Implementing consistent NLO factorization in single inclusive forward hadron production

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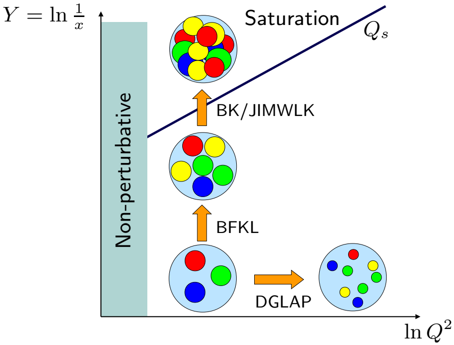
B. D., T. Lappi, Y. Zhu, PRD 93 (2016) 114016 [arXiv:1604.00225] & arXiv:1703.04962

Motivations

Our goal is to study QCD in the saturation regime



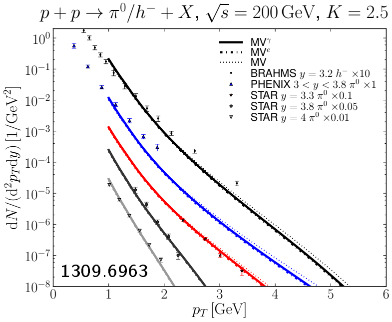
The production of forward particles is a crucial tool to probe small x values

Saturation effects stronger in pA collisions ($Q_s^2 \sim A^{1/3}$)

Here we study the inclusive production of a forward hadron in proton-nucleus collisions: $pA \to hX$

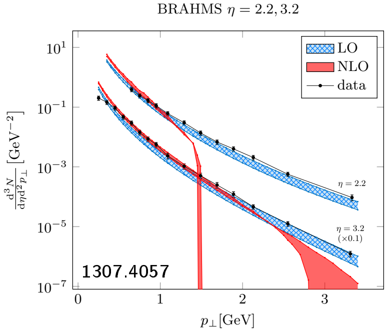
Motivations

Typical calculation at LO:
(Lappi, Mäntysaari)



K factor needed to describe the data

First numerical calculation at NLO:
(Staśto, Xiao, Zaslavsky)

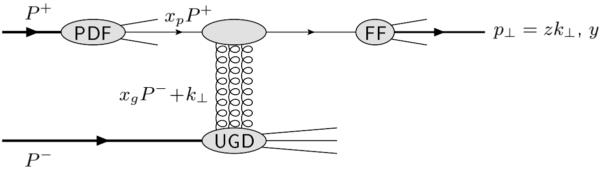


Negative cross section above some $p_\perp$

Several proposals to solve the negativity problem at NLO, for example the kinematical constraint / Ioffe time cutoff (Altinoluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky. Can extend the positivity range but doesn't solve the problem completely.

Formalism

Single inclusive forward hadron production at LO in the $q \to q$ channel:



Dilute projectile: $x_p = \frac{k_\perp}{\sqrt{s}} e^y$, described by collinear PDFs

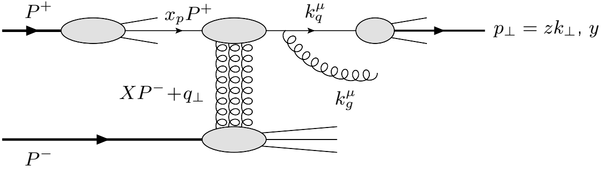
Dense target: $x_g = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{F}$

$$\mathcal{F}(k_\perp) = \int \mathrm{d}^2\mathbf{b}\, \mathcal{S}(k_\perp)\ ,\ \mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})\ ,\ S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

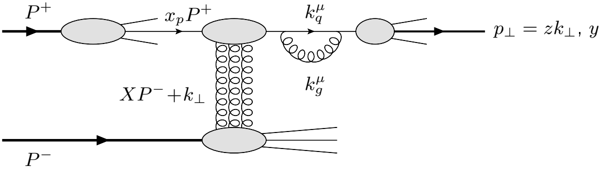
Rapidity (or x) dependence of S : given by the Balitsky-Kovchegov equation

Expression for the NLO cross section: Chirilli, Xiao, Yuan ('CXY')

Example of real $q \to q$ contribution:



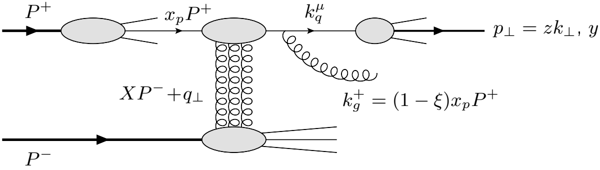
Example of virtual $q \to q$ contribution:



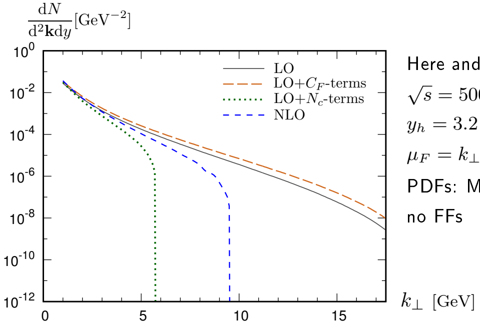
$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

After summing the real and virtual contributions, two types of divergences remain in the NLO cross section:

- The collinear divergence
  - Occurs when the additional gluon is collinear to the incoming or outgoing quark
  - Affects only the NLO corrections proportional to C_F
  - Absorbed in the DGLAP evolution of the PDFs and FFs
- The rapidity divergence
  - Occurs when $\xi \to 1 \Leftrightarrow$ the rapidity of the unobserved gluon $\to -\infty$
    $\Leftrightarrow$ this gluon is collinear to the target
  - Affects only the NLO corrections proportional to N_c
  - Absorbed in the BK evolution of the target



Multiplicity after subtracting the divergences:



Here and in the following:

$\sqrt{s} = 500$ GeV

$y_h = 3.2$

$\mu_F = k_\perp$

PDFs: MSTW2008 NLO

no FFs

The negativity at large $k_\perp$ is apparently caused by the N_c-terms

The LO+N_c contributions can be written as

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^1 \frac{\mathrm{d}\xi}{1-\xi}\left[\mathcal{K}(k_\perp,\xi,x_g) - \mathcal{K}(k_\perp,1,x_g)\right],$$

where

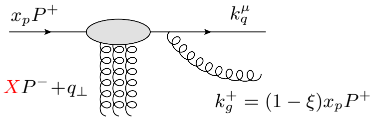
$$\mathcal{K}(k_\perp,\xi,X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi-x_p)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp,\xi,X) - x_p q(x_p)\mathcal{J}_v(k_\perp,\xi,X)\right],$$

and

$$\begin{aligned}\mathcal{J}(k_\perp,\xi,X) &= \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp,X) \\ &\quad - \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp,X)\mathcal{S}(l_\perp,X), \\ \mathcal{J}_v(k_\perp,\xi,X) &= \mathcal{S}(k_\perp,X)\left[\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\right. \\ &\quad \left. - \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(l_\perp,X)\right].\end{aligned}$$

At large $k_\perp$, the function $\mathcal{K}(k_\perp,\xi,X)$ is positive and increasing with ξ, therefore $\mathcal{K}(k_\perp,\xi,x_g) - \mathcal{K}(k_\perp,1,x_g)$ is negative and can be large enough to make the cross section negative

Iancu, Mueller, Triantafyllopoulos: consider the kinematics:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2}\right)$$
$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ at large } k_\perp$$

Thus the LO+N_c terms read

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c,sub}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p) \frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{K}(k_\perp, \xi, X(\xi)) - \mathcal{K}(k_\perp, 1, X(\xi))\right].$$

The limit $\xi < 1 - \frac{x_g}{x_0}$ ensures that $X(\xi) < x_0$, the initial condition for the BK evolution of the target. Using the integral BK equation,

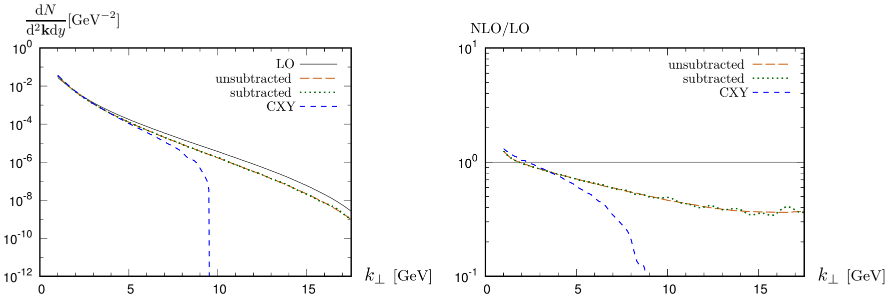
$$\mathcal{S}(k_\perp, x_g) = \mathcal{S}(k_\perp, x_0) + 2\alpha_s N_c \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{J}(k_\perp, 1, X(\xi)) - \mathcal{J}_v(k_\perp, 1, X(\xi))\right],$$

the LO+N_c terms can be rewritten as

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c,unsub}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p) \frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(k_\perp, \xi, X(\xi)),$$

which is explicitly positive at large $k_\perp$.

Results at fixed coupling $\alpha_s = 0.2$:



The choice of the rapidity scale in the NLO terms, although subleading in principle, is important at large $k_\perp$

The 'subtracted' and 'unsubtracted' expressions give the same results

(Initial condition for the BK evolution at $x_0 = 0.01$: MV model

$S(\mathbf{r}, x_0) = \exp\left[-\frac{\mathbf{r}^2 Q_{\mathbf{s},0}^2}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e\right)\right]$, $Q_{\mathbf{s},0}^2 = 0.2$ GeV^2 and $\Lambda_{\mathrm{QCD}} = 0.241$ GeV)

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

In practice the BK equation is usually solved in coordinate space, with some prescription for the running coupling

Fixed coupling BK equation:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right]$$

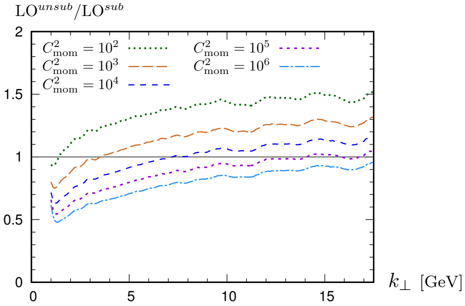
BK equation with Balitsky's prescription for the running coupling:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2) N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right) + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]$$

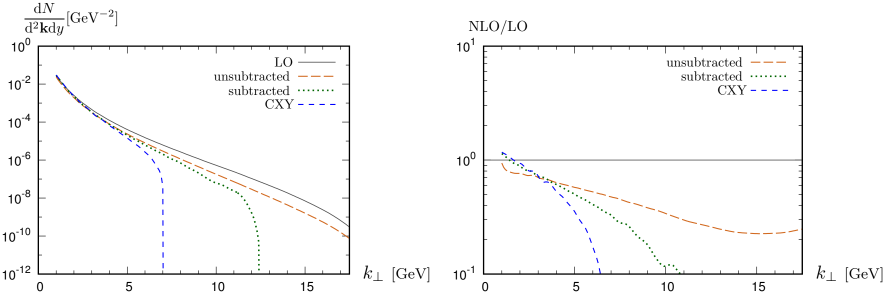
Running coupling: $\alpha_s(\mathbf{r}^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{4C^2}{\mathbf{r}^2\Lambda^2_{\mathsf{QCD}}}\right)}$, $\alpha_s(k_\perp^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{C^2_{\mathsf{mom}} k_\perp^2}{\Lambda^2_{\mathsf{QCD}}}\right)}$

Initial condition at $x_0 = 0.01$: $S(\mathbf{r}, x_0) = \exp\left[-\frac{\mathbf{r}^2 Q^2_{\mathsf{s},0}}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathsf{QCD}}} + e_c \cdot e\right)\right]$,

with $Q^2_{\mathsf{s},0} = 0.06$ GeV^2, $C^2 = 7.2$ and $e_c = 18.9$ obtained by a fit to HERA DIS data (Lappi, Mäntysaari). $C^2_{\mathsf{mom}} = 10^3$ is fixed by comparing the LO limits of the 'subtracted' ($\alpha_s \to 0$) and 'unsubtracted' ($\xi \to 1$) expressions with $\alpha_s \to \alpha_s(k_\perp^2)$:



Results with running coupling:



The 'subtracted' and 'unsubtracted' expressions are no longer equivalent

'Subtracted' expression: closer to the 'CXY' result at small $k_\perp$, still leads to negative results at large $k_\perp$

Conclusions

We have studied a recent proposal for the implementation of NLO factorization in single inclusive forward hadron production

- Change of the rapidity scale in the NLO terms: large effect numerically
- Fixed coupling: positive cross sections at all transverse momenta
- Running coupling: mismatch between the couplings used in coordinate and momentum space

Directions for future work:

- Better understanding of how to deal with the running of the coupling
- Add the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions
- Use NLO BK for the evolution of the target
- The initial condition for the BK evolution of the target must be obtained by a fit (e.g. to HERA DIS data) also performed at NLO

Recall that

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\mathcal{K}\left(k_\perp, \xi, \frac{x_g}{1-\xi}\right),$$

with

$$\mathcal{K}(k_\perp, \xi, X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi - x_p)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp, \xi, X) - x_p q\left(x_p\right)\mathcal{J}_v(k_\perp, \xi, X)\right].$$

We can write $\mathcal{J} = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}}\widetilde{\mathcal{J}}$ and $\mathcal{J}_v = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}}\widetilde{\mathcal{J}}_v$, with

$$\widetilde{\mathcal{J}}(\mathbf{r}, \xi, X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{\mathbf{x}\cdot(\mathbf{x}-\mathbf{r})}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2}\left[S(\mathbf{r}-(1-\xi)\mathbf{x}, X) - S(\xi\mathbf{x}, X)S(\mathbf{r}-\mathbf{x}, X)\right],$$

$$\widetilde{\mathcal{J}}_v(\mathbf{r}, \xi, X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{1}{\mathbf{x}^2}\left[S(\mathbf{r}+(1-\xi)\mathbf{x}, X) - S(\mathbf{x}, X)S(\mathbf{r}-\xi\mathbf{x}, X)\right].$$

(and similarly for the C_F terms)

In these notations the BK equation reads

$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = -2\alpha_s N_c\left[\widetilde{\mathcal{J}}(\mathbf{r}, 1, X) - \widetilde{\mathcal{J}}_v(\mathbf{r}, 1, X)\right]$$

BK equation with Balitsky's prescription for the running coupling:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2)N_\mathsf{c} \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right) + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]$$

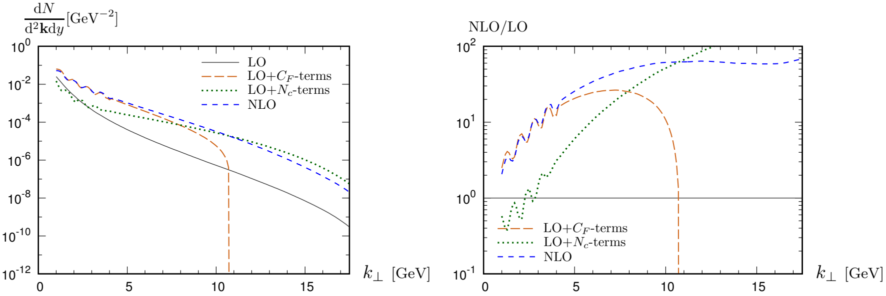
This can be generalized to $\xi \neq 1$ by replacing $\widetilde{\mathcal{J}}_v$ with

$$\widetilde{\mathcal{J}}_v^{\mathsf{rc}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{1}{\mathbf{x}^2}\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\xi\mathbf{x})^2)}\left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right],$$

and by replacing the explicit α_s factors by $\alpha_s(\mathbf{r}^2)$. Not a unique choice but:

- $\xi = 1$: recovers Balitsky's prescription
- Fixed coupling results unchanged

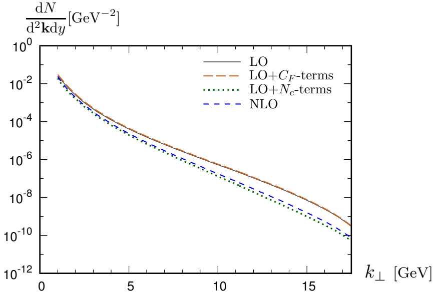
Results with this formulation ('unsubtracted' version):

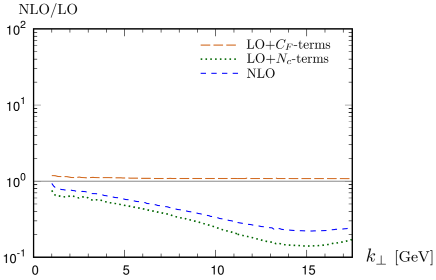


Completely different results compared to fixed or momentum space running coupling: NLO result orders of magnitude larger than the LO one

The 'subtracted' expression gives the same results
(The Balitsky prescription is correctly recovered at $\xi = 1$)

Results with momentum space running coupling ('unsubtracted' version):





Results with fixed coupling: