Spinor helicity methods in DIS at small $x$: 3-parton production

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based on work done with A. Ayala, M. Hentschinski and M.E. Tejeda-Yeomans
QCD at small $x$: gluon saturation

“attractive” bremsstrahlung vs. “repulsive” recombination

\[ S \to \infty, \quad Q^2 \text{ fixed} \]
\[ x_{Bj} \equiv \frac{Q^2}{S} \to 0 \]

energy $\sim 1/x$

\[ \frac{\alpha_s}{Q^2} \frac{xG(x, Q^2)}{\pi r^2} \sim 1 \]

\[ Q_s^2(x, b_t, A) \sim A^{1/3} \left( \frac{1}{x} \right)^{0.3} \]
Probing saturation in high energy collisions

“nucleus-nucleus” (dense-dense)

“proton-nucleus” (dilute-dense)

DIS

structure functions (diffraction)

NLO di-hadron/jet correlations

3-hadron/jet angular correlations

Modification of production spectra

Need quite a bit of modeling

Much less modeling

Multiple scattering via Wilson lines:

\( p_t \) broadening

X-evolution via JIMWLK:

Suppression of spectra/away side peaks
**Particle production high energy collisions**

target (proton, nucleus) as a classical color field

building block: quark propagator in the background color field

\[ S_F(q, p) \equiv (2\pi)^4 \delta^4(p - q) S_F^0(p) + S_F^0(q) \tau_f(q, p) S_F^0(p) \]

with \( S_F^0(p) = \frac{i}{p + i\epsilon} \)

\[ \tau_f(q, p) \equiv (2\pi)\delta(p^+ - q^+) \gamma^+ \int d^2x_t e^{i(q_t - p_t) \cdot x_t} \]

\[ \{ \theta(p^+) [V(x_t) - 1] - \theta(-p^+) [V^+(x_t) - 1] \} \]
angular correlations in 3-parton production in DIS

$$\gamma^* T \rightarrow q \bar{q} g X$$

+ radiation from anti-quark

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans
massless quarks: helicity eigenstates

\[
\begin{align*}
  u_\pm(k) & \equiv \frac{1}{2} (1 \pm \gamma_5) u(k) \\
v_\mp(k) & \equiv \frac{1}{2} (1 \pm \gamma_5) v(k)
\end{align*}
\]

helicity operator

\[
h \equiv \vec{\Sigma} \cdot \hat{p} = \begin{pmatrix}
  \vec{\sigma} \cdot \hat{p} & 0 \\
  0 & \vec{\sigma} \cdot \hat{p}
\end{pmatrix}
\]

\[
\vec{\Sigma} \cdot \hat{p} u_\pm(p) = \pm u_\pm(p) \\
-\vec{\Sigma} \cdot \hat{p} v_\pm(p) = \pm v_\pm(p)
\]

\[
\begin{align*}
  u_+(k) = v_-(k) & = \frac{1}{2^{1/4}} \begin{bmatrix}
    \sqrt{k^+} \\
    \sqrt{k^- e^{i\phi_k}} \\
    \sqrt{k^+} \\
    \sqrt{k^- e^{i\phi_k}}
  \end{bmatrix} \\
  u_-(k) = v_+(k) & = \frac{1}{2^{1/4}} \begin{bmatrix}
    \sqrt{k^- e^{-i\phi_k}} \\
    -\sqrt{k^+} \\
    -\sqrt{k^- e^{-i\phi_k}} \\
    \sqrt{k^+}
  \end{bmatrix}
\end{align*}
\]

with \( e^{\pm i\phi_k} = \frac{k_x \pm ik_y}{\sqrt{2k^+ k^-}} = \sqrt{2} \frac{k_t \cdot \epsilon_{\pm}}{k_t} \)

\[
\begin{align*}
n^\mu & = (n^+ = 0, n^- = 1, n_\perp = 0) \\
\bar{n}^\mu & = (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0)
\end{align*}
\]

\[
\begin{align*}
k^\pm & = \frac{E \pm k_z}{\sqrt{2}} \\
\epsilon_{\pm} & = \frac{1}{\sqrt{2}} (1, \pm i)
\end{align*}
\]
spinor helicity methods

notation:

\[ |i^\pm \rangle \equiv |k_i^\pm \rangle \equiv u_\pm(k_i) = v_\mp(k_i) \quad < i^\pm | \equiv < k_i^\pm | \equiv \bar{u}_\pm(k_i) = \bar{v}_\mp(k_i) \]

basic spinor products:

\[ < i j > \quad \equiv \quad < i^- | j^+ > = \bar{u}_-(k_i) u_+(k_j) = \sqrt{|s_{ij}|} e^{i\phi_{ij}} \quad \cos \phi_{ij} = \frac{k_i \cdot k_j}{\sqrt{|s_{ij}| k_i^+ k_j^+}} \]

\[ [i j] \quad \equiv \quad < i^+ | j^- >= \bar{u}_+(k_i) u_-(k_j) = -\sqrt{|s_{ij}|} e^{-i\phi_{ij}} \quad \sin \phi_{ij} = \frac{k_i^+ k_j^+ - k_j^+ k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}} \]

with

\[ s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j \]

\[- < i j > [i j] \quad \text{and} \quad < ii > = [ii] = 0 \quad \text{and} \quad < ij ] = [ ij ] = 0 \]

charge conjugation \[ < i^+ | \gamma^\mu | j^+ > = < j^- | \gamma^\mu | i^- > \]

Fierz identity \[ < i^+ | \gamma^\mu | j^+ > < k^+ | \gamma^\mu | l^+ > = 2[ik] < lj > \]

any off-shell momentum \[ k^{\mu} \equiv \bar{k}^{\mu} + \frac{k^2}{2k^+} n^{\mu} \quad \text{where} \quad \bar{k}^{\mu} \text{ is on-shell} \quad \bar{k}^2 = 0 \]

any on-shell momentum \[ \not{\!p} = |p^+ > < p^+ | + |p^- > < p^- | \]
Diagram A1

numerator: Dirac Algebra

\[ a_1 \equiv \bar{u}(p) (k)(\not{p} + \not{k}) k_1 (l)(k_1 - \not{l}) v(q) \]

longitudinal photons \quad \text{quark anti-quark gluon helicity: + - +}

\[ a_1^{L;+-+} = -\frac{\sqrt{2}}{[n \bar{k}] l^+} \frac{Q}{l^+} \begin{bmatrix} np \end{bmatrix} < kp > [np] < n \bar{k}_1 > [n \bar{k}_1] < n q > \]

\[ (\begin{bmatrix} n \bar{k}_1 \end{bmatrix} - l^+ \begin{bmatrix} n \bar{n} \end{bmatrix}) \]

\[ < np > = -[np] = \sqrt{2p^+} \quad \text{with} \]

transverse photons: +

\[ a_1^{\perp;+-+} = -\frac{\sqrt{2}}{[n \bar{k}] [pn]} < kp > [pn] < nk_1 > [k_1 n] < \bar{n} k_1 > [k_1 n] < n q > \]
Diagram A3

longitudinal photons

quark anti-quark gluon helicity: + - +

\[
a_3^{L;+--} = \frac{\sqrt{2} Q}{l^+ [n \bar{k}_2]} [p n] ( < n \bar{k}_1 > [\bar{k}_1 n] - < n \bar{k}_2 > [\bar{k}_2 n] ) < \bar{k}_2 \bar{k}_1 > [\bar{k}_1 n] \\
( < n \bar{k}_1 > [\bar{k}_1 n] - l^+ < n \bar{n} > [\bar{n} n] ) < n q > \\
= -2^4 Q (l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3} [z_3 k_1 t \cdot \epsilon - (z_1 + z_3) k_2 t \cdot \epsilon]
\]

the rest is some standard integrals and Wilson lines
structure of Wilson lines: amplitude

\[
\begin{align*}
\left( \begin{array}{c}
\end{array} \right)_{ij} & = \left[ V^\dagger(y_t) V(x_t) t^a \right]_{ij} \\
\left( \begin{array}{c}
\end{array} \right)_{ij} & = \left[ V^\dagger(y_t) t^b V(x_t) \right]_{ij} U^{ba}(z_t)
\end{align*}
\]

new d.o.f: quadrupoles
**Quadrupole:** \[ < Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} \; < Tr \; V(r) \; V^\dagger(\bar{r}) \; V(\bar{s}) \; V^\dagger(s) > \]

*Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan: PLB706 (2011) 219*
3-parton production

linear regime: use ugd's

\[ z_1 = z_2 = 0.2, \quad z_3 = 0.6 \]

\[ p_t = q_t = k_t = 4 \text{ GeV} \]

\[ Q^2 = 16 \text{ GeV}^2 \]

\[ \Delta \phi_{12} = \frac{2\pi}{3} \quad \text{vary} \quad \Delta \phi_{13} \]

\[ k_t^2 \tilde{T}(k_t) \]

multiple scattering: broadening of the peak
x-evolution: reduction of magnitude
3-parton azimuthal angular correlations

- **Multiple scattering:** broadening of the peak
- **x-evolution:** reduction of magnitude
Possible extensions to other processes?

real photons: \( Q^2 \to 0 \)

**ultra-peripheral nucleus-nucleus collisions**
- inclusive 3-jet production
- NLO inclusive di-jet production

**crossing symmetry:**

\[
\gamma^{(*)} T \to q \bar{q} g X \quad \leftrightarrow \quad \begin{cases} 
q T &\to q g \gamma^{(*)} X \\
\bar{q} T &\to \bar{q} g \gamma^{(*)} X \\
g T &\to q \bar{q} \gamma^{(*)} X
\end{cases}
\]

**proton-nucleus collisions** (collinear factorization in proton?)

- di-jet + photon production in pA

\[
pA \to h_1 h_2 \gamma^{(*)} X
\]
Possible extensions to other processes?

MPI (double/triple parton scattering)

\[ \gamma^{(*)} T \rightarrow q \bar{q} g X \quad \longleftrightarrow \quad \begin{cases} 
q \bar{q} T & \rightarrow g \gamma^{(*)} X \\
g \bar{q} T & \rightarrow \bar{q} \gamma^{(*)} X \\
g q T & \rightarrow q \gamma^{(*)} X 
\end{cases} \]

\[ pA \rightarrow h \gamma^{(*)} X \]

if one *assumes* target is accurately described by CGC at small x
this will tell us about DPS (proton “GPD” at large x)
some thoughts/ideas/dreams/……

cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

Munier, Peigne, Petreska, arXiv:1603.01028

\[
Z \frac{dI}{d\ln z} \equiv \frac{\frac{d\sigma^{a+A \rightarrow a+g+X}}{dydy'd^2p_t}}{\frac{d\sigma^{a+A \rightarrow a+X}}{dyd^2p_t}}
\]

the difference between a nuclear target and a proton target is the medium induced energy loss

this is used to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

can also do this for di-jets in DIS
SUMMARY

CGC is a systematic approach to high energy collisions

Leading Log CGC works (too) well

- it has been used to fit a wealth of data; ep, eA, pp, pA, AA
- need to eliminate/minimize late time/hadronization effects

Precision (NLO) studies of less-inclusive observables are needed

Azimuthal angular correlations offer a unique probe of CGC

- 3-hadron/jet correlations provide a more detailed map of CGC dynamics

DIS is the ideal ground for precision CGC studies