Quark correlations in the Color Glass Condensate

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Contents:

1. Motivation.

2. Calculation.

3. Estimates.

4. Conclusions and outlook.

The ridge:

- Two-particle correlations in pp and pPb at the LHC show features that in AA are attributed to final state interactions describable by viscous relativistic hydrodynamics and interpreted as a signal of equilibration.

- EKT and AdS/CFT: hydro works even for large momentum anisotropies.

- What about a non-hydro initial-state explanation? (anyway long range rapidity correlations must come from the very early times...)

The ridge:

- Two-particle correlations in pp and pPb at the LHC show features that in AA are attributed to final state interactions describable by viscous relativistic hydrodynamics and interpreted as a signal of equilibration.
- EKT and AdS/CFT: hydro works even for large momentum anisotropies.
- What about a non-hydro initial-state explanation? (anyway long range rapidity correlations must come from the very early times…)

\[
\frac{dN}{d\phi} = \left( \frac{dN}{d\phi} \right) \left( 1 + \sum_n 2n \cos n(\phi - \Psi_n) \right) \quad v_{\text{ridge}}(\Delta \phi) = G \left( 1 + \sum_{n=2}^{\infty} 2n \cos n\Delta \phi \right)
\]

\[
v_{n,n}(p_T^a, p_T^b) = v_n(p_T^a)v_n(p_T^b)
\]
Several explanations have been proposed in the CGC: assume that the final state carry the imprint of initial-state correlations, and use that the CGC wave function is rapidity invariant over $Y \propto 1/\alpha_s$.

- **Local anisotropy of target fields** (Kovner-Lublinsky, Dumitru-McLerran-Skokov).


- **Spatial variation of partonic density** (Levin-Rezaeian).
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● Spatial variation of partonic density (Levin-Rezaeian).

● Work undergoing to explain n-particle correlations, n>2, and odd harmonics.

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Bose enhancement for gluons:

- The appearance of the ridge in the final state, within the glasma graph approach, can be traced to the Bose enhancement of gluons in the (rapidity invariant) wave function:
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\[ [a^i_a(k), a^i_b(p)] = (2\pi)^2 \delta_{ab} \delta^{ij} \delta^{(2)}(k - p) \]

\[ a^i_a(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta < Y/2|} \frac{d\eta}{2\pi} a^i_a(\eta, k) \]
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rescattering on the target

\[
C \int_{k_1, k_2} \langle in | a^{\dagger i}_a(k_1) a^{\dagger j}_b(k_2) a^k_a(k_1) a^l_b(k_2) | in \rangle \left[ \delta^{ik} - \frac{k^i_1 k^k_1}{p^2} \right] \left[ \delta^{jl} - \frac{k^j_2 k^l_2}{q^2} \right] N(p - k_1) N(q - k_2)
\]
Bose enhancement for gluons:

- The appearance of the ridge in the final state, within the glasma graph approach, can be traced to the Bose enhancement of gluons in the (rapidity invariant) wave function:

\[
[a_a^i(k), a_b^{\dagger j}(p)] = (2\pi)^2 \delta_{ab} \delta^{ij} \delta^{(2)}(k - p)
\]

\[
a_a^i(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta - Y/2|} d\eta \frac{dN}{2\pi} a_a^i(\eta, k)
\]

\[
C \int_{k_1, k_2} \langle in | a_a^{\dagger i}(k_1) a_b^{\dagger j}(k_2) a_a^k(k_1) a_b^l(k_2) | in \rangle \left[ \delta^{ik} - \frac{k_i^2 k_1^2}{p^2} \right] \left[ \delta^{jl} - \frac{k_j^2 k_2^2}{q^2} \right] N(p - k_1) N(q - k_2)
\]

\[
D(k_1, k_2) = S^2 (N_c^2 - 1)^2 \frac{k_1^4 k_2^4 g^4 \mu^2(k_1) \mu^2(k_2)}{k_1^2 k_2^2} \left\{ 1 + \frac{1}{S(N_c^2 - 1)} \left[ \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2) \right] \right\}
\]
Bose enhancement for gluons:

- Question: do quarks in the CGC wave function experience Pauli blocking and, if so, is it short or long range in rapidity?

\[
[a^i_a(k), a^{\dagger j}_b(p)] = (2\pi)^2 \delta_{ab} \delta^{ij} \delta^{(2)}(k - p)
\]

\[
a^i_a(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta < Y/2|} \frac{d\eta}{2\pi} a^i_a(\eta, k)
\]

\[
C \int_{k_1, k_2} \langle in | a^{\dagger i}_a(k_1) a^{\dagger j}_b(k_2) a^k_a(k_1) a^l_b(k_2) | in \rangle \left[ \delta^{ik} - \frac{k^{i}_1 k^{k}_1}{p^2} \right] \left[ \delta^{jl} - \frac{k^{j}_2 k^{l}_2}{q^2} \right] N(p - k_1) N(q - k_2)
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The two-particle inclusive cross section reads:

\[
\frac{d\sigma}{dp^+d^2pdq^+d^2q} = \frac{1}{(2\pi)^6} \langle v|\hat{S}^\dagger \Omega^\dagger \left[ d_{\alpha,s_1}^\dagger (p^+,p) d_{\beta,s_2}^\dagger (q^+,q) d_{\beta,s_2} (q^+,q) d_{\alpha,s_1} (p^+,p) \right] \Omega \hat{S} \Omega^\dagger |v \rangle
\]
The elements:

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\]

operator that diagonalises perturbatively
the Light Cone QCD Hamiltonian
The elements:

- The two-particle inclusive cross section reads:

\[
\frac{d\sigma}{dp+d^2pdq+d^2q} = \frac{1}{(2\pi)^6} \langle \Omega S \Omega^\dagger \Omega S \Omega^\dagger | \langle d\alpha_{s1}(p^+,p) d\beta_{s2}(q^+,q) d\alpha_{s1}(p^+,p) | \rangle \rangle
\]

operator that diagonalises perturbatively the Light Cone QCD Hamiltonian

- eikonal S-matrix
The two-particle inclusive cross section reads:

\[
\frac{d\sigma}{dp^+ d^2 p dq + d^2 q} = \frac{1}{(2\pi)^6} \langle \Omega^{\dagger} \hat{S}^{\dagger} \Omega^{\dagger} \rangle [ d_{\alpha, s_1} (p^+, p) d_{\beta, s_2}^{\dagger} (q^+, q) d_{\beta, s_2} (q^+, q) d_{\alpha, s_1} (p^+, p) ] \Omega^{\dagger} \hat{S} \Omega | v \rangle
\]

operator that diagonalises perturbatively the Light Cone QCD Hamiltonian

\[
\{ d_{s_1}^{\omega} (k^+, k), d_{s_2}^{\dagger \xi} (q^+, q) \} = (2\pi)^3 \delta^{\omega \xi} \delta_{s_1 s_2} \delta (k^+ - q^+) \delta^{(2)}(k - q)
\]

quark creation operator
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The elements:

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$$\frac{d\sigma}{dp^+d^2pdq+d^2q} = \frac{1}{(2\pi)^6} \langle v | \hat{S}^+ \Omega^\dagger [d_{\alpha,s_1}^\dagger (p^+,p) d_{\beta,s_2}^\dagger (q^+,q) d_{\beta,s_2}(q^+,q) d_{\alpha,s_1}(p^+,p)] \Omega S \Omega^\dagger | v \rangle$$

operator that diagonalises perturbatively the Light Cone QCD Hamiltonian

$$\{d_{s_1}^{\omega}(k^+,k), d_{s_2}^{\dagger \xi}(q^+,q)\} = (2\pi)^3 \delta^{\omega\xi} \delta_{s_1s_2} \delta(k^+-q^+)\delta^{(2)}(k-q)$$

quark creation operator

valence state

eikonal S-matrix
The elements:

- The two-particle inclusive cross section reads:

\[
\frac{d\sigma}{dp+dp'dq+dq'} = \frac{1}{(2\pi)^6} \langle v | \hat{S}^\dagger \Omega^\dagger \left[ d_{\alpha,s_1}^\dagger (p^+, p) d_{\beta,s_2}^\dagger (q^+, q) d_{\beta,s_2} (q^+, q) d_{\alpha,s_1} (p^+, p) \right] \Omega \hat{S} \Omega^\dagger | v \rangle
\]

- Dressed valence state of interest here from the LCH (1610.03453):

\[
|v\rangle_4^D = \text{virtual} + \frac{g^4}{2} \int \frac{dk^+ d\alpha d^2 p' d^2 \bar{p}'}{(2\pi)^3} \frac{d\bar{k}^+ d\beta d^2 q' d^2 \bar{q}'}{(2\pi)^3}
\]

\[
\times \left[ \zeta_{\xi,\eta}^{\epsilon,\delta} \right] \left[ (\alpha k^+, p', \bar{p}' ; 1) \zeta_{\gamma,\delta}^\epsilon \delta (\bar{k}^+, q', \bar{q}'; \beta) \right] \left[ d_{\alpha,s_1}^\dagger (\alpha k^+, p') d_{\beta,s_2}^\dagger (\beta \bar{k}^+, q') \right] |v\rangle
\]

\[
\rho_1 \quad \rho_2
\]

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The elements:

- The two-particle inclusive cross section reads:

\[
\frac{d\sigma}{d\rho + d^2 p d\rho + d^2 q} = \frac{1}{(2\pi)^6} \langle \Omega \hat{S}^+ \Omega \Omega^+ \left[ d_{\alpha,s_1}^r(p^+, p) d_{\beta,s_2}^{\dagger}(q^+, q) d_{\beta,s_2}(q^+, q) d_{\alpha,s_1}(p^+, p) \right] \Omega \hat{S}^+ \Omega^+ \rangle.
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\[
|v\rangle_4^D = \text{virtual} + \frac{g^4}{2} \int \frac{d^3 k d^3 p d^3 p'}{(2\pi)^3} \frac{d^3 \bar{k} d^3 \beta d^3 q d^3 \bar{q}}{(2\pi)^3} \times \left[ \zeta_{s_1 s_2}^{\gamma \delta}(k^+, p', \bar{p}', \alpha) \zeta_{r_1 r_2}^{\gamma \delta}(k^+, q', \bar{q}', \beta) \right.
\]

\[
\left. \times d_{s_1}^{t\epsilon}(\bar{q}k^+, p') d_{s_2}^{t\epsilon}(\alpha k^+, p') d_{r_1}^{t\epsilon}(\beta \bar{k}^+, q') d_{r_2}^{t\epsilon}(\beta k^+, q') \right] |v\rangle.
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\]

- Operator that diagonalises perturbatively the Light Cone QCD Hamiltonian:

\[
\{ d_{s_1}^\omega (k^+,k), d_{s_2}^\gamma (q^+,q) \} = (2\pi)^3 \delta^{\omega}\gamma \delta_{s_1s_2} \delta(k^+ - q^+) \delta^2(k - q)
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\]

- \( \rho g \sim 1 \), so only density-enhanced contributions are taken i.e. NOT \( \rho \)

N. Armesto, 06.04.2017 - Quark correlations in the CGC: 2. Calculation.
Correlations in the WF (I):

- The average number of quark pairs in the LC WF is:

\[
\frac{dN}{dp^+d^2pdq+q^2} = \frac{1}{(2\pi)^6} \left\langle \frac{D}{4} \langle v | \bar{d}^\dagger_{\alpha,s_1} (p^+,p) \bar{d}^\dagger_{\beta,s_2} (q^+,q) \bar{d}_{\beta,s_2} (q^+,q) \bar{d}_{\alpha,s_1} (p^+,p) | v \rangle \right\rangle^D_P
\]

\[
= \frac{1}{(2\pi)^4} g^8 \int d^2k d^2\bar{k} d^2l d^2\bar{l} \langle \rho^a(k) \rho^c(\bar{k}) \rho^b(l) \rho^d(\bar{l}) \rangle_P
\]

\[
\times \left\{ \text{tr}(\tau^a \tau^b) \text{tr}(\tau^c \tau^d) \Phi_2(k,l;p) \Phi_2(\bar{k},\bar{l};q) - \text{tr}(\tau^a \tau^b \tau^c \tau^d) \Phi_4(k,l,\bar{k},\bar{l};p,q) \right\}
\]
The average number of quark pairs in the LC WF is:

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\frac{dN}{dp^+ d^2 p d q + d^2 q} = \frac{1}{(2\pi)^6} \left\langle \frac{D}{4} \langle v | d_{\alpha,s_1}^\dagger(p^+, p) d_{\beta,s_2}^\dagger(q^+, q) d_{\alpha,s_1} (p^+, p) | v \rangle^D \right\rangle_P \\
= \frac{1}{(2\pi)^4} g^8 \int d^2 k d^2 \bar{k} d^2 l d^2 \bar{l} \left\langle \rho^a(k) \rho^c(\bar{k}) \rho^b(l) \rho^d(\bar{l}) \right\rangle_P \\
\times \left\{ \text{tr}(\tau^a \tau^b) \text{tr}(\tau^c \tau^d) \right\} \Phi_2(k, l; p) \Phi_2(\bar{k}, \bar{l}; q) - \text{tr}(\tau^a \tau^b \tau^c \tau^d) \Phi_4(k, l, \bar{k}, \bar{l}; p, q) \right\}
\]

\[\Phi_2(k, l; p) \equiv \int_0^1 d\alpha \int \frac{d^2 \vec{p}'}{(2\pi)^2} \sum_{s_1 s_2} \phi_{s_1, s_2}(k, p, \vec{p}'; \alpha) \phi_{s_1, s_2}^*(l, p, \vec{p}'; \alpha) \times \int \frac{d^2 \vec{q}'}{(2\pi)^2} \frac{d^2 \vec{q}}{(2\pi)^2} \phi_{s_1, s_2}(k, p, \vec{p}'; \alpha) \phi_{s_1, s_2}(k, q, \vec{q}'; \beta) \phi_{s_1, s_2}^*(l, p, \vec{q}'; \beta) \phi_{s_1, s_2}^*(l, q, \vec{q}'; \alpha) \]

\[\Phi_4(k, l, \bar{k}, \bar{l}; p, q) \equiv \sum_{s_1, s_2, s_1', s_2'} \int_0^1 d\alpha d\beta \frac{\alpha + \beta e^{m_1-m_2}}{(\beta + \beta e^{m_1-m_2})(\alpha + \alpha e^{m_2-m_1})} \]

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\]

\[
= \frac{1}{(2\pi)^4} g^8 \int d^2 k d^2 \bar{k} d^2 l d^2 \bar{l} \langle \rho^a(k)\rho^c(\bar{k})\rho^b(l)\rho^d(\bar{l}) \rangle_p
\]

\[
\times \left\{ \text{tr}(\tau^a\tau^b)\text{tr}(\tau^c\tau^d)\Phi_2(k,l;p)\Phi_2(\bar{k},\bar{l};q) - \text{tr}(\tau^a\tau^b\tau^c\tau^d)\Phi_4(k,l,\bar{k},\bar{l};p,q) \right\}
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\]

\[
\Phi_4(k,l,\bar{k},\bar{l};p,q) = \sum_{s_1,s_2,s_1,s_2} \int_0^1 d\alpha d\beta \frac{d\alpha d\beta}{(\beta + \beta e^{m_1-m})(\alpha + \alpha e^{m_2-m})}
\]

N. Armesto, 06.04.2017 - Quark correlations in the CGC: 2. Calculation.
Correlations in the WF (II):

- Performing MV projectile averages:
  \[ \langle \rho^a(k)\rho^b(p) \rangle_P = (2\pi)^2 \mu^2(k) \delta^{ab} \delta^{(2)}(k+p) \]

\[ \Phi_2\Phi_2 \]

Uncorrelated \( O(N_c^4) \)

\[ \Phi_4 \]

Correlated \( O(N_c^2) \)

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\[ (\Phi_4^A) \text{ Correlated } O(N_c^3) \]

\[ (\Phi_4^B) \text{ Correlated } O(N_c^3) \]

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Correlations in the WF (II):

- Performing MV projectile averages: \( \langle \rho^a(k)\rho^b(p) \rangle_P = (2\pi)^2\mu^2(k)\delta^{ab}\delta^{(2)}(k+p) \)

\[ \Phi_2\Phi_2 \]

\[ \Phi_4 \]

- At large \( N_c \): correlation determined by \( \Phi_4^{A,B} \): minus sign indicative of Pauli blocking.

(\( \Phi_4^A \)) Correlated \( O(N_c^3) \)

(\( \Phi_4^B \)) Correlated \( O(N_c^3) \)
Correlations in particle production:

\[
\frac{d\sigma}{d\eta_1 d^2 p \, d\eta_2 d^2 q} = \frac{g^8}{(2\pi)^4} \int_{x,y,\bar{x},\bar{y}} \int_{z_1,z_2,\bar{z}_1,\bar{z}_2,\bar{z},\bar{w}} \frac{1}{2} \langle \rho^a(x) \rho^b(\bar{x}) \rho^c(y) \rho^d(\bar{y}) \rangle_P \\
\times \langle \Phi_2(x,y;z_1,z_2,\bar{z};p) \Phi_2(\bar{x},\bar{y};\bar{z}_1,\bar{z}_2,\bar{w};q) \rangle \\
\times \text{tr} \left\{ [\tau^a - S^a_{A}(x) S_F(z_1) \tau^a S^\dagger_F(\bar{z})] [\tau^c - S^c_{A}(y) S_F(\bar{z}) \tau^c S^\dagger_F(z_2)] \right\} \\
\times \text{tr} \left\{ [\tau^b - S^{b\bar{b}}_{A}(\bar{x}) S_F(\bar{z}_1) \tau^{b\bar{b}} S^\dagger_F(\bar{w})] [\tau^d - S^{d\bar{d}}_{A}(\bar{y}) S_F(\bar{w}) \tau^d S^\dagger_F(z_2)] \right\} - \Phi_4(x,y,\bar{x},\bar{y};z_1,z_2,\bar{z}_1,\bar{z}_2,\bar{z},\bar{w};p,q) \\
\times \text{tr} \left\{ [\tau^a - S^a_{A}(x) S_F(z_1) \tau^a S^\dagger_F(\bar{z})] [\tau^c - S^c_{A}(y) S_F(\bar{z}) \tau^c S^\dagger_F(z_2)] \right\} \\
\times \left\{ [\tau^b - S^{b\bar{b}}_{A}(\bar{x}) S_F(\bar{z}_1) \tau^{b\bar{b}} S^\dagger_F(\bar{w})] [\tau^d - S^{d\bar{d}}_{A}(\bar{y}) S_F(\bar{z}) \tau^d S^\dagger_F(z_2)] \right\}_T,
\]
Correlations in particle production:

\[ \frac{d\sigma}{d\eta_1 d^2 p d\eta_2 d^2 q} = \frac{g^8}{(2\pi)^4} \int_{x,y,z_1,z_2,\bar{z},\bar{z}_1,\bar{z}_2,\bar{w}} \left( \frac{1}{2} \langle \rho^a(x) \rho^b(\bar{x}) \rho^c(y) \rho^d(\bar{y}) \rangle \right)_P \]

\[ \times \langle \Phi_2(x, y; z_1, z_2, \bar{z}; p) \Phi_2(\bar{x}, \bar{y}; \bar{z}_1, \bar{z}_2, \bar{w}; q) \rangle \]

\[ \times \text{tr} \left\{ \left[ \tau^a - S^a_A(x) S_F(z_1) \tau^a S_F^\dagger(\bar{z}) \right] \left[ \tau^c - S^c_A(y) S_F(\bar{z}) \tau^c S_F^\dagger(z_2) \right] \right\} \]

\[ \times \text{tr} \left\{ \left[ \tau^b - S^b_A(x) S_F(z_1) \tau^b S_F^\dagger(\bar{w}) \right] \left[ \tau^d - S^d_A(y) S_F(\bar{w}) \tau^d S_F^\dagger(z_2) \right] \right\} \]

\[ - \Phi_4(x, y, \bar{x}, \bar{y}; z_1, z_2, \bar{z}_1, \bar{z}_2, \bar{z}, \bar{w}; p, q) \]

\[ \times \text{tr} \left\{ \left[ \tau^a - S^a_A(x) S_F(z_1) \tau^a S_F^\dagger(\bar{z}) \right] \left[ \tau^c - S^c_A(y) S_F(\bar{w}) \tau^c S_F^\dagger(z_2) \right] \right\} \]

\[ \times \left[ \tau^b - S^b_A(x) S_F(\bar{z}_1) \tau^b S_F^\dagger(\bar{w}) \right] \left[ \tau^d - S^d_A(y) S_F(\bar{w}) \tau^d S_F^\dagger(\bar{z}_2) \right] \right\} \right\}_T, \]

\[ \Phi_2 \Phi_2 \]

\[ \Phi_4 \]

N. Armesto, 06.04.2017 - Quark correlations in the CGC: 2. Calculation.
Correlations in particle production:

\[
\frac{d\sigma}{d\eta_1 d^2 p_1 d\eta_2 d^2 q} = \frac{g^8}{(2\pi)^4} \int_{x,y,z,\bar{z}} \int_{z_1,z_2,\bar{z}_1,\bar{z}_2,\bar{w}} \frac{1}{2} \langle \rho^a(x)\rho^b(\bar{x})\rho^c(y)\rho^d(\bar{y}) \rangle P \\
\times \langle \Phi_2(x, y; z_1, z_2, \bar{z}; p)\Phi_2(\bar{x}, \bar{y}; \bar{z}_1, \bar{z}_2, \bar{w}; q) \rangle \\
\times \text{tr} \left\{ \left[ r^a - S_{A}^{a}(x)S_{F}(z_1) r^a S_{F}(\bar{z}) \right] \left[ r^c - S_{A}^{c}(y)S_{F}(\bar{z}) r^c S_{F}(z_2) \right] \right\} \\
\times \text{tr} \left\{ \left[ r^b - S_{A}^{b}(\bar{x})S_{F}(\bar{z}_1) r^b S_{F}(\bar{w}) \right] \left[ r^d - S_{A}^{d}(\bar{y})S_{F}(\bar{w}) r^d S_{F}(\bar{z}_2) \right] \right\} \\
- \Phi_4(x, y, \bar{x}, \bar{y}; z_1, z_2, \bar{z}_1, \bar{z}_2, \bar{z}, \bar{w}; p, q) \\
\times \text{tr} \left\{ \left[ r^a - S_{A}^{a}(x)S_{F}(z_1) r^a S_{F}(\bar{z}) \right] \left[ r^c - S_{A}^{c}(y)S_{F}(\bar{w}) r^c S_{F}(z_2) \right] \right\} \\
\times \left[ r^b - S_{A}^{b}(\bar{x})S_{F}(\bar{z}_1) r^b S_{F}(\bar{w}) \right] \left[ r^d - S_{A}^{d}(\bar{y})S_{F}(\bar{w}) r^d S_{F}(\bar{z}_2) \right] \right\} T,
\]

- No rescattering of gluons before splitting (we assume that it gives no quark correlations).
Correlations in particle production:

\[
\frac{d\sigma}{d\eta_1 d^2 p d\eta_2 d^2 q} = \frac{g^8}{(2\pi)^4} \int_{x,y,\bar{x},\bar{y}} \int_{z_1,z_2,\bar{z}_1,\bar{z}_2,\bar{w}} \frac{1}{2} \langle \rho^a(x)\rho^b(\bar{x})\rho^c(y)\rho^d(\bar{y}) \rangle_p \times \left\langle \Phi_2(x, y; z_1, z_2, \bar{z}; p)\Phi_2(\bar{x}, \bar{y}; \bar{z}_1, \bar{z}_2, \bar{w}; q) \right\rangle_{T} \times \text{tr} \left\{ [\tau^a - S_{A}^{aa}(x) S_{F}(z_1) \tau^a S_{F}^\dagger(\bar{z})][\tau^c - S_{A}^{ce}(y) S_{F}(\bar{z}) \tau^c S_{F}^\dagger(z_2) ] \right\} \\
\times \text{tr} \left\{ [\tau^b - S_{A}^{bb}(\bar{x}) S_{F}(\bar{z}_1) \tau^b S_{F}^\dagger(\bar{w})][\tau^d - S_{A}^{dd}(\bar{y}) S_{F}(\bar{w}) \tau^d S_{F}^\dagger(\bar{z}_2) ] \right\} \\
- \Phi_4(x, y, \bar{x}, \bar{y}; z_1, z_2, \bar{z}_1, \bar{z}_2; \bar{z}, \bar{w}; p, q) \\
\times \text{tr} \left\{ [\tau^a - S_{A}^{aa}(x) S_{F}(z_1) \tau^a S_{F}^\dagger(\bar{z})][\tau^c - S_{A}^{ce}(y) S_{F}(\bar{w}) \tau^c S_{F}^\dagger(z_2) ] \right\} \\
\times \left[ \tau^b - S_{A}^{bb}(\bar{x}) S_{F}(\bar{z}_1) \tau^b S_{F}^\dagger(\bar{w})][\tau^d - S_{A}^{dd}(\bar{y}) S_{F}(\bar{w}) \tau^d S_{F}^\dagger(\bar{z}_2) ] \right\} \right\rangle_{T},
\]

- The same $N_c$ counting holds: leading correlation given by $\Phi_4$ with a minus sign.
- Expand $S$ in the target colour field $\alpha$:

\[
S(x) = \exp\{igt^a \alpha^a(x)\} \quad \alpha^a(x) = \frac{1}{\sqrt{2}} (x, y) \rho^a_T(y)
\]

- Perform MV averages for the target:

\[
\langle \rho^a_T(k)\rho^b_T(p) \rangle_T = (2\pi)^2 \lambda^2(k) \delta^{ab} \delta^{(2)}(k + p)
\]

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Approximations:

- In order to get large $N_c$ analytical estimates that can answer about the sign of the correlations and their rapidity extent:
  
  $\eta_1 - \eta_2 \gg 1$

  $|p| \sim |q| \sim |p - q| \gg Q_s$

- Translational invariance: $S \equiv (2\pi)^2 \delta^{(2)}(0)$

- Color neutrality on scales $1/Q_s$: $Q_s^2 = g^4 \mu^2, \quad Q_T^2 = g^4 \lambda^2$

  $\mu^2(k), \lambda^2(k) = 0$ for $k < Q_s, \quad \mu^2(k), \lambda^2(k) = \mu^2, \lambda^2$ for $k > Q_s$

- The target should not distort too much the correlations: $Q_T < Q_s$

- We keep only leading logarithms.
The results:

- For the correlations in the WF:

\[
\frac{dN^P(p, q; \eta_1, \eta_2)}{d^2 p d^2 q d\eta_1 d\eta_2} \simeq -\frac{S}{(2\pi)^2} e^{\eta_2 - \eta_1} (\eta_1 - \eta_2)^2 \frac{\mu^4}{p^4 q^4} g^8 \frac{N_c^3}{4} \left\{ \frac{25\pi^2}{2} q^4 \left[ \eta_1 - \eta_2 + \ln \frac{p^2}{Q_s^2} \right]^2 \delta^{(2)}(q - p) \right. \\
\left. + \pi \left[ \frac{3(p^2 + q^2)}{(p - q)^4} \left\{ 5 \left[ p^2 q^2 - (p \cdot q)^2 \right] - (p - q)^2 p \cdot q \right\} \ln \frac{(p - q)^2}{Q_s^2} + (\eta_1 - \eta_2) p \cdot q \right] \right\}
\]

- For the correlations in the produced quarks:

\[
\left[ \frac{d\sigma}{d^2 p d^2 q d\eta_1 d\eta_2} \right]_{\text{correlated}} = -S(2\pi)^2 N_c^5 \frac{Q_s^2 Q_T^2}{4 g^4} e^{\eta_2 - \eta_1} (\eta_1 - \eta_2)^2 \ln \left( \frac{Q_T^2}{\Lambda^2} \right) \frac{\pi^3}{p^4} \\
\times \left\{ \frac{50\pi}{16} \ln \left( \frac{Q_s^4}{Q_T^2 \Lambda^2} \right) \delta^{(2)}(q - p) + \frac{9Q_s^2}{q^4} \left[ \frac{2(p^2 + q^2)^2 + p^2 q^2}{(p - q)^4} \right] \ln \left( \frac{(p - q)^2}{Q_s^2} \right) + \frac{9Q_s^2}{2q^4} \left[ \ln \left( \frac{q^2}{Q_s^2} \right) + \ln \left( \frac{p^2}{Q_s^2} \right) \right] \right\}
\]
The results:

- For the correlations in the WF:

\[
\left[ \frac{dN^P(p, q; \eta_1, \eta_2)}{d^2pd^2qd\eta_1d\eta_2} \right]_{\text{correlated}} \approx - \frac{S}{(2\pi)^2} e^{\eta_2 - \eta_1}(\eta_1 - \eta_2)^2 \frac{\mu^4}{p^4q^4} g^8 \frac{N_c^3}{4} \left\{ \frac{25\pi^2}{2} q^4 \left[ \eta_1 - \eta_2 + \ln \frac{p^2}{Q_s^2} \right]^2 \delta^{(2)}(q - p) \right. \\
\left. + \pi \left[ \frac{3(p^2 + q^2)}{(p - q)^4} \left\{ 5 [p^2q^2 - (p \cdot q)^2] - (p - q)^2 p \cdot q \right\} \ln \frac{(p - q)^2}{Q_s^2} + (\eta_1 - \eta_2)p \cdot q \right] \right\}
\]

- For the correlations in the produced quarks:

\[
\left[ \frac{d\sigma}{d^2pd^2qd\eta_1d\eta_2} \right]_{\text{correlated}} = -S(2\pi)^2 N_c^5 \frac{Q_s^2}{4g^4} e^{\eta_2 - \eta_1}(\eta_1 - \eta_2)^2 \ln \left( \frac{Q_T^2}{\Lambda^2} \right) \frac{\pi^3}{p^4} \\
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\]

- **Negative correlations:** Pauli blocking.
- **Short range in rapidity** (peaked for $\Delta \eta \sim 2$).
- $O(\alpha_s^2 N_c)$ suppressed with respect to gluon ridge.
- **Delta function survives rescattering:** contribution from only antiquarks interacting with the target, to be smeared by $R_{\text{projectile}}$.
- **For production**, a non-perturbative regulator appears: poles $k+l$ for two internal momenta $k$, $l$ appearing in the colour charges of $P,T$. 

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Conclusions:

- We have performed a CGC calculation, in the light cone wave functions approach, for quark-quark correlations, under the Glasma graph approximations used to describe the gluon ridge.

- Quarks experience Pauli blocking.

- Quark correlations are short-range in rapidity.

- Quark correlations are parametrically suppressed by $\alpha_s^2N_c$.

- They could be seen, in principle, in D-D correlations but the mass effects should be taken into account.

- They could contribute to odd harmonics.

- They could be extended to the forward region: hybrid formalism.
Outlook:

- Baryon-baryon correlations show features that seem suggestive of Pauli blocking; a similar effect seen in $e^+e^-$ but modifications of hadronisation reproduced the data.

\[ C(\varphi_1, \eta_1, \varphi_2, \eta_2) = \frac{P_{12}(\varphi_1, \eta_1, \varphi_2, \eta_2)}{P_1(\varphi_1, \eta_1)P_2(\varphi_2, \eta_2)} \]
Outlook:

- Baryon-baryon correlations show features that seem suggestive of Pauli blocking; a similar effect seen [PRL57(1986)3140] in $e^+e^-$ but there modifications of hadronisation reproduced the data.

\[ C(\varphi_1, \eta_1, \varphi_2, \eta_2) = \frac{P_{12}(\varphi_1, \eta_1, \varphi_2, \eta_2)}{P_1(\varphi_1, \eta_1)P_2(\varphi_2, \eta_2)} \]
Outlook:

- Baryon-baryon correlations show features that seem suggestive of Pauli blocking; a similar effect seen \[^{[\text{PRL57(1986)3140}]}\] in $e^+e^-$ but there modifications of hadronisation reproduced the data.

\[
C(\varphi_1, \eta_1, \varphi_2, \eta_2) = \frac{P_{12}(\varphi_1, \eta_1, \varphi_2, \eta_2)}{P_1(\varphi_1, \eta_1)P_2(\varphi_2, \eta_2)}
\]

- Quark correlations could contribute to charge correlations (proposed as signals of Chiral Magnetic Effect).

\[1612.08975\]

\[1610.00263\]

N. Armesto, 06.04.2017 - Quark correlations in the CGC.
Outlook:

- Baryon-baryon correlations show features that seem suggestive of Pauli blocking; a similar effect seen \cite{PRL57(1986)3140} in $e^+e^-$ but there modifications of hadronisation reproduced the data.

\[
C(\phi_1, \eta_1, \phi_2, \eta_2) = \frac{P_{12}(\phi_1, \eta_1, \phi_2, \eta_2)}{P_1(\phi_1, \eta_1)P_2(\phi_2, \eta_2)}
\]

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