Non-linear dynamics in DIS at NLO

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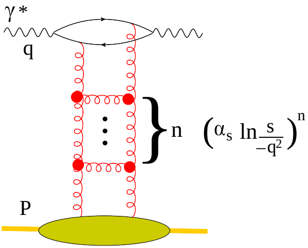
Outline

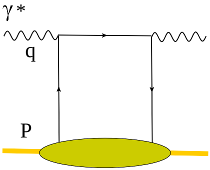
- High-energy Operator Product Expansion: factorization in rapidity space.
- Evolution equation and background field method.
- NLO impact factor and composite Wilson line operators.
- Conclusions.

Leading Log Approximation in scatt. process at high energy

electron-proton/nucleus Deep Inelastic Scattering (DIS) $s = (q+P)^2$

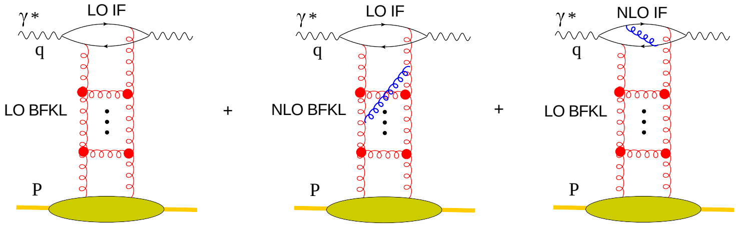
$$\langle P|\mathrm{T}\, j^{\mu}(x) j^{\nu}(y)|P\rangle$$





energy suppressed

- BFKL resum $\left(\alpha_s \ln \frac{s}{-q^2}\right)^n$
- Dynamics is linear and it describes proliferation of gluons
  $\Rightarrow$ Violation of Unitarity



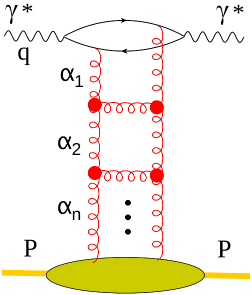
- NLO BFKL: $\alpha_s \left(\alpha_s \ln \frac{1}{x_B}\right)^n$

- NLO Impact factor contains contributions prop. to $\alpha_s \ln \frac{1}{x_B}$ which can be included in the LLA.

- Need systematics of perturbation theory
  $\Rightarrow$ Operator Product Expansion formalism can be the solution.

Semi-classical approach: Background field method

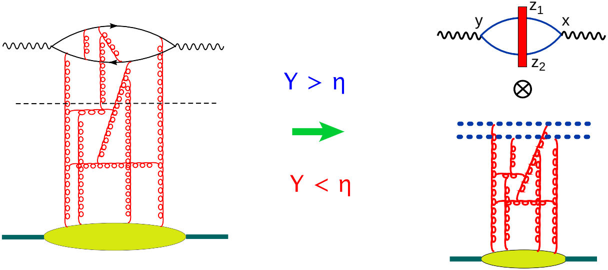
p_1^μ , p_2^μ light-cone vectors $\Rightarrow k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp^\mu$
$\alpha_1 \gg \alpha_2 ... \gg \alpha_n$



Fields are ordered in their rapidities $\Rightarrow$

- fast fields are treated as quantum fields
- slow fields are treated as classical fields

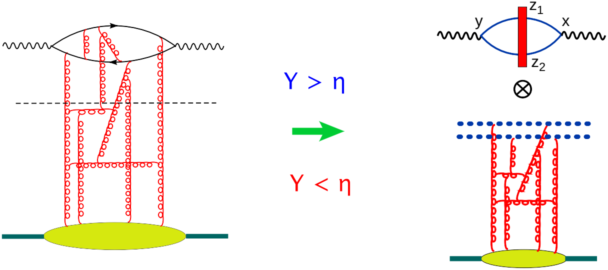
DIS amplitude is factorized in rapidity: η



$|B\rangle$ is the target state.

$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

High-Energy Operator Product Expansion

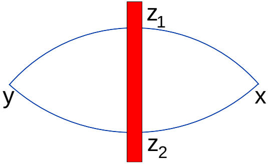


$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

- If we use a model to evaluate $\langle B|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle B|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ with respect to the rapidity parameter η.

LO Impact Factor

Conformal invariance: $(x^+, x_\perp)^2 = -x_\perp^2 \Rightarrow$ after the inversion $x_\perp \to x_\perp / x_\perp^2$ and $x^+ \to x^+ / x_\perp^2$



Conformal vectors:

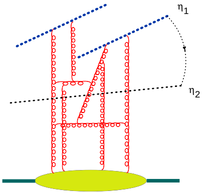
$$\kappa = \frac{\sqrt{s}}{2x_*}(\frac{p_1}{s} - x^2 p_2 + x_\perp) - \frac{\sqrt{s}}{2y_*}(\frac{p_1}{s} - y^2 p_2 + y_\perp)$$

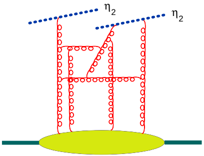
$$\zeta_1 = \left(\frac{p_1}{s} + z_{1\perp}^2 p_2 + z_{1\perp}\right), \qquad \zeta_2 = \left(\frac{p_1}{s} + z_{2\perp}^2 p_2 + z_{2\perp}\right)$$

$\sqrt{\frac{s}{2}}x^+ = x_* \equiv x_\mu p_2^\mu$ (similarly for y); $\mathcal{R} = \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$

$$I^{\rm LO}_{\mu\nu}(z_1, z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \frac{\partial^2}{\partial x^\mu \partial y^\nu}\left[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\right]$$

Evolution Equation





- Separate fields in quantum and classical according to low and large rapidity. Formally we may write:

$$\langle B|\mathcal{O}^{\eta_1}|B\rangle \to \langle \mathcal{O}^{\eta_1}\rangle_A \to \langle \mathcal{O}'^{\eta_2}\otimes \mathcal{O}'^{\eta_1}\rangle_A$$

- Integrate over the quantum fields and get one-loop rapidity evolution of the operator $\mathcal{O}$

$$\langle \mathcal{O}^{\eta_1}\rangle_A = \alpha_s(\eta_1-\eta_2)K_{\rm evol}\otimes\langle \mathcal{O}'^{\eta_2}\rangle_A$$

- Where in principle $\mathcal{O}$ and $\mathcal{O}'$ are different operators.

Non-linear evolution equation

- Linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \, K_{\rm evol} \otimes \mathcal{O}^{\eta_2}$

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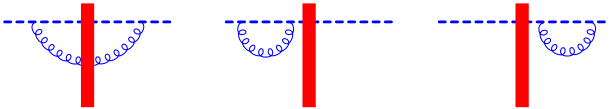
- Non-linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \, K_{\rm evol} \, \otimes \, \{\mathcal{O}^{\eta_2} \mathcal{O}^{\eta_2}\}$

Non-linear evolution equation

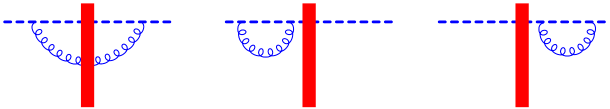


$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta \int \frac{d^2z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

$\Delta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

Non-linear evolution equation



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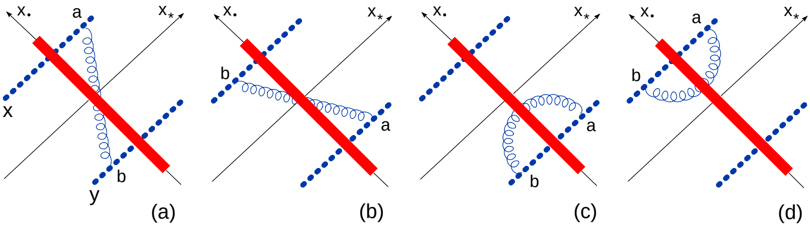
$$\{U_x^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

- Obtain a set of rules that allow one to get the LO evolution of any trace or product of traces of Wilson lines
- Hierarchy of evolution equation: B-JIMWLK equation.

Leading order evolution equation

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

Non-linear evolution equation: Balitsky-Kovchegov equation

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

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- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.

Non linear evolution equation: Balitsky-Kovchegov equation

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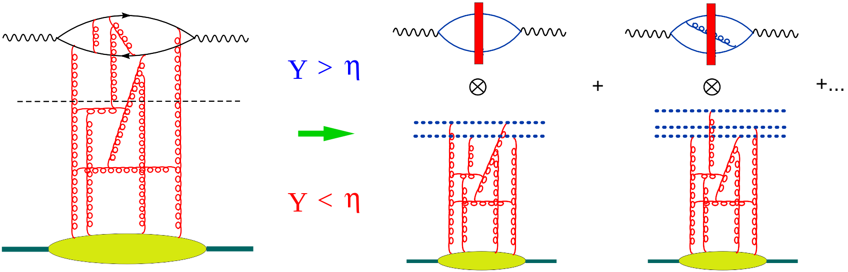
- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.

- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK eqn
  - background field method: describes recombination process.

High-energy expansion in color dipoles at the NLO



The high-energy operator expansion is

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

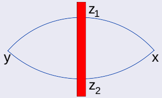
$$+ \int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

LO and NLO Impact Factor

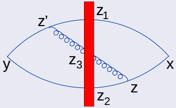
$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2 \; I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

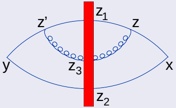
$$+ \int d^2z_1 d^2z_2 d^2z_3 \; I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

LO Impact Factor diagram: $I^{\rm LO}$



NLO Impact Factor diagrams: $I^{\rm NLO}$





NLO Photon Impact Factor

$$\left[\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A\right]^{\rm LO} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A$$

$$\left[\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A\right]^{\rm NLO} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3\, \left[I_1^{\mu\nu}(z_1,z_2,z_3) + I_2^{\mu\nu}(z_1,z_2,z_3)\right]$$
$$\times[{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}]$$

where $I_2^{\mu\nu}(z_1,z_2,z_3)$ is finite and conformal, while

$$I_1^{\mu\nu}(z_1,z_2,z_3) = \frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\, e^{i\frac{s\alpha}{4}\mathcal{Z}_3}$$

is rapidity divergent.

$$\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A$$

$$+ \int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3\, I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta)[{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}] + \dots$$

$\Rightarrow$

$$\left[\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A\right]^{\rm NLO} - \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\left[\langle {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A\right]^{\rm LO}$$

$$= \int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3\, I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta)[{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}]$$

$$\left[\langle {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A\right]^{\rm LO} = \frac{\alpha_s}{2\pi^2}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}[{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}]\int_0^{e^\eta}\frac{d\alpha}{\alpha}$$

How to get the NLO Impact factor

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3\ I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta)[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]$$
$$= \int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3\ \Big\{ I_2^{\mu\nu}(z_1,z_2,z_3) + \frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\Big[\int_0^{+\infty}\frac{d\alpha}{\alpha}\, e^{i\frac{s\alpha}{4}\mathcal{Z}_3} - \int_0^{e^\eta}\frac{d\alpha}{\alpha}\Big]\Big\}$$
$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]$$

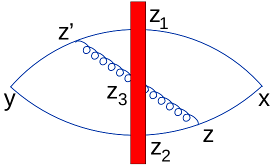
$$\Big[\int_0^{+\infty}\frac{d\alpha}{\alpha}\, e^{i\frac{s\alpha}{4}\mathcal{Z}_3} - \int_0^{e^\eta}\frac{d\alpha}{\alpha}\Big] \to -\ln\frac{\sigma s}{4}\mathcal{Z}_3 - \frac{i\pi}{2} + C$$

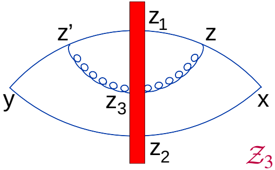
where $\sigma = e^\eta$ and C is the Euler constant

$$\mathcal{Z}_3 \equiv \frac{(x-z_3)_\perp^2}{x^+} - \frac{(y-z_3)_\perp^2}{y^+}$$

$\mathcal{Z}_3$ is not conformal invariant in the transverse 2-d coordinate space, but QCD at tree level has to be conformal invariant.

NLO Impact Factor

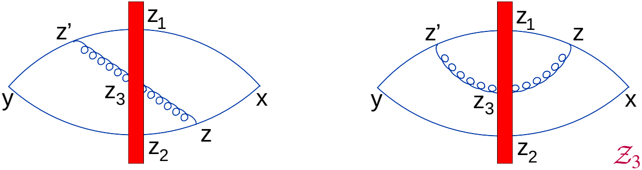




$$\mathcal{Z}_3 \equiv \frac{(x-z_3)_\perp^2}{x^+} - \frac{(y-z_3)_\perp^2}{y^+}$$

$$I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta) \;=\; -\,I^{\rm LO}_{\mu\nu} \times \frac{\alpha_s}{2\pi}\frac{z_{13}^2}{z_{12}^2 z_{23}^2}\ln\frac{\sigma s}{4}\mathcal{Z}_3 \;+\; \text{conf.}$$

The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff $\eta = \ln\sigma$ is not invariant.



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The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff $\eta = \ln\sigma$ is not invariant.

However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$[\mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]^{\rm conf} \;=\; \mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$+\,\frac{\alpha_s}{4\pi}\int d^2 z_3\;\frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\frac{1}{N_c}\mathrm{tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]\ln\frac{a z_{12}^2}{z_{13}^2 z_{23}^2} \;+\; O(\alpha_s^2)$$

the impact factor becomes conformal at the NLO.

Operator expansion in conformal dipoles

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm tr}[\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf}$$
$$+\int d^2z_1d^2z_2d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

$$I^{\rm NLO}_{\mu\nu} = \ - I^{\rm LO}_{\mu\nu}\ \frac{\alpha_s N_c}{4\pi}\int dz_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\ln\frac{z_{12}^2 e^{2\eta} a s^2}{z_{13}^2 z_{23}^2}\mathcal{Z}_3^2 + {\rm conf.}$$

The new NLO impact factor is conformally invariant.

In conformal $\mathcal{N}=4$ SYM theory one can construct the composite conformal dipole operator order by order in perturbation theory.

NLO structure functions for DIS off a large nucleus

$$(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\ \Big\{ I^{\mu\nu}_{\rm LO}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}$$

$$+\int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)} - 2C\Big) I^{\mu\nu}_{\rm LO} +\ I^{\mu\nu}_2\Big]$$

$$\times[{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}{\rm tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}$$

where

$$(I_2)_{\mu\nu}(z_1,z_2,z_3)\ = \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Bigg\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)}+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}$$

$$+\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big] + \frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big]$$

$$+(\zeta_1\leftrightarrow\zeta_2)\Bigg\}$$

NLO evolution of composite "conformal" dipoles in QCD

$$\frac{2ad}{da}[\mathrm{tr}\{\hat U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{conf}} = \frac{\alpha_s}{2\pi^2}\int d^2z_3\ \Big([\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\} - N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\mathrm{conf}}$$

$$\times\ \frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2-z_{23}^2}{z_{13}^2z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2}+\frac{67}{9}-\frac{\pi^2}{3}\Big)\Big]$$

$$+\ \frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\ \Big\{\Big[-2+\frac{z_{14}{}^2z_{23}^2+z_{24}{}^2z_{13}^2-4z_{12}^2z_{34}^2}{2(z_{14}{}^2z_{23}^2-z_{24}{}^2z_{13}^2)}\ln\frac{z_{14}{}^2z_{23}^2}{z_{24}{}^2z_{13}^2}\Big]$$

$$\times\ [\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - \mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\hat U_{z_4}\hat U^\dagger_{z_2}\hat U_{z_3}\hat U^\dagger_{z_4}\} - (z_4\to z_3)]$$

$$+\ \frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}{}^2}\Big[2\ln\frac{z_{12}^2z_{34}^2}{z_{14}^2z_{23}^2}+\Big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{14}^2z_{23}^2}\Big)\ln\frac{z_{13}^2z_{24}{}^2}{z_{14}{}^2z_{23}^2}\Big]$$

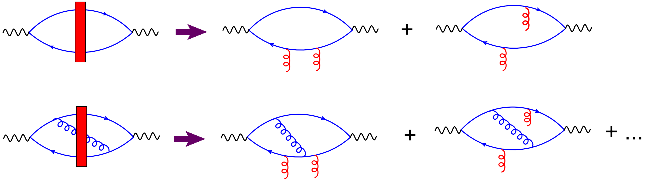
$$\times\ [\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\mathrm{tr}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - \mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_4}\hat U_{z_3}\hat U^\dagger_{z_2}\hat U_{z_4}\hat U^\dagger_{z_3}\} - (z_4\to z_3)]\Big\}$$

$b = \frac{11}{3}N_c - \frac{2}{3}n_f$ I. Balitsky and G.A.C

$K_{\mathrm{NLO\ BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part + Conformal analytic ($\mathcal{N}=4$) part

Linearized $K_{\mathrm{NLO\ BK}}$ reproduces the known result for the forward NLO BFKL kernel Fadin and Lipatov (1998).

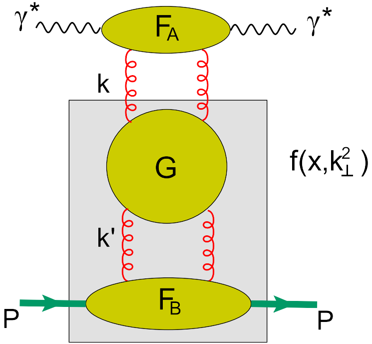
2-gluon approx. and BFKL pomeron in DIS



$$I^{\text{LO}}\hat{\mathcal{U}}(x_\perp, y_\perp)$$
$$I^{\text{NLO}}\left\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y)\right\}$$

where $\mathcal{U}(x,y) = 1 - \frac{1}{N_c}\text{tr}\{U_x U_y^\dagger\}$ and we neglected the non-linear term $\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)$

NLO DIS in the k_T factorization form



- $f(x,k_\perp^2) \propto \int \frac{d^2k'}{k_\perp'^2} F_B(k_\perp'^2)\, k_\perp^2 G(x,k_\perp,k'_\perp)$
- $\mathcal{V}(z) \equiv z^{-2}\mathcal{U}(z)$

$$2a\frac{d}{da}\mathcal{V}_a(z) = \frac{\alpha_s N_c}{2\pi^2}\int d^2z' \left[\frac{2\mathcal{V}_a(z')}{(z-z')^2} - \frac{z^2\mathcal{V}_a(z)}{z'^2(z-z')^2}\right]$$

$$\int d^4x\, e^{iqx}\langle p|T\{\hat{j}_\mu(x)\hat{j}_\nu(0)\}|p\rangle = \frac{s}{2}\int\frac{d^2k_\perp}{k_\perp^2}\, I_{\mu\nu}(q,k_\perp)\mathcal{V}_{a_m=x_B}(k_\perp)$$

NLO DIS in the k_T factorization form

NLO Evolution of the unintegrated gluon distribution

$$
\begin{aligned}
2a\frac{d}{da}\mathcal{V}_a(k) \;=\; & \frac{\alpha_s N_c}{\pi^2}\int d^2k'\Big\{\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2} - \frac{(k,k')\mathcal{V}_a(k)}{k'^2(k-k')^2}\Big] \\
& \times\Big(1+\;\frac{\alpha_s b}{4\pi}\Big[\ln\frac{\mu^2}{k^2}+\frac{N_c}{b}\Big(\frac{67}{9}-\frac{\pi^2}{3}-\;\frac{10n_f}{9N_c}\Big)\Big]\Big)-\;\frac{b\alpha_s}{4\pi} \\
& \times\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2}\ln\frac{(k-k')^2}{k'^2}-\frac{k^2\mathcal{V}_a(k)}{k'^2(k-k')^2}\ln\frac{(k-k')^2}{k^2}\Big] \\
& +\frac{\alpha_s N_c}{4\pi}\Big[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2}+\;F(k,k')+\Phi(k,k')\Big]\;\mathcal{V}_a(k')\Big\} \\
& +3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{V}_a(k)
\end{aligned}
$$

I. Balitsky and G.A.C.

Photon Impact Factor for BFKL pomeron in momentum space

k_T-factorization form

I. Balitsky and G.A.C.

$$I^{\mu\nu}(q,k_\perp)$$
$$= \frac{N_c}{32}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\left(\frac{k_\perp^2}{Q^2}\right)^{\frac{1}{2}-i\nu}\left\{\left[\left(\frac{9}{4}+\nu^2\right)\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_1(\nu)\right)P_1^{\mu\nu}\right.\right.$$
$$\left.+\left(\frac{11}{4}+3\nu^2\right)\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_2(\nu)\right)P_2^{\mu\nu}\right]$$
$$\left.+\frac{\frac{1}{4}+\nu^2}{2k_\perp^2}\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_3(\nu)\right)\left[\tilde{P}^{\mu\nu}\bar{k}^2+\bar{P}^{\mu\nu}\tilde{k}^2\right]\right\}$$

$$P_1^{\mu\nu} = g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}$$
$$P_2^{\mu\nu} = \frac{1}{q^2}\left(q^\mu-\frac{p_2^\mu q^2}{q\cdot p_2}\right)\left(q^\nu-\frac{p_2^\nu q^2}{q\cdot p_2}\right)$$
$$\bar{P}^{\mu\nu} = \left(g^{\mu1}-ig^{\mu2}-p_2^\mu\frac{\bar{q}}{q\cdot p_2}\right)\left(g^{\nu1}-ig^{\nu2}-p_2^\nu\frac{\bar{q}}{q\cdot p_2}\right)$$
$$\tilde{P}^{\mu\nu} = \left(g^{\mu1}+ig^{\mu2}-p_2^\mu\frac{\tilde{q}}{q\cdot p_2}\right)\left(g^{\nu1}+ig^{\nu2}-p_2^\nu\frac{\tilde{q}}{q\cdot p_2}\right)$$

Photon Impact Factor for BFKL pomeron in momentum space

k_T-factorization form

I. Balitsky and G.A.C.

$$
\begin{aligned}
\mathcal{F}_{1(2)}(\nu) &= \Phi_{1(2)}(\nu) + \chi_\gamma \Psi(\nu), \quad \mathcal{F}_3(\nu) = F_6(\nu) + \left(\chi_\gamma - \frac{1}{\bar{\gamma}\gamma}\right)\Psi(\nu), \\
\Psi(\nu) &\equiv \psi(\bar{\gamma}) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma), \\
F_6(\gamma) &= F(\gamma) - \frac{2C}{\bar{\gamma}\gamma} - 1 - \frac{2}{\gamma^2} - \frac{2}{\bar{\gamma}^2} - 3\frac{1+\chi_\gamma - \frac{1}{\gamma\bar{\gamma}}}{2+\bar{\gamma}\gamma}, \\
\Phi_1(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar{\gamma}\gamma} + 1 + \frac{25}{18(2-\gamma)} + \frac{1}{2\bar{\gamma}} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2} \\
\Phi_2(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar{\gamma}\gamma} + 1 + \frac{1}{2\bar{\gamma}\gamma} - \frac{7}{2(2+3\bar{\gamma}\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar{\gamma}\gamma}, \\
F(\gamma) &= \frac{2\pi^2}{3} - \frac{2\pi^2}{\sin^2\pi\gamma} - 2C\chi_\gamma + \frac{\chi_\gamma - 2}{\bar{\gamma}\gamma}
\end{aligned}
$$

where $\gamma = \frac{1}{2} + i\nu$

Conclusions

- Dynamics of QCD at high-energy is non-linear.
- Guiding line for the systematics of the high-energy OPE is conformal invariance in 2-d coordinates space order by order.
- The NLO Impact Factor obtained through the OPE in terms of composite Wilson lines operators is conformal invariant and energy independent.
- NLO Impact factor for pomeron exchange in momentum space has been obtained (aloso available in coordinate space and Mellin space).
- To get the DIS cross section at NLO at highe energy one has to convolute the NLO impact factor with the evolution equation of $\mathrm{tr}\{U_{x_1}U^\dagger_{x_2}\}\mathrm{tr}\{U_{x_3}U^\dagger_{x_4}\}$ at LO and the LO Impact factor with NLO BK equation.