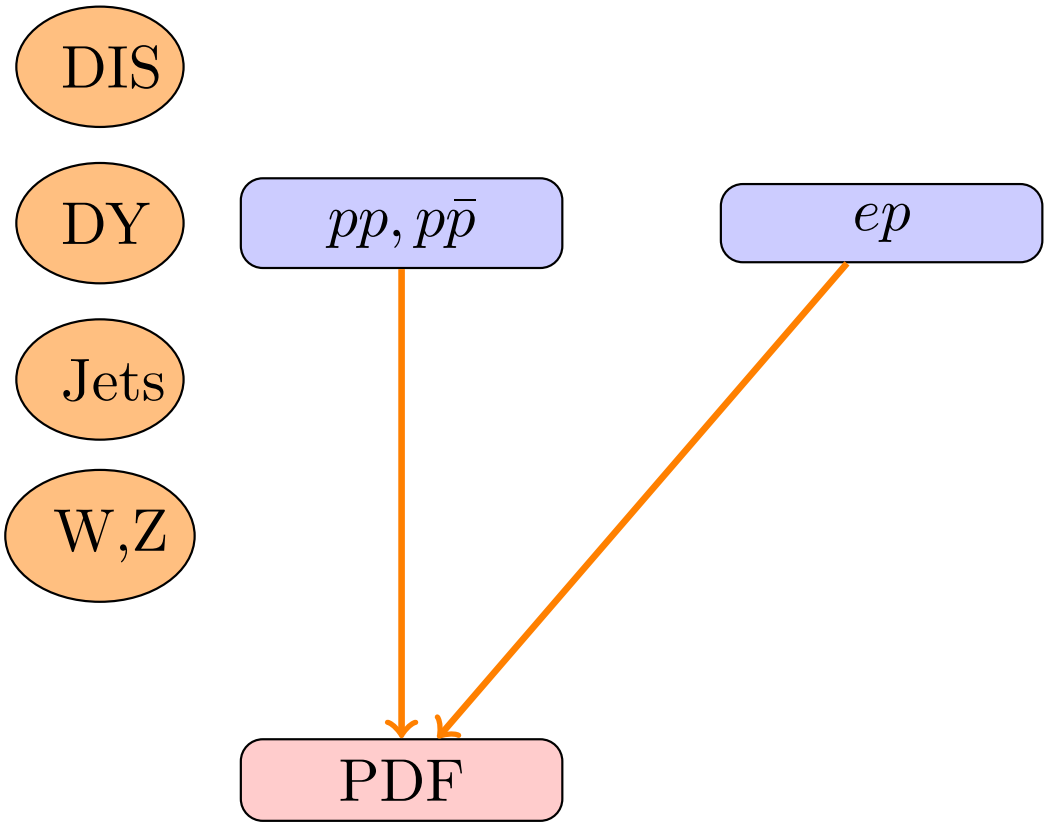
The JAM masterplan

Jefferson Lab Angular Momentum Collaboration

$pp, p\bar{p}$

ep

The JAM masterplan



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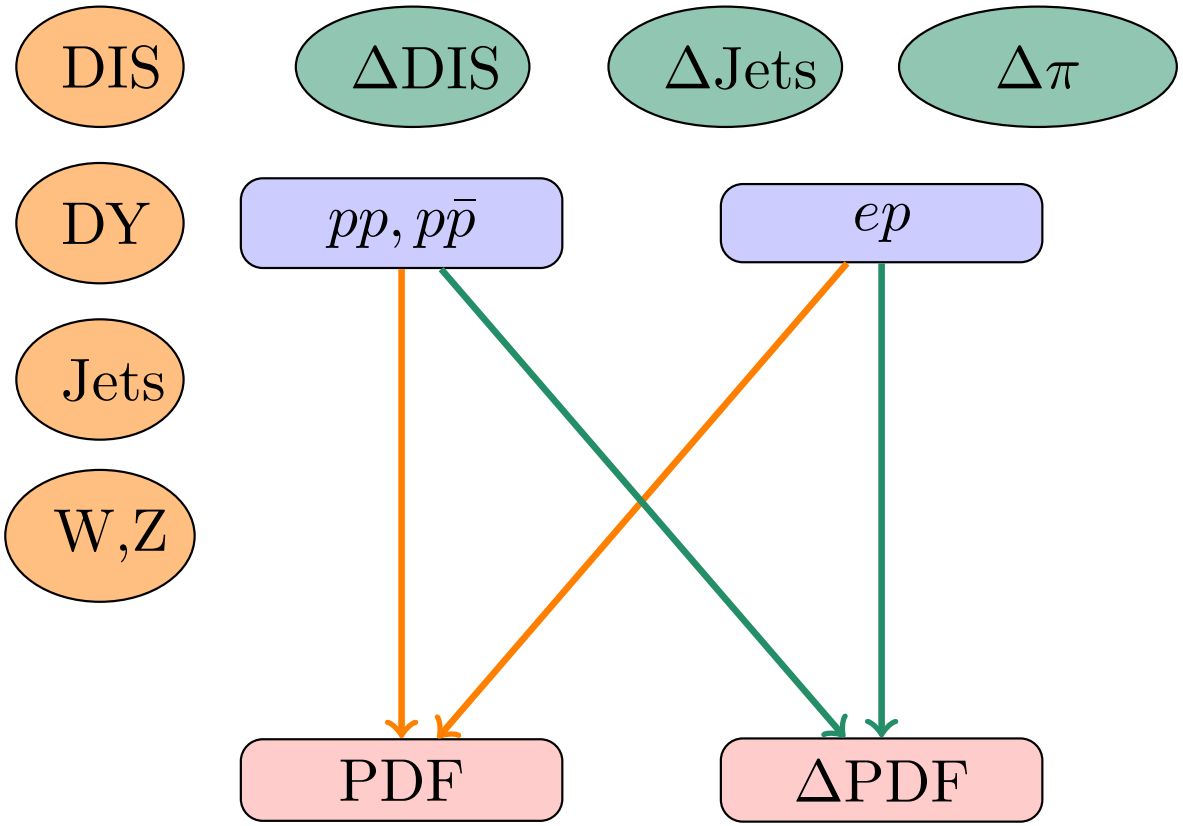

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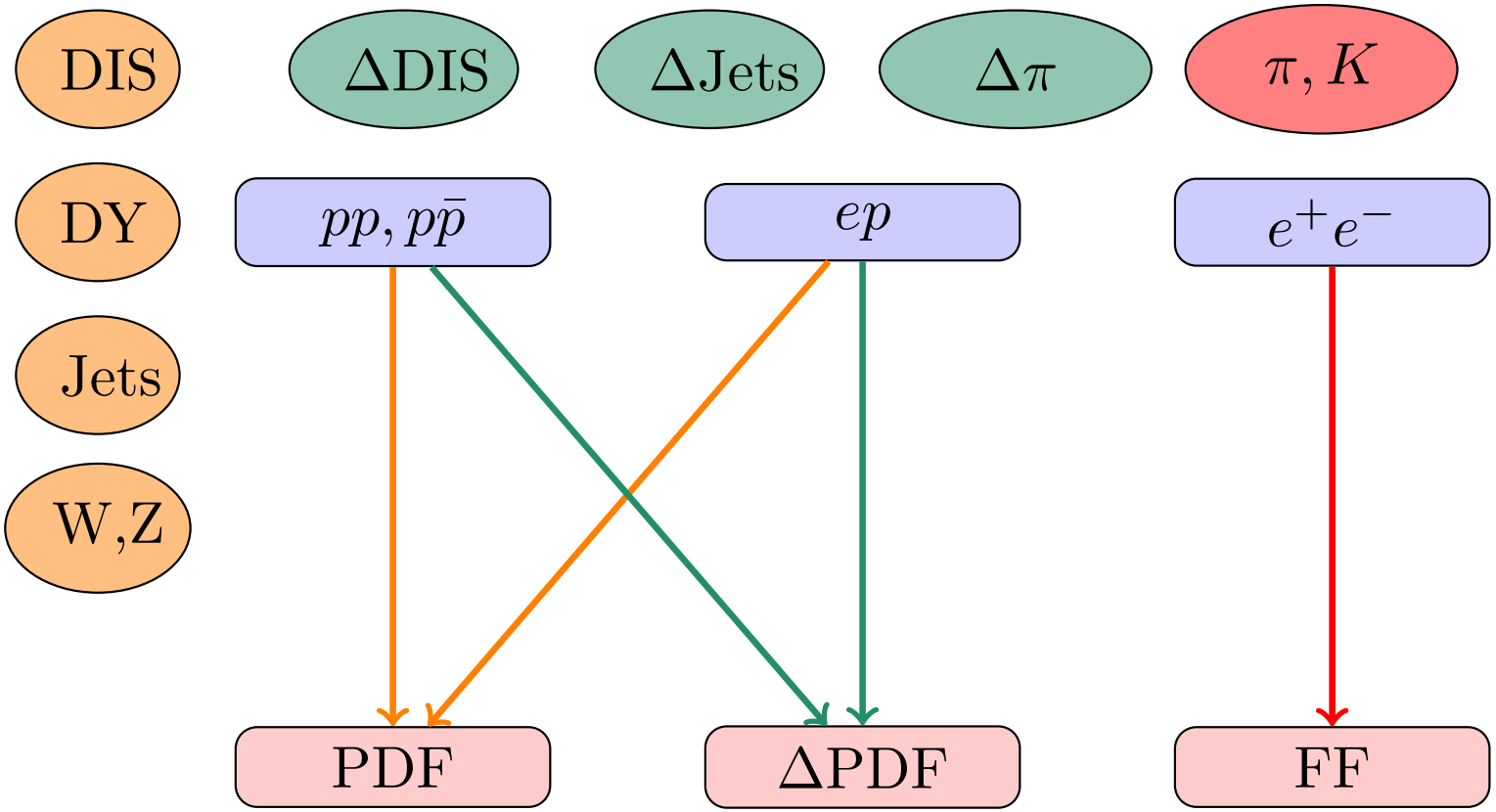

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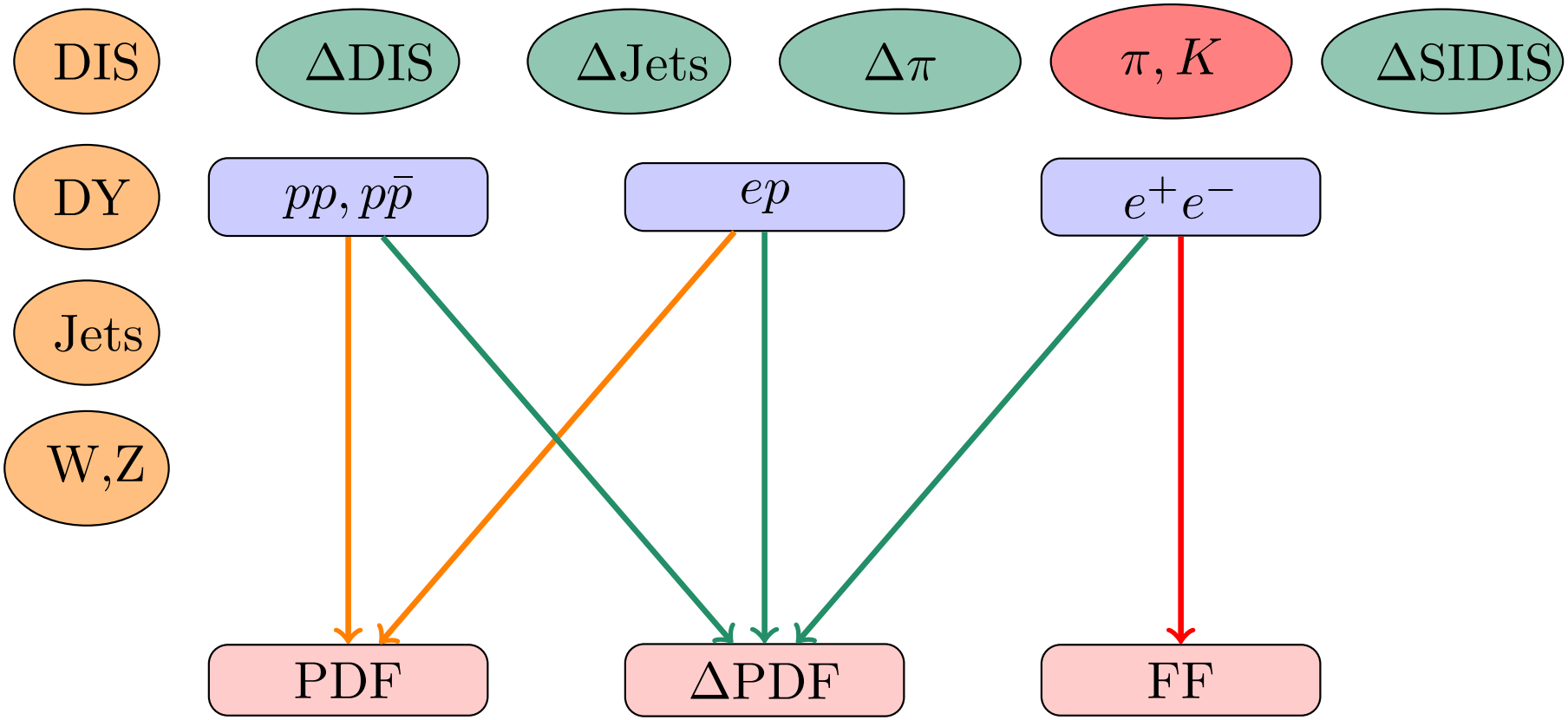

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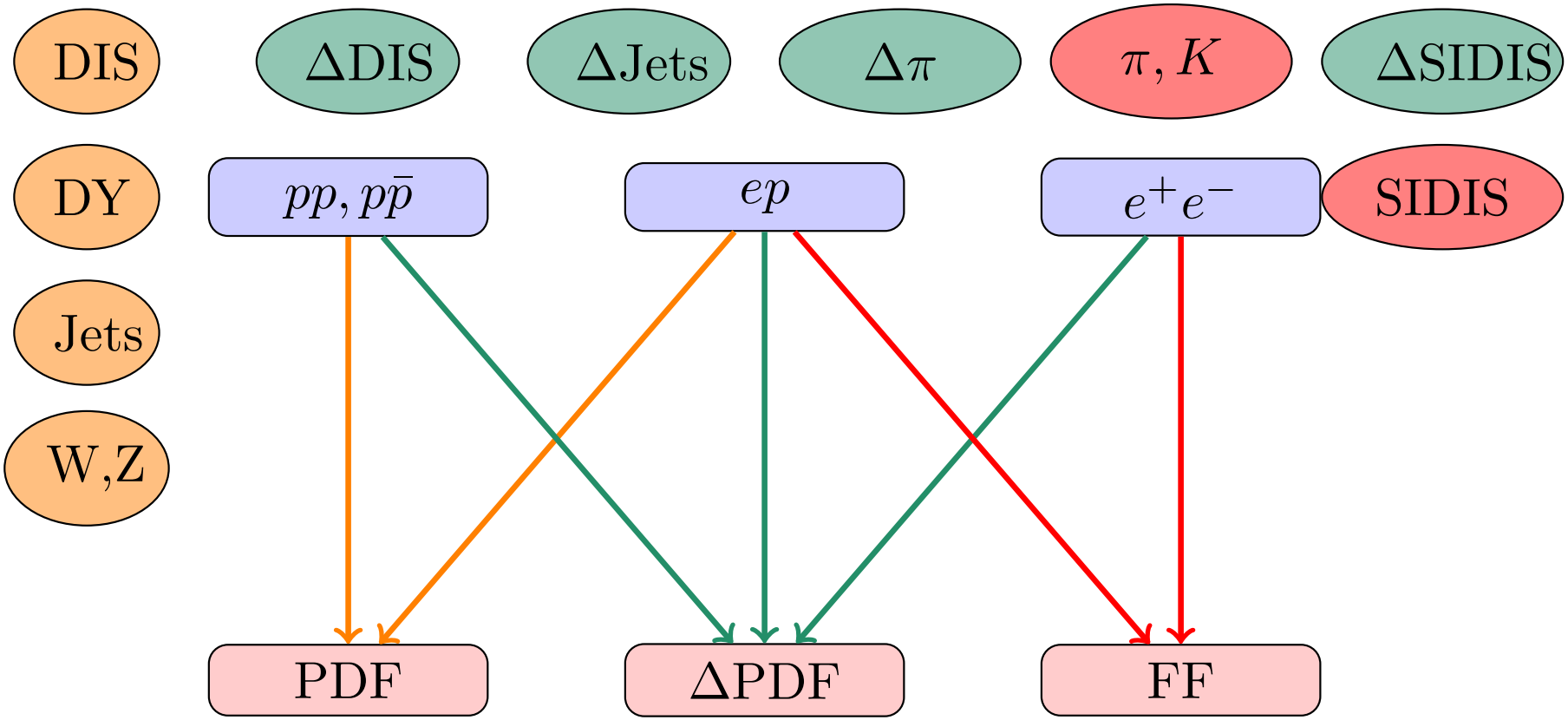

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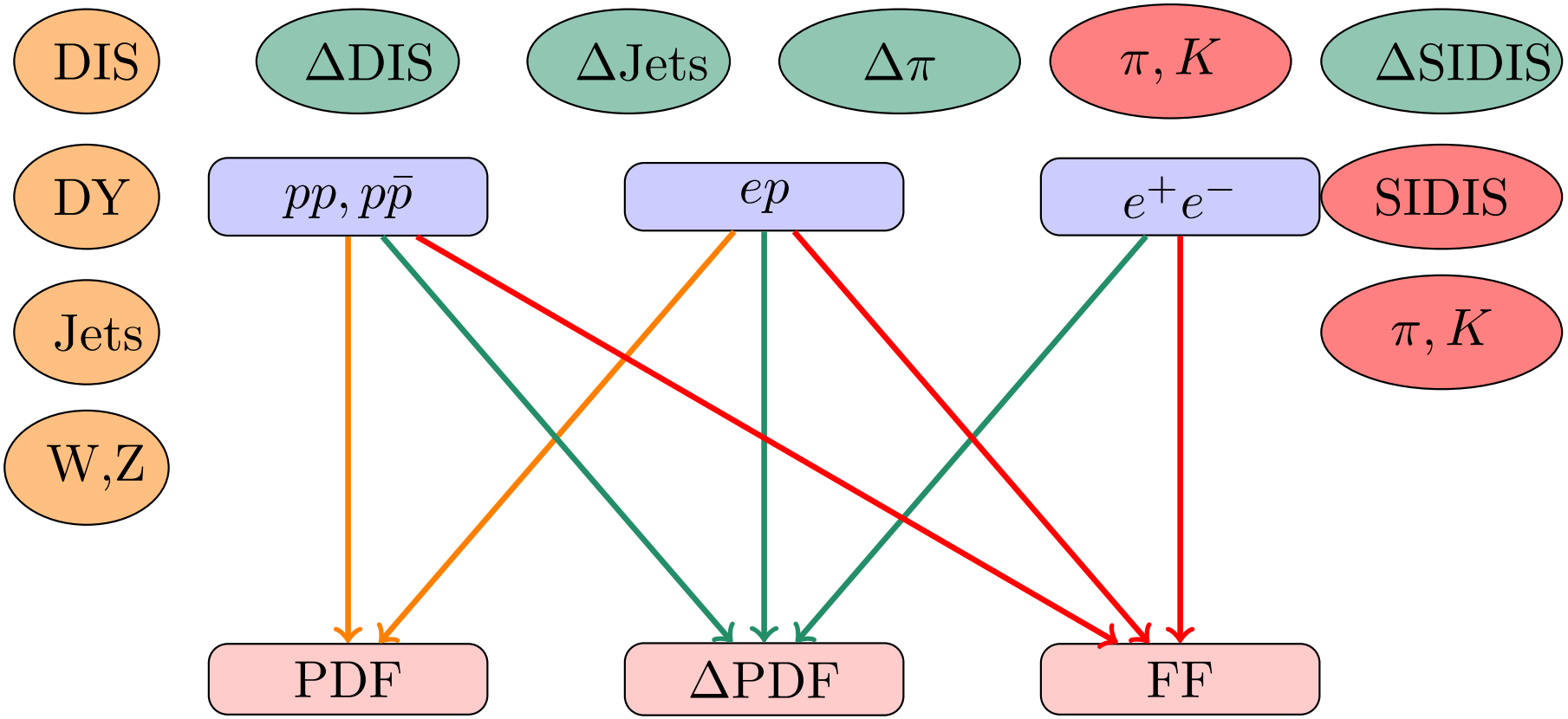

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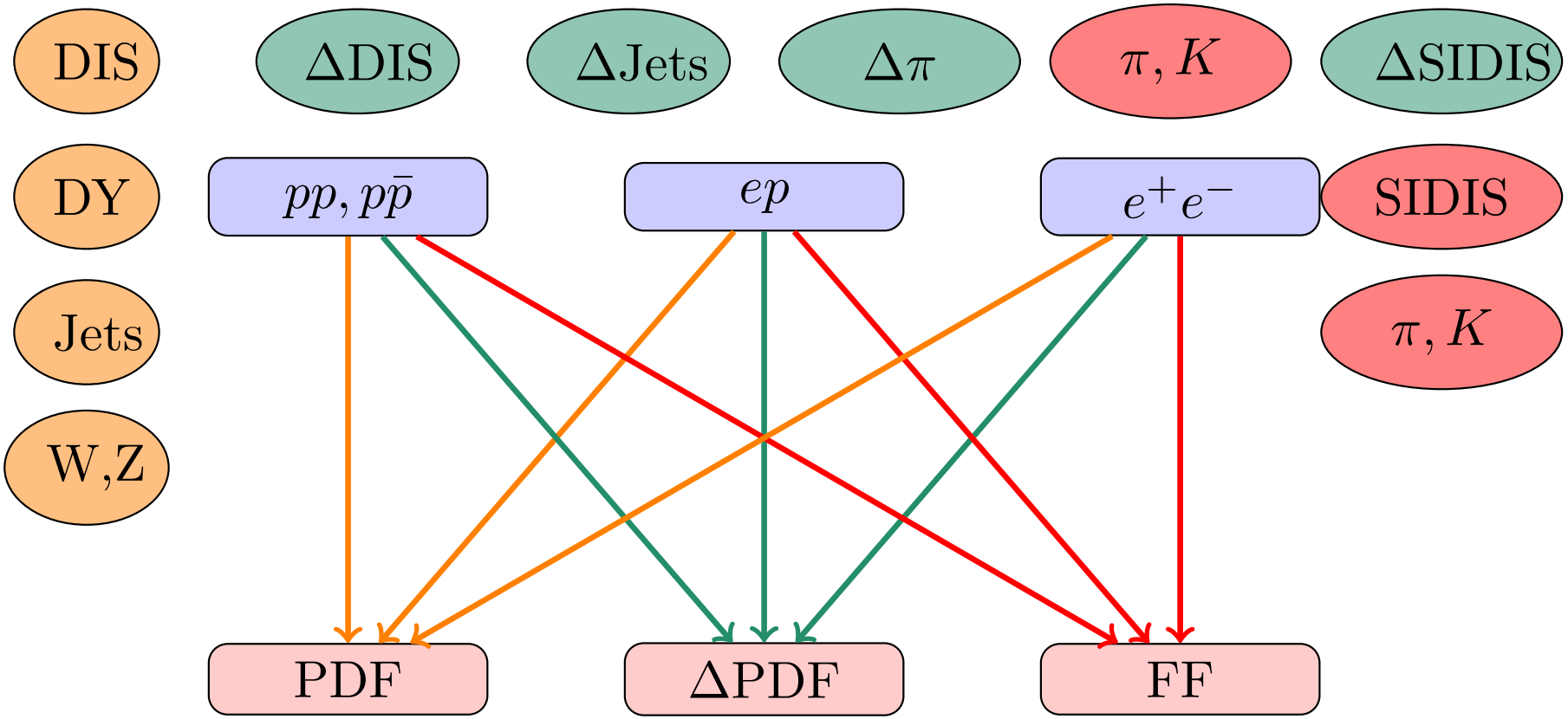

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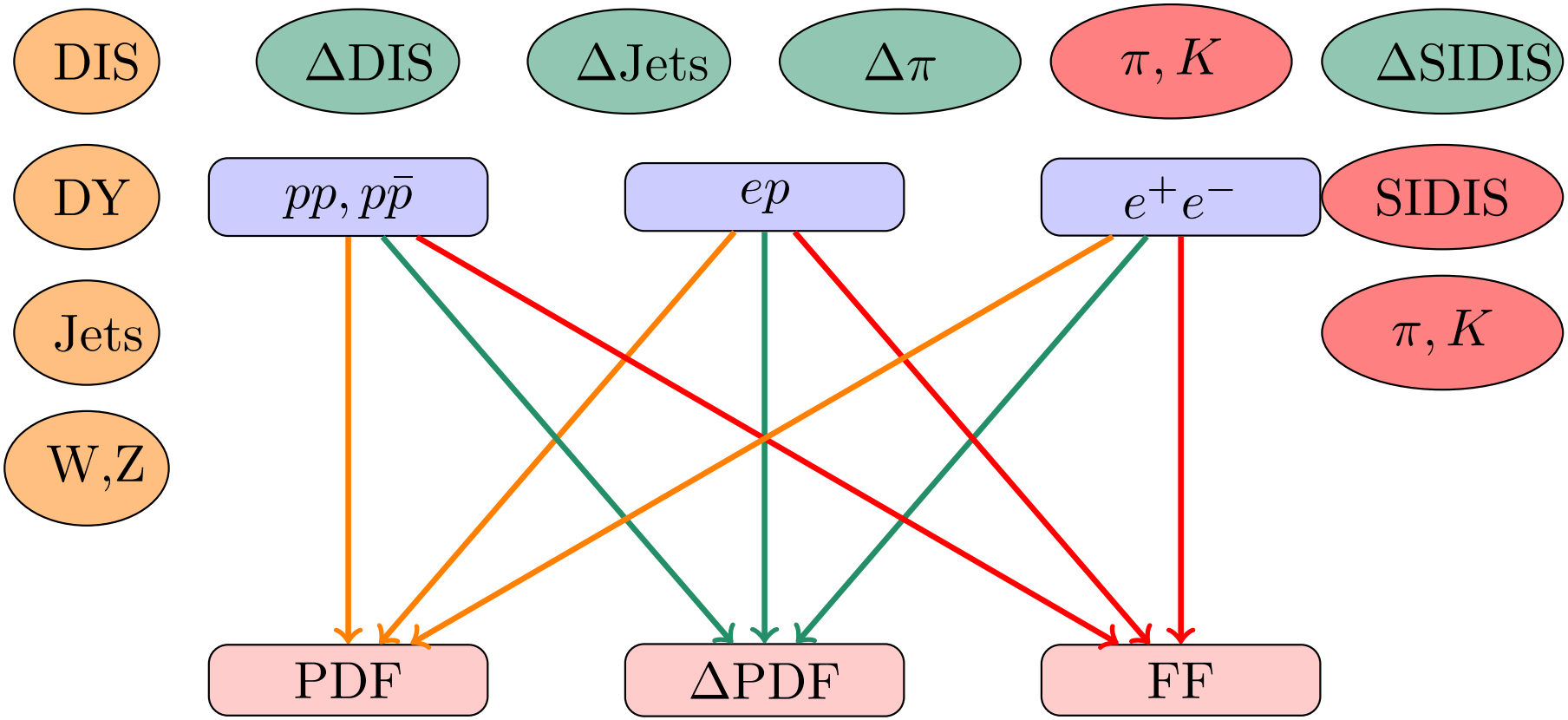

- Test of universality of collinear framework

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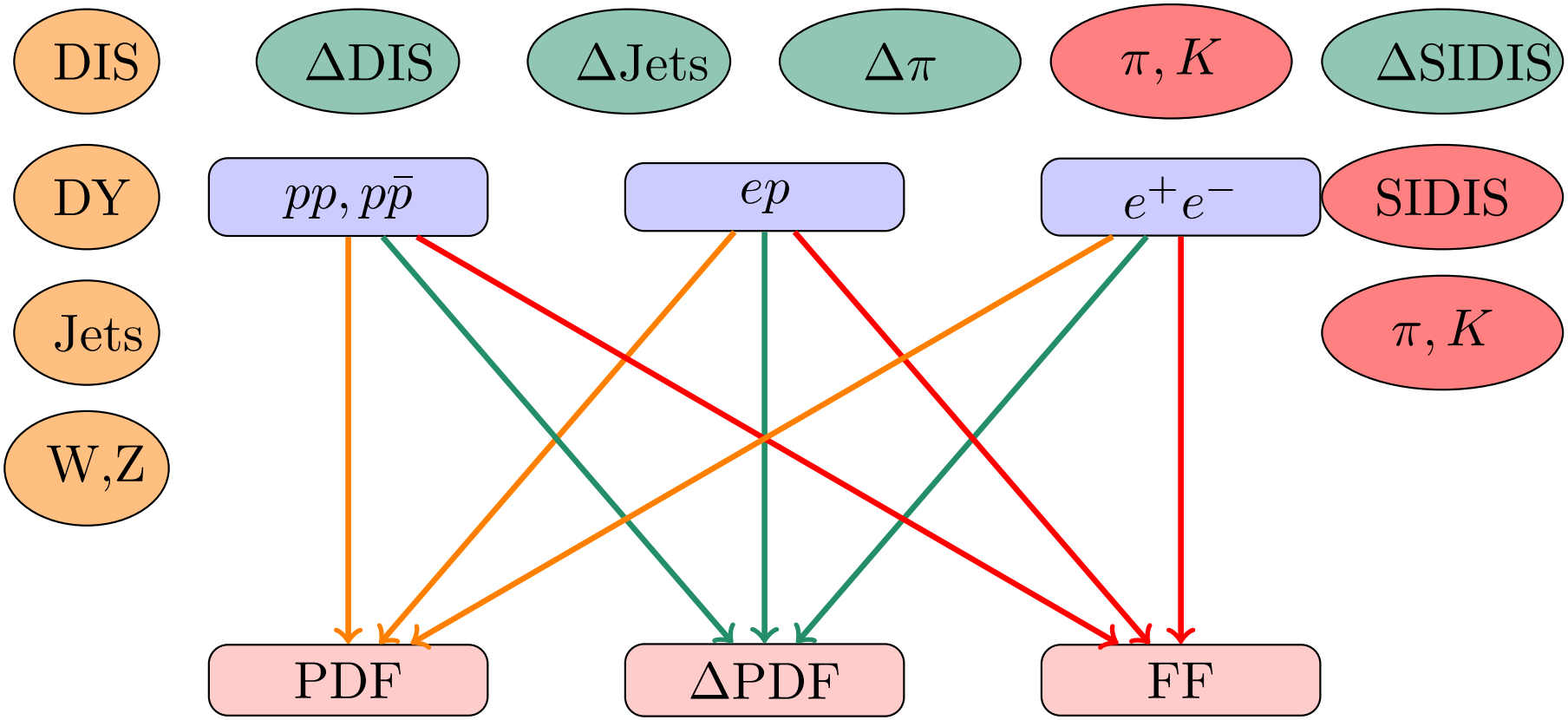
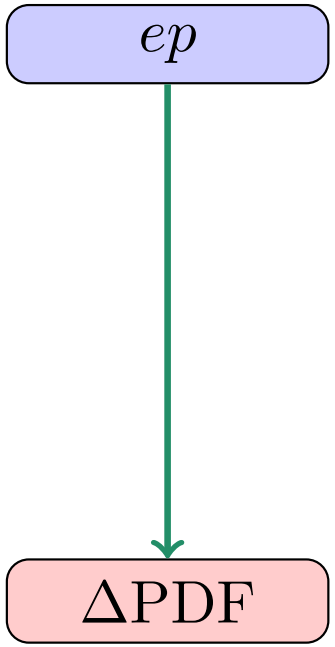


- Test of universality of collinear framework
- Collinear distributions are part of the TMD framework

Milestones



Iterative Monte Carlo analysis of spin-dependent parton distributions

Nobuo Sato,[1] W. Melnitchouk,[1] S. E. Kuhn,[2] J. J. Ethier,[3] and A. Accardi[4,1]
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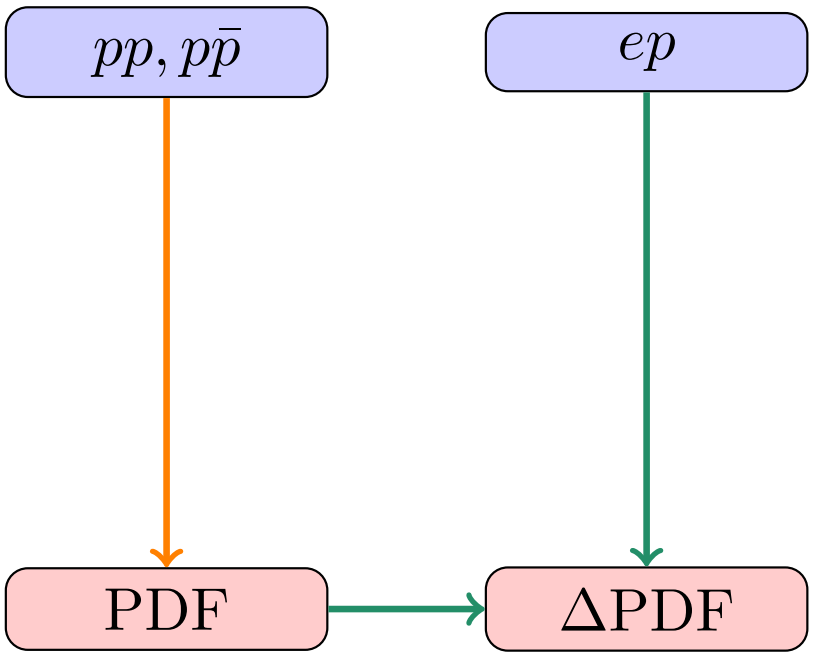

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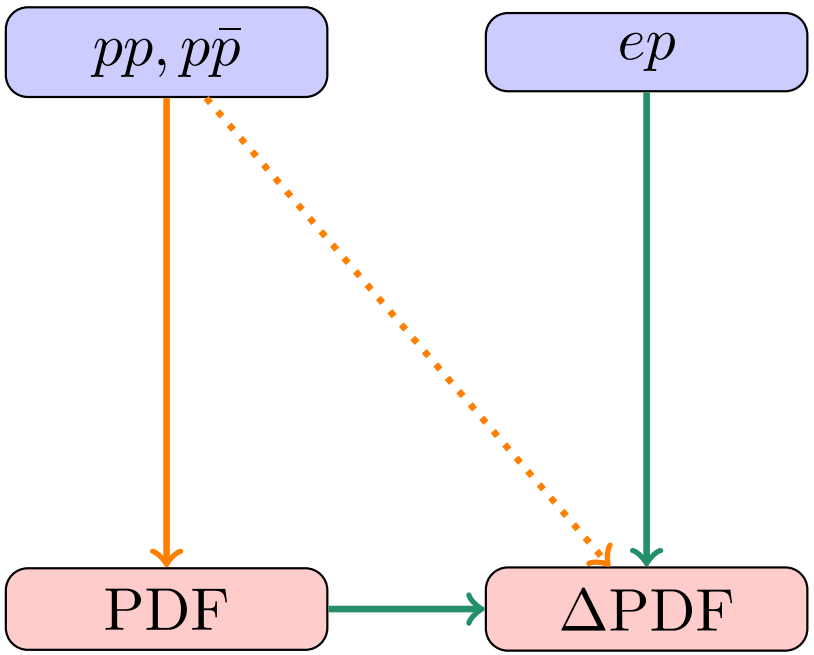

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First Monte Carlo analysis of fragmentation functions from single-inclusive e^+e^- annihilation

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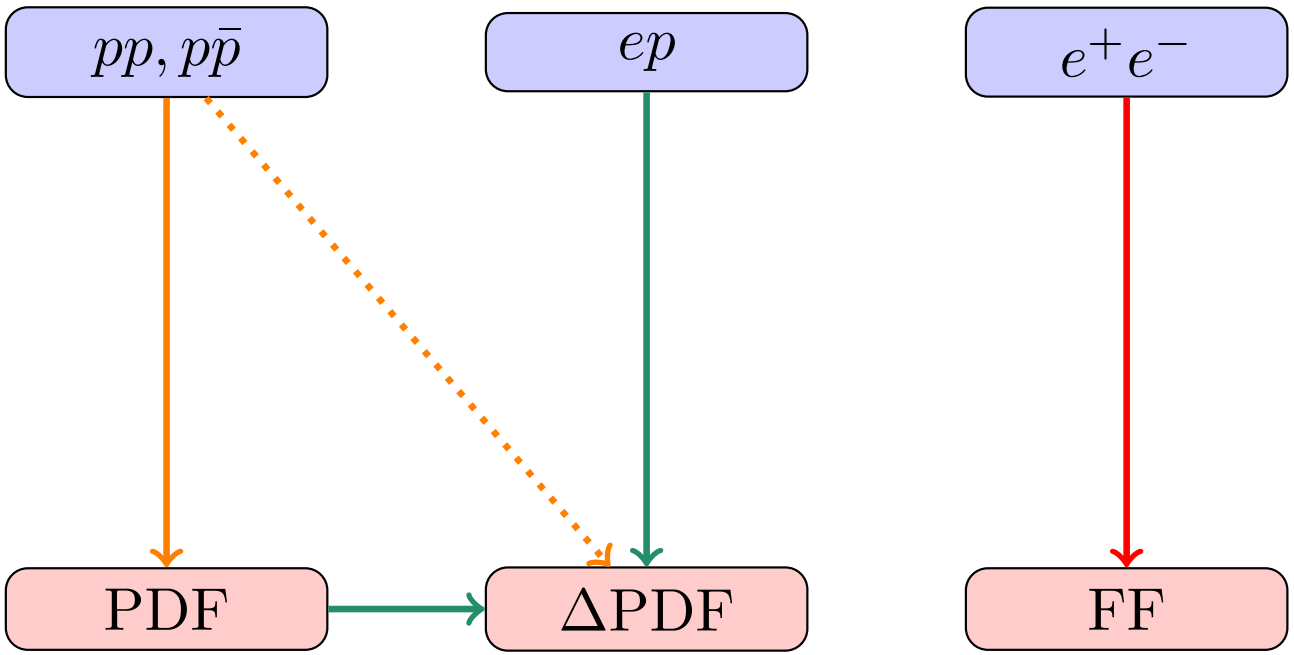

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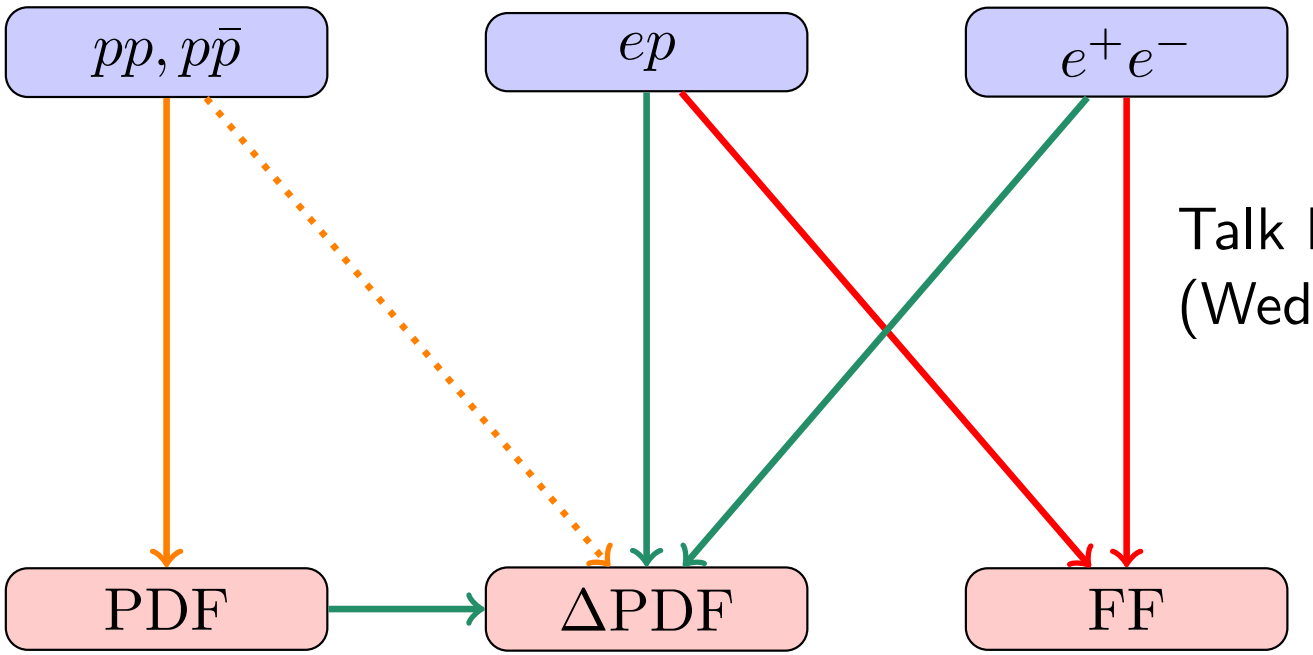

Talk by J. Ethier (Wed. WG1)

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  - $D^{\mathrm{favor}}(z) = T_1(z) + T_2(z)$
  - $D^{\mathrm{unfavor}}(z) = T_1(x)$
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- 24 kaon parameters and 18 pion parameters

JAM framework

The goal is to estimate:

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; [\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]]^2$$

Monte Carlo methods

- $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \{\boldsymbol{a}_k\}$
- $\mathrm{E}[\mathcal{O}] \approx \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k)$
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Maximum Likelihood

- Maximize $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \boldsymbol{a}_0$
- $\mathrm{E}[\mathcal{O}] \approx \mathcal{O}\left(\boldsymbol{a}_0\right)$
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Methodology

original data

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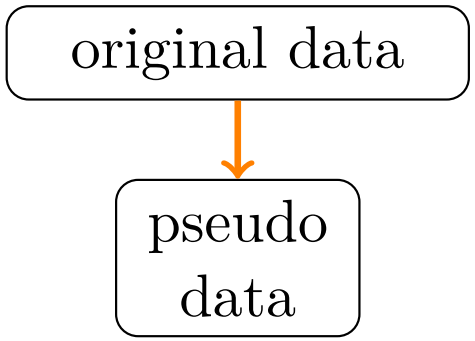

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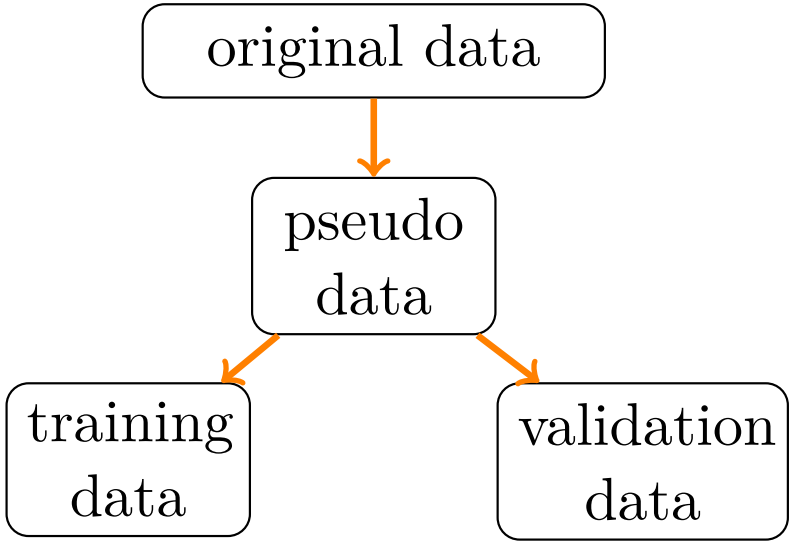

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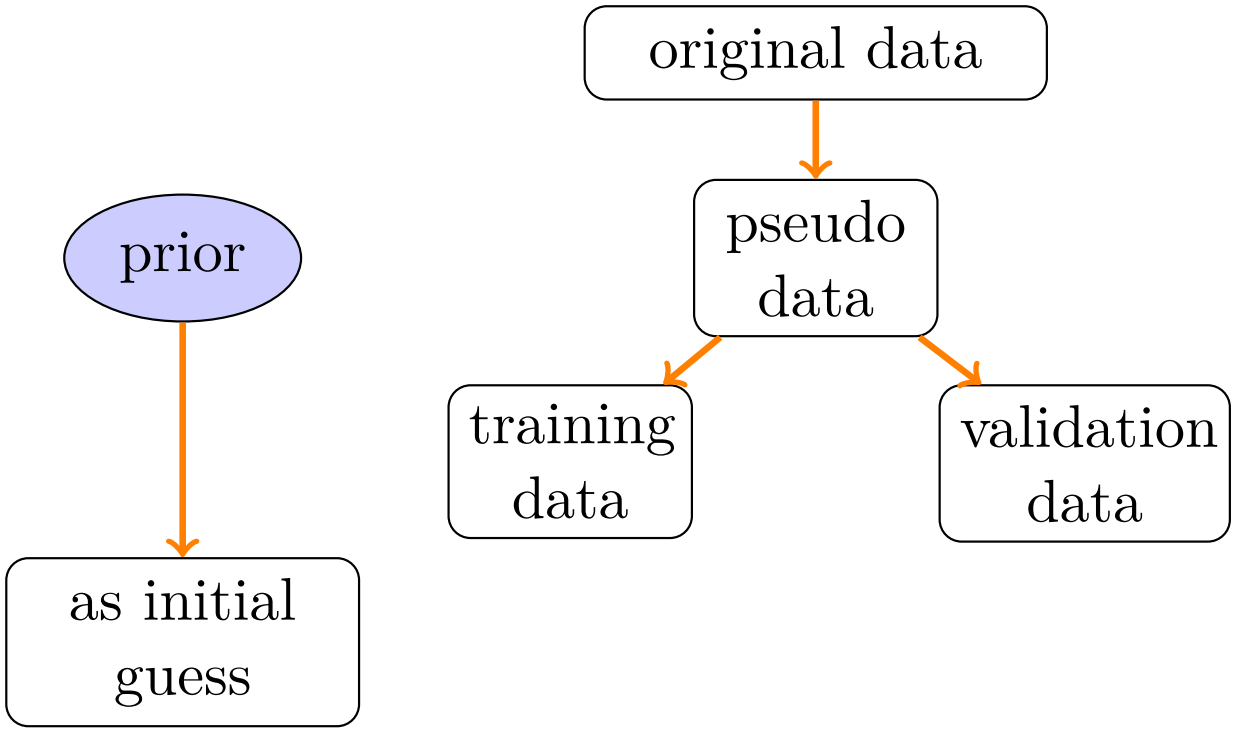

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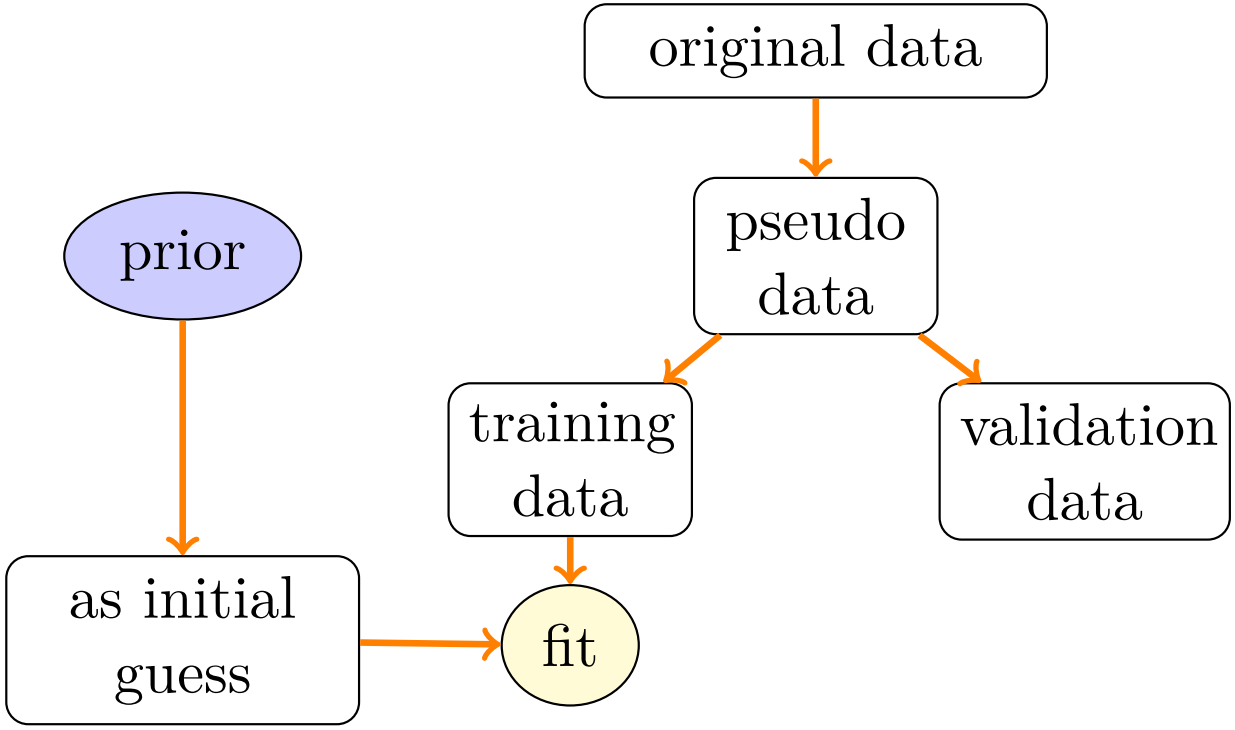

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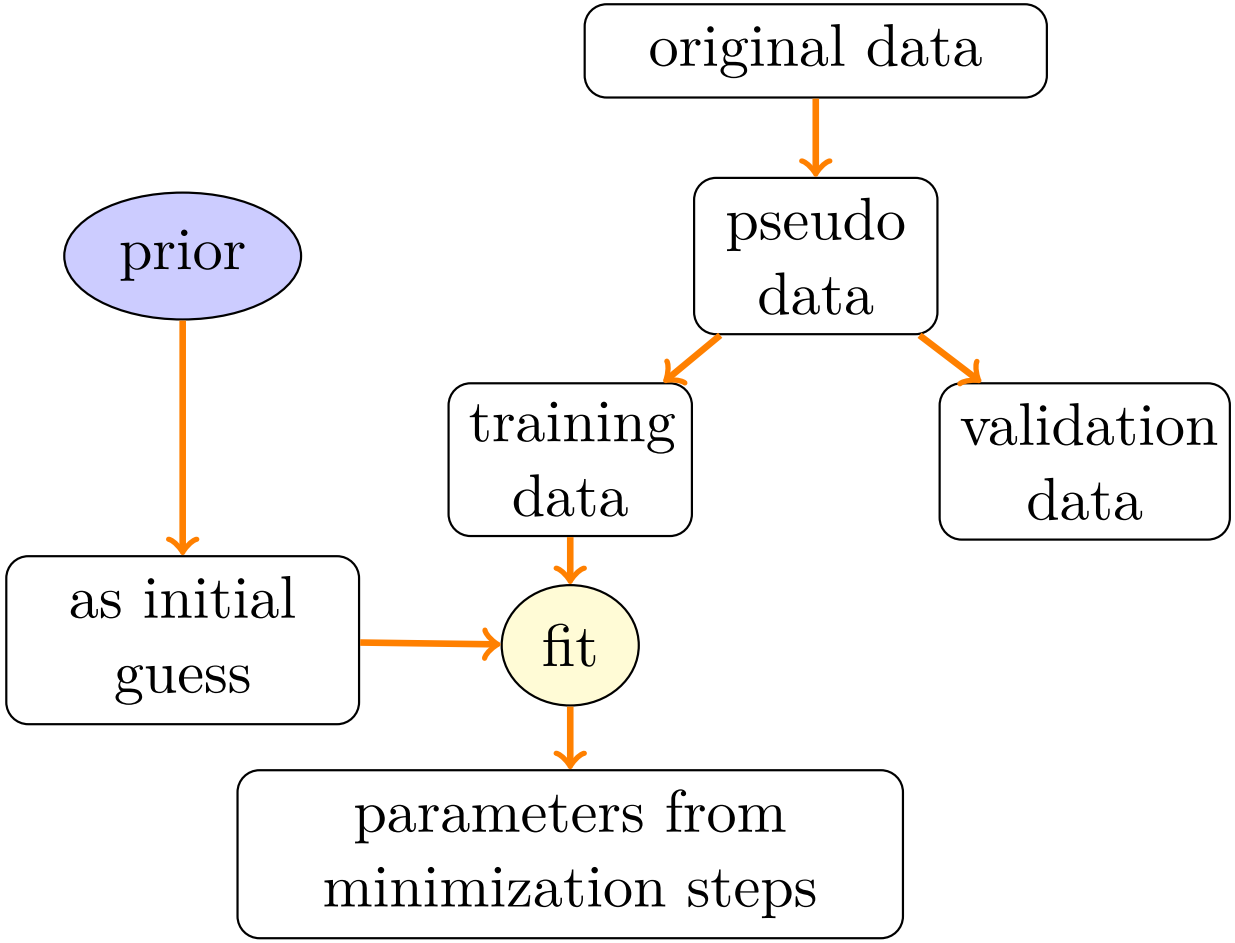

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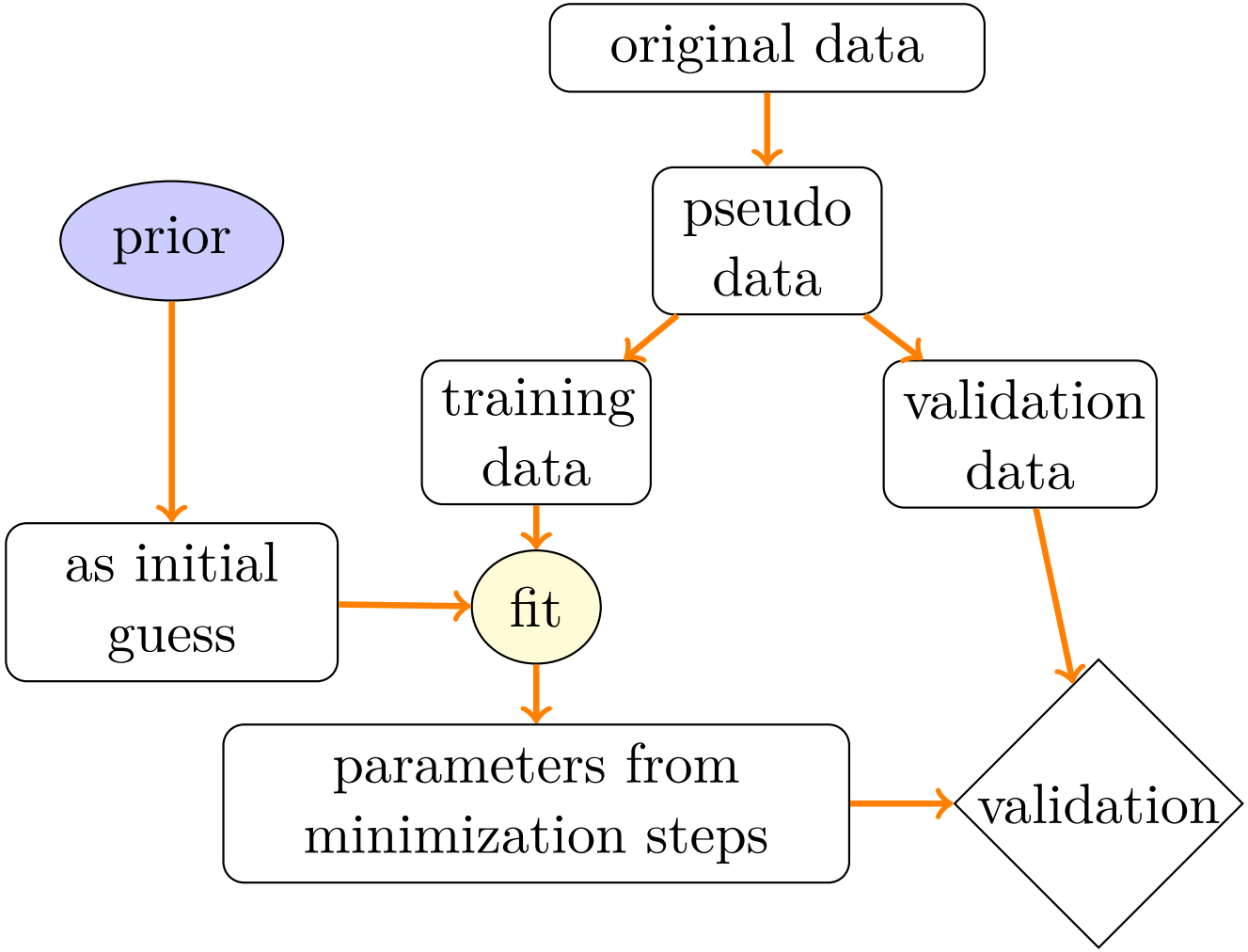

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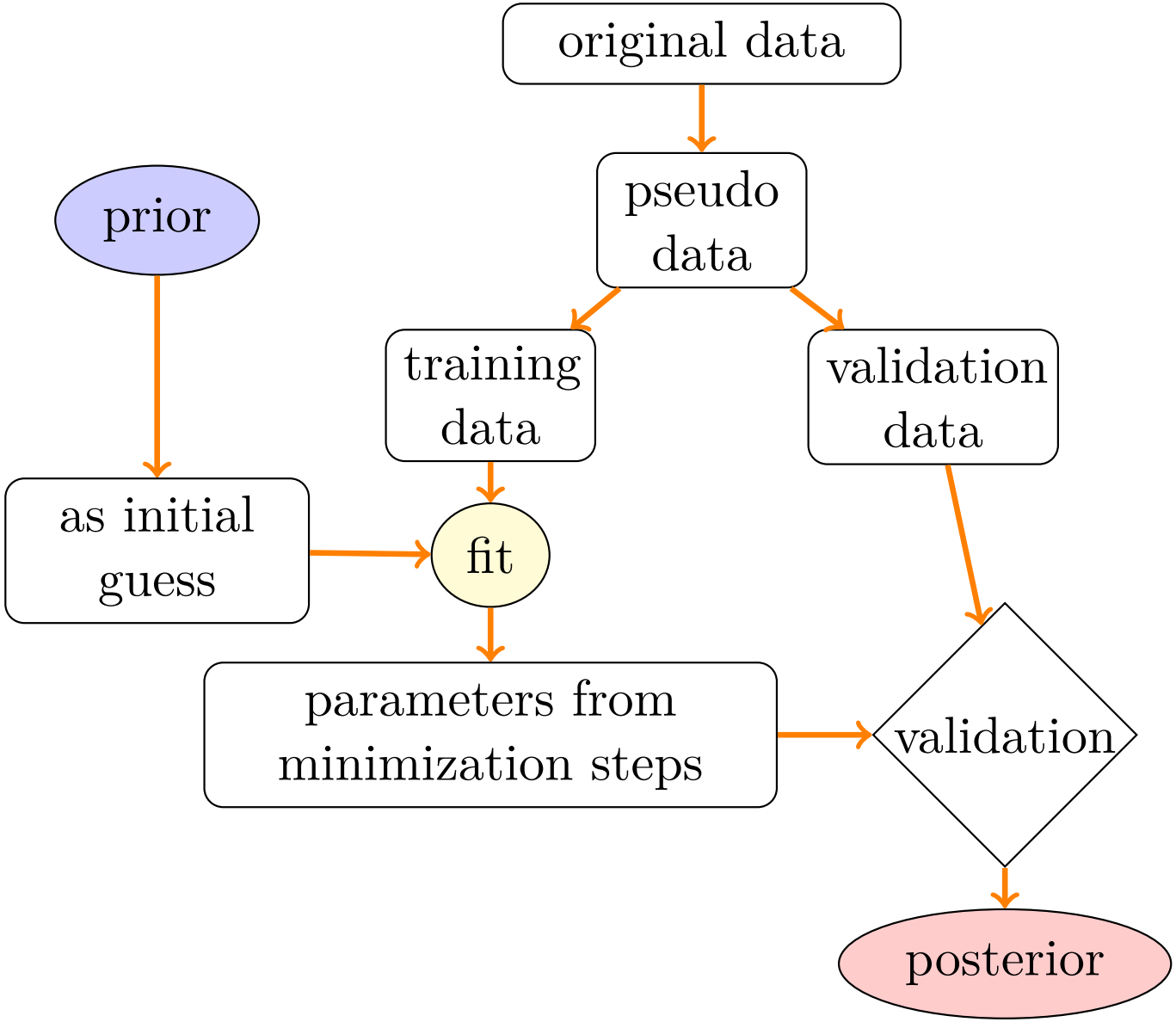

Iterative Monte Carlo analysis (IMC)

sampler

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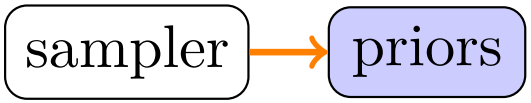

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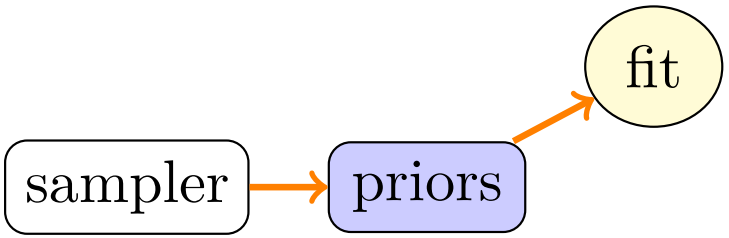

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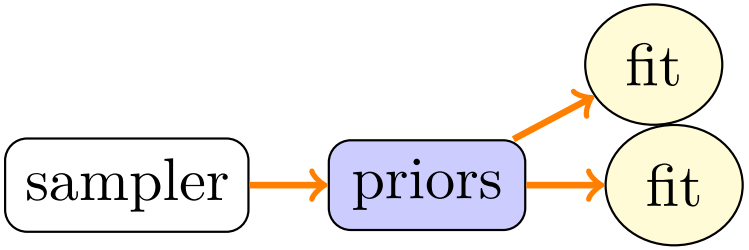

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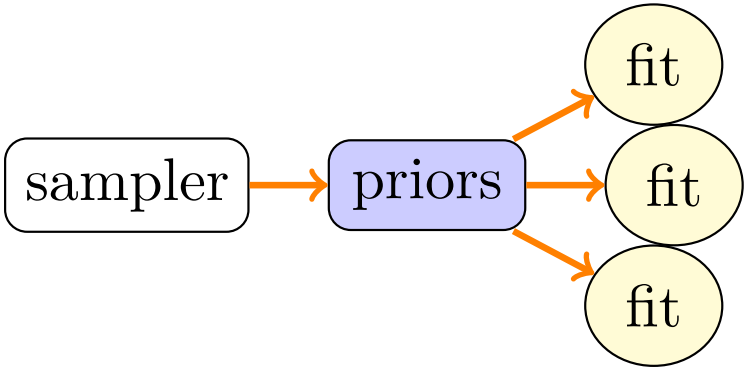

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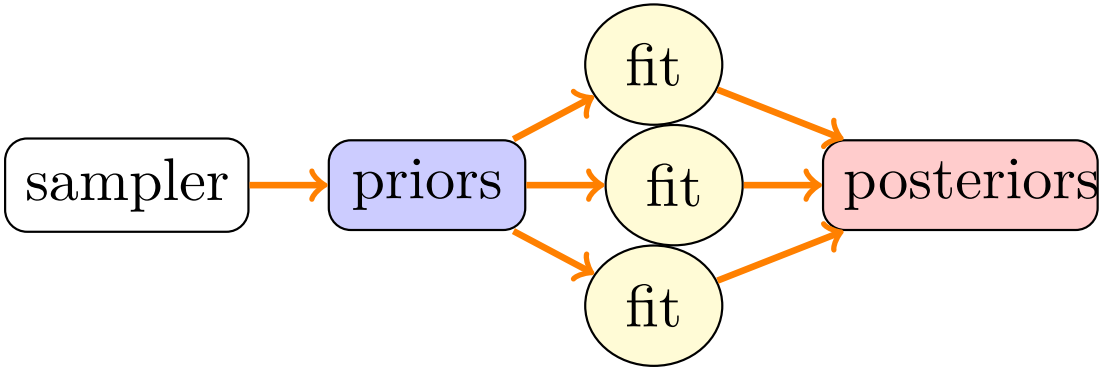

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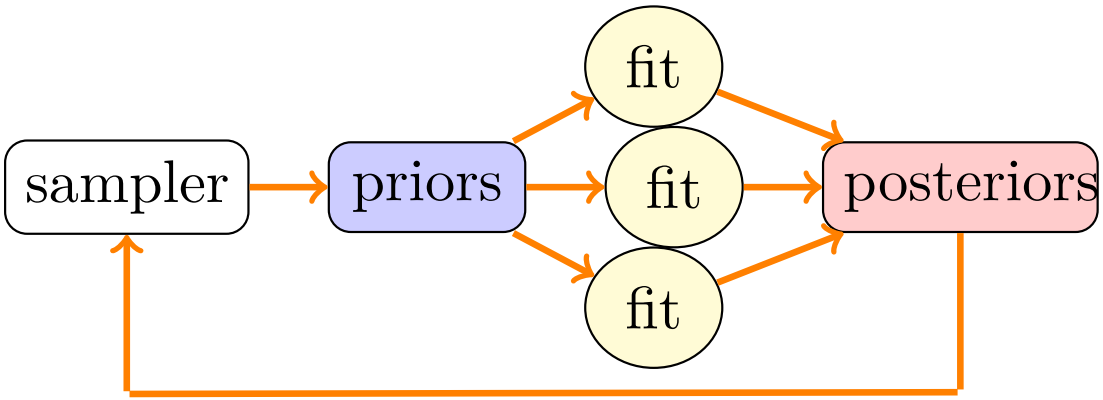

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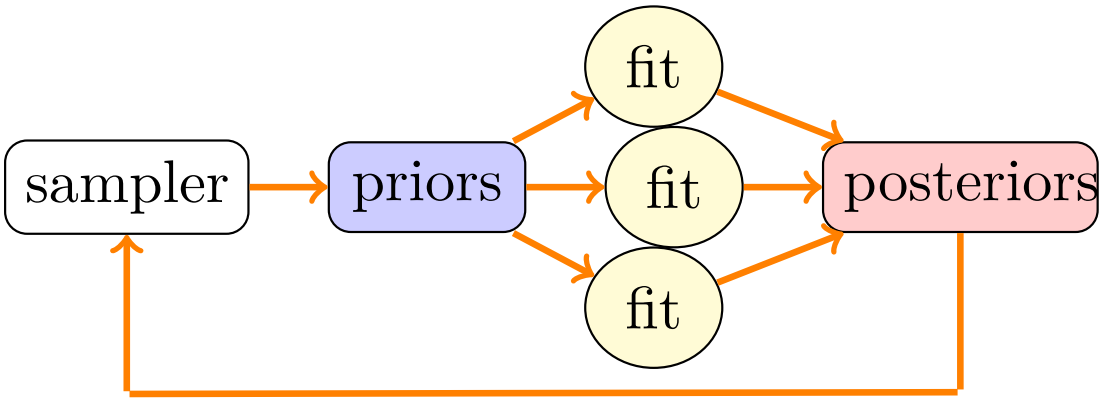

- Multiple iterations until convergence of posterior distribution

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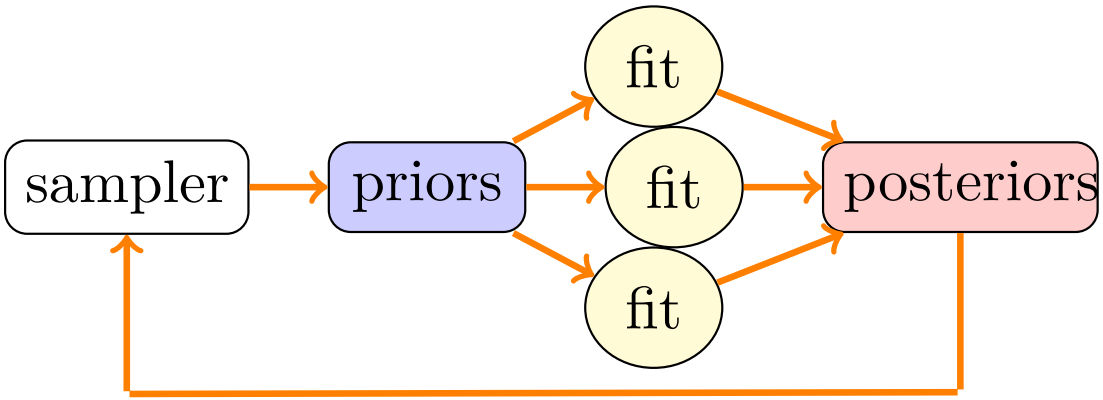

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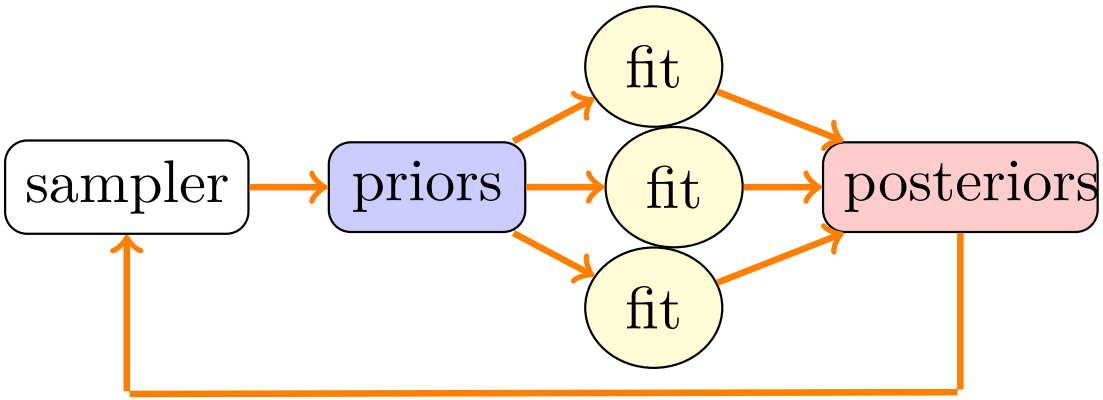

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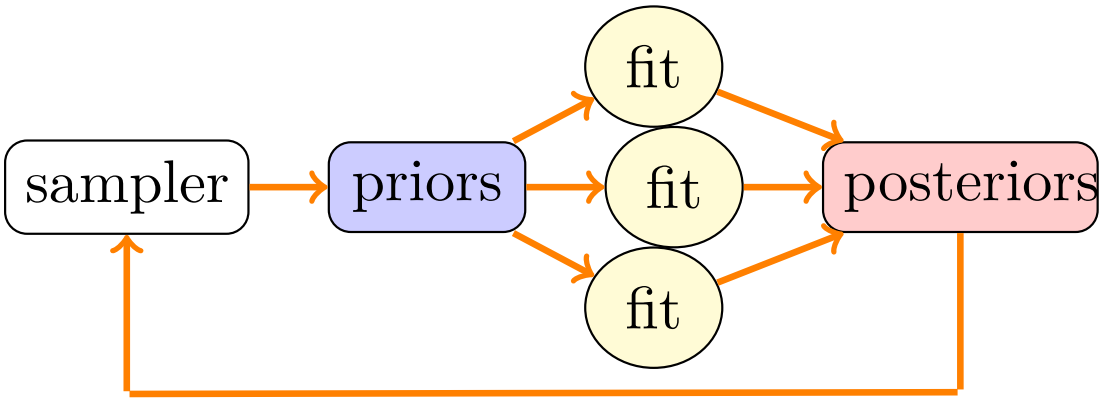

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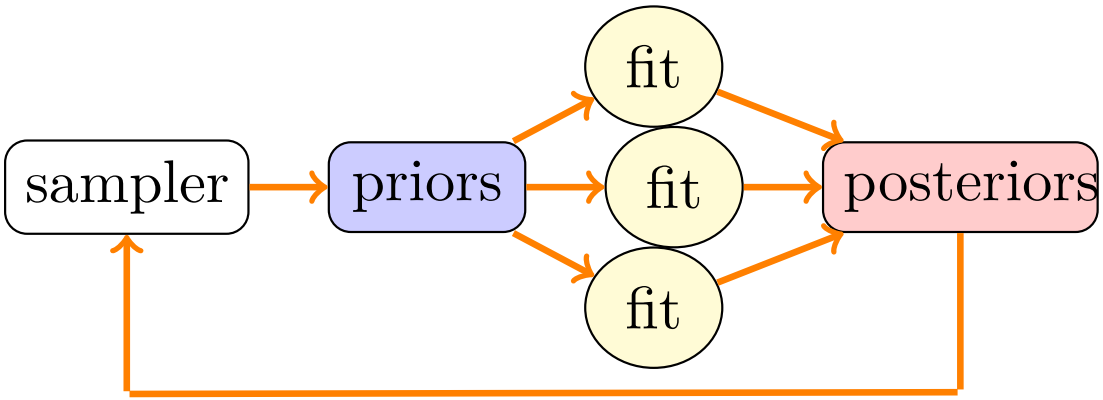

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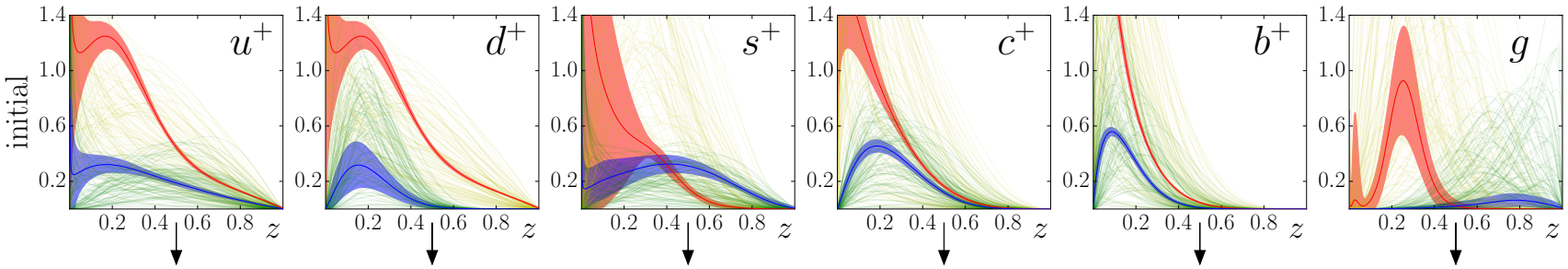
- Robust estimation of uncertainties

IMC in action



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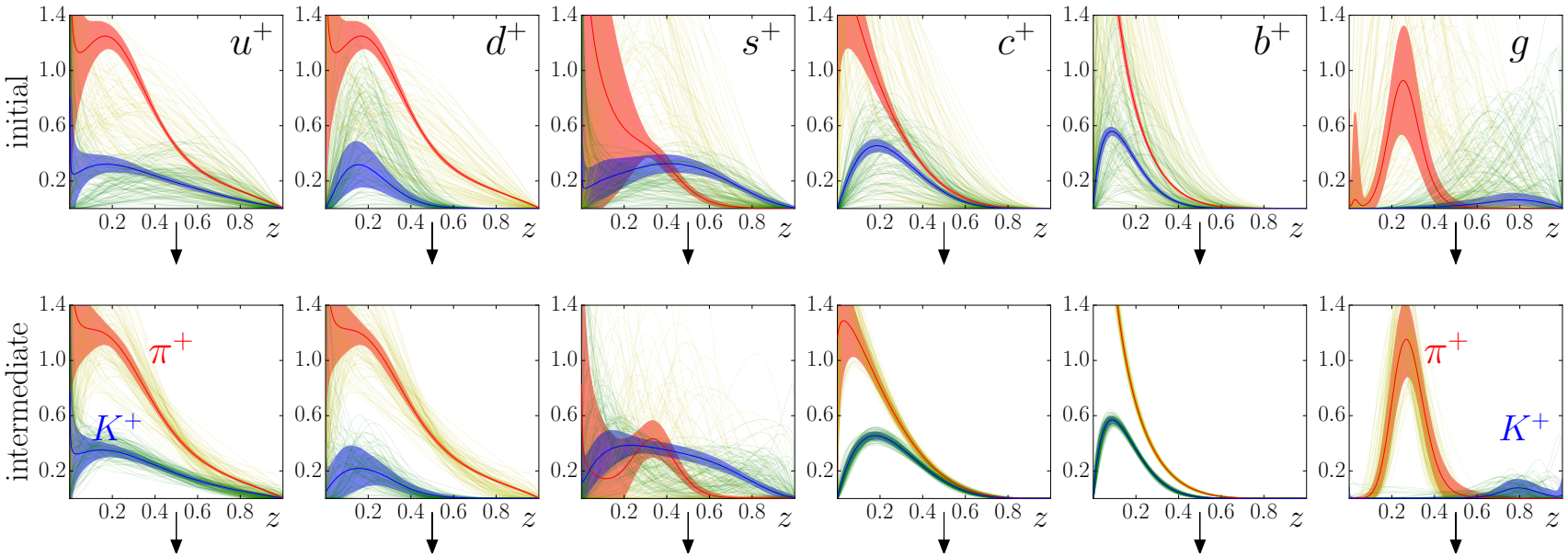

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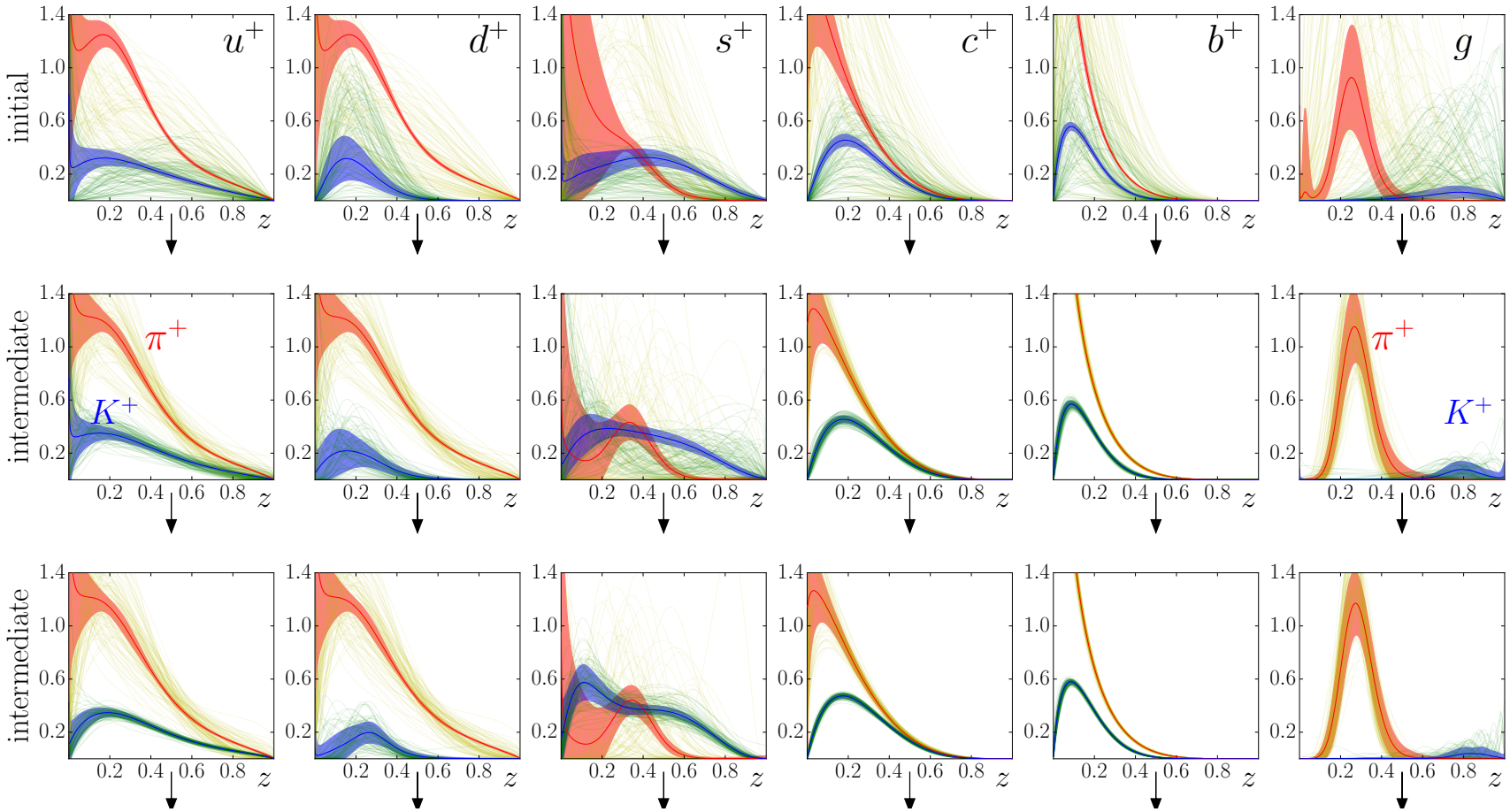

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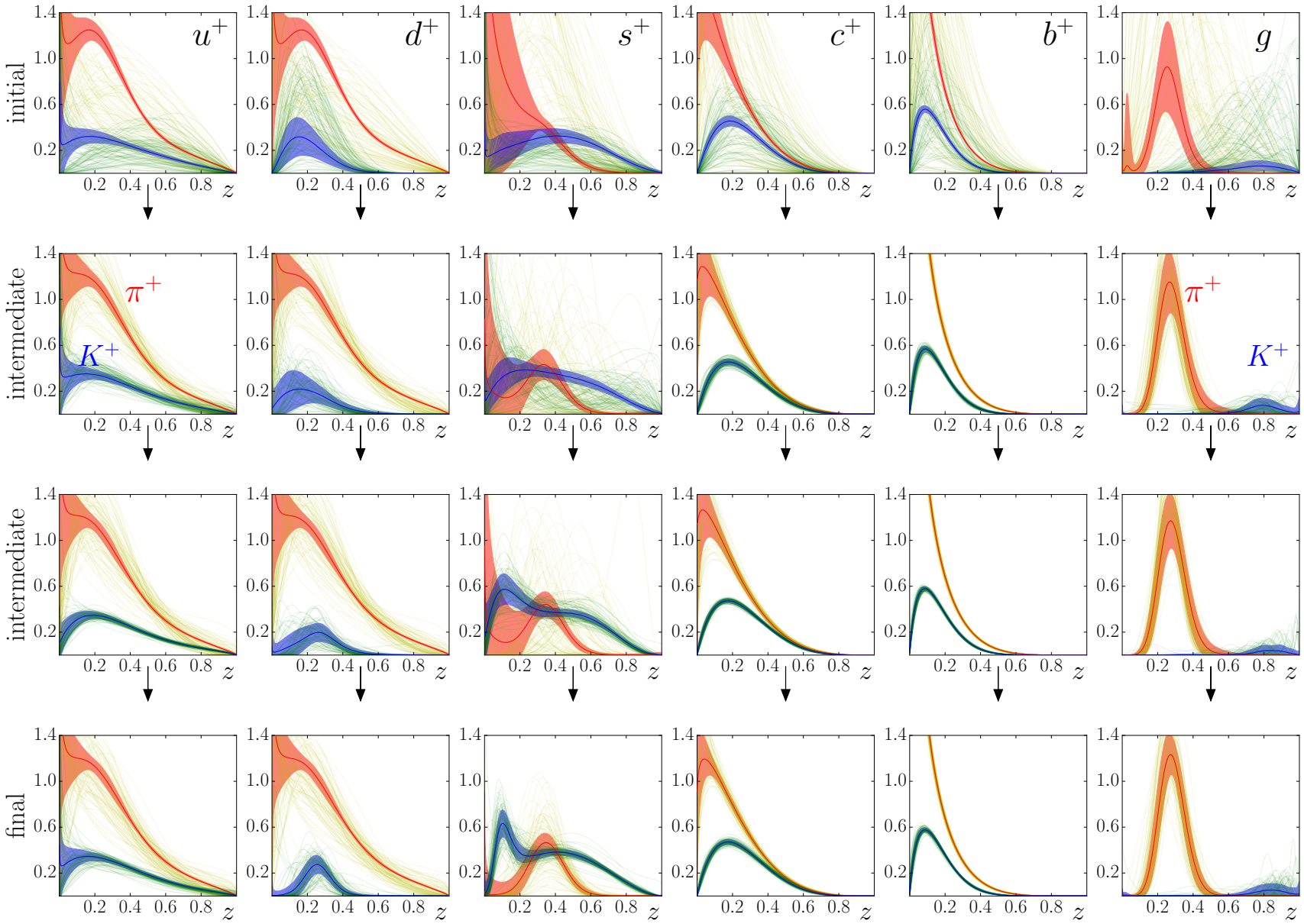
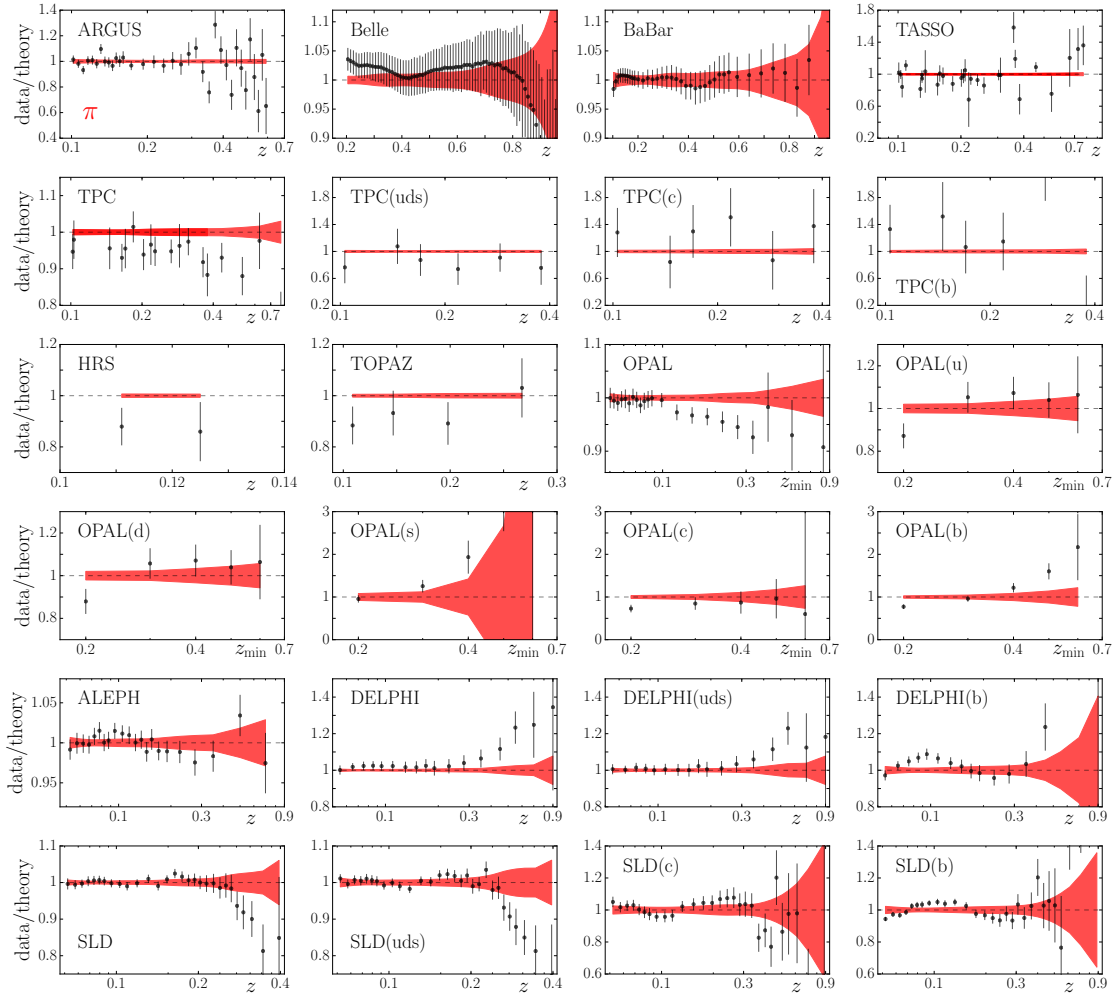


π analysis $(\chi^2/N_{\rm pts} = 1.31)$



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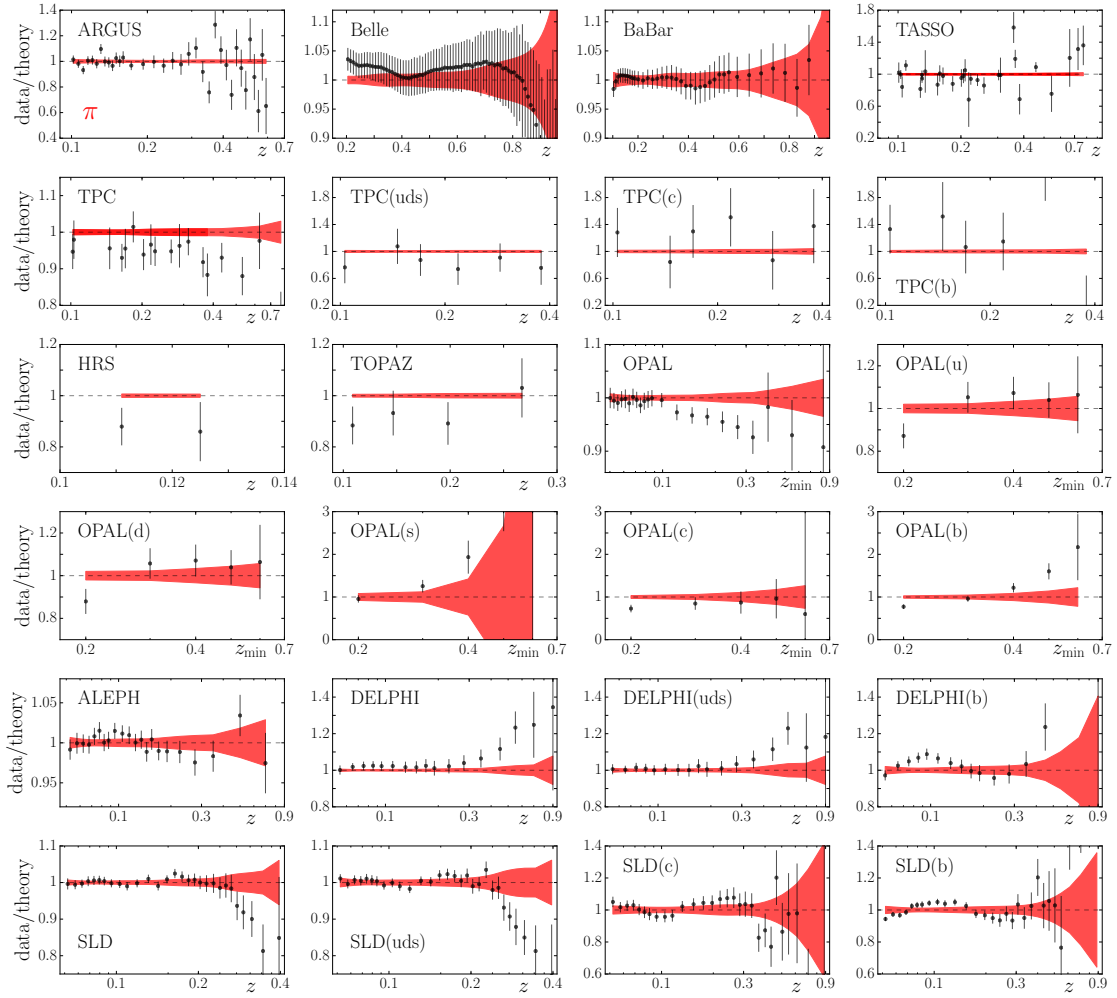

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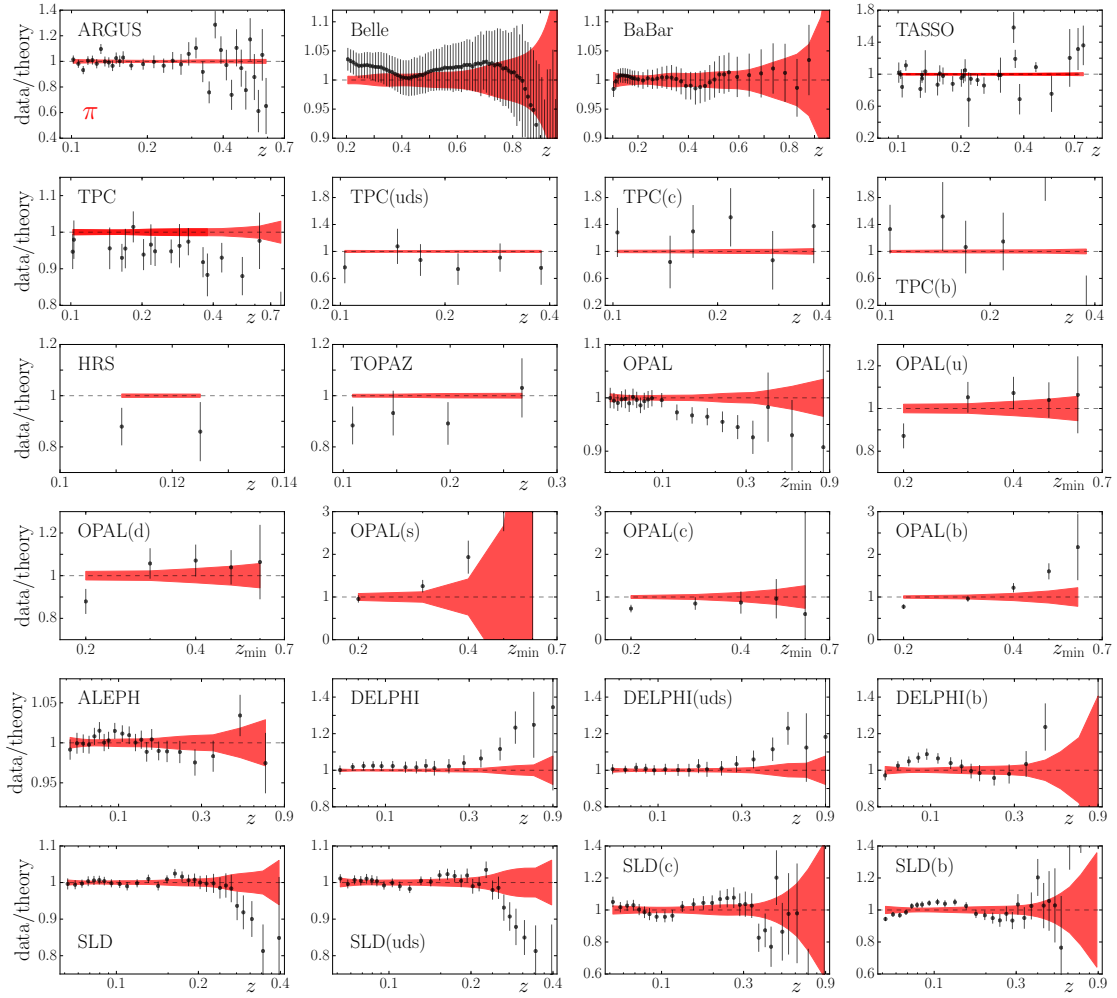

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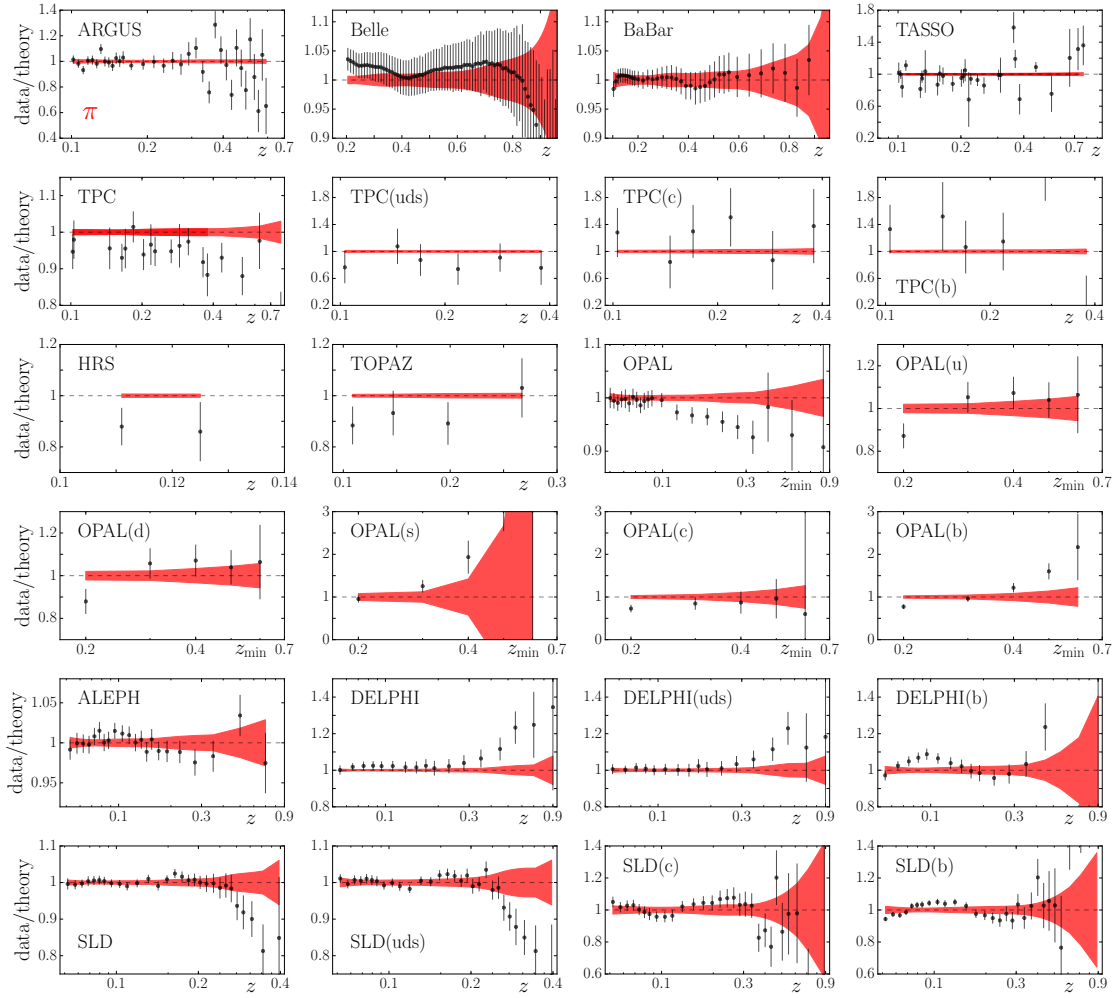

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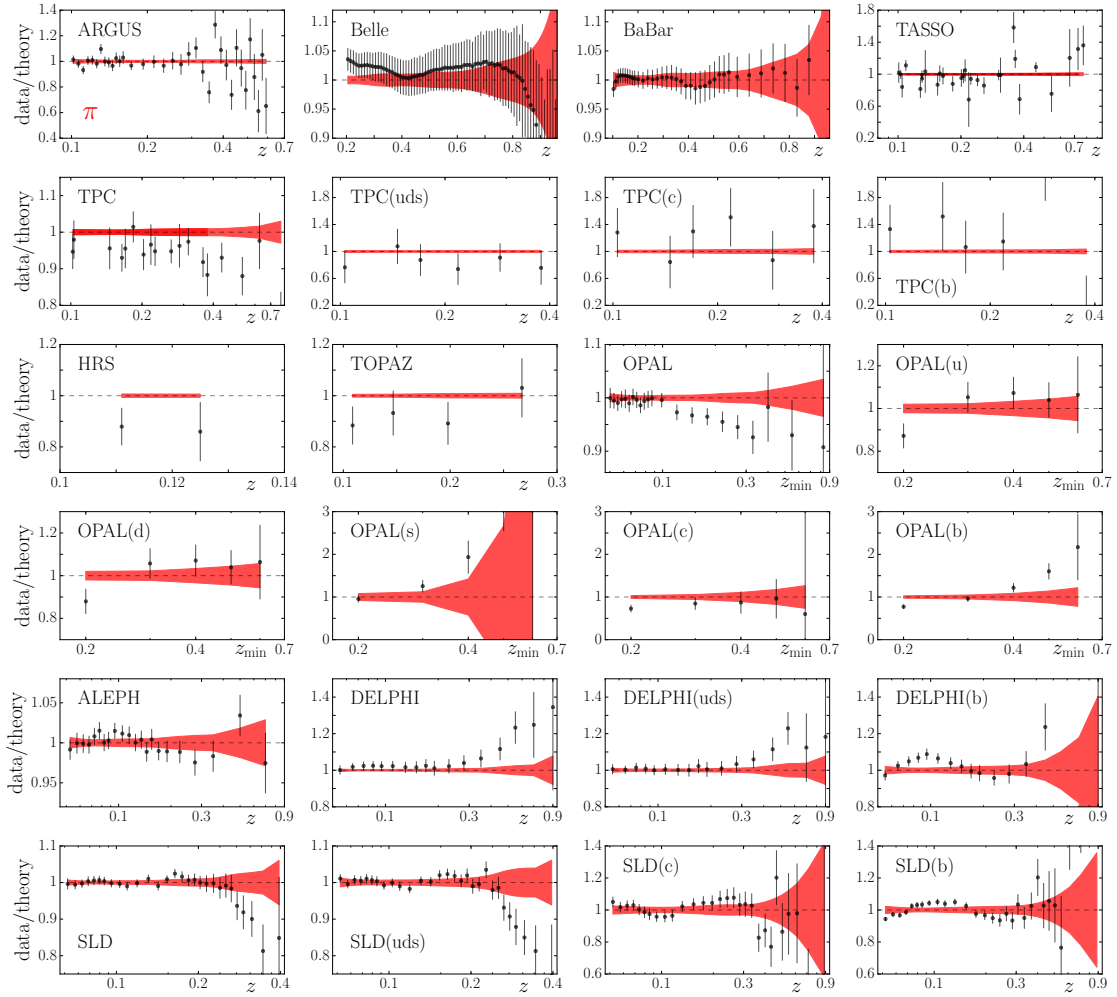

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- Data inconsistencies at large z for $Q^2 = M_z^2$

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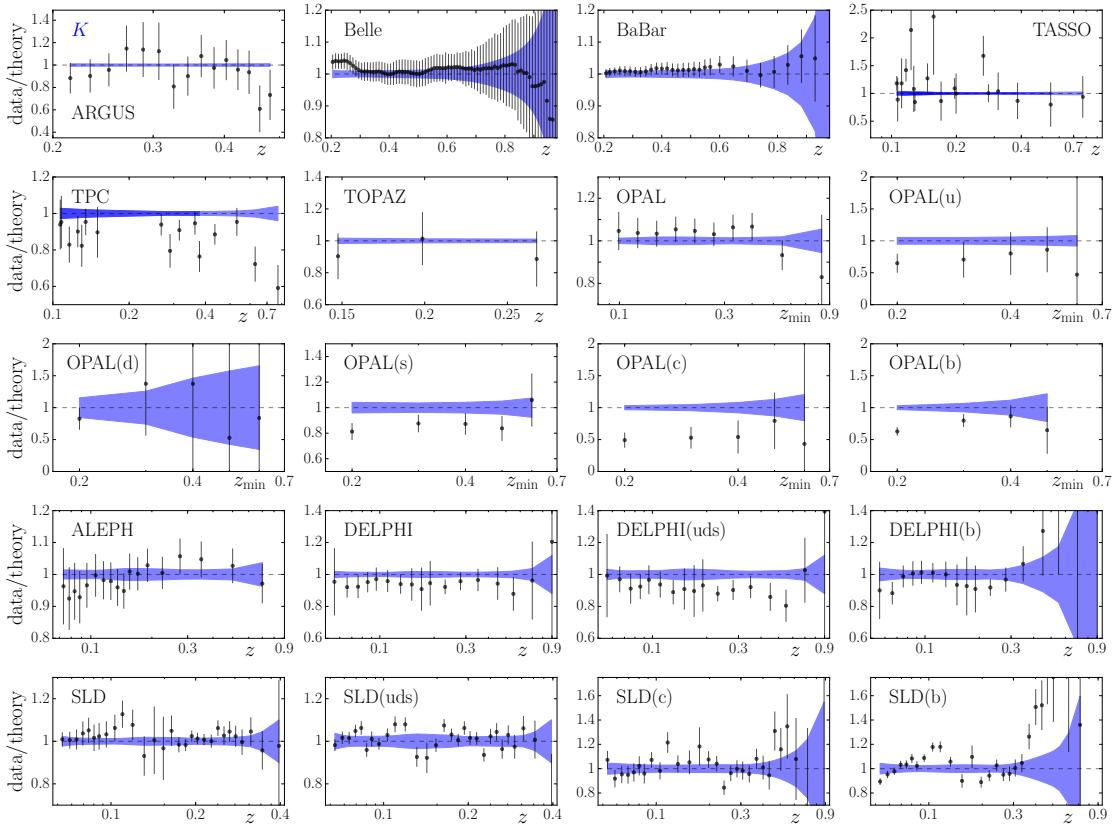

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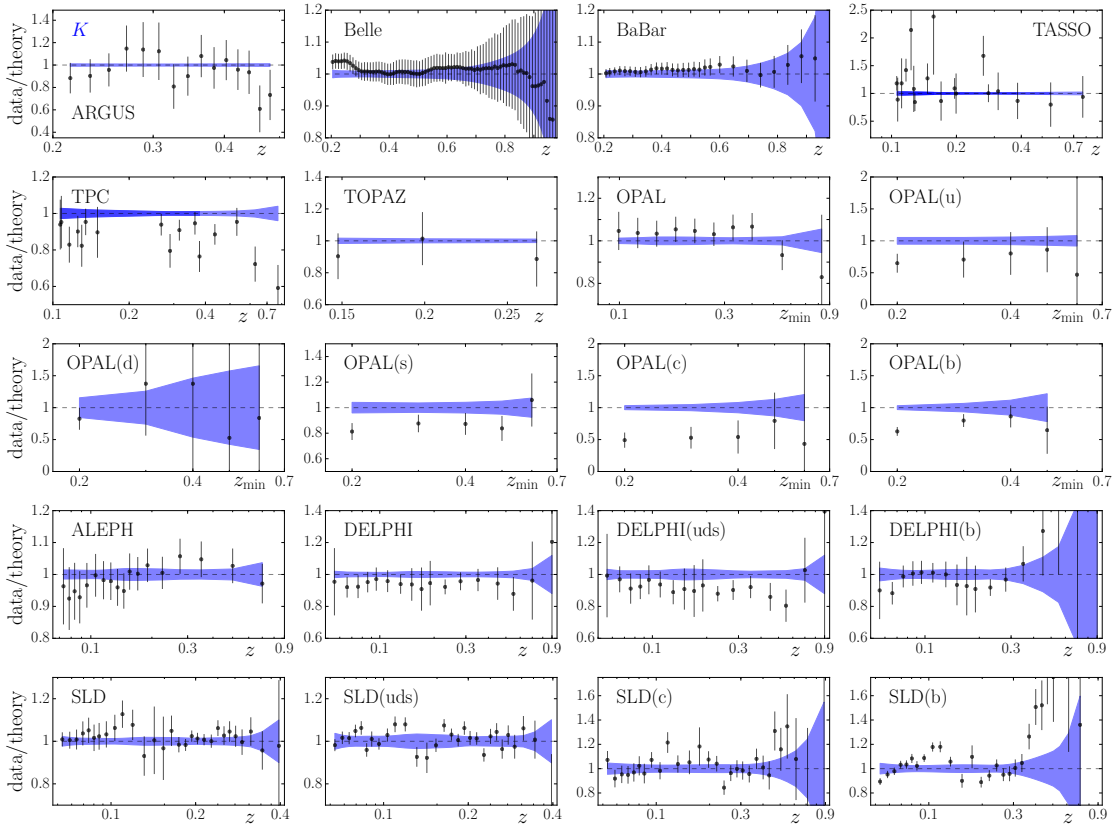

- $z_{\rm cut} > 0.2$ for low energies to avoid hadron mass corrections

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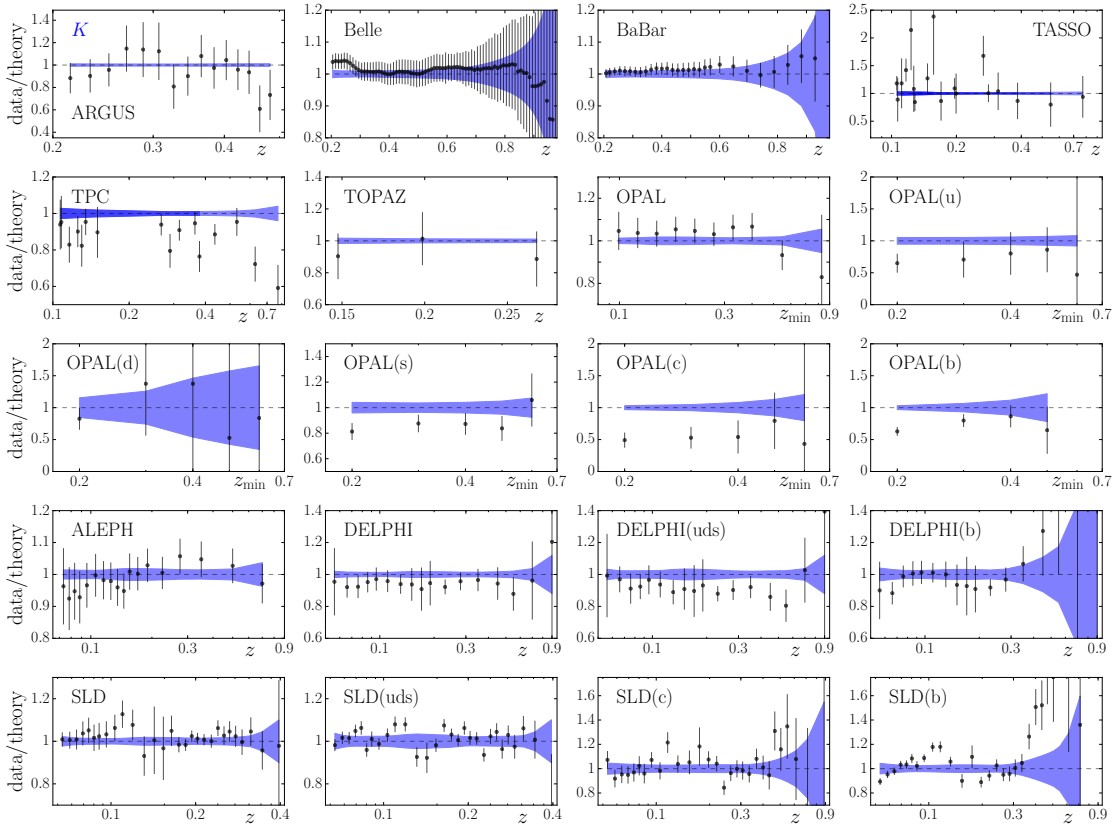

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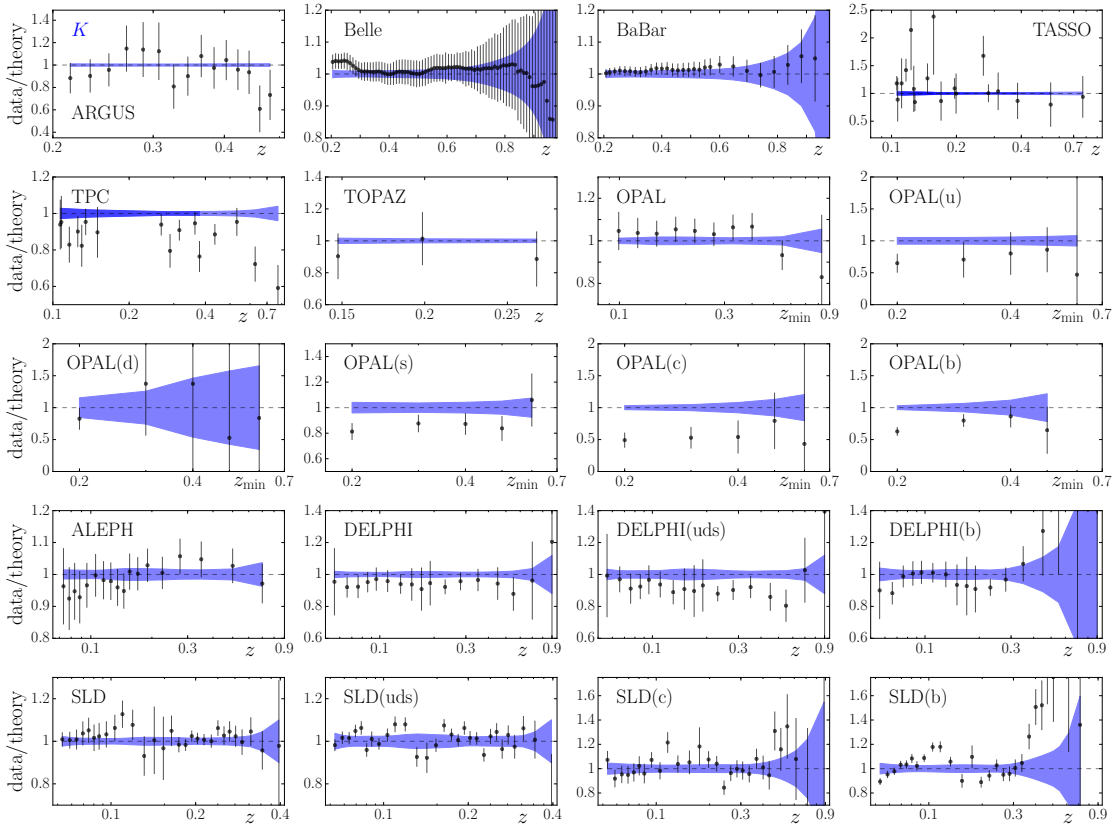

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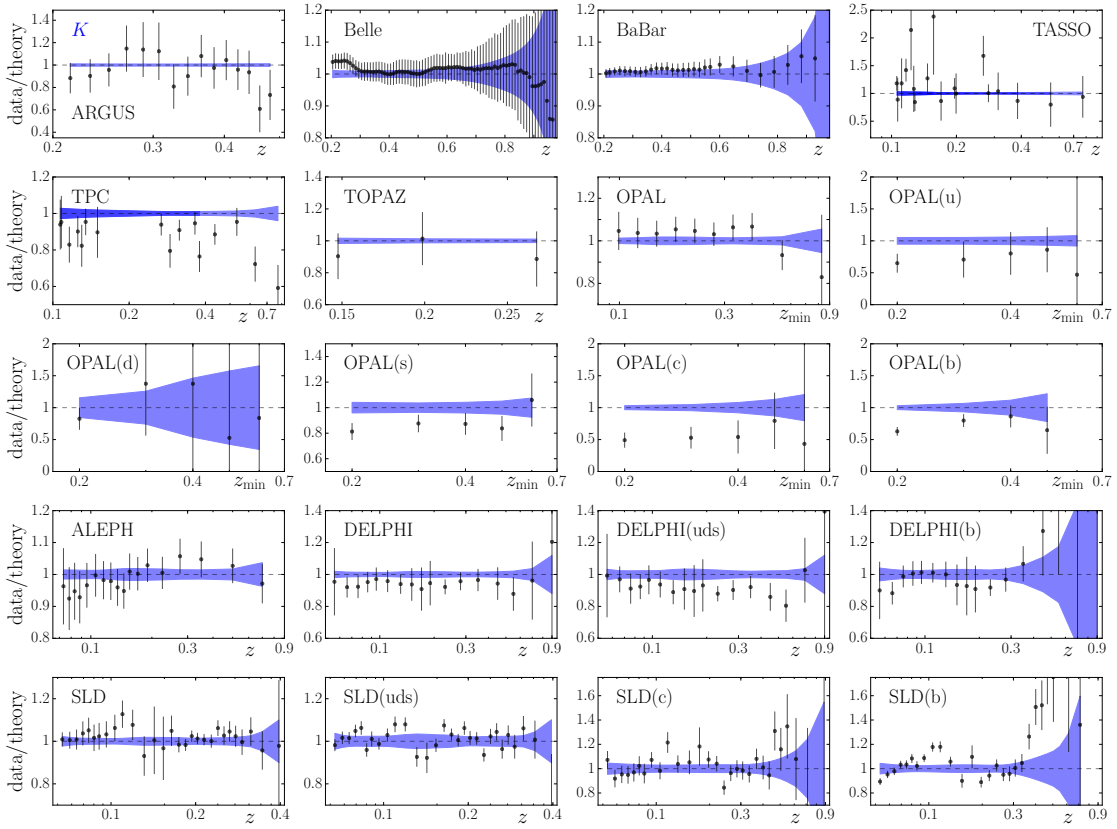

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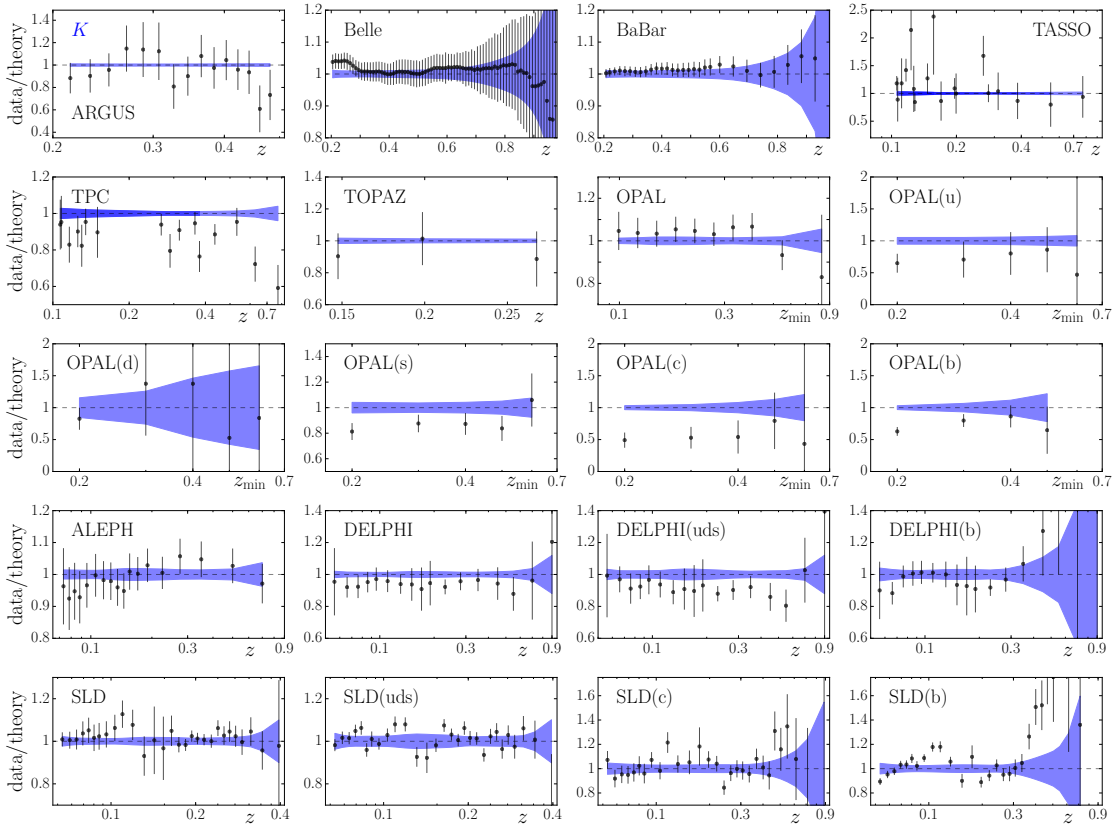

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- Consistent shapes across all z
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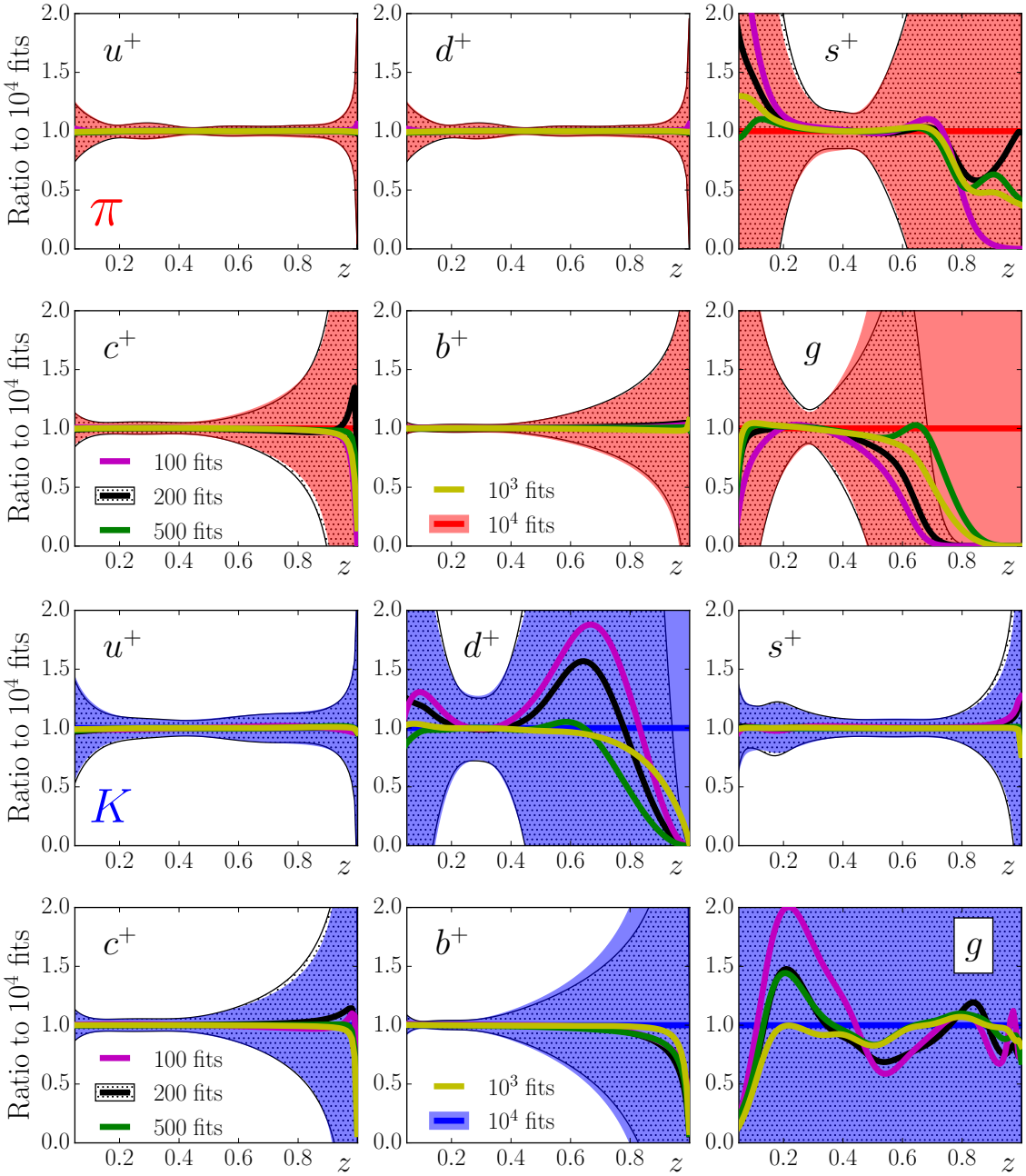
Fragmentation functions



- Larger constraints on favored FFs than unfavored FFs

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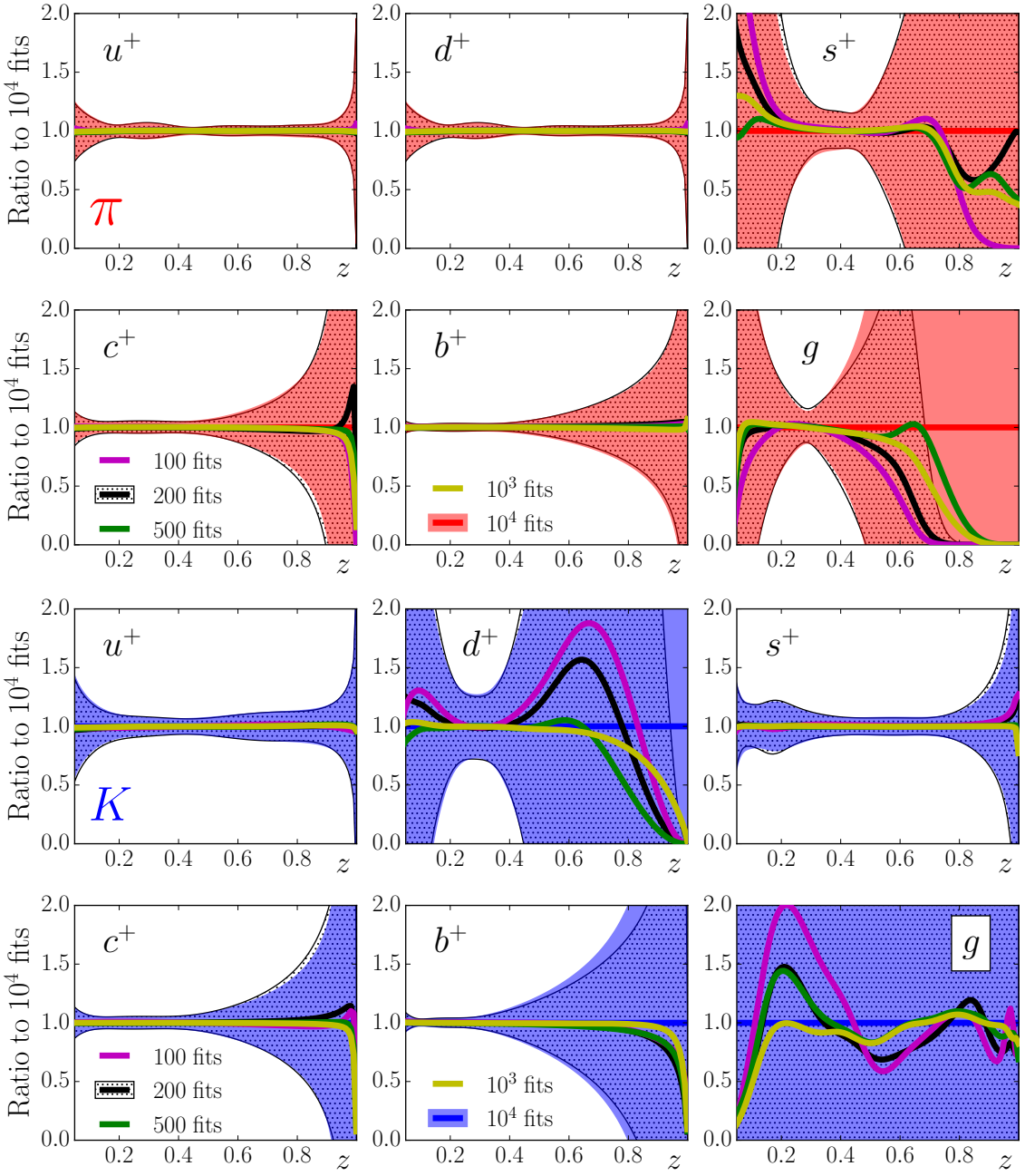
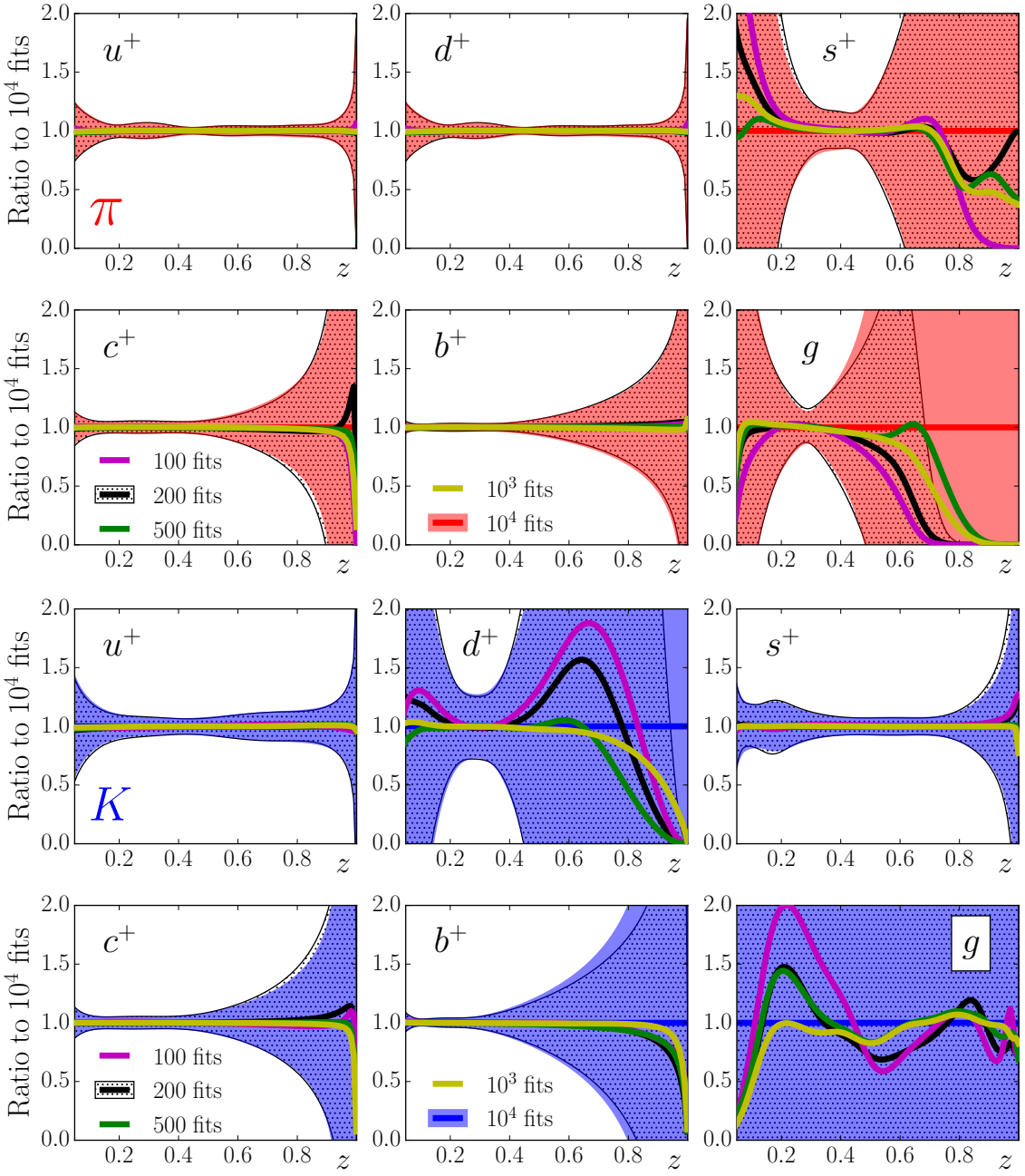


- Larger constraints on favored FFs than unfavored FFs
- More sensitivity on D_g^π than D_g^K

Fragmentation functions



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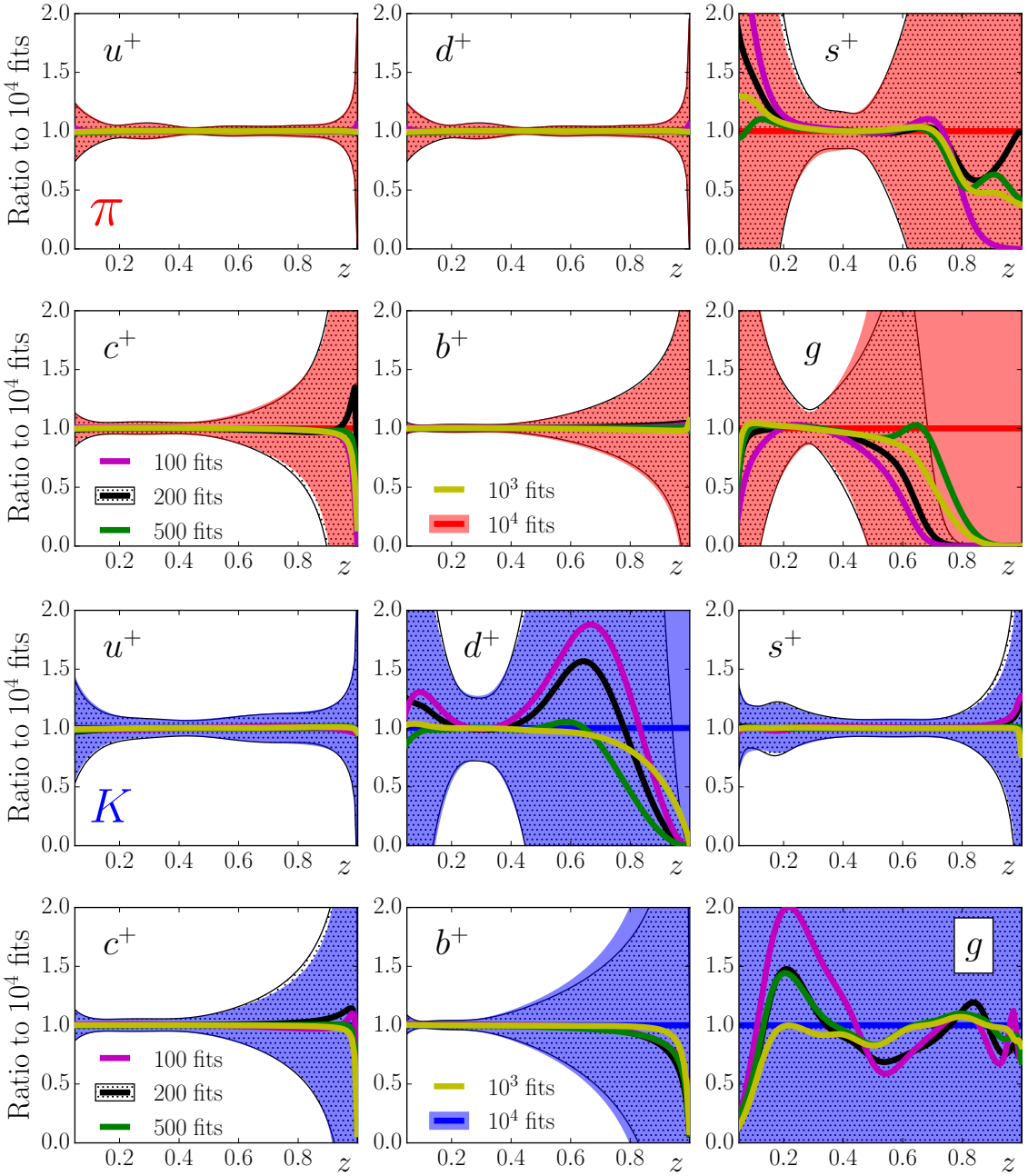

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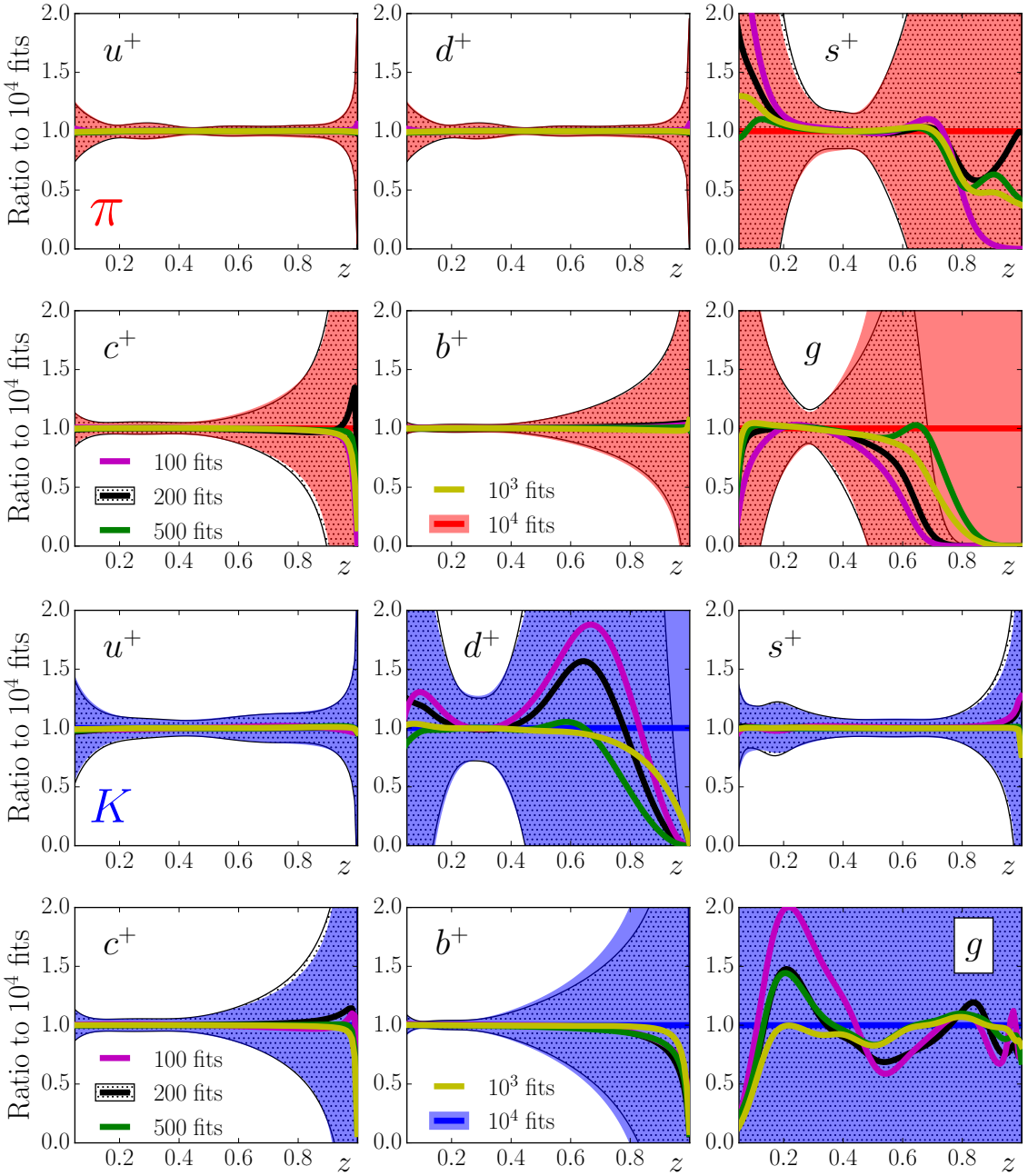

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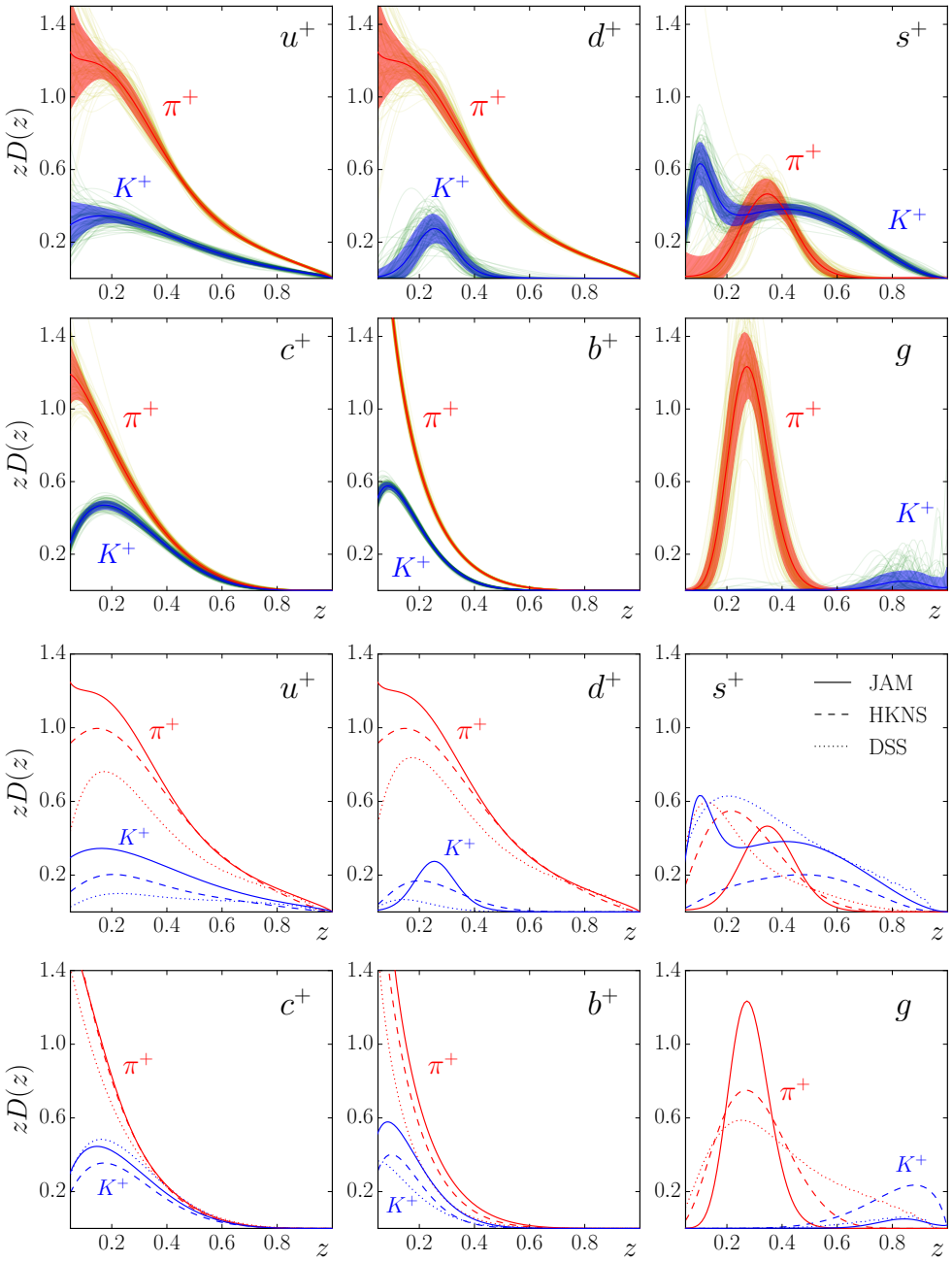
- Convergence attained with ~ 200 posteriors

Fragmentation functions



- Similar behavior of unfavored $D^{\pi}_{s^+}$ and $D^{K}_{d^+}$

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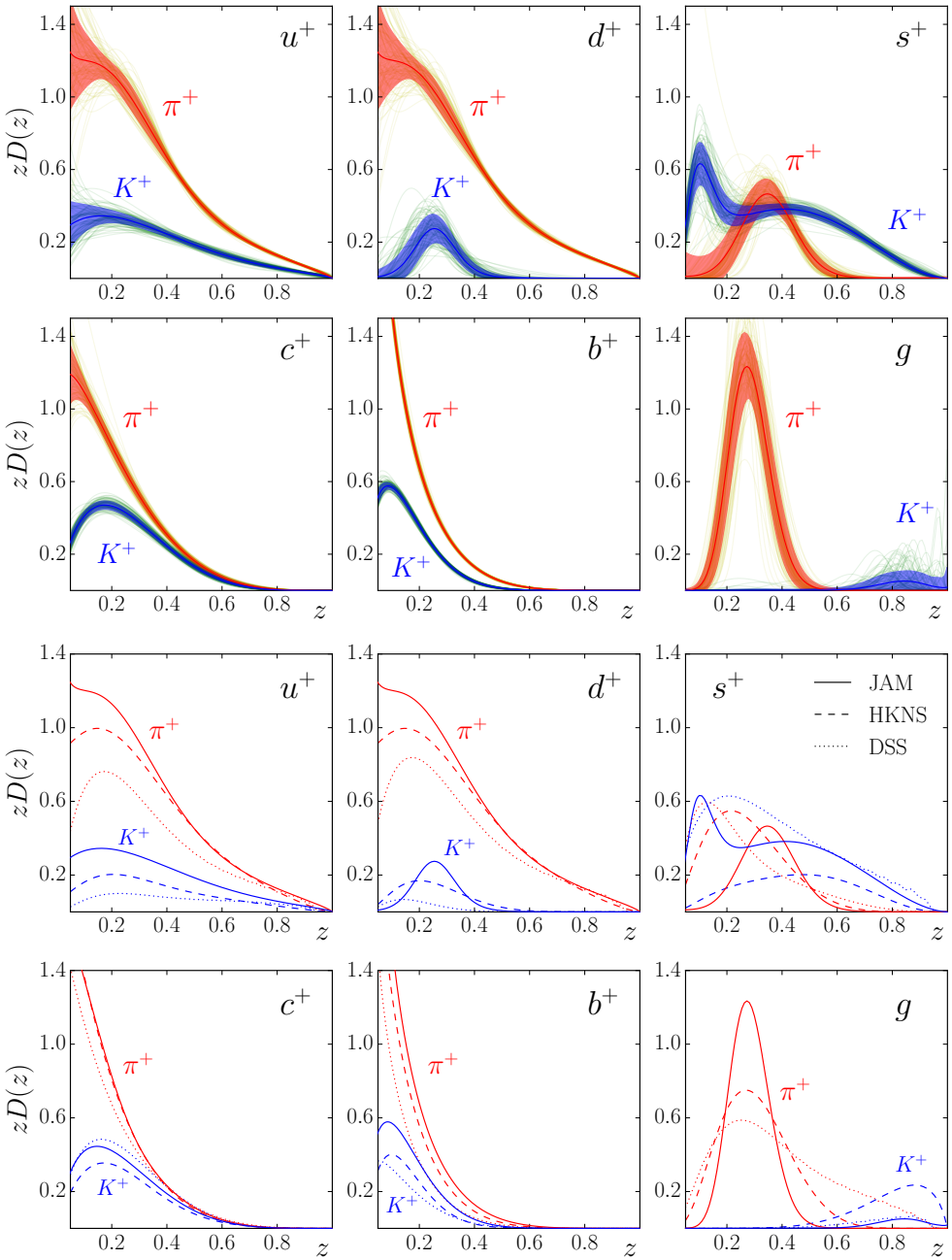

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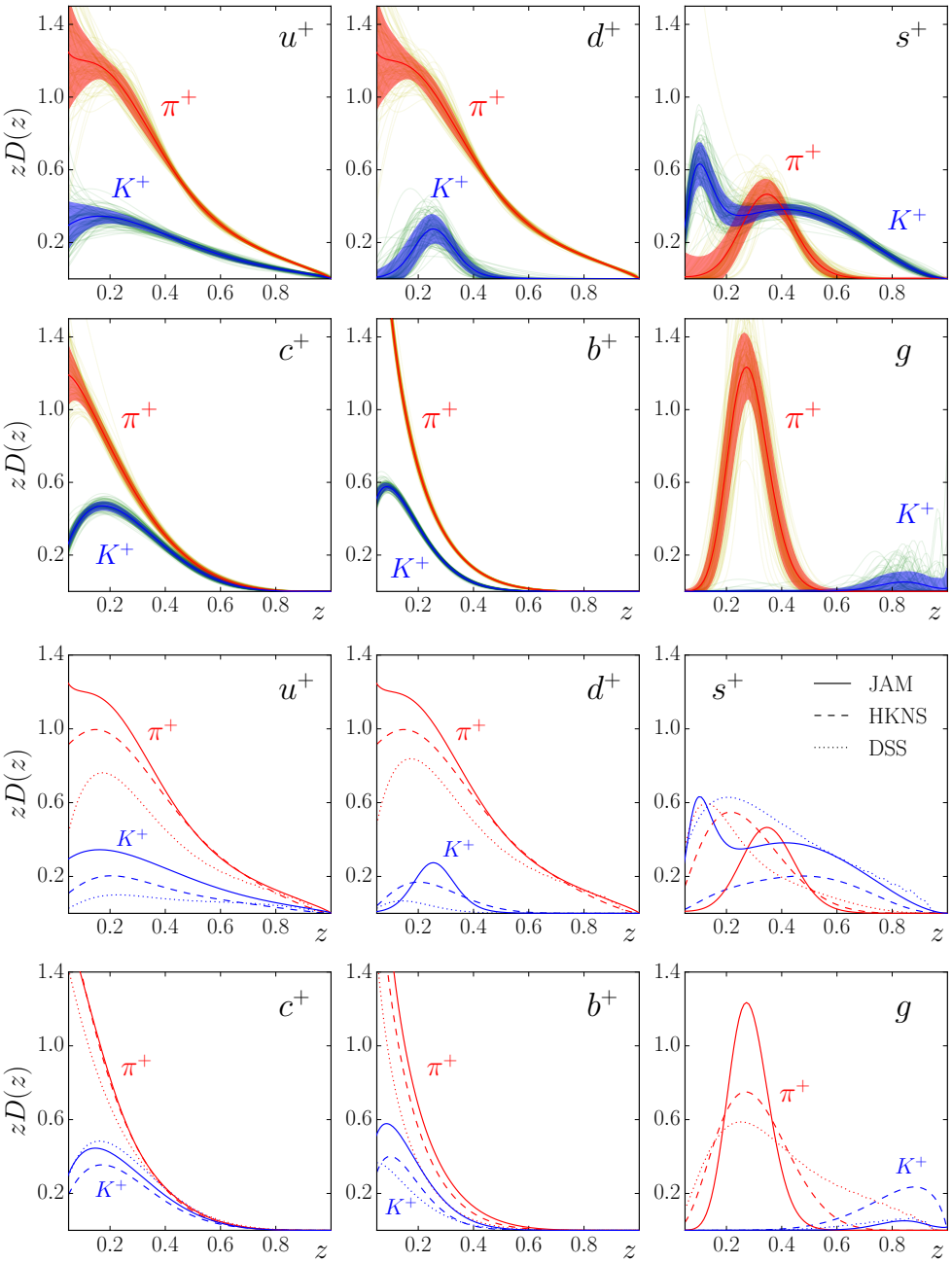

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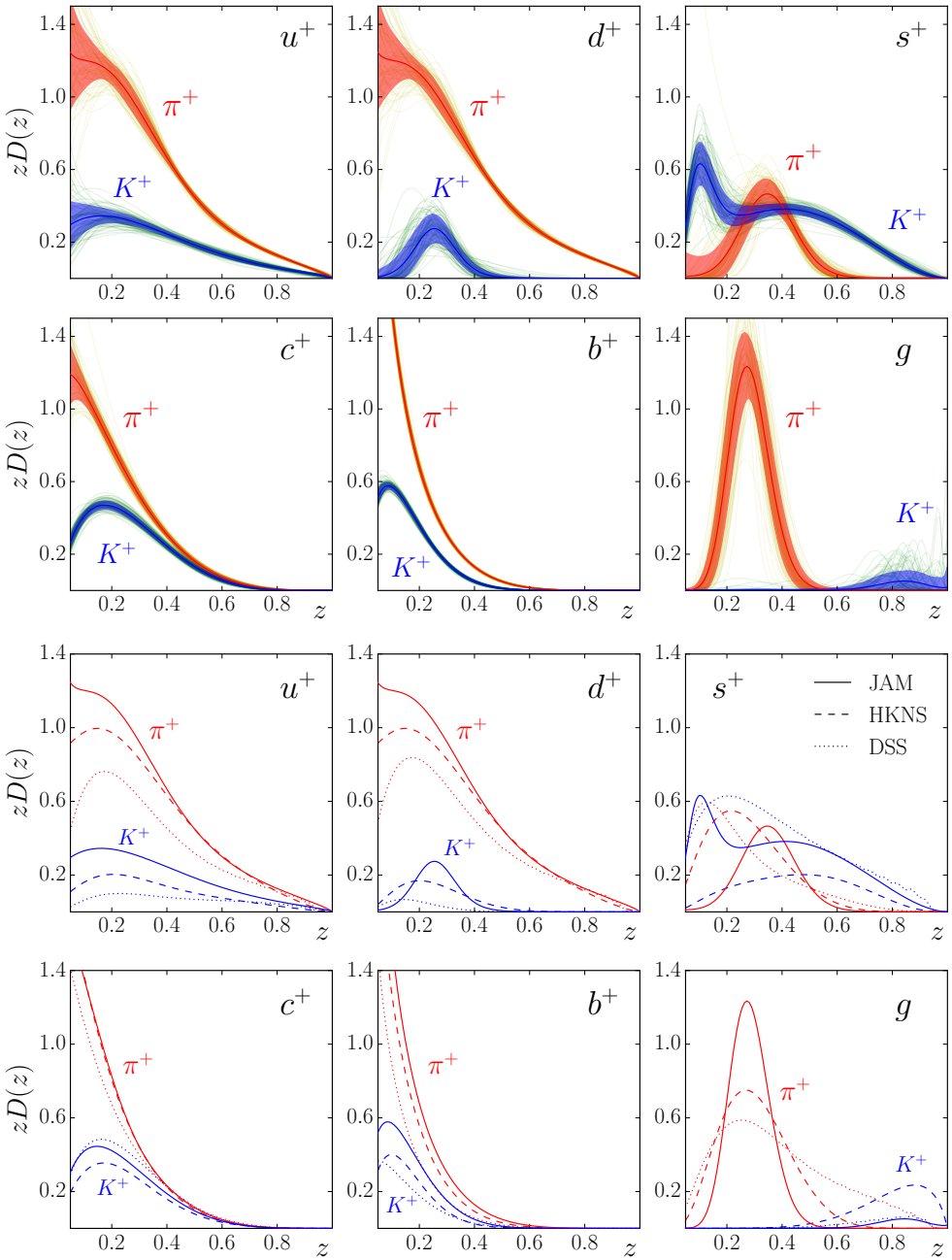

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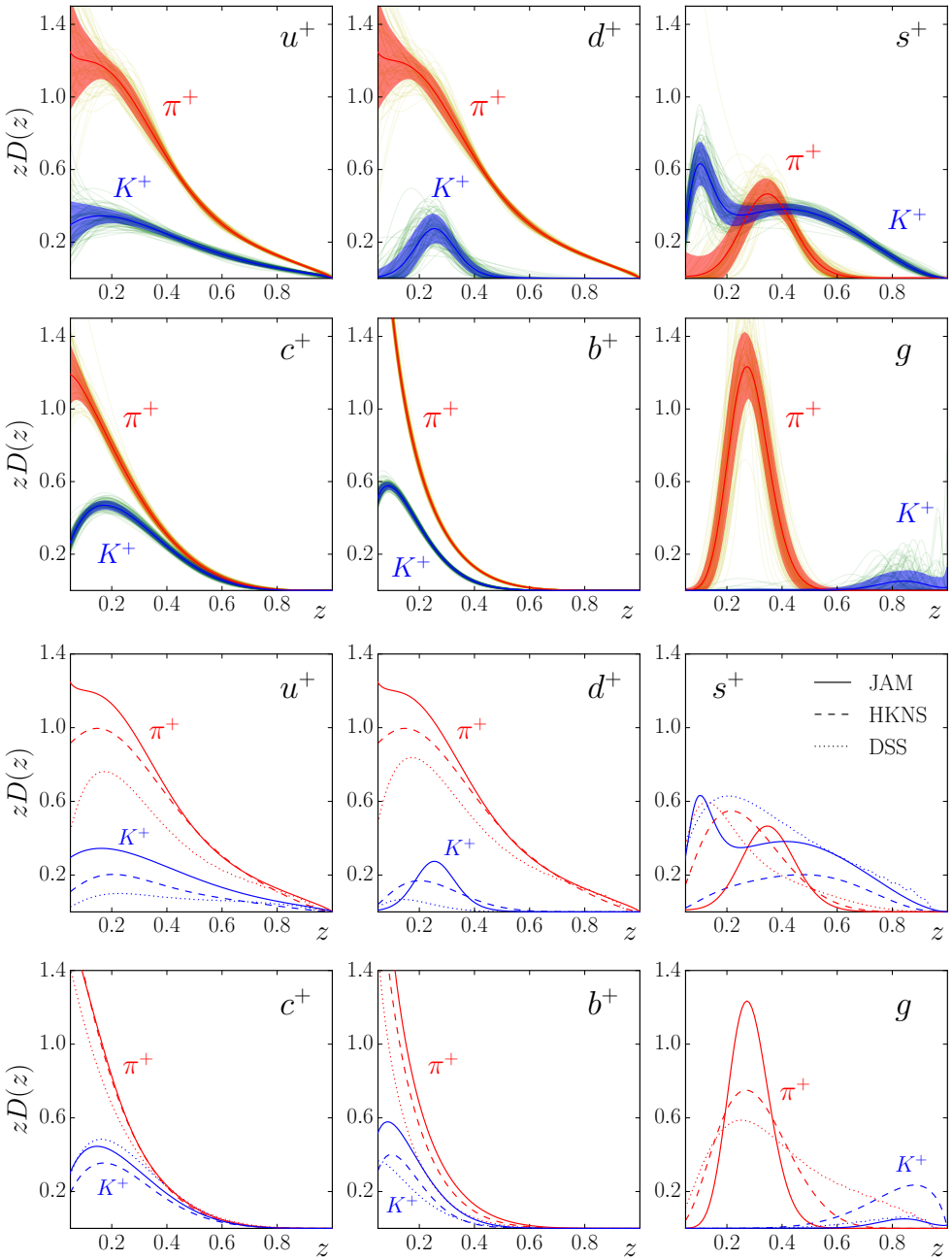

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- Favored $D^{\pi}_{u^+}$ and $D^{K}_{s^+}$ have similar shape at large–z
- JAM $D^{K}_{s^+}$ is more similar to DSS. **Will it change the sign of Δs^+?** (see talk by J.Ethier on Wed. WG1)

Summary and outlook

- **The collinear master plan:** Baseline PDFs and FFs needed for TMD analysis
  - New MC fitting methodology
  - New IMC analysis for polarized PDFs and FFs
  - Combine polarized PDFs and FF analysis (see talk by J. Ethier)
  - Universal analysis for all collinear distributions (ultimate goal)

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- **MCEG for SIDIS:** (talk by N. Sato @ WG7)
  - Pythia8 validation of Hermes multiplicities (in progress)
  - Extraction of FFs from Pythia8: test of DGLAP and Pythia8's parton shower