Fluctuations of the multiplicity of produced particles in pA collisions

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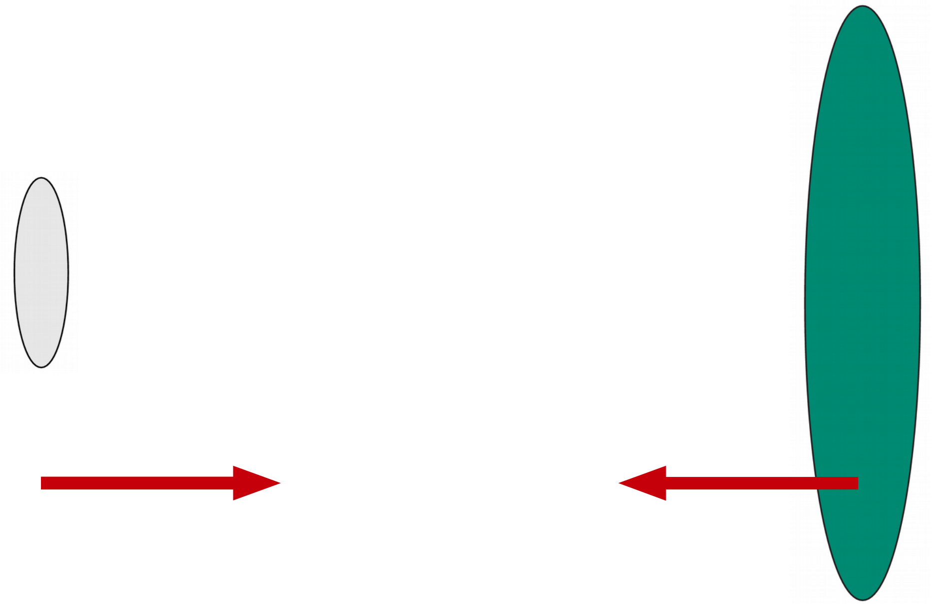
Based on work with T. Liou and A.H. Mueller (Phys. Rev. D, 2017)
and numerical work with L. Dominé, C. Lorcé and S. Pekar (to appear)

Observable

Consider proton-nucleus collisions



What is the probability P_n to observe n particles [=k times the mean multiplicity] **in the fragmentation region of the proton, in a particular event?**

Outline

- Picture of particle production in pA collisions
- How a hadronic state dresses at high energies: the color dipole model
- Probability distribution of the particle multiplicity

Particle production in a dilute-dense collision

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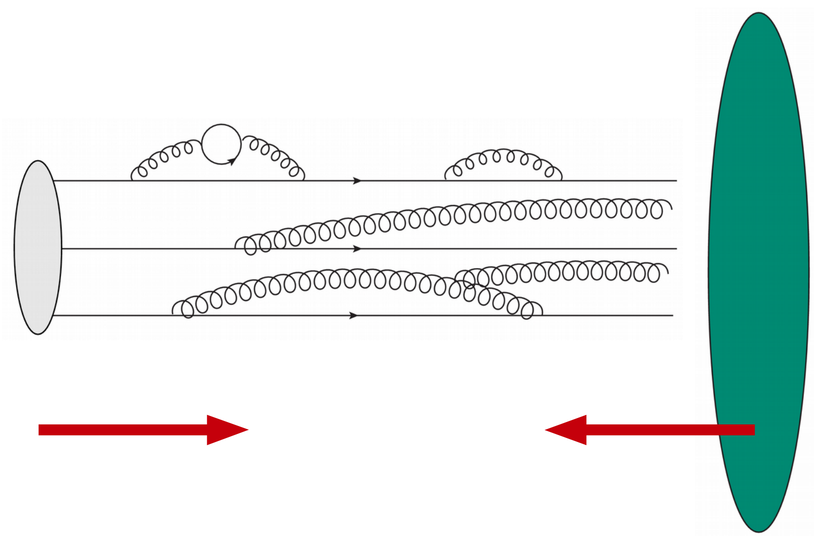

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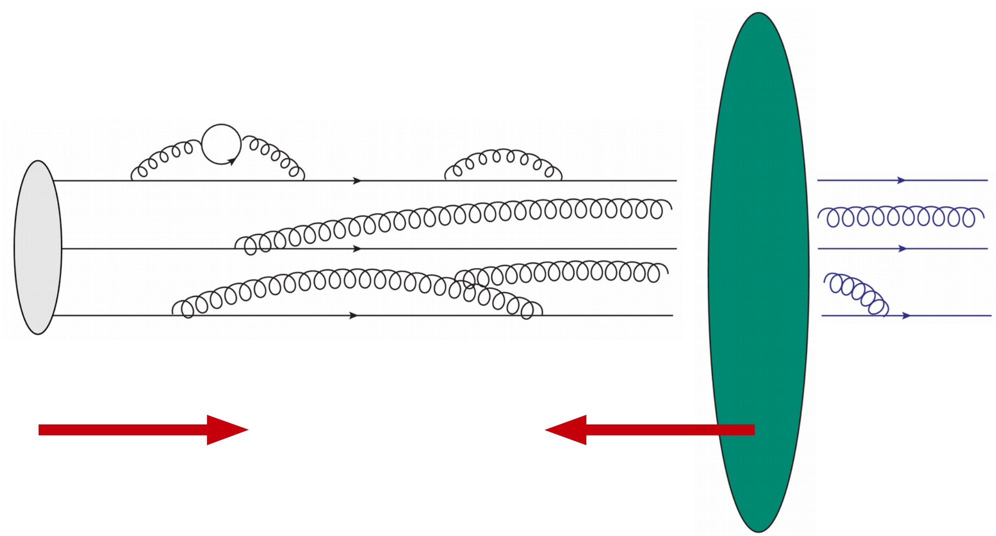

<u>Final state</u>: The gluons which have a transverse momentum less than Q_A are **freed** thanks to multiple scatterings with the nucleus and go the the final state (in the forward region of the proton), while the others essentially do not interact and just recombine.

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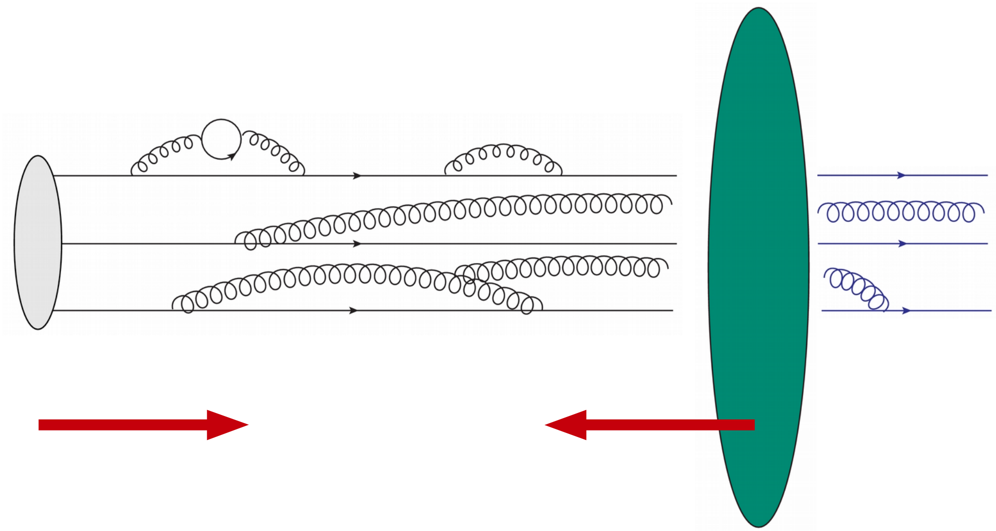

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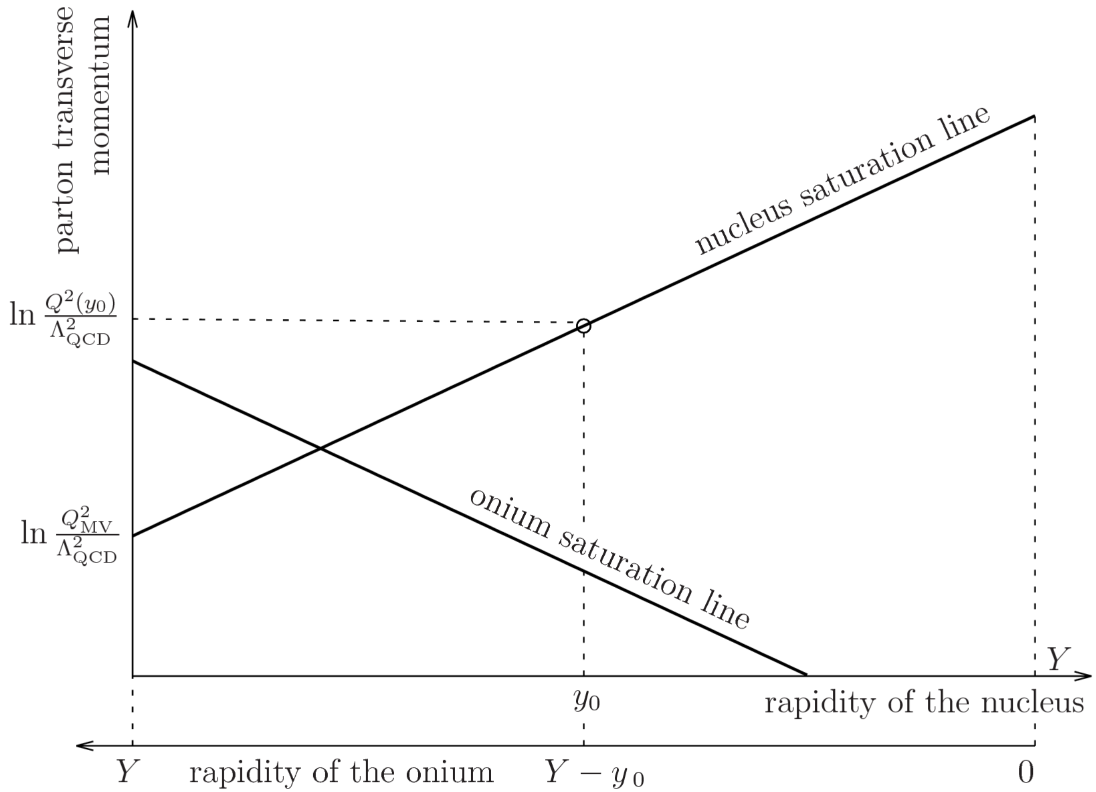
The multiplicity measured in the proton fragmentation region in an event is the gluon number density at the scale Q_A in the corresponding realization of the QCD evolution.

Particle production in a dilute-dense collision

We choose a frame in which the saturation scale of the nucleus is much larger than that of the proton, and **look at central rapidity in that frame**.



The multiplicity of produced particles at central rapidity is related to the gluon density in the proton by

$$\frac{dN}{dy}(y \simeq 0) = x\, G(x, Q_A^2(y_0))$$

$$\text{where} \quad x = e^{-(Y-y_0)} \frac{Q_A(y_0)}{\text{proton mass}}$$

Let's try and understand the event-by-event fluctuations of the gluon density!

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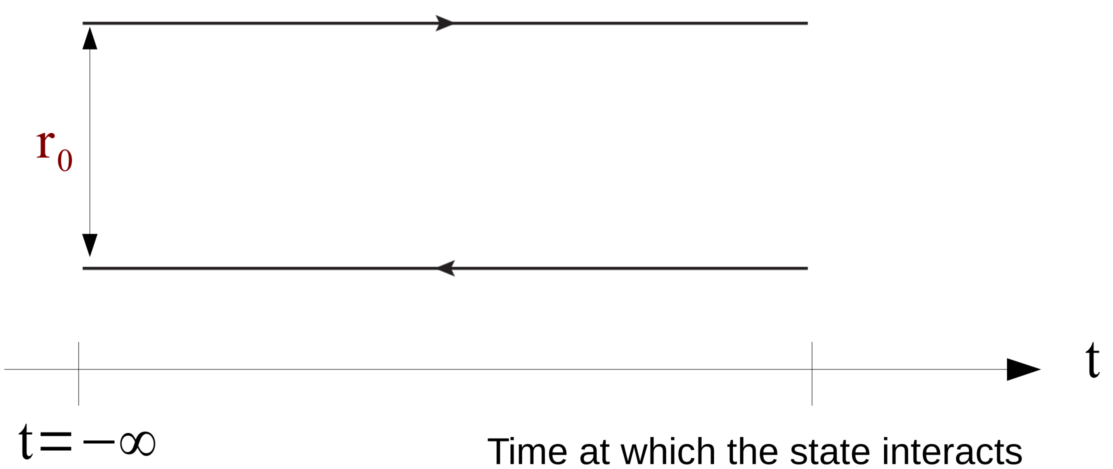
QCD calculation: the color dipole model

To simplify, we consider a **color neutral q-$\bar{q}$ pair** (=onium) of given transverse size.



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Probability of observing a gluon fluctuation when one increases the rapidity from **0** to **dy**:

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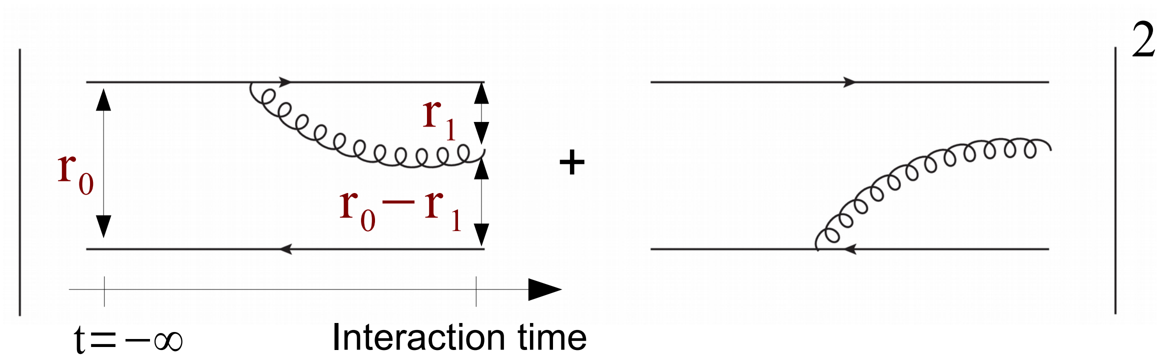

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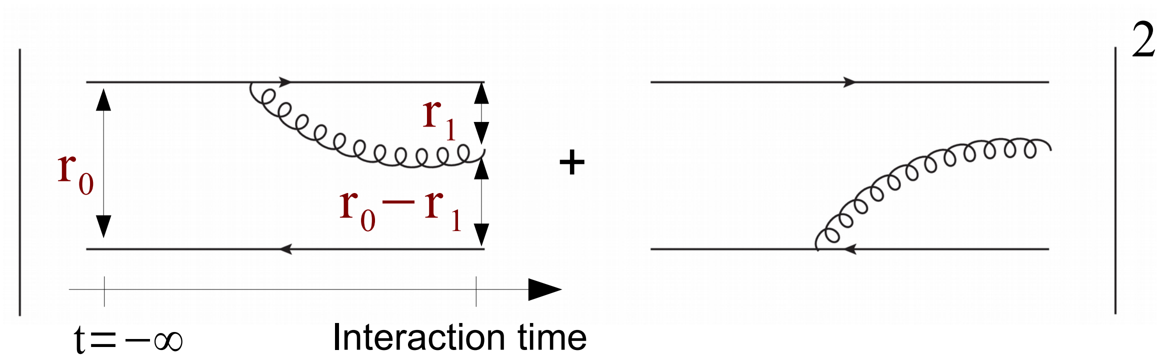

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$$= dy \, \frac{\alpha_s (N_c^2 - 1)}{\pi N_c} \, \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \, \frac{d^2 r_1}{2\pi}$$

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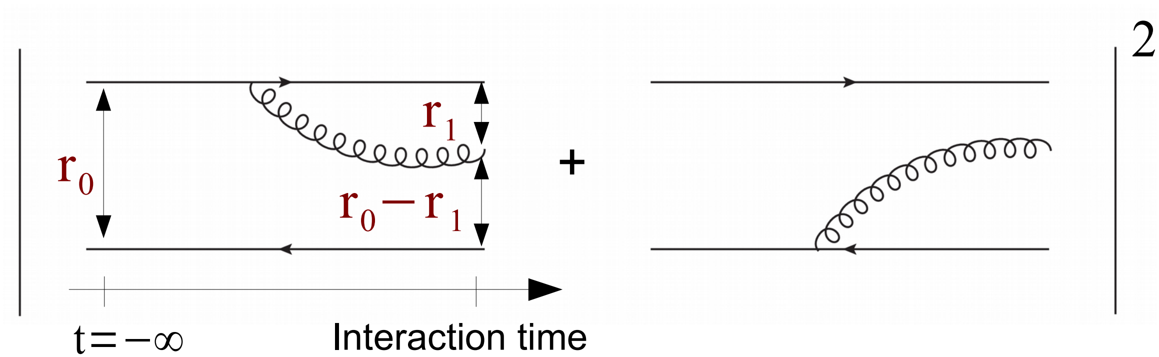

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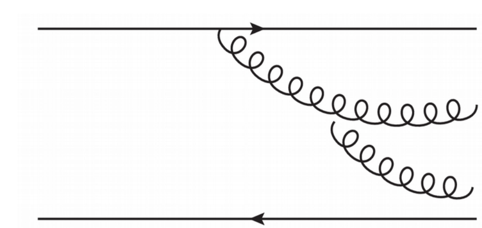
For finite rapidities, one needs to consider higher-orders:



QCD calculation: the color dipole model

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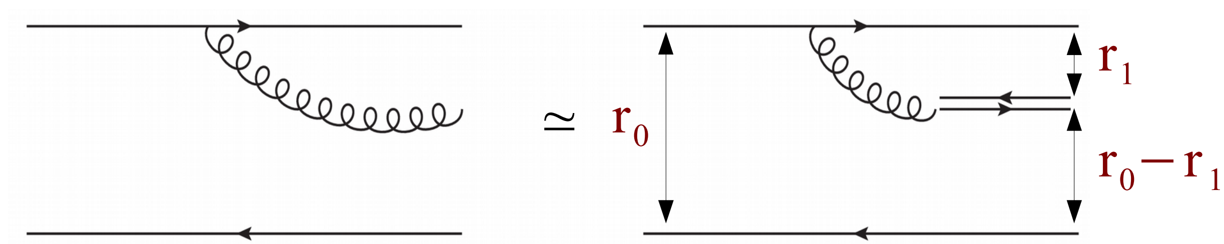

Gluon emission = color-dipole splitting, with probability

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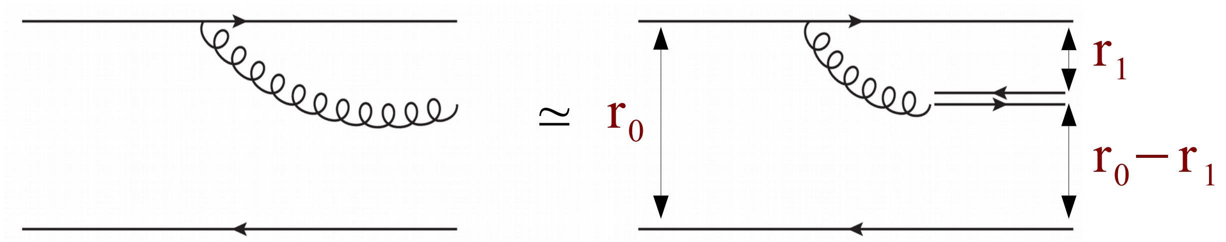

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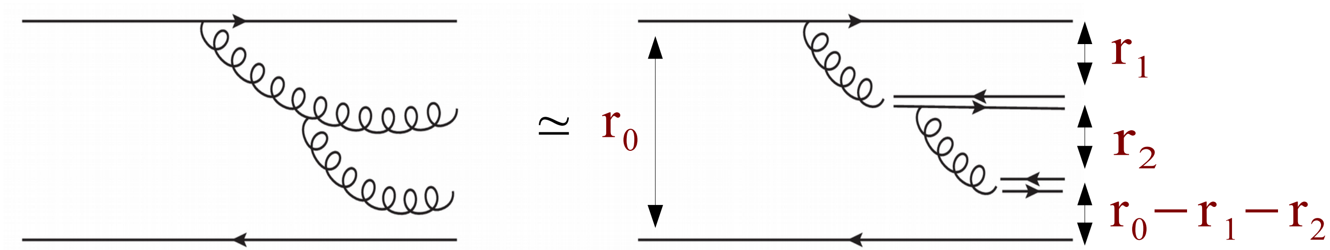

Two successive dipole branchings

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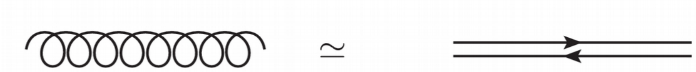

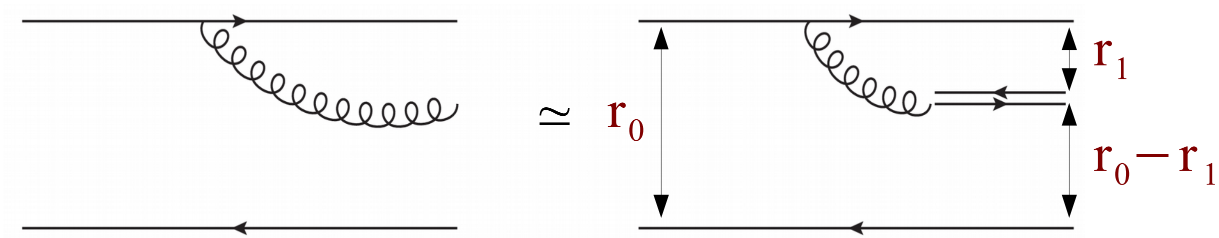

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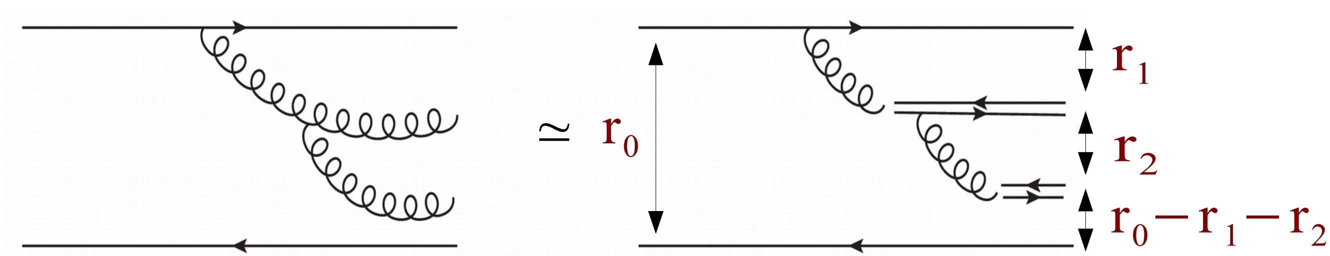

Two successive dipole branchings

$x\,G(x, k^2) \sim$ **density of dipoles of size 1/k generated after rapidity evolution y=log(1/x)**

How the dipole model works

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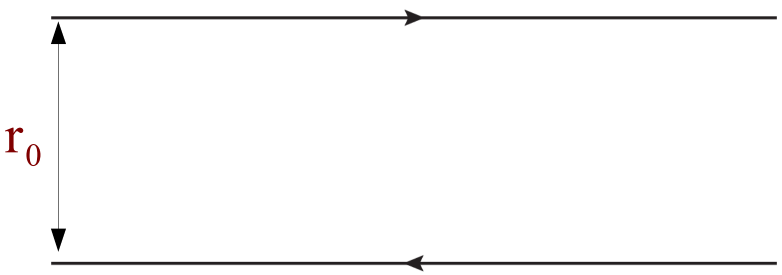

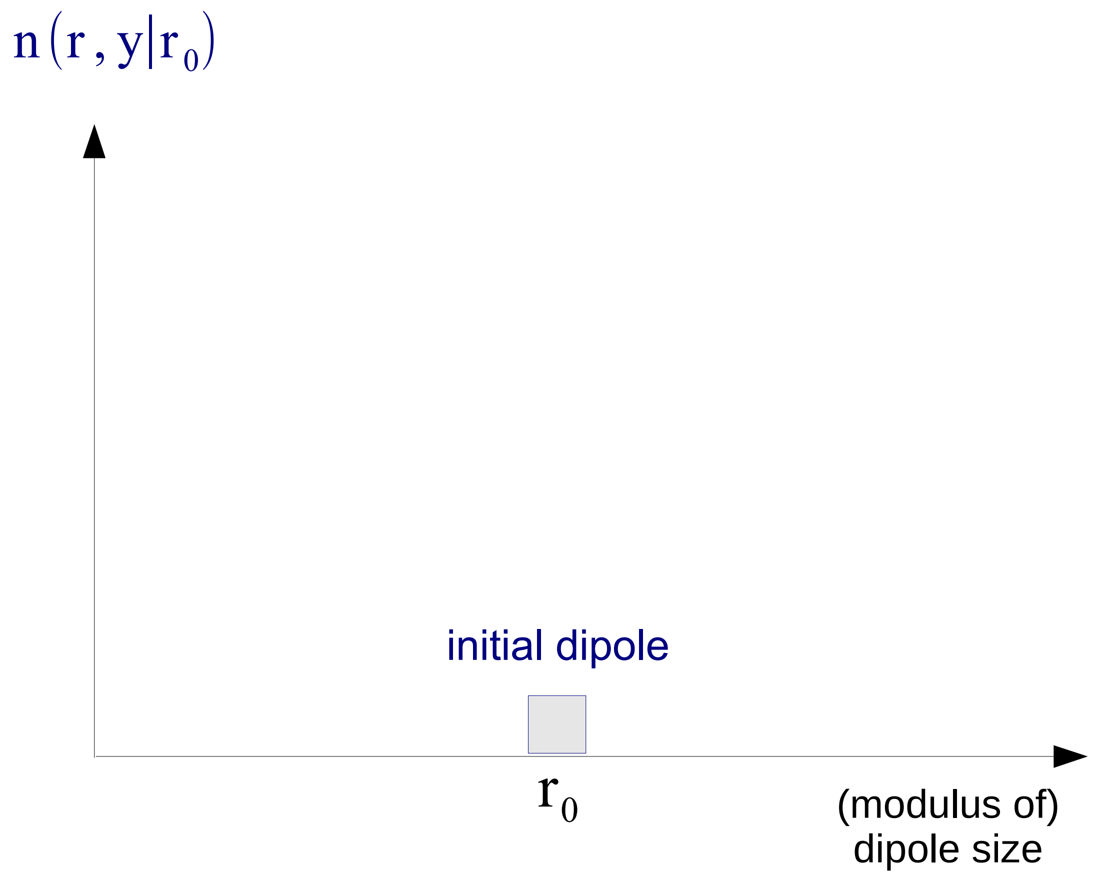

$$y = 0$$

How the dipole model works

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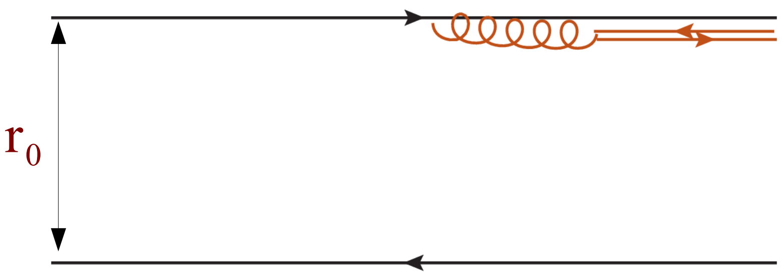
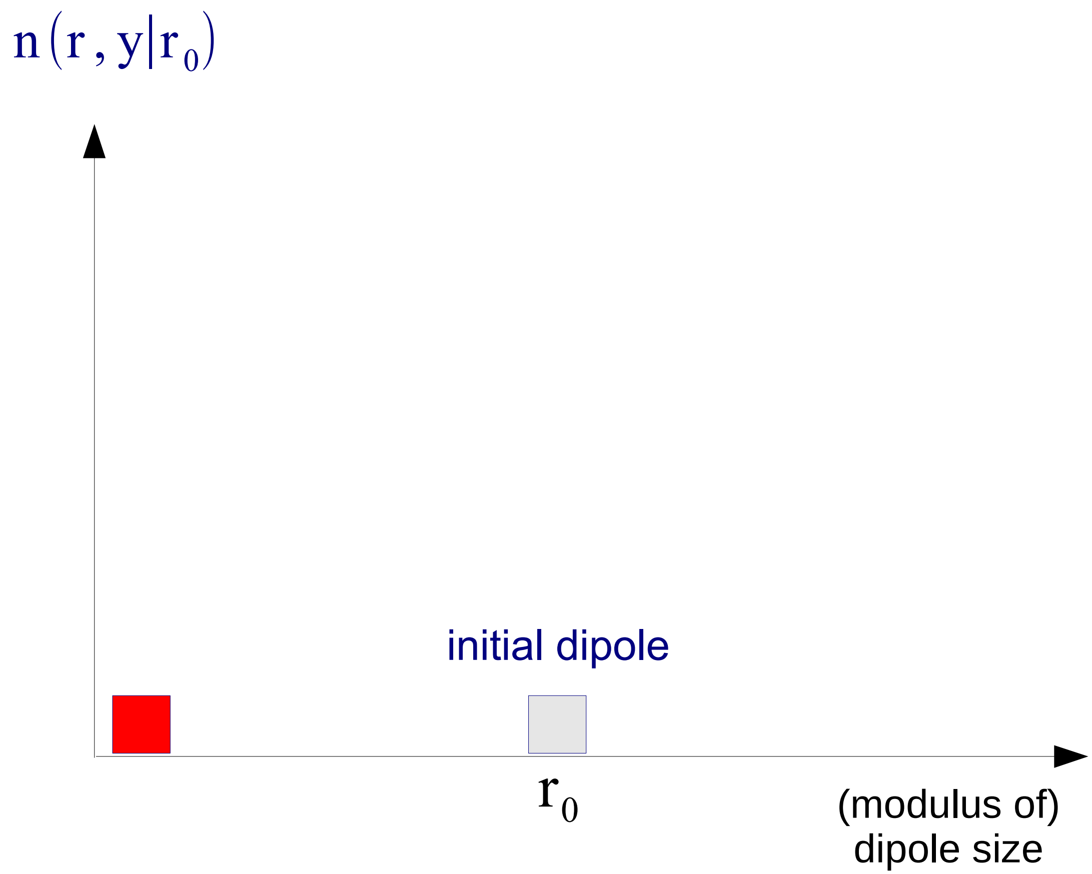
Singular when the gluon is close
to the quark or to the antiquark
Collinear singularity





The small-dipole size region gets very easily populated

$y > 0$

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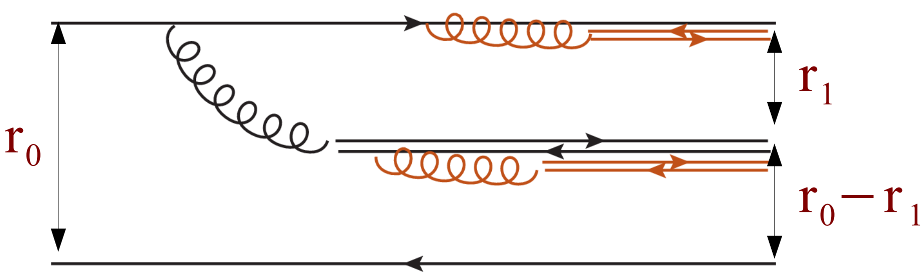

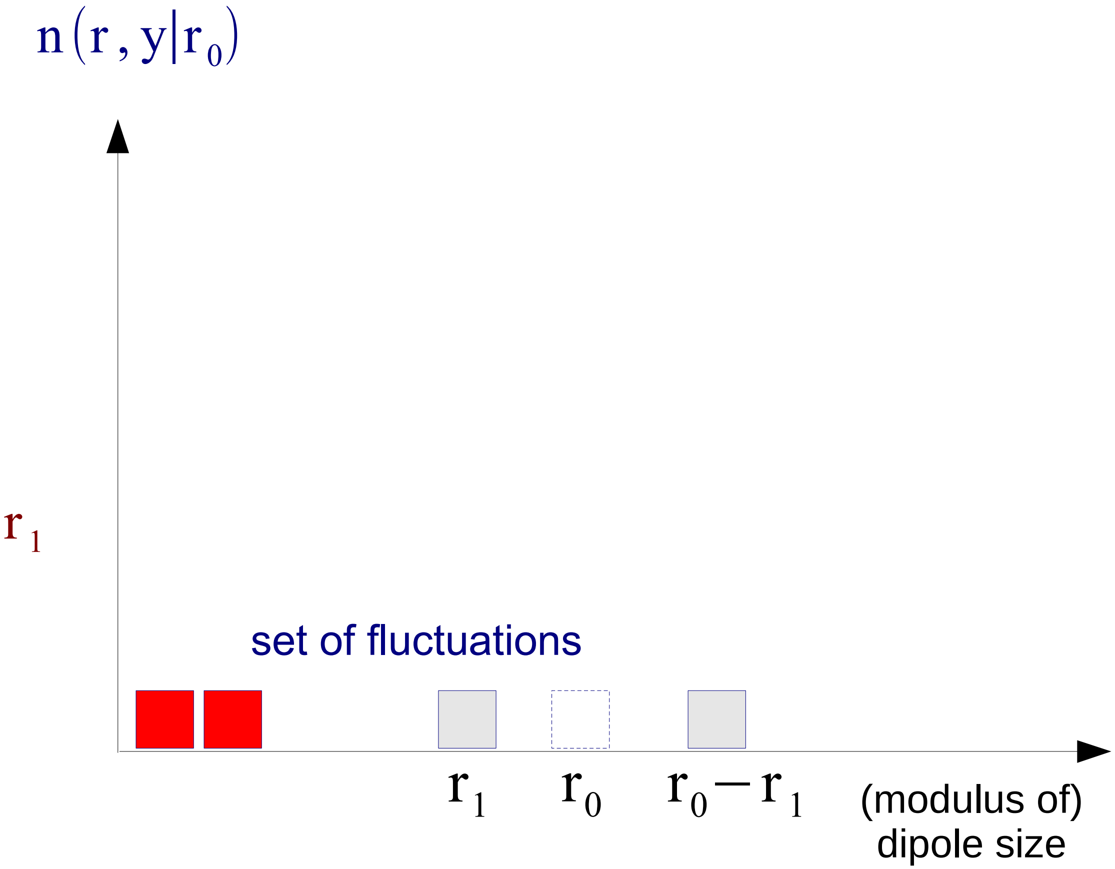

Dipoles of size of the order of the size of the initial dipole (are larger) need sone finite rapidity to get produced

$$y \sim \frac{1}{\alpha_s N_c / \pi}$$

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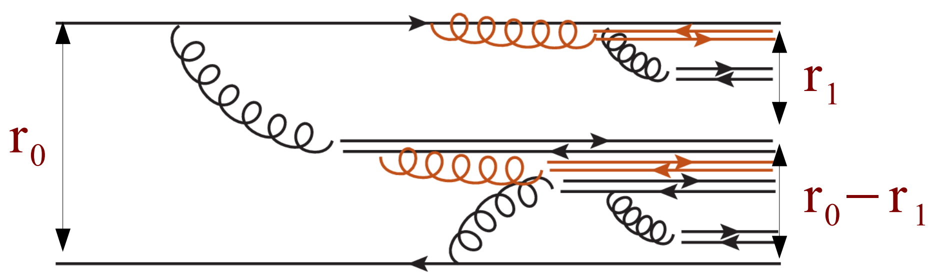
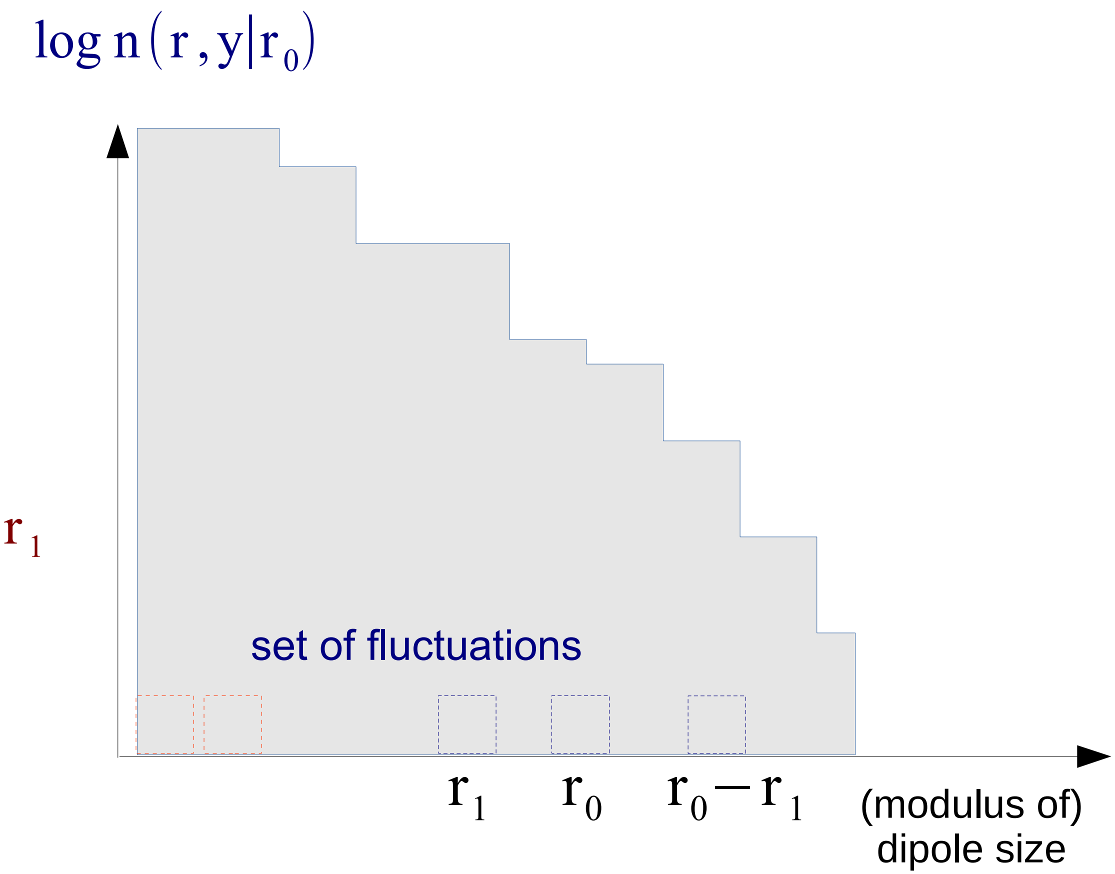


The total number of dipoles grows exponentially with rapidity through a branching process



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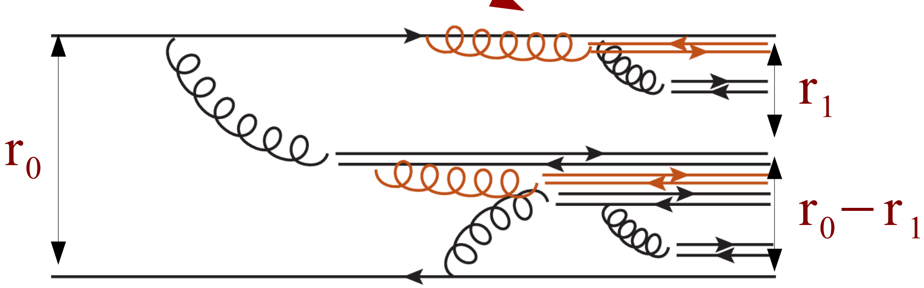
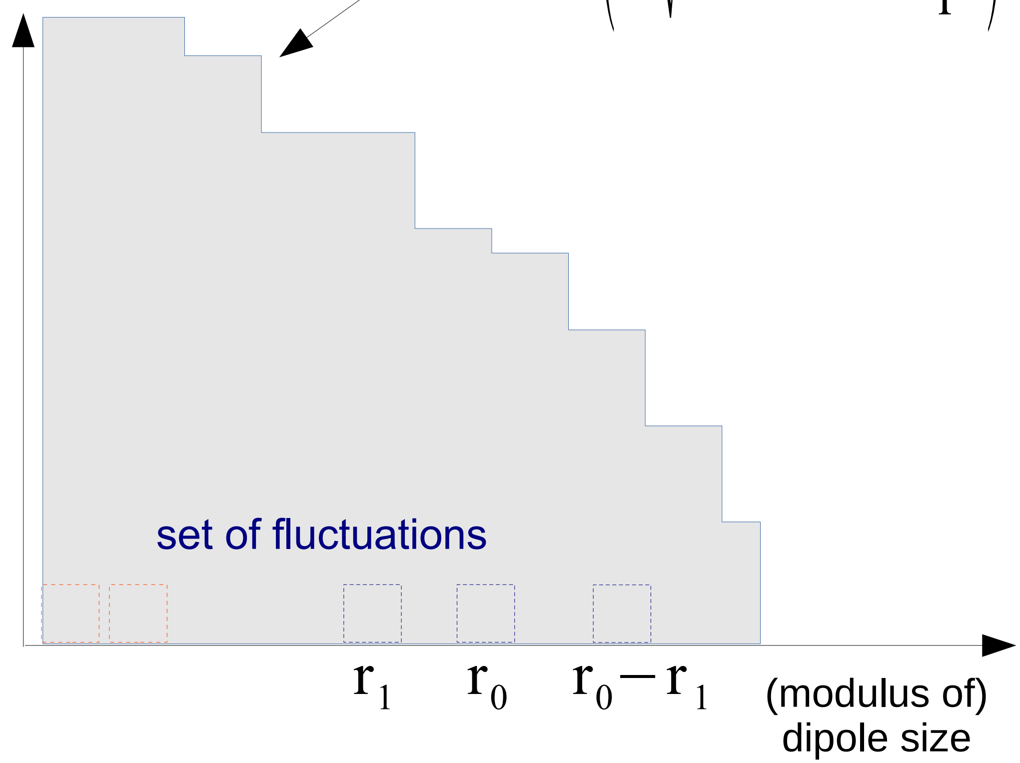


The total number of dipoles grows exponentially with rapidity through a branching process

$\log n(r, y|r_0)$

$$n \sim \exp\left(2\sqrt{\frac{\alpha_s N_c}{\pi}\,y \log\frac{r_0^2}{r^2}}\right)$$



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Dipole-nucleus scattering: total multiplicity

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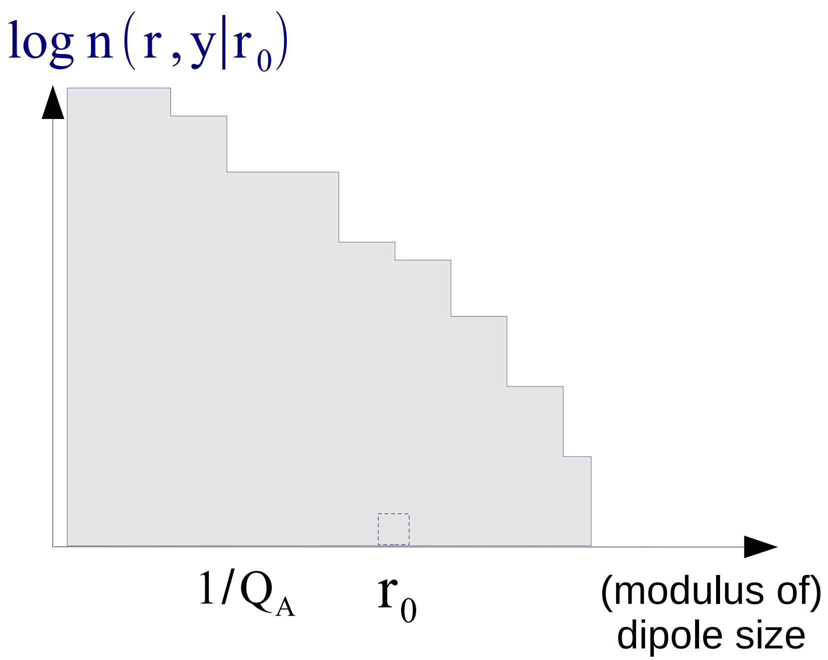

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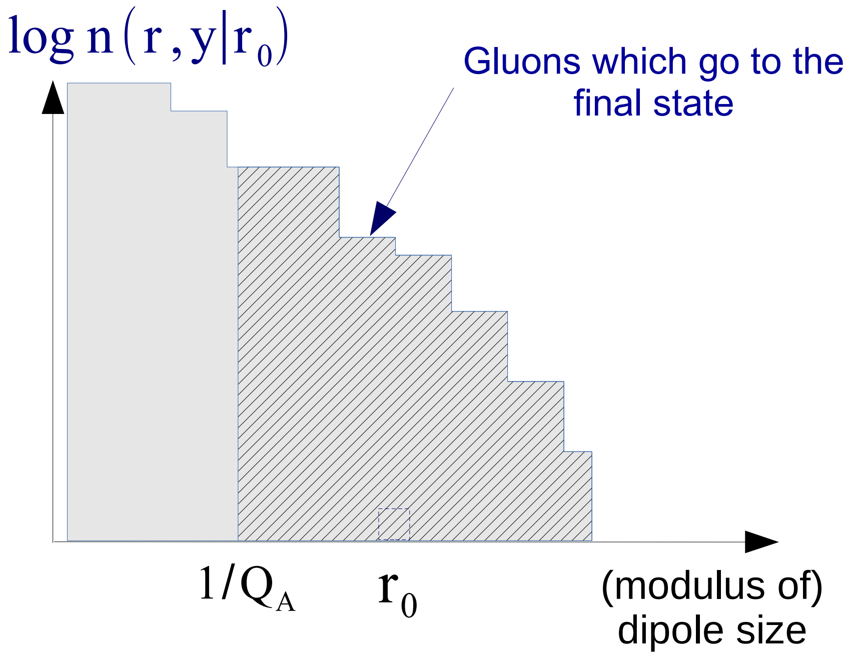

The **gluons which go to the final state**, i.e. which are freed in the scattering, correspond to **dipoles which have a size larger than the inverse saturation momentum of the nucleus (assumed to be much smaller than r_0).**

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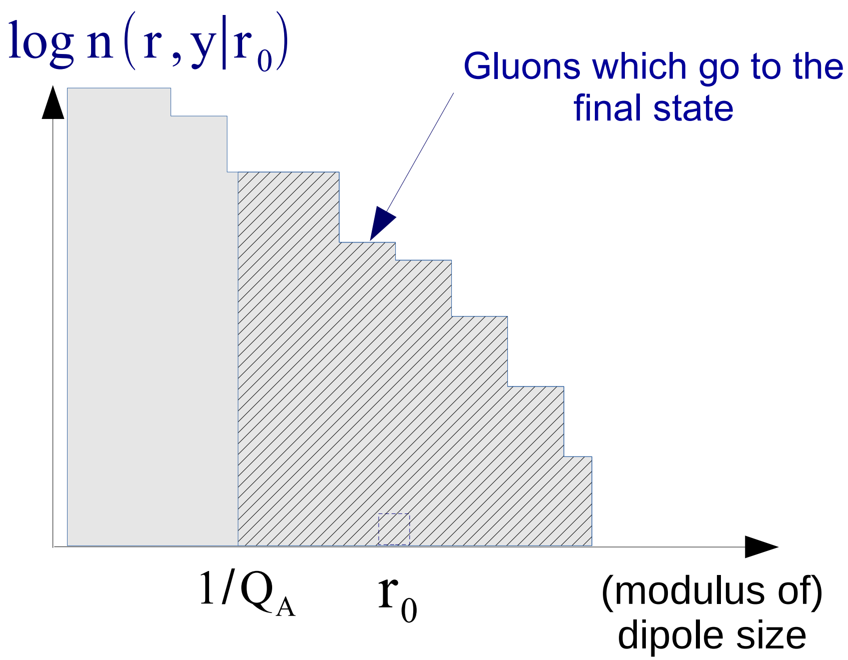

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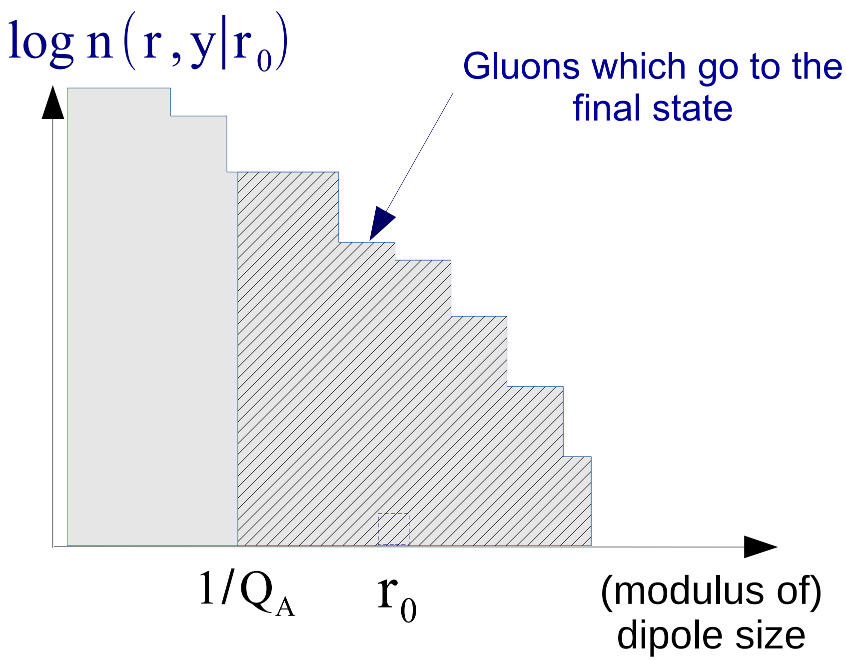

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Solution:
(n large)

$$P_n \sim \exp\left(- \frac{\log^2 n}{\frac{4\alpha_s N_c}{\pi}(Y - y_0)} \right)$$

Salam (1996)

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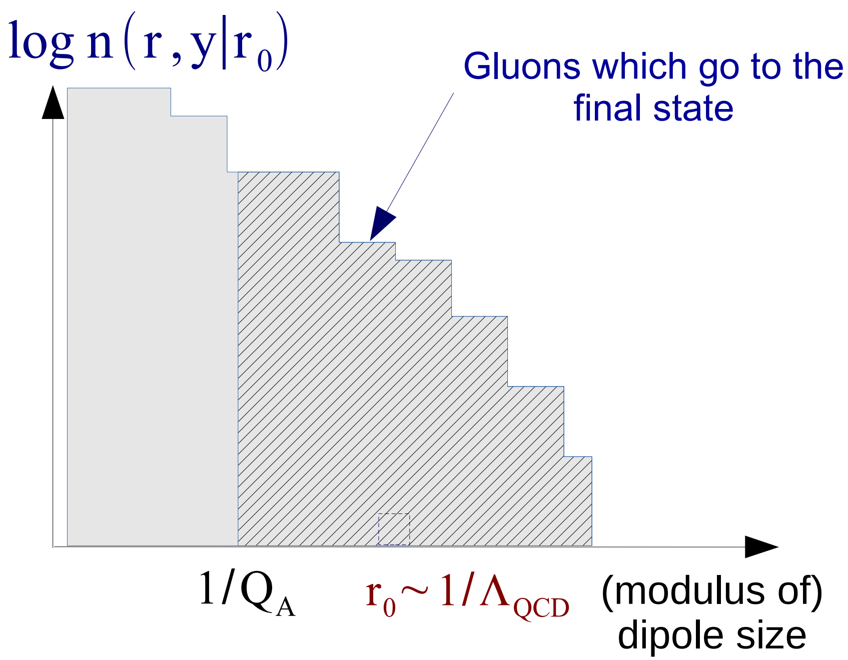

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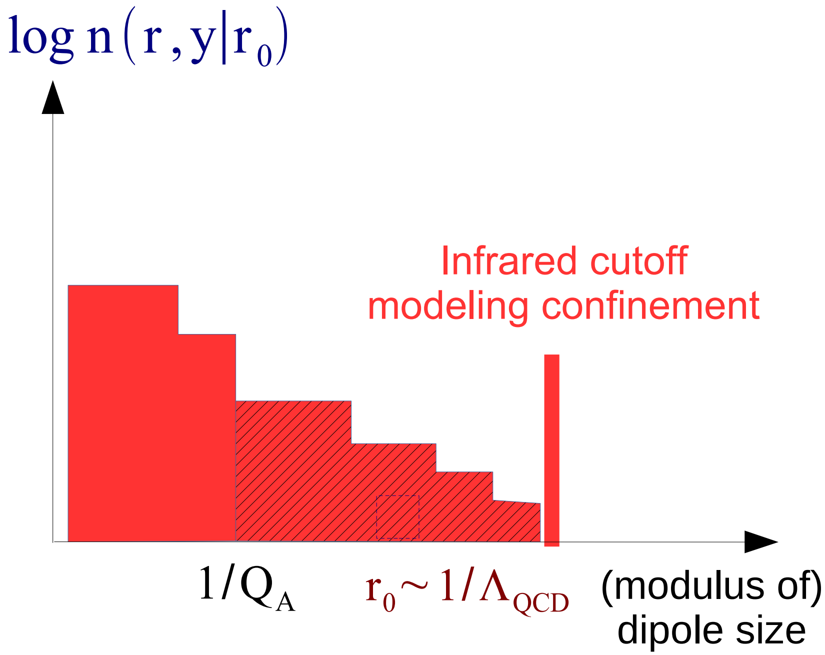

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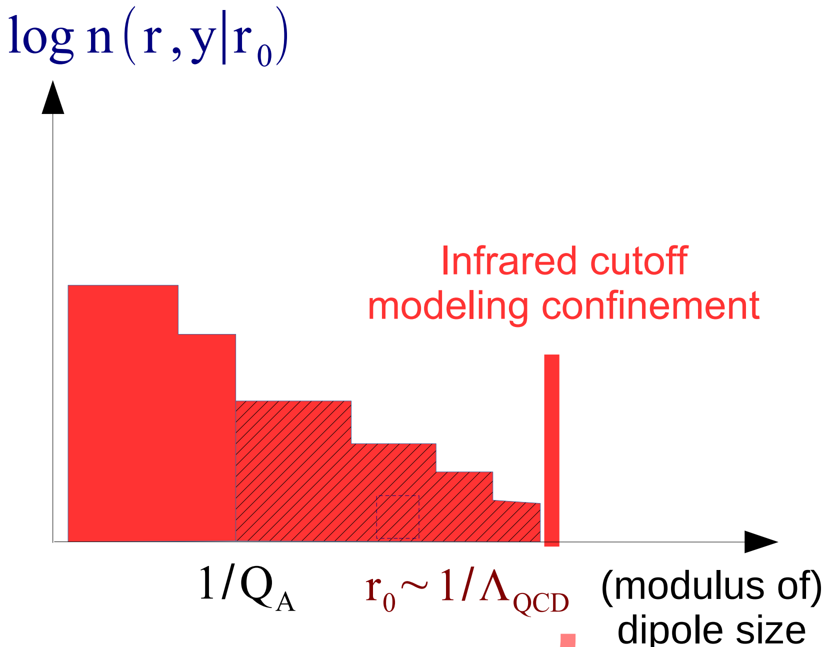

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(NB: arbitrary! so try different possible implementations)

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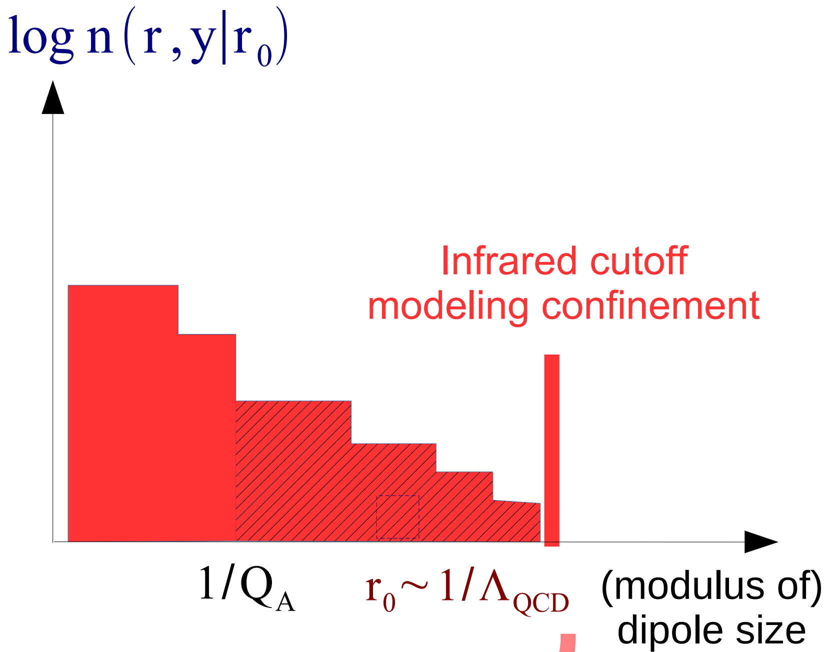

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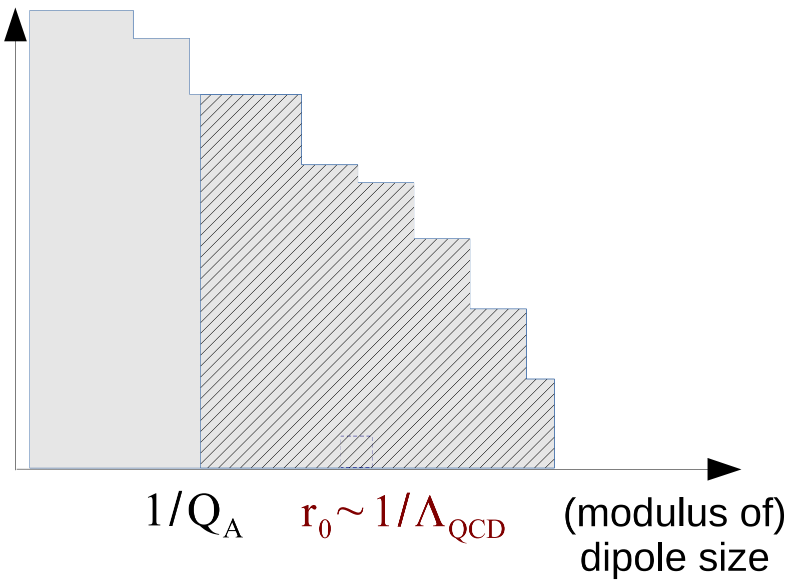
$$P_n \sim \frac{1}{c\,\bar{n}} \exp\left(- \frac{n}{c\,\bar{n}} \right)$$

so far, c undetermined in our calculation

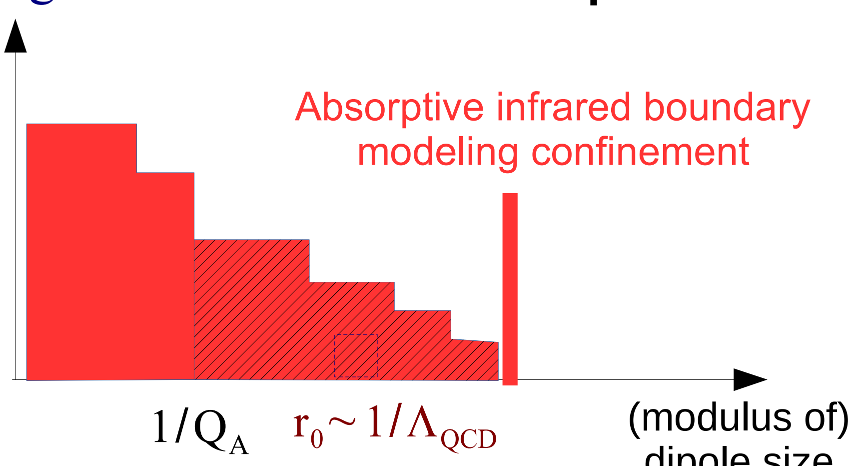
Dipole-nucleus scattering: total multiplicity

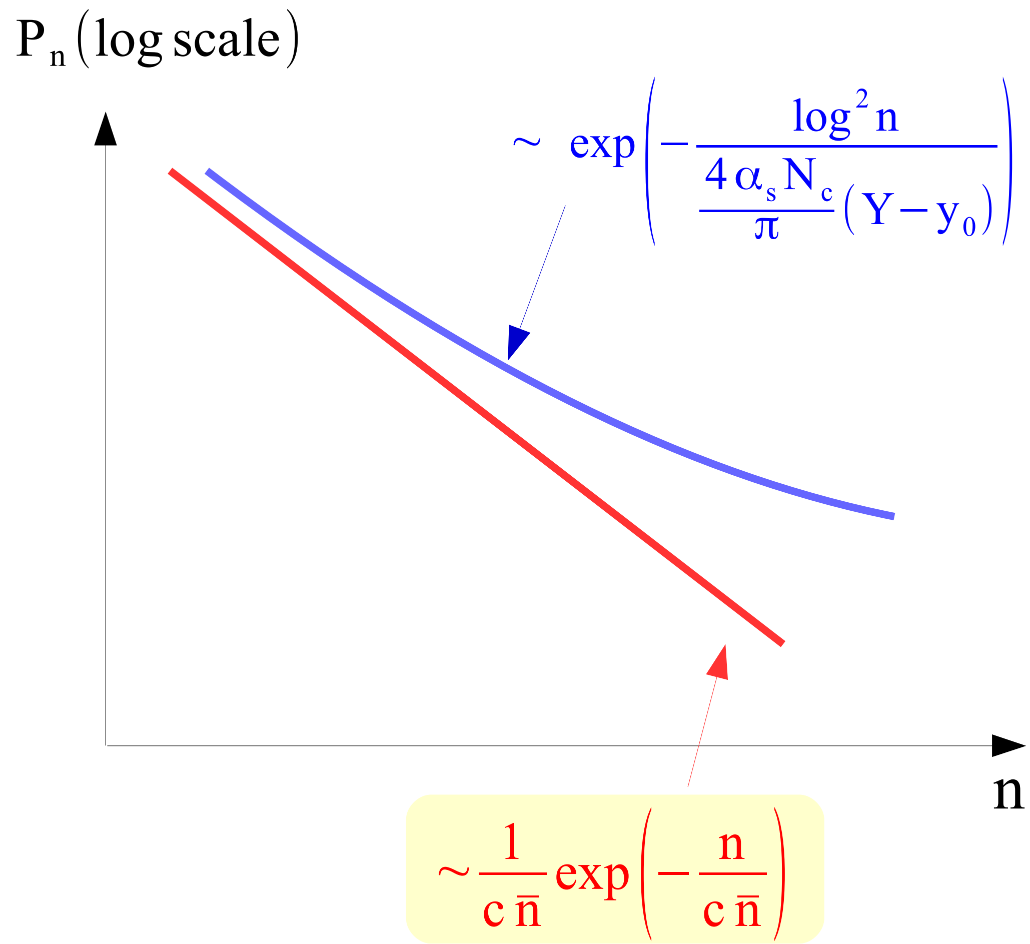
$$\frac{dN}{dy}(y\simeq 0)=x\,G\left(x,Q_A^2(y_0)\right)\quad\text{where}\quad x=e^{-(Y-y_0)}\frac{Q_A(y_0)}{\text{onium mass}}$$

$\log n$ **Perturbative calculation**



$\log n$ **Realistic model for pA**

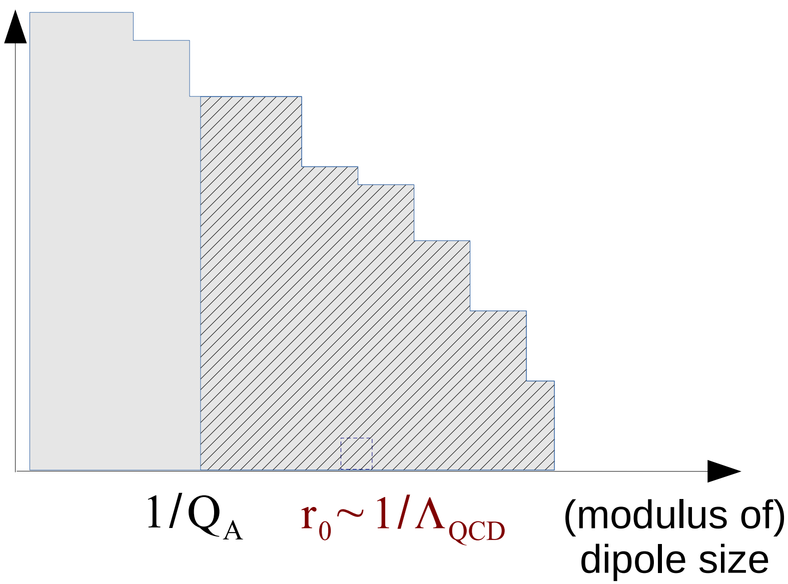




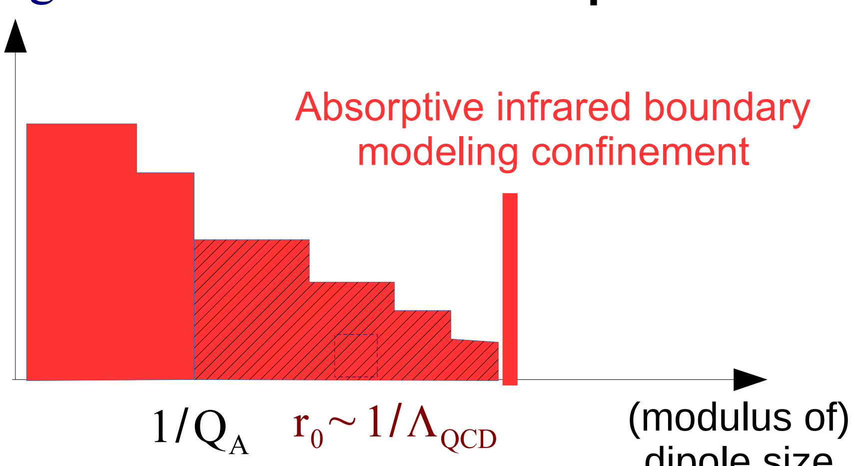
Dipole-nucleus scattering: total multiplicity

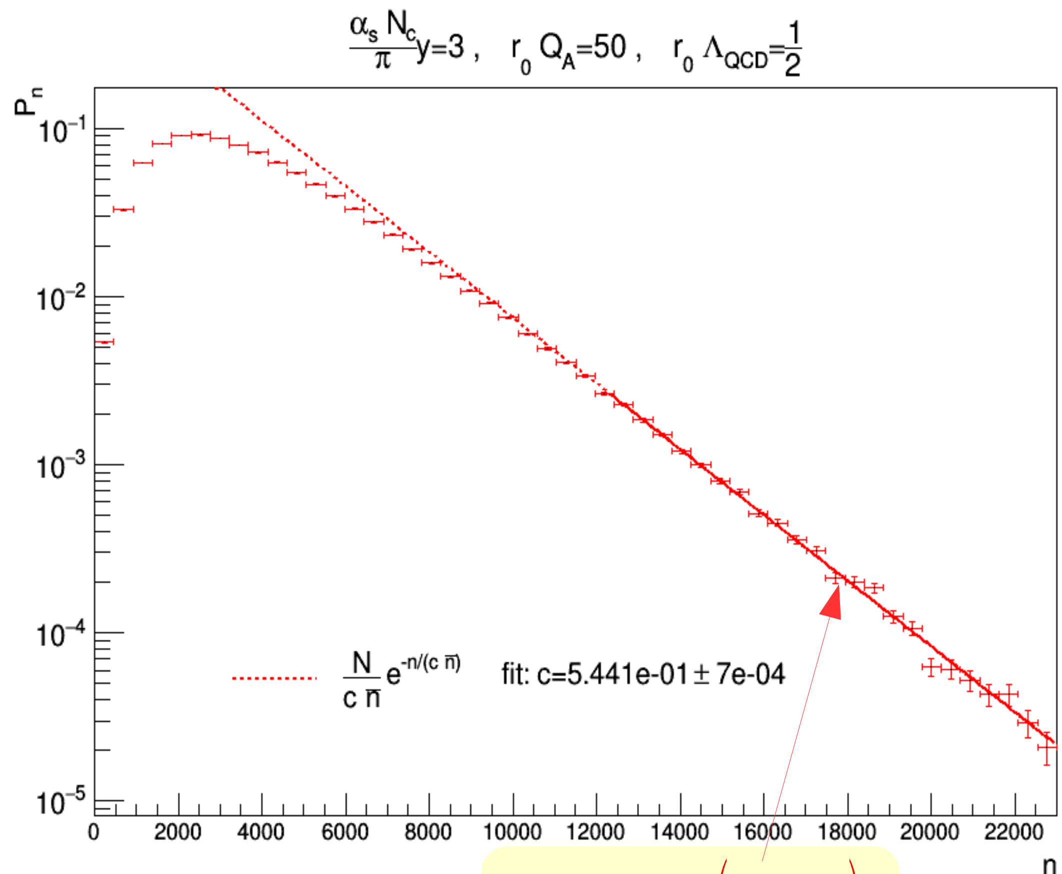
$$\frac{dN}{dy}(y \simeq 0) = x\,G\left(x, Q_A^2(y_0)\right) \quad \text{where} \quad x = e^{-(Y-y_0)} \frac{Q_A(y_0)}{\text{onium mass}}$$

$\log n$ **Perturbative calculation**



$\log n$ **Realistic model for pA**





$$\sim \frac{1}{c\,\bar{n}} \exp\left(-\frac{n}{c\,\bar{n}}\right)$$

Work in progress:

numerical simulations: ***c ~ 0.5 - 0.6***

c seems quite insensitive to the detailed implementation of confinement!

Summary

- At high energies, **hadrons look like dense states of gluons** (= “color glass condensates”), very far from the valence picture. This is a property of QCD.

- The (stochastic) evolution of (ideal) onia wave functions towards high energy can be computed in QCD. The **color dipole model** is a convenient implementation of this evolution.

- **Fluctuations of the multiplicity in pA scattering in the proton fragmentation region can be related to the *event-by-event fluctuations of the total integrated gluon density in the proton.***
  pA data at the LHC is a great opportunity to study these fluctuations!

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Outlook

- Better understand the fluctuations in the dipole model+confinement - test robustness of our solution *(in progress)*

- Try and build a realistic model for phenomenology (proton instead of onium etc...)