Spinor helicity methods in DIS at small x: 3-parton production

Jamal Jalilian-Marian

Baruch College, New York, USA

and

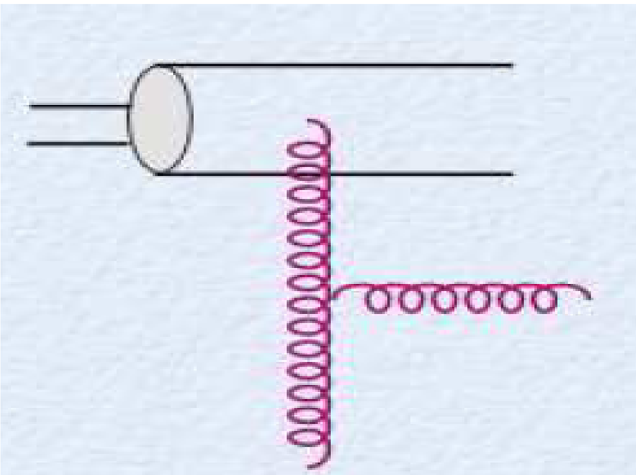
Ecole Polytechnique, Palaiseau, France

DIS 2017
3-7 April, 2017
Birmingham, UK

based on work done with A. Ayala, M. Hentschinski and M.E. Tejeda-Yeomans

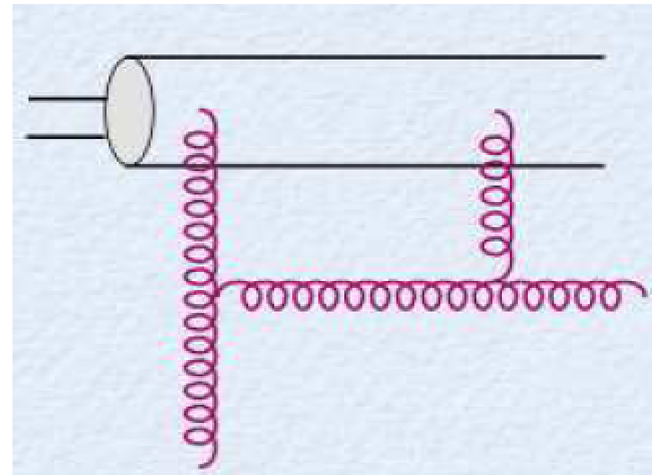
QCD at small x: gluon saturation

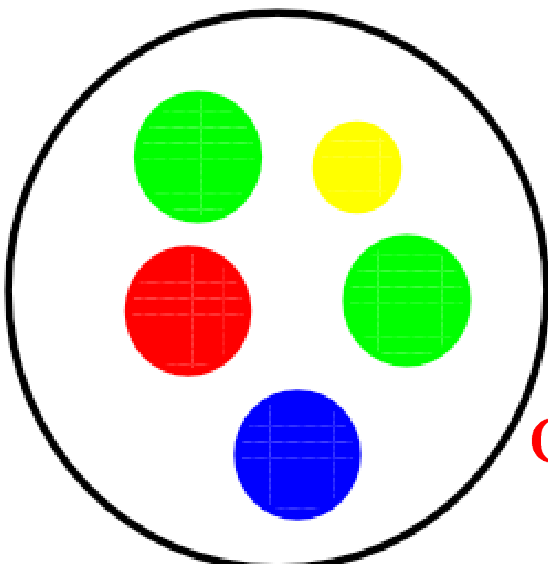
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



$$S \to \infty, \;\; Q^2 \;\; fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$

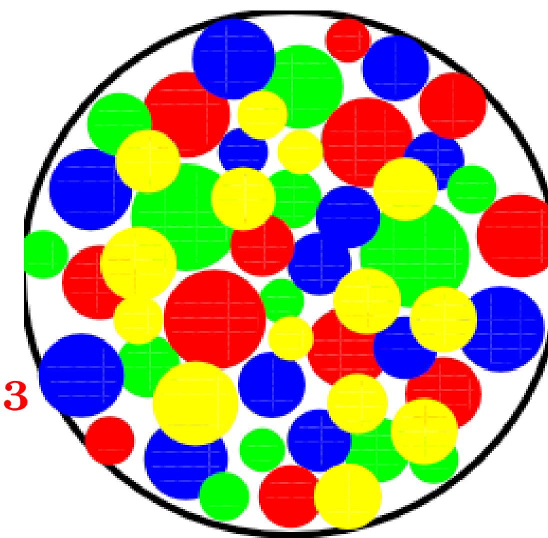




energy ~ 1/x

$$\frac{\alpha_s}{Q^2} \, \frac{xG(x, Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \, (\frac{1}{x})^{0.3}$$



Probing saturation in high energy collisions

"nucleus-nucleus" (dense-dense)

need quite a bit of modeling

↓

much less modeling

"proton-nucleus" (dilute-dense)

DIS

structure functions (diffraction)

NLO di-hadron/jet correlations

3-hadron/jet angular correlations

modification of production spectra

multiple scattering via Wilson lines:

p_t broadening

X-evolution via JIMWLK:

suppression of spectra/away side peaks

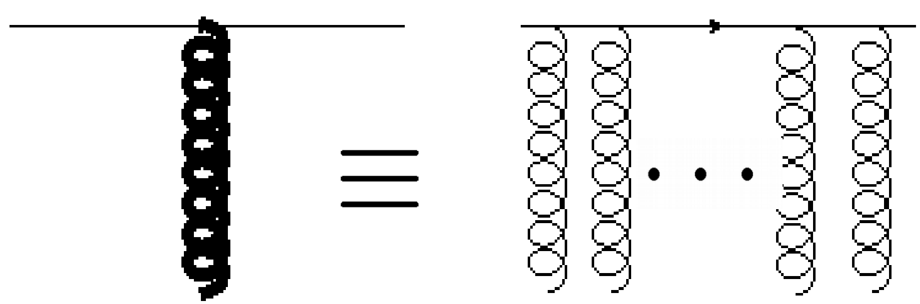
Particle production high energy collisions

target (proton, nucleus) as a classical color field

building block: quark propagator in the background color field

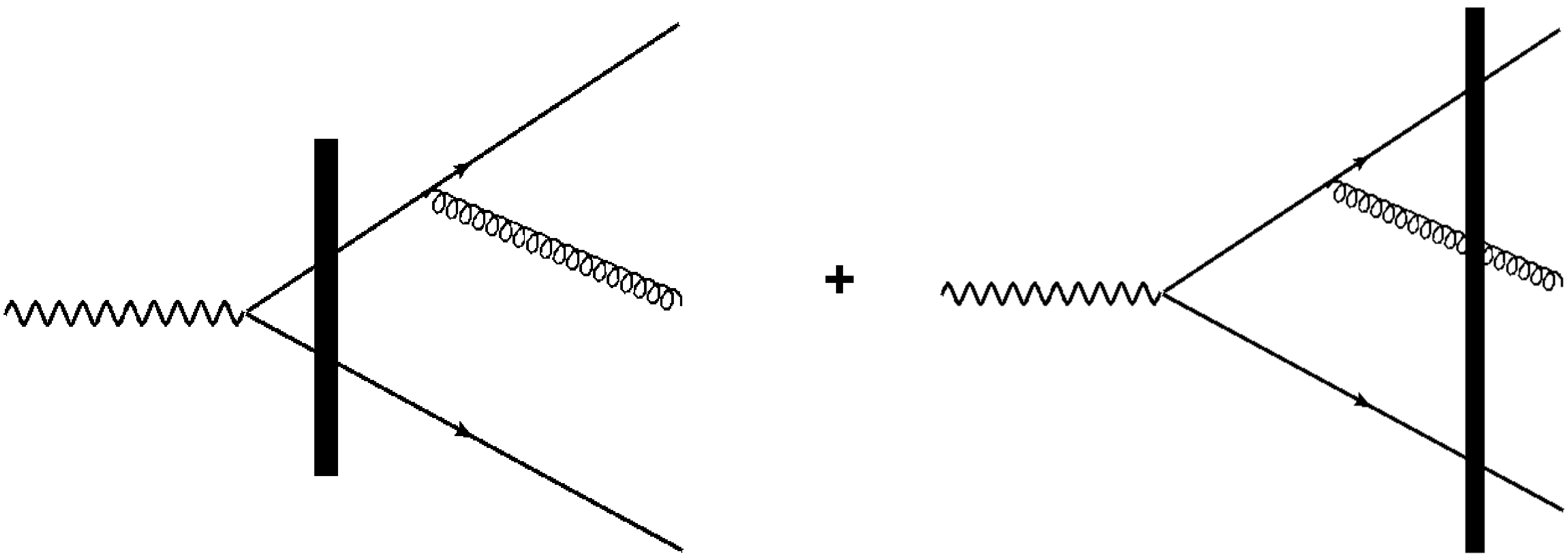
$$\mathrm{S}_F(q,p) \equiv \underbrace{(2\pi)^4\delta^4(p-q)\,S_F^0(p)}_{\text{no interaction}} + S_F^0(q)\,\underbrace{\tau_f(q,p)}_{\text{interaction}}\,S_F^0(p) \qquad \text{with} \qquad S_F^0(p) = \frac{i}{\not{p} + i\epsilon}$$

$$\tau_f(q,p) \equiv (2\pi)\delta(p^+ - q^+)\,\gamma^+ \int d^2x_t\, e^{i(q_t - p_t)\cdot x_t} \left\{\theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1]\right\}$$



angular correlations in 3-parton production in DIS

$$\gamma^{\star}\mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$



+ radiation from anti-quark

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans
PLB761 (2016) 229 and arXiv:1701.07143

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

helicity operator

$$\mathrm{h} \equiv \vec{\Sigma} \cdot \hat{p} = \begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix}$$

$$\begin{aligned} \vec{\Sigma} \cdot \hat{p}\, u_\pm(p) &= \pm u_\pm(p) \\ -\vec{\Sigma} \cdot \hat{p}\, v_\pm(p) &= \pm v_\pm(p) \end{aligned}$$

$$u_+(k) = v_-(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{2k^+ k^-}} = \sqrt{2}\, \dfrac{k_t \cdot \epsilon_\pm}{k_t}$

$$\begin{aligned} n^\mu &= (n^+ = 0, n^- = 1, n_\perp = 0) \\ \bar{n}^\mu &= (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0) \end{aligned}$$

and

$$\begin{aligned} k^\pm &= \frac{E \pm k_z}{\sqrt{2}} \\ \epsilon_\pm &= \frac{1}{\sqrt{2}}(1, \pm i) \end{aligned}$$

spinor helicity methods

notation:

$$|i^{\pm}>\equiv|k_i^{\pm}>\equiv u_{\pm}(k_i)=v_{\mp}(k_i) \qquad <i^{\pm}|\equiv<k_i^{\pm}|\equiv\overline{u}_{\pm}(k_i)=\overline{v}_{\mp}(k_i)$$

basic spinor products:

$$<i\,j> \;\equiv\; <i^-|j^+>=\overline{u}_-(k_i)\,u_+(k_j)\;=\;\sqrt{|s_{ij}|}e^{i\phi_{ij}} \qquad \cos\phi_{ij} \;=\; \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$[i\,j] \;\equiv\; <i^+|j^->=\overline{u}_+(k_i)\,u_-(k_j)\;=\;-\sqrt{|s_{ij}|}e^{-i\phi_{ij}} \qquad \sin\phi_{ij} \;=\; \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$\begin{aligned} s_{ij} &= (k_i+k_j)^2 = 2k_i\cdot k_j \\ &= -<ij>[ij] \end{aligned}$$

and

$$\begin{aligned} <ii> &= [ii]=0 \\ <ij] &= [ij>=0 \end{aligned}$$

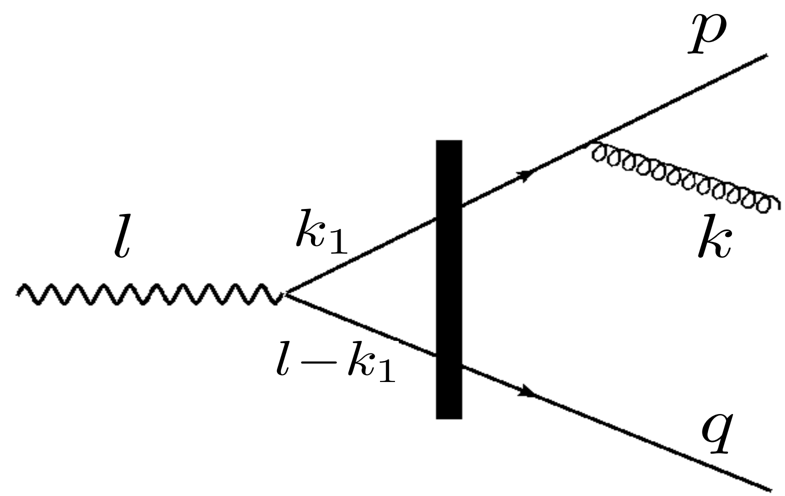
charge conjugation $<i^+|\gamma^\mu|j^+>=<j^-|\gamma^\mu|i^->$

Fierz identity $<i^+|\gamma^\mu|j^+><k^+|\gamma^\mu|l^+>=2[ik]\;<lj>$

any off-shell momentum $k^\mu\equiv\bar{k}^\mu+\frac{k^2}{2k^+}\,n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2=0$

any on-shell momentum $\not{p}=|p^+><p^+|+|p^-><p^-|$

Diagram A1



numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\,(k)\,(\not{p}+\not{k})\;\not{k}_1\;(l)\,(\not{k}_1-\not{l})\;v(q)$$

$$\not{l} = l^+\not{\overline{n}} - \frac{Q^2}{2l^+}\not{n}$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$
$$(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}])$$

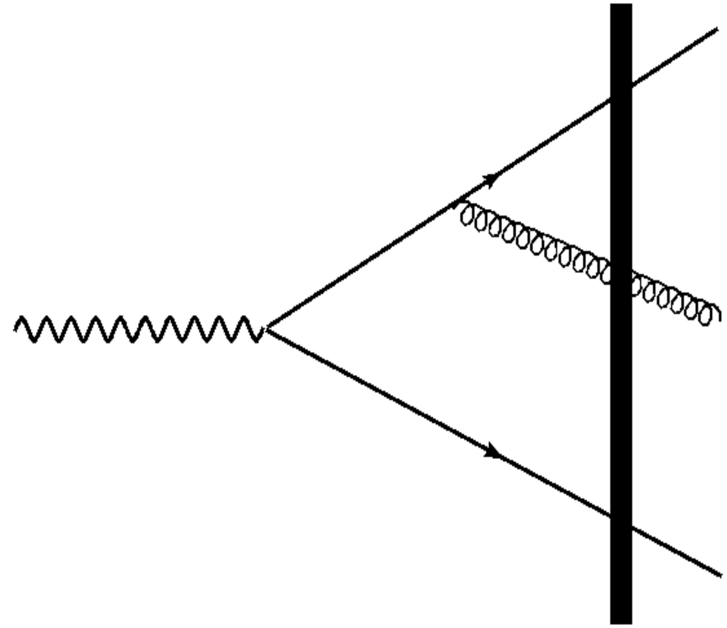
with

$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1n] < \bar{n}k_1 > [k_1n] < nq >$$

Diagram A3



longitudinal photons

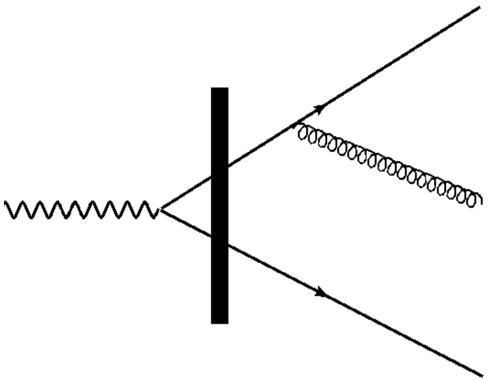
quark anti-quark gluon helicity: + - +

$$
\begin{aligned}
a_3^{L;+-+} &= \frac{\sqrt{2}Q}{l^+\,[n\bar{k}_2]}[pn]\left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n] \\
&\quad \left(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq > \\
&= -2^4 Q (l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3}\left[z_3\, k_{1\,t}\cdot\epsilon - (z_1+z_3)k_{2\,t}\cdot\epsilon\right]
\end{aligned}
$$

the rest is some standard integrals and Wilson lines

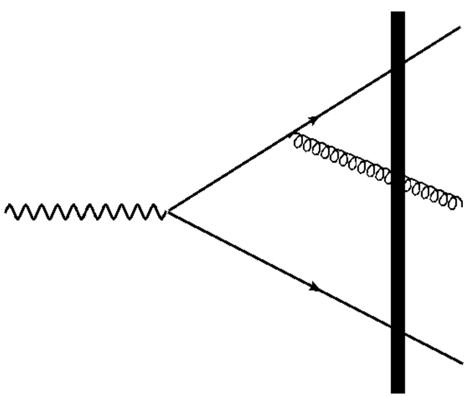
structure of Wilson lines: amplitude

$$\left(\vphantom{\int} \right)_{ij}$$



$$= \left[V^\dagger(y_t)\, V(x_t)\, t^a\right]_{ij}$$

$$\left(\vphantom{\int} \right)_{ij}$$

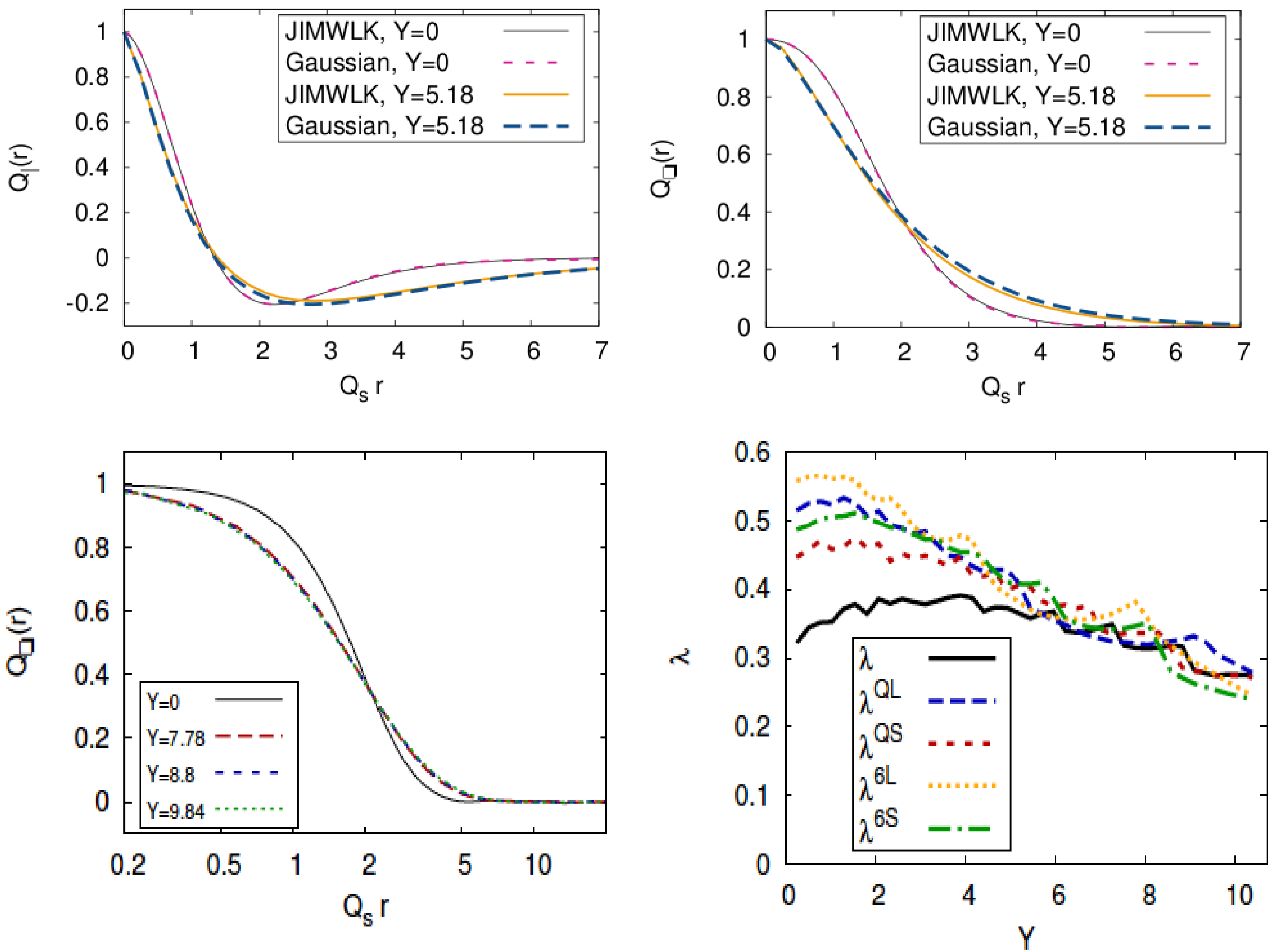


$$= \left[V^\dagger(y_t)\, t^b\, V(x_t)\right]_{ij}\; U^{ba}(z_t)$$

new d.o.f: quadrupoles

Quadrupole: $< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$

Dumitru, Jalilian-Marian, Lappi, Schenke, *Venugopalan: PLB706 (2011) 219*



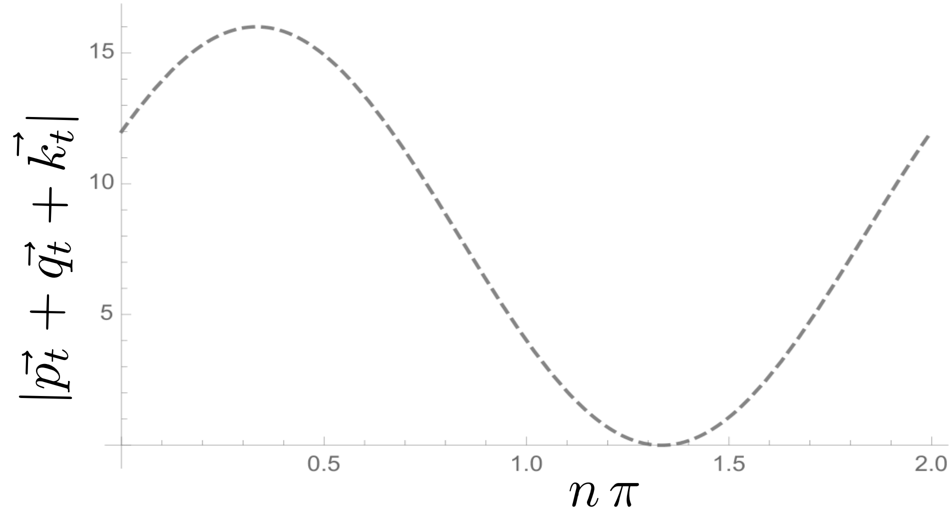
3-parton production

<u>linear regime</u>: use ugd's

$$z_1 = z_2 = 0.2, z_3 = 0.6$$

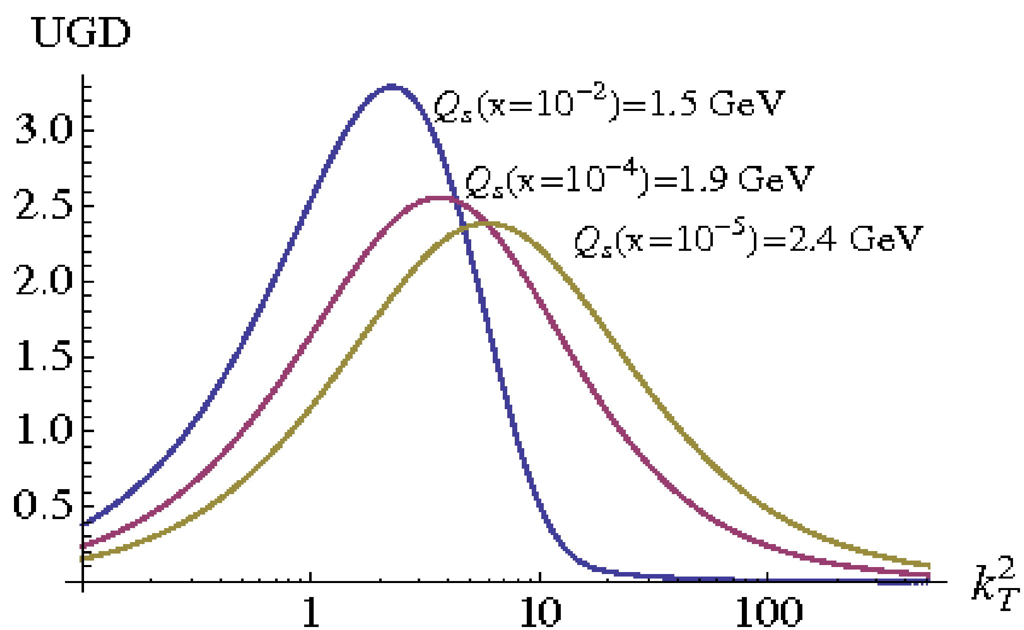
$$p_t = q_t = k_t = 4\, GeV$$

$$Q^2 = 16\, GeV^2$$



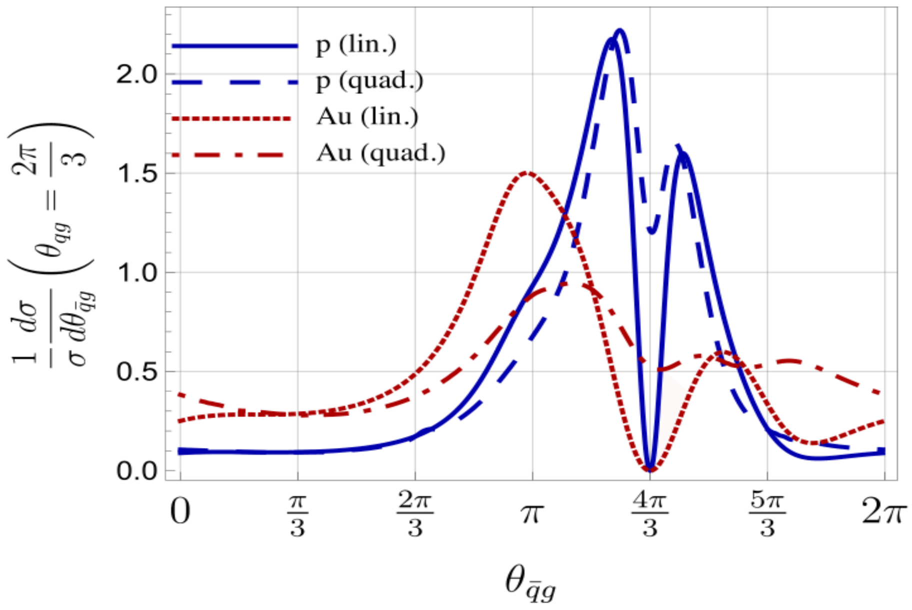
$$\Delta\phi_{12} = \frac{2\pi}{3} \qquad \textbf{vary} \qquad \Delta\phi_{13}$$

$$k_t^2\, \tilde{T}(k_t)$$



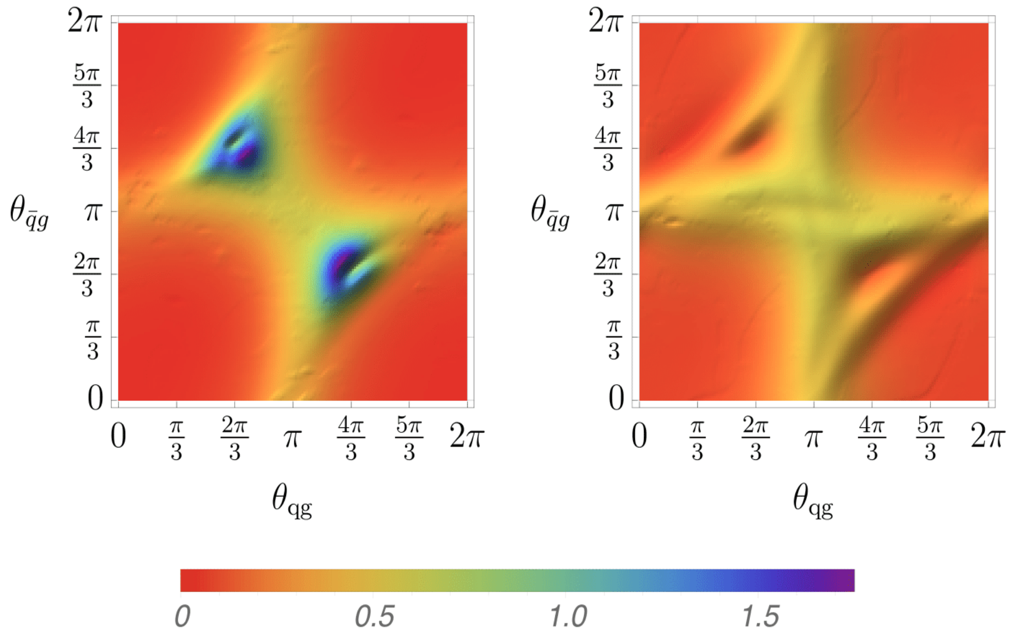
multiple scattering: broadening of the peak
x-evolution: reduction of magnitude

3-parton azimuthal angular correlations



multiple scattering:
<u>broadening</u> of the peak

x-evolution:
<u>reduction</u> of magnitude



Possible extensions to other processes?

real photons: $Q^2 \to 0$

ultra-peripheral nucleus-nucleus collisions

inclusive 3-jet production

NLO inclusive di-jet production

crossing symmetry:

$$\gamma^{(\star)}\, T \longrightarrow q\, \bar{q}\, g\, X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\, T \longrightarrow q\, g\, \gamma^{(\star)}\, X \\ \bar{q}\, T \longrightarrow \bar{q}\, g\, \gamma^{(\star)}\, X \\ g\, T \longrightarrow q\, \bar{q}\, \gamma^{(\star)}\, X \end{array} \right\}$$

proton-nucleus collisions (collinear factorization in proton?)

di-jet + photon production in pA

$$pA \longrightarrow h_1\, h_2\, \gamma^{(\star)}\, X$$

Possible extensions to other processes?

MPI (double/triple parton scattering)

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,\bar{q}\,T \longrightarrow g\,\gamma^{(\star)}\,X \\ g\,\bar{q}\,T \longrightarrow \bar{q}\,\gamma^{(\star)}\,X \\ g\,q\,T \longrightarrow q\,\gamma^{(\star)}\,X \end{array} \right\}$$

$$pA \longrightarrow h\,\gamma^{(\star)}\,X$$

if one *assumes* target is accurately described by CGC at small x
this will tell us about DPS (proton "GPD" at large x)

some thoughts/ideas/dreams/......

cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

cold matter energy loss?
Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

Munier, Peigne, Petreska, arXiv:1603.01028

$$z\frac{dI}{dz} \equiv \frac{\frac{d\sigma^{a+A\to a+g+X}}{dydy'd^2p_t}}{\frac{d\sigma^{a+A\to a+X}}{dyd^2p_t}}$$

the difference between a nuclear target and a proton target is the medium induced energy loss

this is used to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

can also do this for di-jets in DIS

SUMMARY

CGC is a systematic approach to high energy collisions

Leading Log CGC works (too) well

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

need to eliminate/minimize late time/hadronization effects

Precision (NLO) studies of less-inclusive observables are needed

Azimuthal angular correlations offer a unique probe of CGC

3-hadron/jet correlations provide a more detailed map of CGC dynamics

DIS is the ideal ground for precision CGC studies