Small x resummation effects in forward Drell-Yan structure functions

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06-04-2017
Plan: to make full use of forward Drell-Yan process at the LHC as a probe of high energy scattering in QCD

- Forward Drell-Yan: kinematics, observables
- Drell-Yan structure functions
- Lam-Tung relation in QCD
- Dipole picture of forward Drell-Yan scattering
- Small x resummation and twist decomposition
- Results
- Conclusions

Work done with
D. Brzemiński, Mariusz Sadzikowski and Tomasz Stebel
**Forward Drell-Yan at LHC: kinematical reach and use**

- Forward Drell-Yan may be used to measure parton densities down to $x < 10^{-6}$ at $M^2 \sim 10 \text{ GeV}^2$

- Possible effects of small $x$ resummation, multiple scattering and higher twists (small $x$ enhancement of multiple gluon exchange): competition of $1/M^2$ and $x^{-\lambda}$ terms

- Needed to be controlled theoretically to avoid systematic errors of parton determination

- Potentially → strong small $x$ effects and contribution of higher twists.

**Advantage:** 4 independent structure functions
Drell-Yan kinematics

\[ \gamma^*(q) \]

\[ x_F, q^2 = M^2, q_T \]

\[ k_1 \]

\[ (\theta, \phi) \]

\[ k_2 \]

\[ P_1 \]

\[ P_2 \]
Drell-Yan structure functions:

- **Lepton angular distributions:** 4 Drell-Yan structure functions \( W_a \) – frame dependent

\[
\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{em}^2}{2(2\pi)^4 M^4} \left[ (1 - \cos^2 \theta)W_L + (1 + \cos^2 \theta)W_T \
+ (\sin^2 \theta \cos 2\phi)W_{TT} + (\sin 2\theta \cos \phi)W_{LT} \right]
\]

- **Helicity structure functions** → elements of virtual photon production helicity density matrix

- **Photon decays into leptons** → interference between different polarisations possible:
  
  \( W_T : T(+) \rightarrow T(+) \)  \( W_L : L \rightarrow L \)
  
  \( W_{LT} : T \rightarrow L, L \rightarrow T \)  \( W_{TT} : T(+) \rightarrow T(-) \)
Lam-Tung relation

- Hence: DY helicity structure functions: projections of DY amplitudes on virtual photon polarization states

- Lam-Tung relation (1980, 1982): vanishing combination of DY structure functions at leading twist up to NNLO in ollinear QCD

\[ W_L - 2W_{TT} = 0 \]

- Advantage of Lam-Tung relation: it is invariant under frame rotations w.r.t. axis perpendicular to reaction plane

- Lam-Tung relation breaking by higher order QCD effects related to parton $k_T$

- At twist 4 – non-zero contribution $\rightarrow$ enhanced higher twist contributions
Leading diagrams of Drell-Yan

- **Leading Order**
- **NLO**
Leading diagrams of forward Drell-Yan

- Asymmetric kinematics: $x_2 \gg x_1$
- Dominance of the quark sea $\rightarrow$ driven by gluon evolution
- Good approximation: gluon evolution followed by splitting to quark (anti-quark) in the last step
Forward Drell-Yan in dipole formulation

- Large energy limit: conservation of transverse positions in scattering
- “Effective color dipole” emerges from interference of photon emission before and after scattering, $\gamma^*$ carries fraction $z$ of $p^+$ of incident quark

- “Crossed” photon wave function:

- Interference of photon helicity states through leptonic tensor
Forward Drell-Yan in dipole formulation

\[ \sigma_{T,L}^f(qp \to \gamma^* X) = \int d^2 r \ W_{T,L}^f(z, r, M^2, m_f) \sigma_{qq}(x_2, zr) \]

\[ W_T^f = \frac{\alpha_{em}}{\pi^2} \left\{ [1 + (1 - z)^2] \eta^2 K_1^2(\eta r) + m_f^2 z^4 K_0^2(\eta r) \right\} \]
\[ W_L^f = \frac{2\alpha_{em}}{\pi^2} M^2 (1 - z)^2 K_0^2(\eta r) \]

Formalism proposed and developed by:

- Brodsky, Hebecker, Quack (1997)
- B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov (2001)
- Raufeisen, Peng, Nayak (2002): plot
- V. Goncalves et al, A. Szczurek et al.
Inclusive forward Drell-Yan at LHC: higher twist corrections

- Golec-Biernat, Lewandowska, Staśto, 2010 (plot): first analysis of twist content of forward Drell-Yan within the GBW saturation model for dipole cross-section, using the technique of Bartels, Golec-Biernat and Peters done for the inclusive cross-section (in qT and the lepton azimuthal angle)

- Predictions for the LHC (plot) large higher twist corrections within kinematical range of LHC (LHCb)
Forward Drell-Yan cross-section in kT factorisation

\[ L^{\sigma\sigma'} = \epsilon^{(\sigma)}_{\mu} L^{\mu\nu} \epsilon^{(\sigma')\dagger}_{\nu}, \quad L^{\mu\nu} = -g^{\mu\nu} + \frac{\kappa^{\mu} \kappa^{\nu}}{\kappa^2} \]

\[ \Phi_{\sigma\sigma'}(q_T, k_T, z) = \sum_{\lambda_1, \lambda_2 = \pm} A^{(\sigma)}_{\lambda_1, \lambda_2}(\vec{q}_T) A^{(\sigma')\dagger}_{\lambda_1, \lambda_2}(\vec{q}_T) \]

\[ \frac{d\sigma}{d x_F d M^2 d \Omega d^2 q_T} = \frac{\alpha_{\text{em}}}{(2\pi)^2 (P_1 \cdot P_2)^2 M^2 x_F^2 (1 - z)} L^{\sigma\sigma'}(\Omega) \int_0^1 dz \varphi(x_F/z) \]

\[ \times \int d^2 k_T \frac{2\pi \alpha_s}{3} \frac{f(x_g, k_T^2)}{k_T^4} \Phi_{\sigma\sigma'}(q_T, k_T, z) \]
Mellin representation of forward Drell-Yan structure functions:

- Standard procedure: position-space $\rightarrow$ Mellin moments space

\[ W_i = \int_{x_F}^{1} dz \, \varphi(x_F/z) \int_C \frac{ds}{2\pi i} \tilde{\sigma}(-s) \left( \frac{z^2 Q_0^2}{\eta_z^2} \right)^s \hat{\Phi}_i(q_T, s, z) \]

- Convolution of Mellin transform of dipole cross section and impact factor

\[ \eta_z^2 = M^2 (1 - z) \]

\[ \hat{\Phi}_i(q_T, s, z) = \frac{2(2\pi)^4 M^4}{\alpha_{em}^2} \int d^2r \left( \frac{\eta_z^2}{4z^2} r \right)^s \Phi_i(q_T, r, z) \]

- Dipole cross section encodes QCD dynamics, e.g. small $x$ evolution, higher twists

\[ \tilde{\sigma}(-s) = \int_0^\infty \frac{d\rho^2}{\rho^2} \left( \rho^2 \right)^{-s} \hat{\sigma} (\rho) \]
Two descriptions of dipole cross-section in kT factorisation:

- Phenomenological: eikonal multiple gluon ladder exchange – GBW model → twist 2n contribution enhanced by $x^{-n\lambda}$ at small $x$
- BFKL description: based on LL BFKL cross-section and its twist decomposition
- Higher twist effects extracted from singularities of the BFKL kernel in Mellin space, $\chi(\gamma)$, at integer values of anomalous dimensions $s \rightarrow \gamma$

\[
\sigma(\gamma) \sim \exp (c \log(1/x) \alpha_s \chi(\gamma))
\]

\[
\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \sim 1 / (\gamma - n)
\]

→ essential singularities of Mellin cross-section

- Saddle point treatment of $\gamma$–integral of BFKL amplitudes
  → power suppression $x^\delta$ of higher twist terms in LL BFKL amplitudes
Previous findings: Mellin representation of DY impact factors

- Mellin transforms of impact factors for all DY structure functions found, e.g.:

\[
\hat{\Phi}_L(q_T, s, z) = \frac{2}{z^2} \left\{ \frac{2\Gamma^2(s + 1)}{1 + \frac{q_T^2}{\eta_z^2}} \right\} \frac{2F_1}{2F_1} \left( \frac{s + 1, s + 1, 1, -\frac{q_T^2}{\eta_z^2}}{s + 1, s + 2, 1, -\frac{q_T^2}{\eta_z^2}} \right)
- \Gamma(s + 1)\Gamma(s + 2) \frac{2F_1}{2F_1} \left( \frac{s + 1, s + 2, 1, -\frac{q_T^2}{\eta_z^2}}{s + 1, s + 2, 1, -\frac{q_T^2}{\eta_z^2}} \right)
\]

\[
\hat{\Phi}_{TT}(q_T, s, z) = \frac{1}{2z^2} \left\{ \frac{2\pi}{\Gamma(1-s)\sin \pi s} \frac{q_T^2}{\eta_z^2} \right\} \left( 1 + \frac{q_T^2}{\eta_z^2} \right)^{-s-3} \left[ \Gamma(s + 2) \left( 1 + \frac{q_T^2}{\eta_z^2} \right) \frac{2F_1}{2F_1} \left( -s + 1, s + 1, 1, -\frac{q_T^2}{q_T^2 + \eta_z^2} \right) \right]
- \left( 1 + 2\frac{q_T^2}{\eta_z^2} \right) \frac{2F_1}{2F_1} \left( -s + 1, s + 2, 1, -\frac{q_T^2}{q_T^2 + \eta_z^2} \right)
- \left( 1 + 2\frac{q_T^2}{\eta_z^2} \right) \frac{2F_1}{2F_1} \left( -s + 1, s + 2, 2, -\frac{q_T^2}{\eta_z^2} \right)
- \frac{4q_T^2}{1 + \frac{q_T^2}{\eta_z^2}} \Gamma(s + 1)\Gamma(s + 2) \frac{2F_1}{2F_1} \left( s + 1, s + 2, 2, -\frac{q_T^2}{\eta_z^2} \right)
\]

- Useful in BFKL approach and for twist analysis
Mellin treatment of BFKL DY amplitudes

- BFKL essential singularities at twist poles via Laurent expansion

\[
W_j = \int_{x_F}^{1} dz \varphi(x_F / z) \sigma_j(q_T, z, Y)
\]

\[
\sigma_j(q_T, z, Y) = \int_{\mathcal{C}} \frac{ds}{2\pi i} \left( \frac{z^2 Q_0^2}{M^2(1 - z)} \right)^{-s} \tilde{\sigma}(s, Y) \hat{F}_j(q_T, -s, z)
\]

\[
\sigma_j(q_T, z, Y) = \sum_{n=1}^{\infty} \sigma_j^{(2n)}(q_T, z, Y)
\]

\[
\sigma_j^{(2n)}(q_T, z, Y) = -R_p^2 e^{-nt} \int_0^{2\pi} d\theta h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) \exp \left( \epsilon e^{i\theta} t + \frac{\bar{\alpha} s Y}{\epsilon} e^{-i\theta} \right)
\]

\[
\chi_{\text{reg}}^{(n)} = \chi \left( -n + \epsilon e^{i\theta} \right) - \frac{e^{-i\theta}}{\epsilon}
\]
Mellin treatment of BFKL DY amplitudes

\[ h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) = \epsilon e^{i\theta} \left( \frac{z^2}{1-z} \right)^{n-\epsilon \exp i\theta} \hat{\Phi}_j(q_T, n-\epsilon e^{i\theta}, z) \Gamma(-n+\epsilon e^{i\theta}) e^{\bar{\alpha}_s Y} \chi_{reg}^{(n)} \]

\[ h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) = \sum_{m=0}^{\infty} a_m^{(2n)j} (\epsilon e^{i\theta})^m \]

\[ \sigma_j^{(2n)}(q_T, z, Y) = -2\pi R_p^2 \left( \frac{Q_0^2}{M^2} \right)^n \sum_{m=0}^{\infty} a_m^{(2n)j} \left( \frac{\bar{\alpha}_s Y}{t} \right)^{m/2} I_m \left( 2\sqrt{\bar{\alpha}_s Y t} \right) \]

\[ W_j^{(2n)} = -\sigma_0' \left( \frac{Q_0^2}{M^2} \right)^n \sum_{m=0}^{\infty} \int_0^1 dz a_m^{(2n)j} \phi(x_F/z) \left( \frac{\bar{\alpha}_s Y}{t} \right)^{m/2} I_m \left( 2\sqrt{\bar{\alpha}_s S Y t} \right) \]

where \( t \sim \log(M^2) \)

- Integrals over the contour angle parameter \( \theta \) performed and expansion coefficients \( a_m^{(2n)j} \) evaluated analytically \( \rightarrow \) analytic form of expansions of BFKL forward Drell-Yan structure functions
Results for forward DY BFKL structure functions

- In LL BFKL subleading twist contributions found to decrease exponentially with rapidity.
- Exponential increase in rapidity found only for the leading twist
- Example of results for $q_T$-integrated structure functions
- Lam-Tung relation fulfilled in double logarithmic limit, broken beyond

\[
\tilde{W}_j^{(2)} = -\sigma'_0 \left( \frac{Q_0^2}{4M^2} \right) \int_{x_F}^{1} dz \, f_j(z) \frac{z^2}{1-z} \varphi(x_F/z)
\times \sum_{m=0}^{\infty} \tilde{a}^{(2)j}_m \left( \frac{\tilde{\alpha}_s Y}{\ln(4M^2/Q_0^2)} \right)^{\frac{m}{2}} I_{|m|} \left( 2\sqrt{\tilde{\alpha}_s Y \ln \frac{4M^2}{Q_0^2}} \right)
\]

\[
\tilde{a}_0^{(2)L} = -\frac{4}{3}, \quad \tilde{a}_1^{(2)L} = -\frac{4}{3} \left( -2 + 2\gamma_E + \ln \frac{1-z}{z^2} + \psi(5/2) \right)
\]

\[
\tilde{a}_0^{(2)TT} = -\frac{2}{3}, \quad \tilde{a}_1^{(2)TT} = \frac{2}{3} \left( -3 + \gamma_E - \ln \frac{1-z}{z^2} + \ln(64) + 2\psi(5/2) \right)
\]

\[
\tilde{a}_0^{(2)LT} = 0, \quad \tilde{a}_1^{(2)LT} = 0.5236
\]
Results: inclusive Drell-Yan at LHC: LHCb data from BFKL and GBW dipole cross sections

- Good description in terms of GBW, BFKL somewhat above data
Results: inclusive Drell-Yan at the LHC: ATLAS data

- Good description in terms of BFKL, GBW somewhat below data
- However: central kinematics $\rightarrow$ expected terms beyond forward DY approximation
Similar behavior of $W_T$ and $W_L$, significant differences in “interference” structure functions.
Lam-Tung relation in GBW model

- Lam-Tung relation: at leading twist up to NNLO:

\[ W_L - 2W_{TT} = 0 \]

- Holds in the GBW model at twist 2
- At twist 4 – non-zero contribution \( \rightarrow \) enhanced higher twist contributions

\[
W^{(4)}_L - 2W^{(4)}_{TT} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz \frac{q(x_F/z)^2}{[q_T^2 + M^2(1 - z)]^4}
\]

- qT-integrated cross-section also shows breaking of Lam-Tung relation

\[
\int \left( W^{(4)}_L - 2W^{(4)}_{TT} \right) d^2q_T = 2\pi \sigma_0 M^2 \left( W^{(4)}_L - 2\tilde{W}^{(4)}_{TT} \right)
\]

\[
= 2\pi \sigma_0 \frac{Q_0^4}{M^2} \left\{ \frac{1}{18} \phi(x_F) \left[ -19 + 12\gamma_E + 12 \ln \left( \frac{M^2(1 - x_F)}{Q_0^2} \right) \right] \right. + \\
+ \left. \frac{2}{3} \int_{x_F}^1 dz \frac{\phi(x_F/z)^2 - \phi(x_F)}{1 - z} \right\}
\]
Lam-Tung relation from GBW and BFKL

- Striking difference in Lam – Tung relation breaking
- Subleading twist effect in GBW / vs leading twist in BFKL
- Importance of parton $k_T$ effects

$M^2 = 5 \text{ GeV}^2$

$M^2 = 20 \text{ GeV}^2$
Lam-Tung relation from GBW and BFKL in $q_T$ integrated structure functions

- Lam–Tung relation breaking in BFKL occurs at any mass
Conclusions

- We applied Mellin representation of forward Drell-Yan impact factors to analysis of forward Drell-Yan structure functions.
- Models were tested against LHC data: good description of angular averaged cross-sections was found.
- Assuming BFKL / saturation picture - explicit form was found of twist expansion of forward DY structure functions: differential and integrated.
- Lam-Tung relation preserved at twist-2, broken beyond → Lam-Tung combination of DY structure functions at small x is a probe of parton $k_T$ and higher twist effect.
- Essentially different predictions obtained for higher twists and Lam-Tung relation breaking from GBW and BFKL.