

The growth with energy of vector meson photo-production cross-sections and low x evolution

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based on:

I. Bautista, A. Fernandez Tellez, MH.;
[arXiv:1607.05203] (PRD **94** 054002)



Outline

Introduction

Ingredients of our study

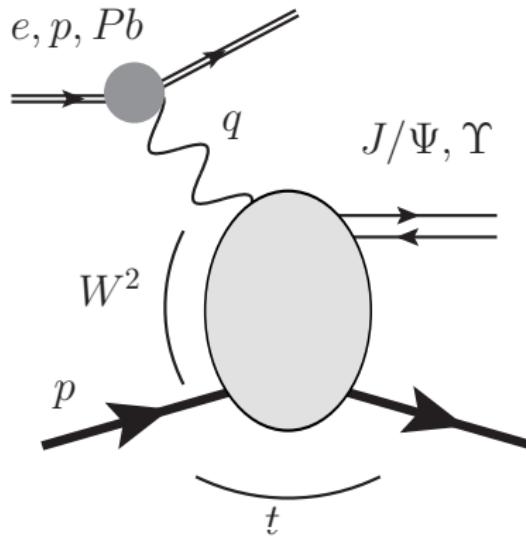
NLO BFKL gluon density

Impact factor $\gamma \rightarrow V$

Amplitude: Real part from imaginary part

Results & Conclusions

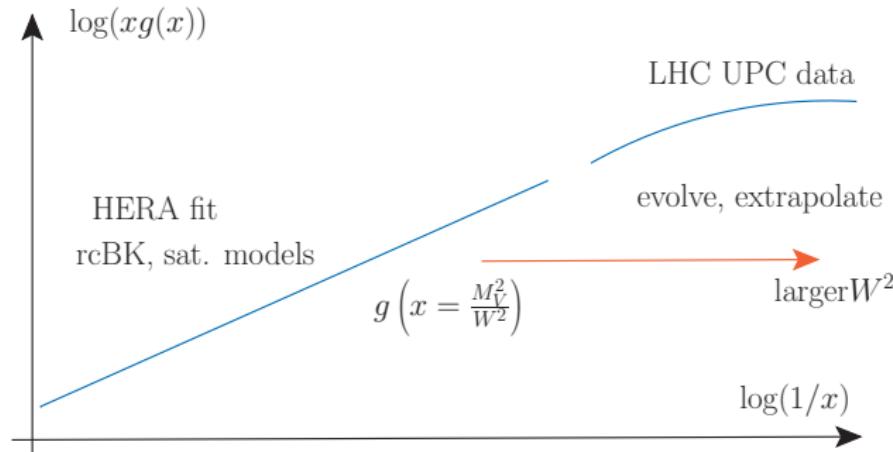
photo-production of J/Ψ and Υ : explore proton at ultra-small x



- ▶ measured at HERA (ep) and LHC (pp , ultra-peripheral pPb)
- ▶ charm and bottom mass provide hard scale → pQCD
- ▶ exclusive process, but allows to relate to inclusive gluon

reach values down to $x = 4 \times 10^{-6} \rightarrow$ (unique ?) opportunity to explore the low x gluon

DGLAP vs. saturation (I)

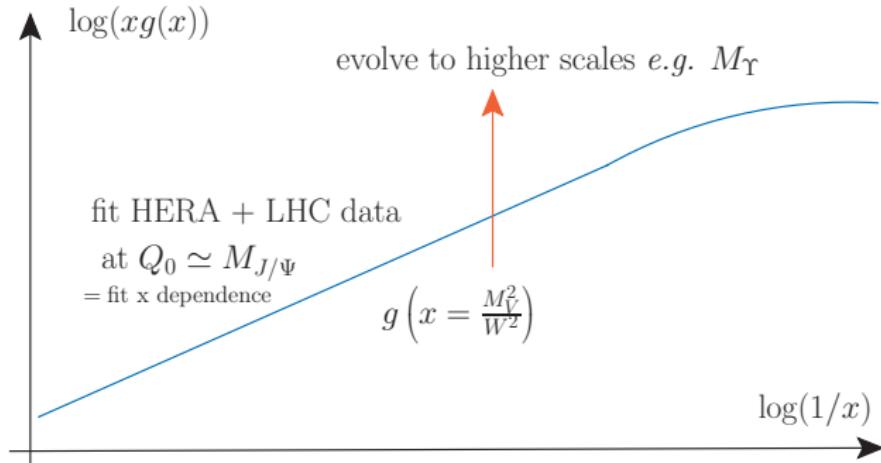


- ▶ describes data or not → re-fit
- ▶ if yes: do we really see saturation effects?

i.e. BK type evolution

$$\frac{d}{d \ln 1/x} G(x) = K \otimes G(x) - \underbrace{G \otimes G}_{\text{present, relevant?}}$$

DGLAP vs. saturation (II)



- ▶ $J/\Psi \rightarrow \Upsilon \simeq$ evolution $2.4 \text{ GeV}^2 \rightarrow 22.4 \text{ GeV}^2$
- ▶ high density effects die away in collinear limit
- ▶ DGLAP unstable at ultra-small x and small scales ...
- ▶ convinced: pdf studies highly valuable → constrain pdf's at ultra-small x
- ▶ useful benchmark for saturation searches (?)

Will argue in the following ...

- ▶ a far better dilute (!) benchmark might (?) be given by BFKL evolution (→ applies for UPCs@LHC, might be different for a future (US-)EIC ... → phase space!)
- ▶ why? BFKL \equiv low x evolution *without* high density/saturation effects
- ▶ available up to NLO [Fadin, Lipatov; PLB 429 (1998) 127]; [Ciafaloni, Camici; PLB 430 (1998) 349], resummation schemes for coll. logs exist & to some degree well explored [Salam; hep-ph/9806482], ...
- ▶ not only explored in $n = 0$ sector → additional constrains from e.g. angular decorrelation studies of jets → see talks on Wednesday

The framework of this BFKL study

procedure:

a) calculate diff. Xsec. at $t = 0$

→ exclusive scattering amplitude can be expressed through *inclusive* gluon distribution

b) parametrize t dependence $\frac{d\sigma(t)}{dt} = \frac{d\sigma(t=0)}{dt} \cdot e^{-|t|B_D(W)}$,

slope $B_D(W) = b_0 + 4\alpha' \ln \frac{W}{W_0}$ + fix parameters by (HERA) data
 (here: values proposed by [Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795])

→ cross-section: $\sigma^{\gamma p \rightarrow Vp}(W) = \underbrace{\frac{1}{B_D(W)}}_{\text{phenomenological}} \underbrace{\left. \frac{d\sigma^{\gamma p \rightarrow Vp}}{dt} \right|_{t=0}}_{\text{BFKL / theory}}$

The setup: diff. Xsec. at $t = 0$

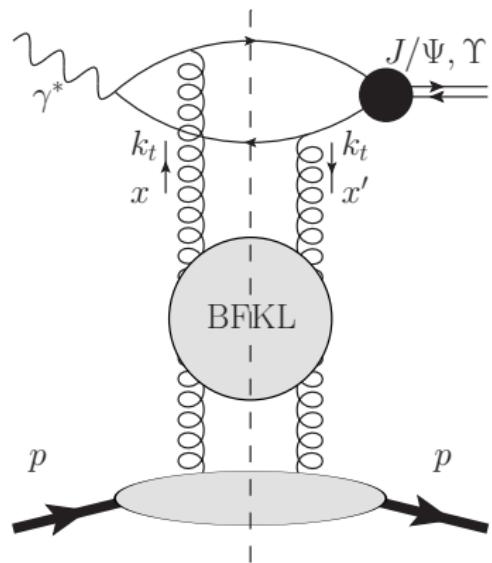
a) imaginary part of scattering amplitude:

- ▶ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ▶ impact factor $\gamma \rightarrow J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies

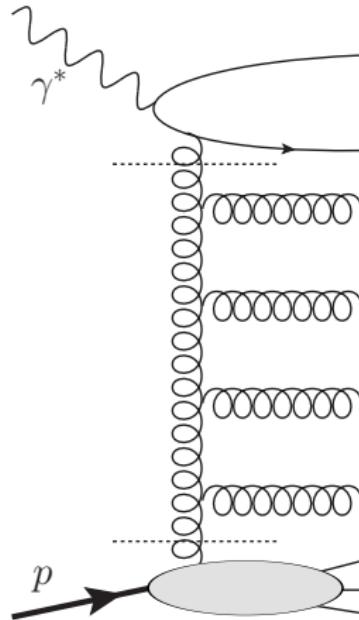
[Kowalski, Motyka, Watt; hep-ph/0606272]

b) real part:

- ▶ $\Im m\mathcal{A}(W^2, t)$ dominant, real part can be numerically large
 - recover real part using dispersion relation



The underlying NLO BFKL fit to DIS data



$$F_2(x, Q^2) = \int_0^\infty dk^2 \int_0^\infty \frac{d\mathbf{q}^2}{\mathbf{q}^2} \Phi_2 \left(\frac{\mathbf{k}^2}{Q^2} \right) \mathcal{F}_{\text{BFKL}}^{\text{DIS}}(x, \mathbf{k}^2, \mathbf{q}^2) \Phi_p \left(\frac{\mathbf{q}^2}{Q_0^2} \right)$$

virtual photon: quarks mass-less, $n_f = 4$ fixed

$$\text{proton impact factor: } \Phi_p \left(\frac{\mathbf{q}^2}{Q_0^2}, \delta \right) = \frac{\mathcal{C}}{\pi \Gamma(\delta)} \left(\frac{\mathbf{q}^2}{Q_0^2} \right)^\delta e^{-\frac{\mathbf{q}^2}{Q_0^2}}$$

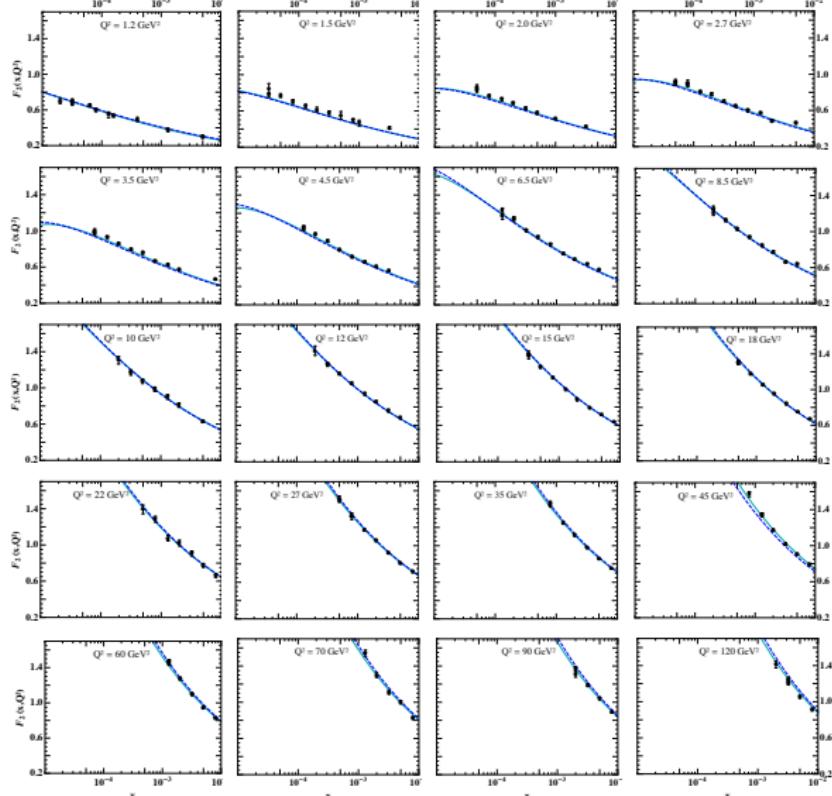
free parameters of proton impact factor from fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

→ allows for definition of unintegrated gluon density
 [Chachamis, Deak, MH, Rodrigo, Sabio Vera; 1507.05778]

$$G(x, \mathbf{k}^2, Q_0^2) = \int \frac{d\mathbf{q}^2}{\mathbf{q}^2} \mathcal{F}_{\text{BFKL}}^{\text{DIS}}(x, \mathbf{k}^2, \mathbf{q}^2) \Phi_p \left(\frac{\mathbf{q}^2}{Q_0^2} \right)$$

	virt. photon impact factor	Q_0/GeV	δ	\mathcal{C}	$\Lambda_{\text{QCD}}/\text{GeV}$
fit 1	leading order (LO)	0.28	8.4	1.50	0.21
fit 2	LO with kinematic improvements	0.28	6.5	2.35	0.21

Good description of combined HERA [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]



data: [H1 & ZEUS collab. 0911.0884]

Solve BFKL equation in conjugate (γ) Mellin space

$$G(x, \mathbf{k}^2, M) = \frac{1}{\mathbf{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\bar{M}^2}{M^2}, \gamma\right) \left(\frac{\mathbf{k}^2}{Q_0^2}\right)^\gamma$$

re-introduce two scales: hard scale of process (M) and scale of running coupling (\bar{M})

\hat{g} : operator in γ space!

$$\begin{aligned} \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\bar{M}^2}{M^2}, \gamma\right) &= \frac{\mathcal{C} \cdot \Gamma(\delta - \gamma)}{\pi \Gamma(\delta)} \cdot \left(\frac{1}{x}\right)^{\chi\left(\gamma, \frac{\bar{M}^2}{M^2}\right)}. \\ &\quad \left\{ 1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log\left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log \frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\}, \end{aligned}$$

resummed NLO BFKL eigenvalue with optimal scale setting (\rightarrow modifies $\chi_1(\gamma)$):

$$\begin{aligned} \chi\left(\gamma, \frac{\bar{M}^2}{M^2}\right) &= \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1(\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi'_0(\gamma) \chi_0(\gamma) \\ &\quad + \chi_{RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}) - \frac{\bar{\alpha}_s^2 \beta_0}{8N_c} \chi_0(\gamma) \log \frac{\bar{M}^2}{M^2}. \end{aligned}$$

Impact factor from boosted Gaussian wave function

use relation dipole amplitude \leftrightarrow unintegrated gluon e.g. [Kutak, Stasto; hep-ph/040811]

$$2 \int d^2 \mathbf{b} \mathcal{N}(x, r, b) = \frac{4\pi}{N_c} \int \frac{d^2 \mathbf{k}}{\mathbf{k}^2} \left(1 - e^{i\mathbf{k} \cdot \mathbf{r}}\right) \alpha_s G(x, \mathbf{k}^2).$$

$$\begin{aligned} \Im \mathcal{A}_T^{\gamma^* p \rightarrow V p}(W, t=0) &= 2 \int d^2 \mathbf{b} \int d^2 \mathbf{r} \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_T(r) \cdot \mathcal{N}(x, r, b) \\ &= \alpha_s (\overline{M} \cdot Q_0) \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \int_0^1 \frac{dz}{4\pi} \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, Q_0, \gamma\right) \cdot \Phi_{V,T}(\gamma, z, M) \cdot \left(\frac{M^2}{Q_0^2}\right)^\gamma \end{aligned}$$

yields (using boosted Gaussian wave-functions with Brodsky-Huang-Lepage prescription)

$$\begin{aligned} \Phi_{V,T}(\gamma, z, M) &= e \hat{e}_f 8\pi^2 \mathcal{N}_T \frac{\Gamma(\gamma)\Gamma(1-\gamma)}{m_f^2} \left(\frac{m_f^2 \mathcal{R}^2}{8z(1-z)}\right)^2 e^{-\frac{m_f^2 \mathcal{R}^2}{8z(1-z)}} e^{\frac{m_f^2 \mathcal{R}^2}{2}} \left(\frac{8z(1-z)}{M^2 \mathcal{R}^2}\right)^\gamma \\ &\quad \left[U\left(2-\gamma, 1, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)}\right) + [z^2 + (1-z)^2] \frac{(2-\gamma)}{2} U\left(3-\gamma, 2, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)}\right) \right], \end{aligned}$$

[$U(a, b, z)$ hypergeometric function of the second kind or Kummer's function]

Real part from imaginary part using dispersion relation ...

$$\mathcal{A}(W^2, t) = \xi^{(\tau=+)}(\lambda) \cdot \text{Im} \mathcal{A}(W^2, t), \quad \xi^{\tau=+}(\lambda) = \left(i + \tan \frac{\lambda\pi}{2} \right)$$

commonly used: $\lambda = \text{const}$ → constant ratio of real & imaginary part

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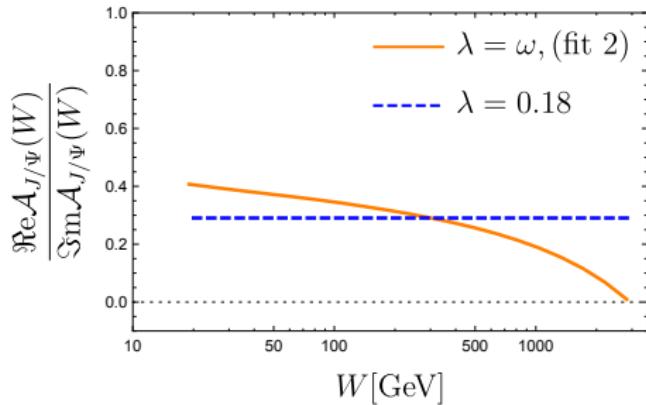
commonly used: $\lambda = \text{const}$ → constant ratio of real & imaginary part
more precise: reconstruct real part using ω -Mellin transform, $\omega \leftrightarrow W^2$:

$$\mathcal{A}(W^2, t) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x} \right)^\omega \left(i + \tan \frac{\omega\pi}{2} \right) a(\omega, t), \quad x = \frac{M_V^2}{W^2 - m_p^2}$$

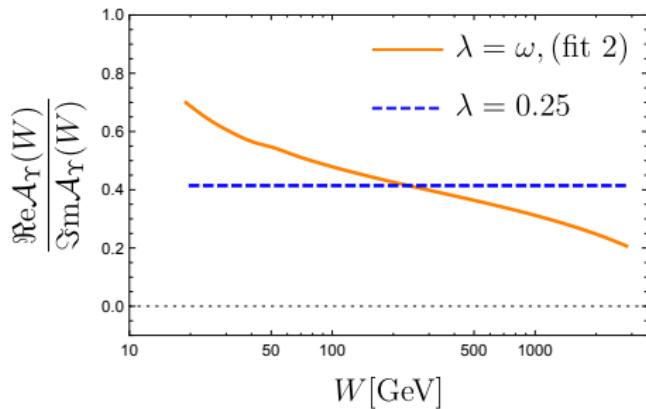
partial wave $a(\omega, t)$ can be fixed from imaginart part

$$a(\omega, 0) = \alpha_s \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{M^2}{Q_0^2} \right)^\gamma \int_0^1 \frac{dz}{4\pi} \Phi_{V,T}(\gamma, z) \frac{\mathcal{C} \cdot \Gamma(\delta - \gamma)}{\pi \Gamma(\delta)} \cdot \begin{Bmatrix} \frac{1}{\omega - \chi \left(\gamma, \frac{M^2}{M^2} \right)} \\ + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma) / (8N_c)}{\left[\omega - \chi \left(\gamma, \frac{M^2}{M^2} \right) \right]^2} \left[-\psi(\delta - \gamma) - \frac{d \ln [\Phi_{V,T}(\gamma, z)]}{d\gamma} \right] \end{Bmatrix}$$

energy dependence of $r(W) = \text{Re}\mathcal{A}/\text{Im}\mathcal{A}$

 $J/\Psi:$ 

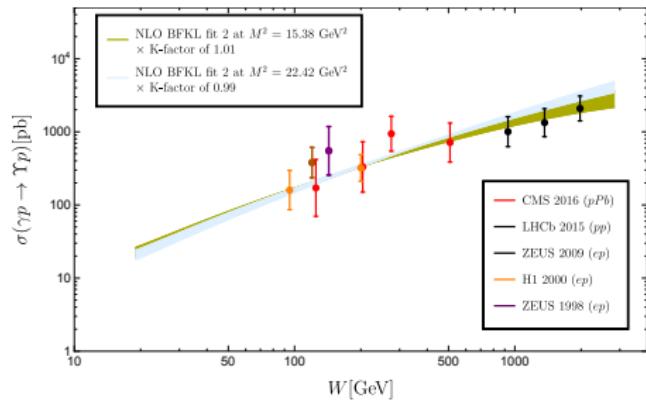
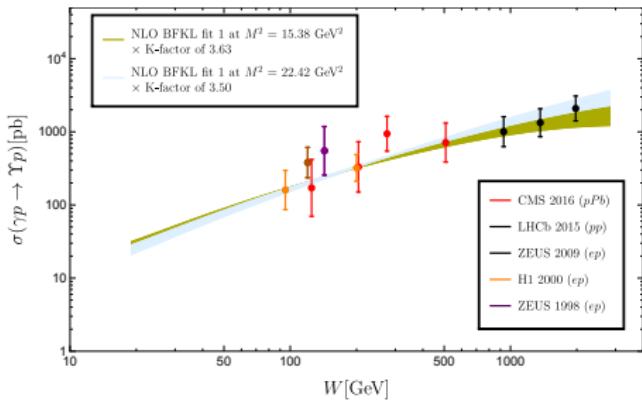
ratio decreases with energy

 $\Upsilon:$ 

sizeable effect on overall
energy dependence
enhances change of λ with
energy

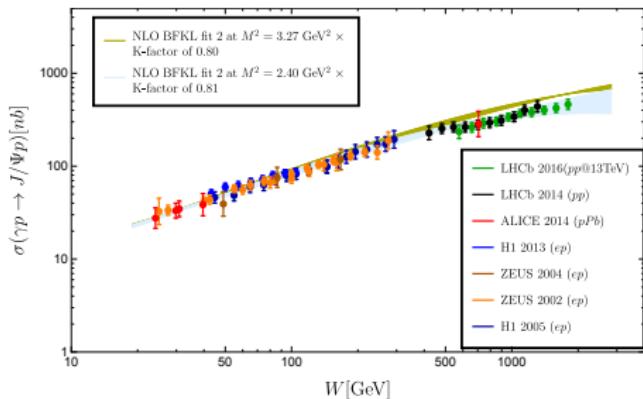
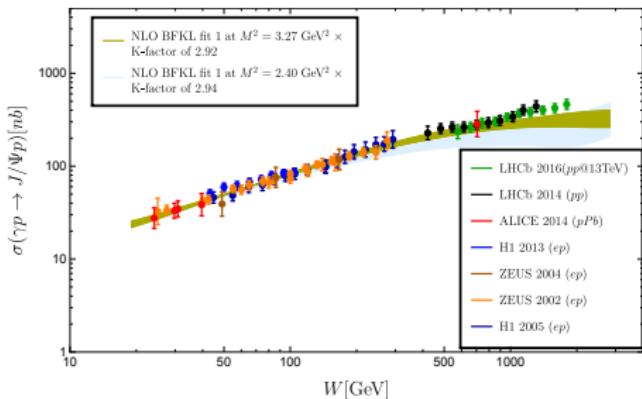
→ slows down the growth

comparison to data: Υ production



- ▶ provide study for two hard scales:
photoproduction scale: $M_{pp} = M_V/2$
impact factor motivated: $M_{if}^2 = 8R_V^{-2}$
- ▶ fix normalization by low energy H1 data point \rightarrow K-factor; no further adjustments

comparison to data: J/Ψ production



- NEW (wrt. [\[Bautista, Fernando Tellez, MH; 1607.05203\]](#)): 13 TeV LHCb data
- fix normalization by low energy ALICE data point \rightarrow K-factor
believe: related to HERA fit (massless, $n_f = 4$, $(\mathcal{C}_1/\mathcal{C}_2)^2 = 2.45$)
- often included (not here): GPD motivated factor (" $x' \neq x$ "); known for collinear [\[Shuvaev, Golec-Biernat, Martin, Ryskin, hep-ph/9902410\]](#)
 - to be calculated for k_T factorized BFKL impact factor
 - \sim kinematic improvements for $\gamma \rightarrow V$

Concluding remarks

BFKL fits [MH, Salas, Sabio Vera; 1301.5283] somehow approach its limits, but so far works very well → keep in mind: *most simple combination of existing NLO BFKL fit & existing VM impact factor*

first suggestion: no need for saturation effects, linear NLO evolution sufficient

why so hard to manifest saturation? two possible reasons:

- a) BFKL simply appropriate framework,... saturation effects not (yet) present
- b) observable $\sim \mathcal{N}^{\text{dipole}} \Leftrightarrow G_{\text{ugd}}^{\text{BFKL}} \Rightarrow$ high density effects (if at all present) only through evolution
→ but evolve not even an order of magnitude w.r.t. HERA data;

A possible way out ...

observables with higher order correlators of Wilson lines
→ inclusive observables (no gap) + resolved final states
(e.g. inclusive di- & tri-hadrons/jets)

$$\mathcal{N} \sim 1 - \frac{1}{N_c} \text{tr} [V(\mathbf{x}) V^\dagger(\mathbf{y})] \quad \leftrightarrow \quad G^{\text{BFKL}}(x, \mathbf{k})$$

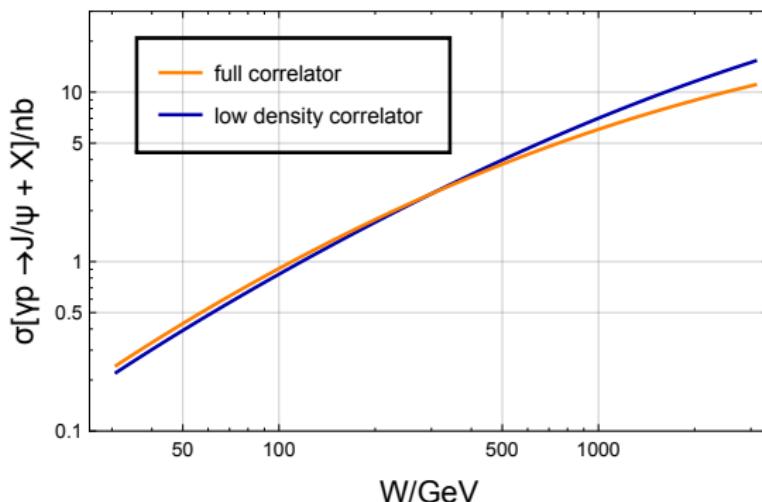
$$Q^{(4)} \sim 1 - \frac{1}{N_c} \text{tr} [V(\mathbf{x}) V^\dagger(\mathbf{y}) V(\mathbf{y}') V^\dagger(\mathbf{x}')] \quad \leftrightarrow \quad G + \#G^2 + \#G^4 + \dots$$

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prelim. study using
bCGC model

... work in progress,
no conclusion to be
drawn yet

Supplementary material

Pomeron: effective degree of freedom which describes the rise of cross-sections with energy

BFKL Pomeron:

microscopic description in terms of quarks & gluons

\rightarrow requires process with hard scale $Q^2 \gg Q_0^2 \Rightarrow \alpha_s(Q^2) \ll 1$

requires:

- expansion of perturbative amplitudes in $1/s$
- + resummation of enhanced terms $(\alpha_s(Q^2) \ln s)^n \sim 1$ to all orders in α_s

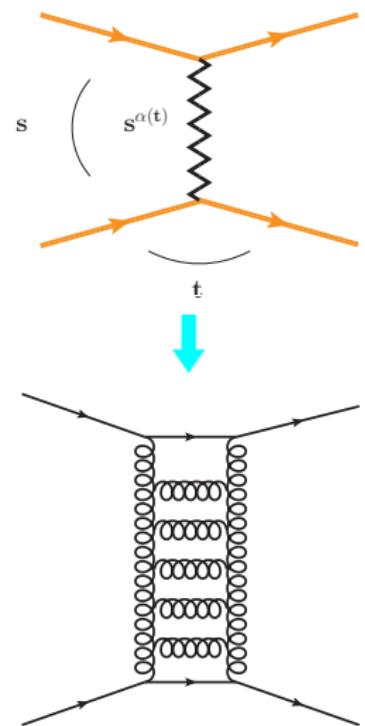
→ BFKL equation

LL: [Fadin, Kuraev, Lipatov, PLB 60 (1975) 50]

[Balitsky, Lipatov, SJNP (1978) 822]

NLL: [Fadin, Lipatov; PLB 429 (1998) 127];

[Ciafaloni, Camici; PLB 430 (1998) 349]



Phenomenology of BFKL evolution

- ▶ at LHC: first success in the description of angular decorrelation of multi-jet observables

[Ducloué, Szymanowski, Wallon; 1312.2624], [Celiberto, Ivanov, Murdaca, Papa; 1504.08233]

[Caporale, Chachamis, Murdaca, Sabio Vera; 1508.07711]

→ see talks by Francesco G. Celiberto

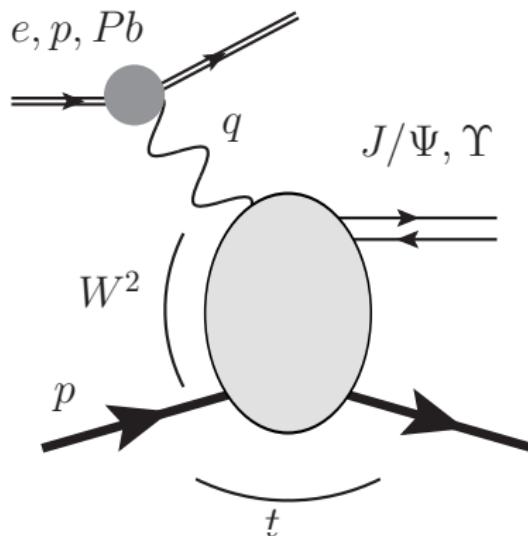
- ▶ test conformal spin $n \neq 0$ components of the BFKL kernel interesting in its own right, perturbatively stable & test of the underlying framework
- ▶ $n = 0$ component: rise of perturbative cross-sections
→ essentially only studied in fits to inclusive HERA DIS data

[Kowalski, Lipatov, Ross; 1005.0355; 1205.6713]

[Hentschinski, Salas, Sabio Vera; 1209.1353, 1301.5283]

[Levin, Potashnikova; 1307.7823]

Description within BFKL framework



- ▶ exclusive process: vacuum quantum # exchange between VM & proton
- ▶ Appropriate theoretical framework: non-forward BFKL $t \neq 0$
- ▶ kernels known up to NLO, but not explored in phenomenological studies & not sufficiently well understood how to use them

use procedure of e.g. DGLAP study [Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795]
 → relate *exclusive* photo-production to *inclusive* gluon distribution

Solve BFKL equation in conjugate (γ) Mellin space

$$\begin{aligned} \chi_{RG}(\bar{\alpha}_s, \gamma, a, b) = & \bar{\alpha}_s(1 + a\bar{\alpha}_s)(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)) - \frac{\bar{\alpha}_s^2}{2}\psi''(1 - \gamma) - \frac{b\bar{\alpha}_s^2 \cdot \pi^2}{\sin^2(\pi\gamma)} \\ & + \frac{1}{2} \sum_{m=0}^{\infty} \left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1 + a\bar{\alpha}_s)}{1 - \gamma + m} + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1 + a\bar{\alpha}_s)} \right) \end{aligned}$$

resums (anti-) collinear ‘logs’ (= γ -poles) of $\bar{\alpha}_s\chi_0(\gamma) + \bar{\alpha}_s\chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2\chi'_0(\gamma)\chi_0(\gamma)$

[Salam; [hep-ph/9806482](#)], [Sabio Vera; [hep-ph/0505128](#)]

optimal scale setting $\rightarrow \gamma$ -dependent running coupling

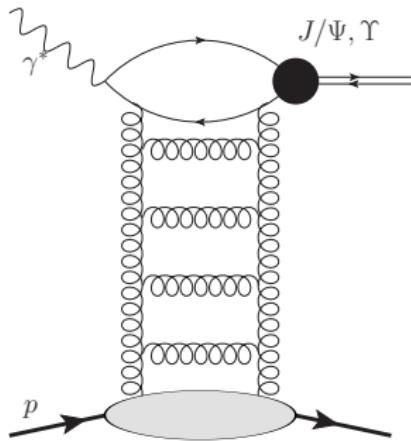
$$\bar{\alpha}_s(\overline{M} \cdot Q_0, \gamma) = \frac{4N_c}{\beta_0 \left[\log\left(\frac{\overline{M} \cdot Q_0}{\Lambda^2}\right) + \frac{1}{2}\chi_0(\gamma) - \frac{5}{3} + 2\left(1 + \frac{2}{3}Y\right) \right]},$$

also use parametrization of running coupling in the infra-red [Webber; [hep-ph/9805484](#)]

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}} + f\left(\frac{\mu^2}{\Lambda^2}\right), \quad f\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{4\pi}{\beta_0} \frac{125 \left(1 + 4\frac{\mu^2}{\Lambda^2}\right)}{\left(1 - \frac{\mu^2}{\Lambda^2}\right) \left(4 + \frac{\mu^2}{\Lambda^2}\right)^4},$$

relate 2 pictures of the BFKL Pomeron

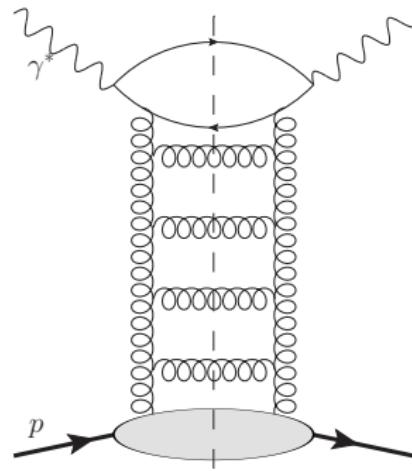
a) exclusive photo-production of vector mesons:



'uncut' Pomeron: diffractive/elastic scattering (amplitude level)

$$\mathcal{A}(s, t)$$

b) proton structure functions:



'cut' Pomeron: high multiplicity events (total X-sec.)

$$\sigma_{\text{tot}} = \frac{1}{s} \Im m \mathcal{A}(s, t = 0)$$

Vector mesons & dipole models ...

factorization into light-front wave function & dipole amplitude

e.g. [Kowalski, Motyka, Watt; hep-ph/0606272]

$$\Im \mathcal{M}_{T,L}^{\gamma^* p \rightarrow V p}(W, t=0) = 2 \int d^2 \mathbf{r} \int d^2 \mathbf{b} \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_{T,L} \mathcal{N}(x, r, b),$$

light-front wave function overlap

$$(\Psi_V^* \Psi)_T = \frac{\hat{e}_f e N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(m_f r) \phi_T(r, z) - [z^2 + (1-z)^2] m_f K_1(m_f r) \partial_r \phi_T(r, z) \right\}$$

scalar parts of VM wave function: boosted Gaussian wave-functions with Brodsky-Huang-Lepage prescription

$$\phi_T^{1s}(r, z) = \mathcal{N}_T z(1-z) \exp \left(-\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2} \right).$$

free parameters fixed through normalization condition & leptonic decay width $\Gamma_{e^- e^+}$:

Meson	m_f/GeV	\mathcal{N}_T	$\mathcal{R}^2/\text{GeV}^{-2}$	M_V/GeV	$8\mathcal{R}^{-2}/\text{GeV}^2$	$\frac{1}{4} M_V^2/\text{GeV}^2$
J/ψ	$m_c = 1.27$	0.596	2.45	3.097	3.27	2.40
Υ	$m_b = 4.2$	0.481	0.57	9.460	15.38	22.42

use parameters obtained by [Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1408.1344]

Existing description of data

... work pretty well

- ▶ J/Ψ : power-law fit to HERA data $\sigma \sim W^{0.67}$ [LHCb Collaboration; 1401.3288]
- ▶ collinear factorization: NLO fits [Jones, Martin, Ryskin, Teubner; 1307.7099]
- ▶ saturation models: IPsat, bCGC, rcBK
[Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1405.6977, 1408.1344]
- ▶ see also [Fiore, Jenkovszky, Libov, Machado; 1408.0530], [Cisek, Schäfer, Szczerba; 1405.2253]

BFKL special:

don't fit W -dependence, but calculate from perturbative low x evolution
don't evoke saturation (= effects beyond BFKL)

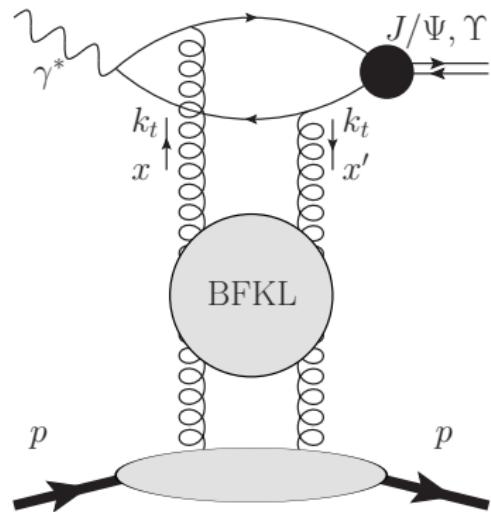
LHC: reach ultra-small x values $\simeq 4 \cdot 10^{-6}$ not constrained by HERA

Comparison to data

- ▶ provide results for both HERA fits (standard (fit 1) & kinematic improved (fit 2) LO impact factor)
- ▶ hard scale M^2 :
 - photoproduction scale $M_{\text{pp}} = M_V/2$
 - $(M_{\text{pp}}^2)_{J/\Psi} = 2.40 \text{ GeV}^2$
 - $(M_{\text{pp}}^2)_{\Upsilon} = 22.42 \text{ GeV}^2$
 - impact factor motivated: $M_{\text{if}}^2 = 8\mathcal{R}_V^{-2}$ – eliminates $(\dots)^{\gamma}$ factor & minimizes NLO running coupling correction related to impact factor
 - $(M_{\text{if}}^2)_{J/\Psi} = 3.27 \text{ GeV}^2$
 - $(M_{\text{if}}^2)_{\Upsilon} = 15.38 \text{ GeV}^2$
- ▶ (hard) running coupling scale $\overline{M} = M$, but vary in range $[M^2/2, M^2 \cdot 2]$ to check stability of result
- ▶ fix normalization by low energy ALICE (J/Ψ) and H1 (Υ) data point
 \rightarrow K-factor

Observations:

- ▶ K -factor: small for fit 2, sizeable for fit 1 – likely related to the impact factors in used in the HERA fit (massless, $n_f = 4$, $(\mathcal{C}_1/\mathcal{C}_2)^2 = 2.45$)
- ▶ common correction not included: GPD motivated factor to take into account $x' \neq x$; currently calculated for collinear pdf [Shuvaev, Golec-Biernat, Martin, Ryskin, hep-ph/9902410] → to be calculated for k_T factorized BFKL impact factor



very good description of W -dependence

$W_{J/\Psi} > 471 \text{ GeV} \& W_\Upsilon > 669 \text{ GeV} \equiv$ beyond region of incl. HERA fit
 (from $x = 4.3 \cdot 10^{-5}$ to $x = 3.5 \cdot 10^{-6}$) → direct test of BFKL evolution

Caveats

- ▶ both BFKL HERA fit & VM photoproduction use LO impact factor
→ large corrections at NLO possible
- ▶ BFKL HERA fit for $n_f = 4$ mass-less quarks

both effects should affect the normalization, not so much W -dependence

- ▶ unintegrated gluon density can develop instability at ultra-small x :

$$G(x, \mathbf{k}^2, M) = \frac{1}{\mathbf{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \hat{g}(x, \gamma) \left(\frac{\mathbf{k}^2}{Q_0^2} \right)^\gamma$$

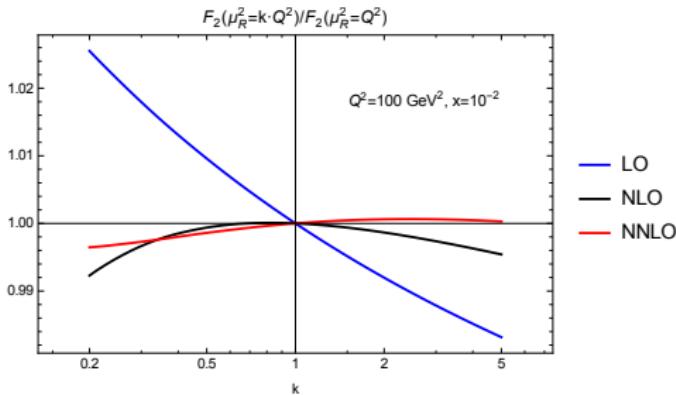
$$\hat{g}(x, \gamma) \sim \left(\frac{1}{x} \right)^{\chi\left(\gamma, \frac{M^2}{M^2}\right)} \cdot \left\{ 1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log\left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log \frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\},$$

- ▶ will enter at some point region $\alpha_s^2 \ln(1/x) \sim 1$ → control of such terms will become necessary

(physical) DGLAP evolution [MH, Stratmann; arXiv:1311.2825, to be completed]

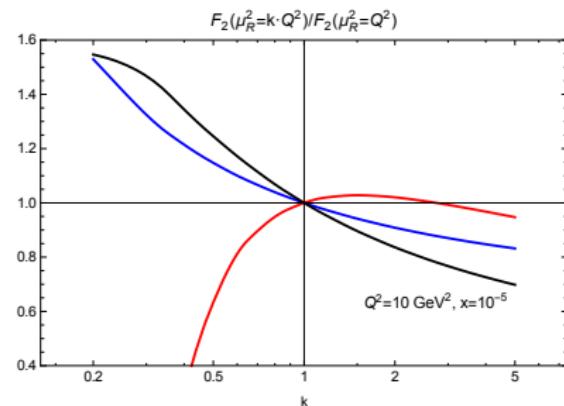
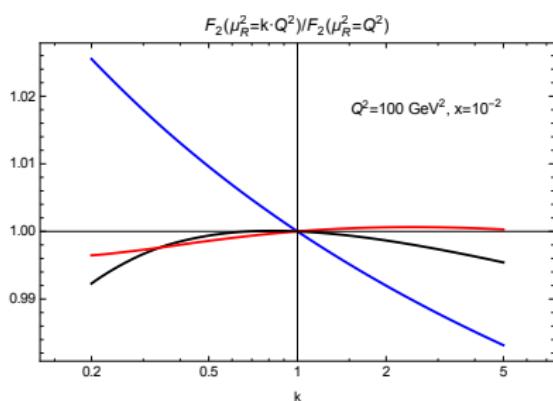
direct evolution of e.g. doublet (F_2, F_L); renormalization scale dependence

$\mu_R = k \cdot Q^2$ stable for $Q^2 = 30 \text{ GeV}^2 \rightarrow 100 \text{ GeV}^2 \dots$



(physical) DGLAP evolution [MH, Stratmann; arXiv:1311.2825, to be completed]

direct evolution of e.g. doublet (F_2, F_L); renormalization scale dependence
 $\mu_R = k \cdot Q^2$ stable for $Q^2 = 30 \text{ GeV}^2 \rightarrow 100 \text{ GeV}^2$...



for $Q^2 = 2 \text{ GeV}^2 \rightarrow 10 \text{ GeV}^2$ at small x highly unstable
 reason: un-resummed $\ln 1/x$

convinced:

- ▶ pdf studies highly valuable ... → constrain pdf's at ultra-small x
- ▶ benchmark for saturation searches (?)