NLO exclusive diffractive processes with saturation

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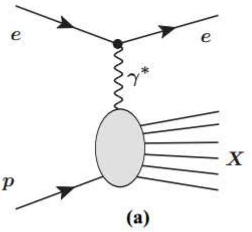
RB, A.V.Grabovsky, L.Szymanowski, S.Wallon
JHEP 409 (2014) 026 and JHEP 1611 (2016) 149
RB, A.V.Grabovsky, D.Yu.Ivanov, L.Szymanowski, S.Wallon
arXiv:1612.08026 [hep-ph]

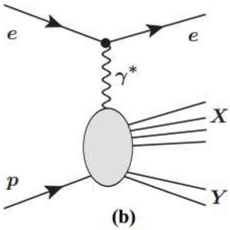
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



DIS events

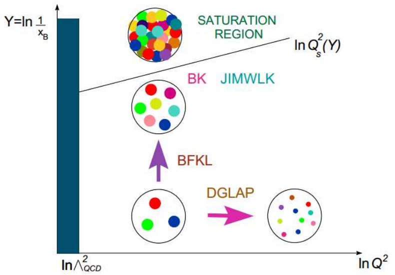


DDIS events

DIS : Deep Inelastic Scattering, DDIS : Diffractive DIS

Diffractive DIS

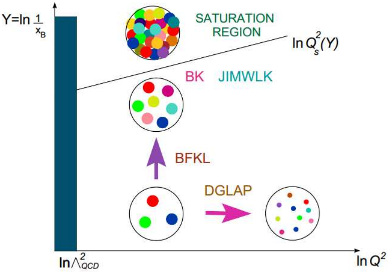
Theoretical approaches for DDIS using pQCD



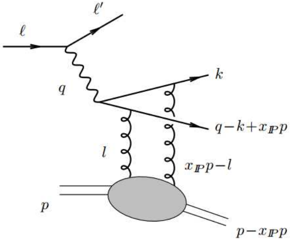
- Collinear factorization approach
  - Relies on a QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

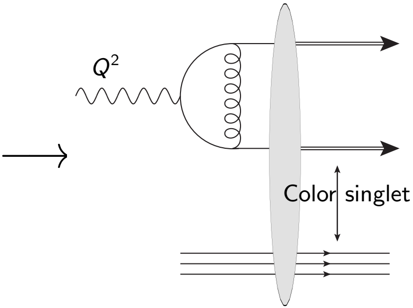
A recent analysis of diffractive dijet production in DIS at HERA seems to favor k_t factorization in the small diffractive mass regime [ZEUS collaboration, 2015]

Diffractive DIS



- Shockwave (CGC) approach
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron exchange is described as the action of a color singlet Wilson line operator on the target states

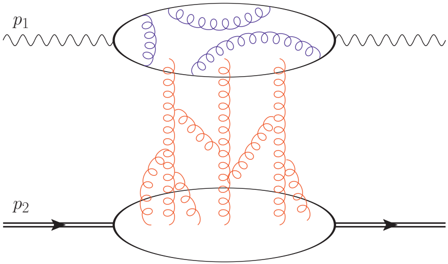




Bartels, Wüsthoff

The shockwave (CGC) framework

Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

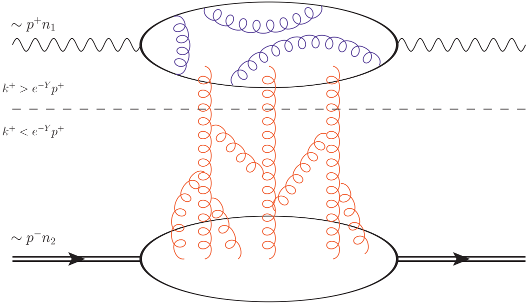
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

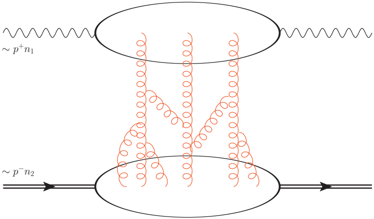
Rapidity separation

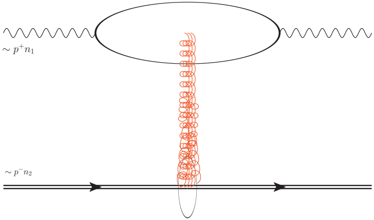


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

Large longitudinal boost to the projectile frame





$$b^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} b^+\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda b^-\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + \left(\sqrt{\frac{m_t^2}{s}}\right)$$

Propagator through the external shockwave field

$$G\left(z_2,\, z_0\right) = -\int d^4 z_1 \theta\left(z_2^+\right)\delta\left(z_1^+\right)\theta\left(-z_0^+\right) G\left(z_2 - z_1\right)\gamma^+ G\left(z_1 - z_0\right) U_1$$

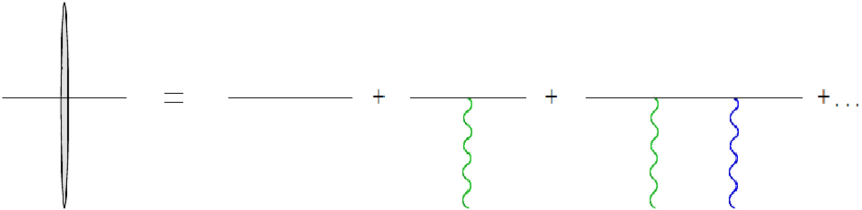
$$G\left(q,\, p\right) = (2\pi)\,\theta\left(p^+\right)\int d^D q_1 \delta\left(q + q_1 - p\right)\delta(q_1^+) G\left(q\right)\gamma^+ \tilde{U}_{\vec{q}_1} G\left(p\right)$$

Wilson lines :

$$U_i^\eta = U_{\vec{z}_i}^\eta = P\exp\left[ig\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+\right]$$

$$U_i^\eta = 1 + ig\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+ + (ig)^2\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_j\right)\theta(z_{ji}^+) dz_i^+ dz_j^+$$

...



B-JIMWLK hierarchy of equations

Dipole operator

$$\mathbf{U}^{\eta}_{12} = \frac{1}{N_c}\mathrm{Tr}(U^{\eta}_{1} U^{\eta\dagger}_{2}) - 1$$

B-JIMWLK equation

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

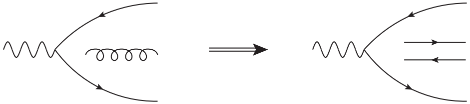
$$\frac{\partial\, \mathbf{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathbf{U}^{\eta}_{13} + \mathbf{U}^{\eta}_{32} - \mathbf{U}^{\eta}_{12} - \mathbf{U}^{\eta}_{13}\mathbf{U}^{\eta}_{32}\right]$$

$$\frac{\partial\, \mathbf{U}^{\eta}_{13}\mathbf{U}^{\eta}_{32}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in Balitsky's equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL part — Triple pomeron vertex

Non-linear term : saturation

First step

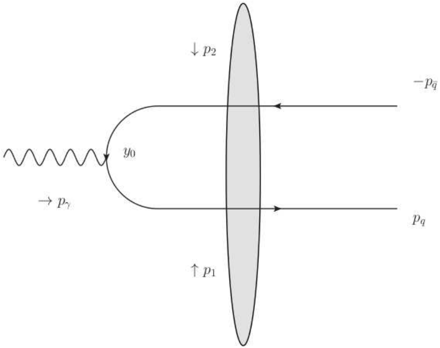
NLO open $q\bar{q}$ production and LO $q\bar{q}g$ production

Assumptions

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach in lightcone gauge
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$p_g^+ < \alpha p_\gamma^+$$

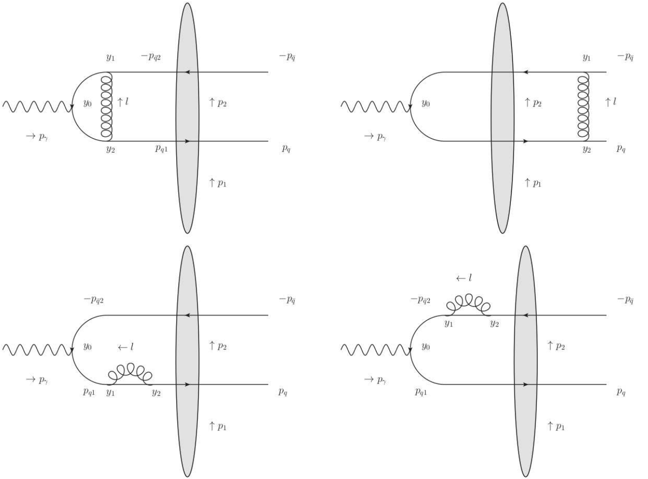
Leading Order



$$\mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 \Phi_0^\alpha(\vec{p}_1, \vec{p}_2)\,\delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2})\,\tilde{\mathbf{U}}_{12}^\alpha$$

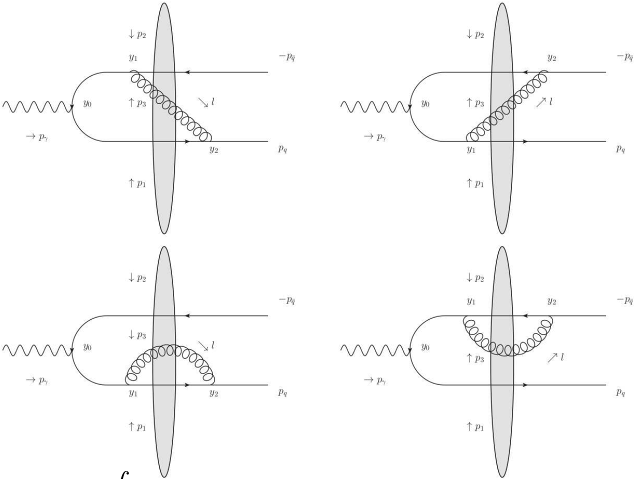
$p_{ij} \equiv p_i - p_j$

First kind of virtual corrections



$$\mathcal{A}_{V1} \propto \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 \Phi^\alpha_{V1}\left(\vec{p}_1,\ \vec{p}_2\right) \delta\left(\vec{p}_{q1}+\vec{p}_{\bar{q}2}\right) \left(\frac{N_c^2-1}{N_c}\right) \tilde{\mathbf{U}}^\alpha_{12}$$

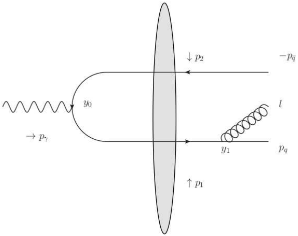
Second kind of virtual corrections

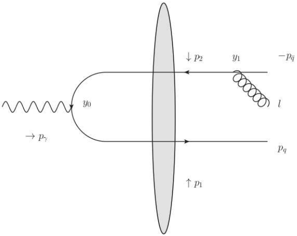


$$\mathcal{A}_{V2} \propto \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^{\alpha}_{V2}\left(\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3\right)\delta\left(\vec{p}_{q1}+\vec{p}_{\bar{q}2}-\vec{p}_3\right)$$

$$\left[\delta\left(\vec{p}_3\right)\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}^{\alpha}_{12} + N_c\left(\tilde{\mathbf{U}}^{\alpha}_{13}\tilde{\mathbf{U}}^{\alpha}_{32}+\tilde{\mathbf{U}}^{\alpha}_{13}+\tilde{\mathbf{U}}^{\alpha}_{32}-\tilde{\mathbf{U}}^{\alpha}_{12}\right)\right]$$

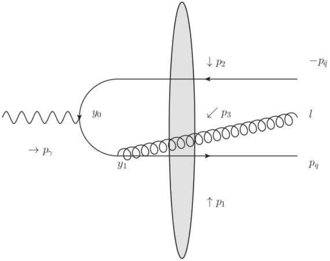
First kind of real corrections

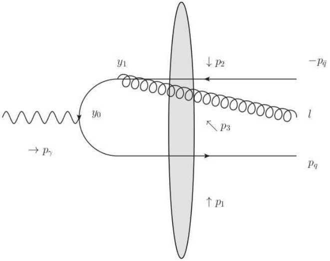




$$\mathcal{A}_{R1} = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \, \Phi^\alpha_{R1}(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g) \left(\frac{N_c^2 - 1}{N_c}\right) \tilde{\mathbf{U}}^\alpha_{12}$$

Second kind of real corrections



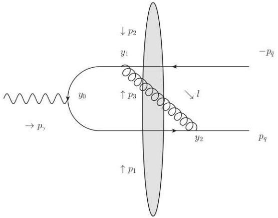


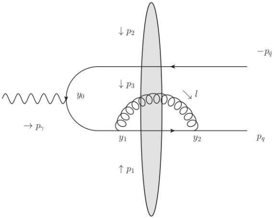
$$\begin{aligned}\mathcal{A}_{R2} \quad = \quad & \varepsilon_\alpha N_c \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \Phi^\alpha_{R2}\left(\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3\right)\delta\left(\vec{p}_{q1}+\vec{p}_{\bar{q}2}+\vec{p}_{g3}\right) \\ & \left[\left(\frac{N_c^2-1}{N_c}\right)\tilde{\mathbf{U}}^\alpha_{12}\delta\left(\vec{p}_3\right)+N_c\left(\tilde{\mathbf{U}}^\alpha_{13}\tilde{\mathbf{U}}^\alpha_{32}+\tilde{\mathbf{U}}^\alpha_{13}+\tilde{\mathbf{U}}^\alpha_{32}-\tilde{\mathbf{U}}^\alpha_{12}\right)\right]\end{aligned}$$

Divergences

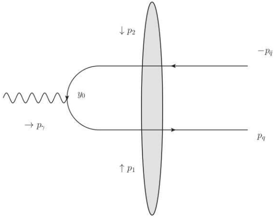
- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence





Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation

$$\frac{\partial \tilde{\mathbf{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathbf{U}}_{13}^{\alpha} \tilde{\mathbf{U}}_{32}^{\alpha} + \tilde{\mathbf{U}}_{13}^{\alpha} + \tilde{\mathbf{U}}_{32}^{\alpha} - \tilde{\mathbf{U}}_{12}^{\alpha} \right)$$

$$\times \left[2\frac{(\vec{k}_1 - \vec{p}_1)\cdot(\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1-\frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2\right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\tilde{\mathbf{U}}_{12}^{\alpha} \Phi_0 \rightarrow \Phi_0 \tilde{\mathbf{U}}_{12}^{\eta} + \log\left(\frac{e^{\eta}}{\alpha}\right) \mathcal{K}_{BK} \Phi_0 \left(\tilde{\mathbf{U}}_{13} \tilde{\mathbf{U}}_{32} + \tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} \right)$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the α dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$(\Phi'^{\mu}_{V2} \otimes \mathbf{UU}) = \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$
$$\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3) \left[\tilde{\mathbf{U}}_{13} + \tilde{\mathbf{U}}_{32} - \tilde{\mathbf{U}}_{12} - \tilde{\mathbf{U}}_{13}\tilde{\mathbf{U}}_{32}\right] \Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

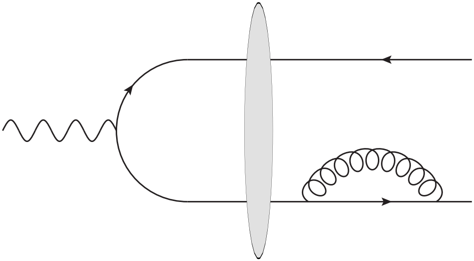
Then $\tilde{\mathbf{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*,\ \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}},\ p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

UV divergence

Tadpole diagrams



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \int \frac{d^D k}{(k^2 + i0)^2} \propto \frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}}$$

Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

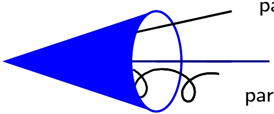
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\varphi_{ik}$

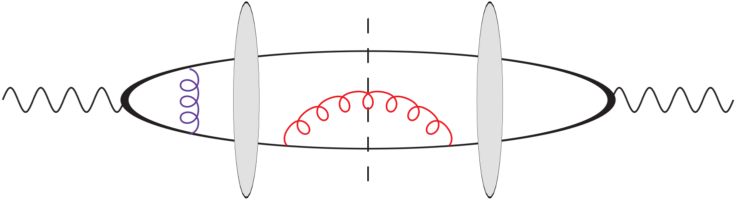
$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Remaining divergence



- Soft divergence $p_g \to 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1}\Phi_{R1}^*$$

Remaining divergence

Soft real emission

$$\left(\Phi_{R1}\Phi_{R1}^*\right)_{soft} \propto \left(\Phi_0\Phi_0^*\right)\int_{\text{outside the cones}}\left|\frac{p_q^\mu}{(p_q.p_g)}-\frac{p_{\bar q}^\mu}{(p_{\bar q}.p_g)}\right|^2\frac{dp_g^+}{p_g^+}\frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$\left(\Phi_{R1}\Phi_{R1}^*\right)_{col} \propto \left(\Phi_0\Phi_0^*\right)\left(\mathcal{N}_q+\mathcal{N}_{\bar q}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k=\frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})}\int_{\alpha p_\gamma^+}^{p_{jet}^+}\frac{dp_g^+dp_k^+}{2p_g^+2p_k^+}\int_{\text{in cone k}}\frac{d^d\vec{p}_g d^d\vec{p}_k}{(2\pi)^d\,\mu^{d-2}}\frac{\mathrm{Tr}\left(\hat{p}_k\gamma^\mu\hat{p}_{jet}\gamma^\nu\right)d_{\mu\nu}(p_g)}{2p_{jet}^+\left(p_k^-+p_g^--p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences (both UV and soft)

Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) (S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}} \mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}} \vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi \ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j \vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2} \vec{p}_j^{\,2}}\right) \ln\left(\frac{4E^2}{x_{\bar{j}} x_j (p_\gamma^+)^2}\right)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}} x_j \mu^2}{(x_{\bar{j}} \vec{p}_j - x_j \vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}} x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4 \vec{p}_j^{\,2} \vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right) \ln\left(\frac{x_j \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}} \vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Cancellation of divergences

Total divergence

$$
\begin{aligned}
div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\
&= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\
&+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right]
\end{aligned}
$$

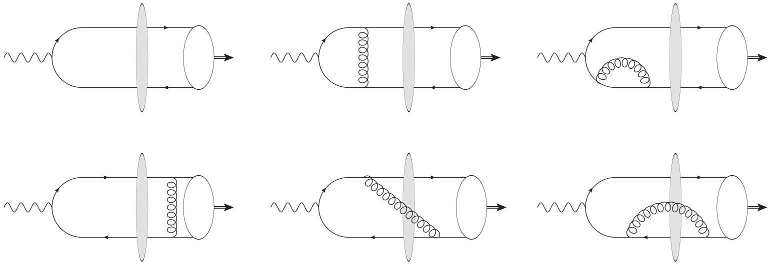
Our cross section is thus finite

Constructing a finite amplitude

Exclusive diffractive production of a light neutral vector meson

Towards an extension of [Munier, Staśto, Mueller] and [Ivanov, Kotsky, Papa]

Exclusive diffractive production of a light neutral vector meson



$$\begin{aligned}\mathcal{A}_0 &= -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c}\int_0^1 dx\varphi_\parallel(x)\int\frac{d^d\vec{p}_1}{(2\pi)^d}\frac{d^d\vec{p}_2}{(2\pi)^d}\\ &\times (2\pi)^{d+1}\,\delta\left(p_V^+ - p_\gamma^+\right)\delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)\\ &\times \Phi_0^\beta\left(x,\ \vec{p}_1,\ \vec{p}_2\right)\tilde{\mathbf{U}}_{12}^\eta.\end{aligned}$$

Leading twist for a longitudinally polarized meson
Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial \varphi(x, \mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz \varphi(z, \mu_F^2) \mathcal{K}(x, z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned}\mathcal{K}(x, z) &= \frac{x}{z}\left[1 + \frac{1}{z-x}\right]\theta(z - x - \alpha) \\ &+ \frac{1-x}{1-z}\left[1 + \frac{1}{x-z}\right]\theta(x - z - \alpha) \\ &+ \left[\frac{3}{2} - \ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z - x).\end{aligned}$$

It is equivalent to the usual ERBL kernel

Infrared finiteness

The amplitude we obtain is finite. For example the dipole $\gamma_L^* \to V_L$ contribution reads

$$
\begin{aligned}
\Phi_1^+(x) &= \int_0^x dz \left(\frac{x-z}{x}\right) \Phi_0^+(x-z) \\
&\times \left[1 + \left(1 + \left[\frac{1}{z}\right]_+\right) \ln\left(\frac{\left(((\bar{x}+z)\vec{p}_1 - (x-z)\vec{p}_2)^2 + (x-z)(\bar{x}+z)Q^2\right)^2}{\mu_F^2 (x-z)(\bar{x}+z)Q^2}\right)\right] \\
&+ \frac{1}{2}\Phi_0^+(x)\left[\frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right) + 3 - \frac{\pi^2}{6} - \frac{3}{2}\ln\left(\frac{\left((\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2\right)^2}{x\bar{x}\mu_F^2 Q^2}\right)\right] \\
&+ \frac{(p_\gamma^+)^2}{2x\bar{x}} \int_0^x dz \left[(\phi_5)_{LL}|_{\vec{p}_3=\vec{0}} + (\phi_6)_{LL}|_{\vec{p}_3=\vec{0}}\right]_+ + (x \leftrightarrow \bar{x}, \vec{p}_1 \leftrightarrow \vec{p}_2).
\end{aligned}
$$

No end point singularity, even for a transverse photon and even in the photoproduction limit.

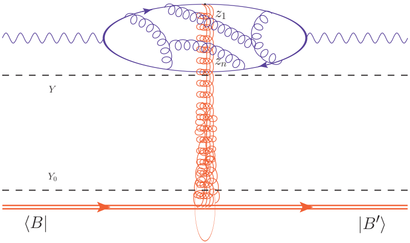
Phenomenological prospects

Practical use of the formalism

- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states

$$\langle P'|\tilde{\mathbf{U}}^{\eta}_{12}|P\rangle, \quad \langle P'|\tilde{\mathbf{U}}^{\eta}_{13}\tilde{\mathbf{U}}^{\eta}_{32}|P\rangle$$

- Solve the NLO B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle B'|U_{\vec{z}_1} ... U_{\vec{z}_n}|B\rangle$$

Residual parameter dependence

Required parameters

- Renormalization scale μ_R
- Factorization scale μ_F in the case of meson production
- Typical target rapidity Y_0
- Typical projectile rapidity Y

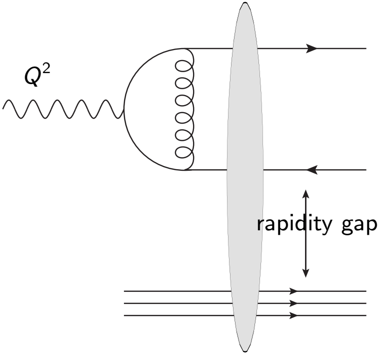
In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so only one arbirary parameter s_0 defined by

$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right)$$

is required.

General amplitude

- Most general kinematics
- The hard scale can be Q^2, t, M_X^2...
- The target can be either a proton or an ion, or another impact factor.
- Finite results for $Q^2 = 0$
- One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$.

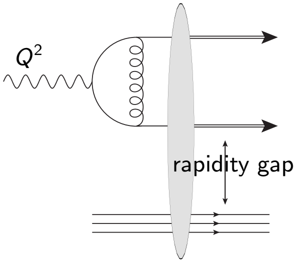


The general amplitude

HERA data fits, ultraperipheral *pp* and *pA* collisions at the LHC, EIC/LHeC predictions...

Phenomenological applications : exclusive dijet production at NLO accuracy

- HERA data for exclusive dijet production in diffractive DIS can be fitted with our results
- We can also give predictions for the same process in a future electron-ion or electron-proton collider (EIC, LHeC...)
- For $Q^2 = 0$ we can give predictions for ultraperipheral *pp* and *pA* collisions at the LHC



Amplitude for diffractive dijet production

HERA data fits, ultraperipheral *pp* and *pA* collisions at the LHC, EIC/LHeC predictions...

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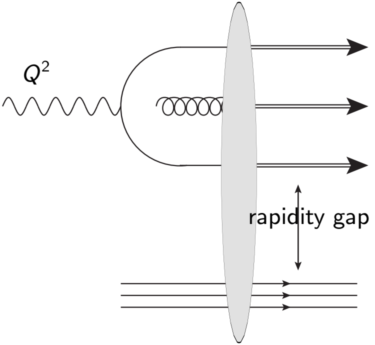

Amplitude for diffractive trijet production

[Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans]

HERA data fits, ultraperipheral *pp* and *pA* collisions at the LHC, EIC/LHeC predictions...

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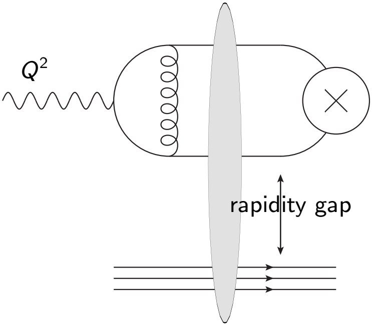

Amplitude for diffractive V production

HERA data fits, large-t ultraperipheral pp and pA collisions at the LHC, EIC/LHeC predictions...

Conclusion

- We provided the full computation of the impact factor for the exclusive diffractive production of a forward dijet and of a light neutral vector meson with NLO accuracy in the shockwave approach
- It leads to an enormous number of possible phenomenological applications to test QCD in its Regge limit and towards saturation in past, present and future ep, eA, pp and pA colliders
- Several theoretical extensions could be obtained with slight modifications to our result