Matching TMD factorization and collinear factorization

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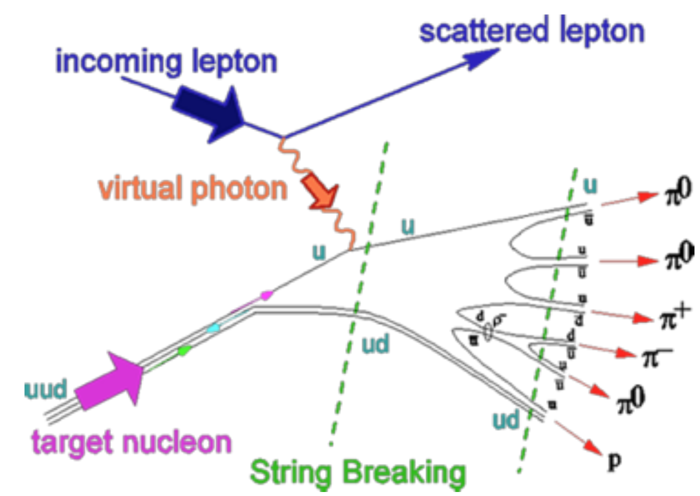
Leonard Gamberg

PennState Berks

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

Overview comments



- Present improved implementation for combining transverse-momentum-dependent (TMD) factorization and collinear factorization in SDIS

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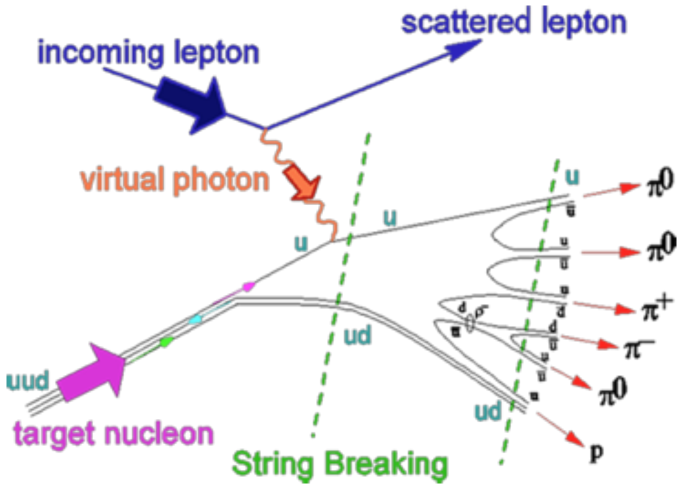
- The result is a modified version of the "$W+Y$" *matching* prescription traditionally used in the Collins-Soper-Sterman (CSS) formalism

- Address the "*standard matching prescription*" traditionally used in the CSS formalism relating low and high q_T behavior of cross section @ moderate Q

  - Collins Soper Sterman NPB 1985
  - A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)

Overview comments



- In particular address the role of Y term matching of low and high q_T behavior of cross section @ moderate Q
  - Collins Soper Sterman NPB 1985
  - A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)
- Introduce method to combine TMD and Collinear Factorization formalism

Comments Message

- The standard $W + Y$ prescription was arranged to apply for large Q situations where there is *a broad range* of transverse momentum s.t. $m << q_T <<Q$
    - That is where q_T/Q is small s.t. *TMD factorization* is valid & …
    - m/q_T is sufficiently small (i.e. $q_T \sim Q$) s.t. *collinear factorization* is valid
- N.B. keeping full accuracy when $m << q_T <<Q$, give rise to situation where both pure TMD and pure collinear factorization ***have degraded accuracy*** "***outside design regions***"
    - TMD factorization degrade as q_T increases $q_T/Q \sim O(1)$ or $q_T \sim Q$ s.t. collinear factorization should hold
    - Other hand, as q_T decreases, $m/q_T \sim O(1)$ or $q_T \sim m$ s.t. TMD factorization holds
- *Generally get results valid over all q_T need to combine info TMD & collinear factorization*

Comments Message

✦ **Collinear fact. valid in two ways**

1. for cross sections differential in q_T w/ $q_\mathrm{T} \sim$ Q (OPE)
2. also valid when we integrate over q_T

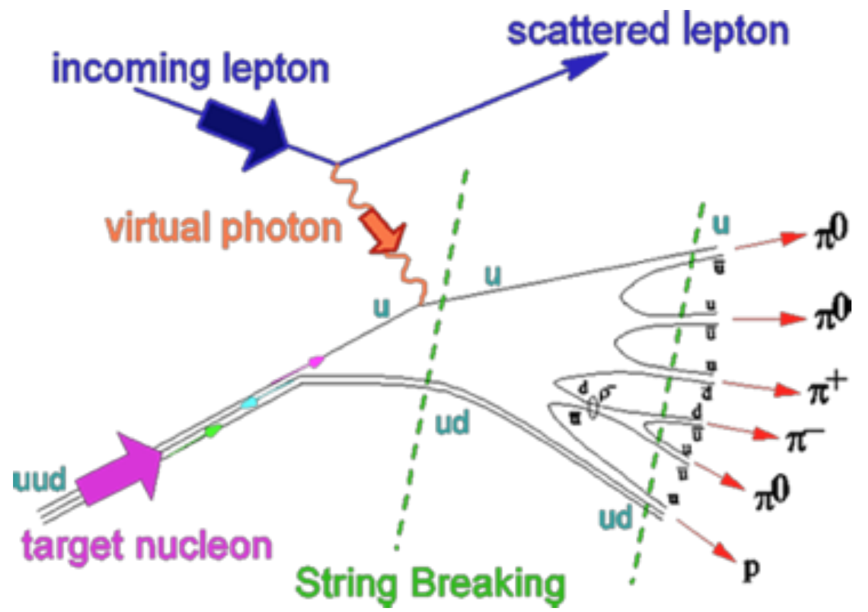
$$\int dq_T \; d\sigma(q_T, Q)$$

✦ However CSS did not specifically address the issue of *matching to collinear factorization* for the cross section integrated over q_T

Comments Message

$$\int dq_T \; d\sigma(q_T, Q)$$

- We develop a prescription to which *matches* the integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two which suppressed by powers of 1/Q
- **Importantly, the exact definitions of the TMD PDFs and FFs are unmodified from the usual ones of factorization derivations**
- We preserve transverse-coordinate space version of the $W_{\rm TMD}$ term, but only modify the way in which it is used



$$\frac{d\sigma(q_T, Q)}{d^2 q_T dQ \ldots} \equiv d\sigma(q_T, Q)$$

Short hand notation throughout talk

Start w/ review of CSS $W + Y$ definition

✦ Collins Soper Sterman NPB 1985

✦ Collins 2011 Cambridge Press

Birds eye view

- Standard CSS formalism separates the cross section into a sum of two terms *W* & *Y* such that *their sum* gives the cross section up to an error that **relative to the cross section is** power suppressed

$$O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

Start w/ review of CSS $W + Y$ definitions

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$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- ***W*** describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$
- W is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence
- The " ***Y*** -term " is described in terms of "collinear approximation" to the cross section: it is the correction term for large $q_T \sim Q$

Start w/ review of CSS $W + Y$ definitions

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$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- The CSS construction of $W + Y$ and the specific approximations are applied thru the **operation-approximators** $\boldsymbol{T_{\mathrm{TMD}}}$ and $\boldsymbol{T_{\mathrm{coll}}}$ that apply in their ***"design" regions*** $m \sim q_T \ll Q$ and $m \ll q_T \sim Q$ respectively which we emphasize by the ***range*** of the argument above

$$m \lesssim q_T \lesssim Q$$

"Matching-1" $W + Y$-schematic

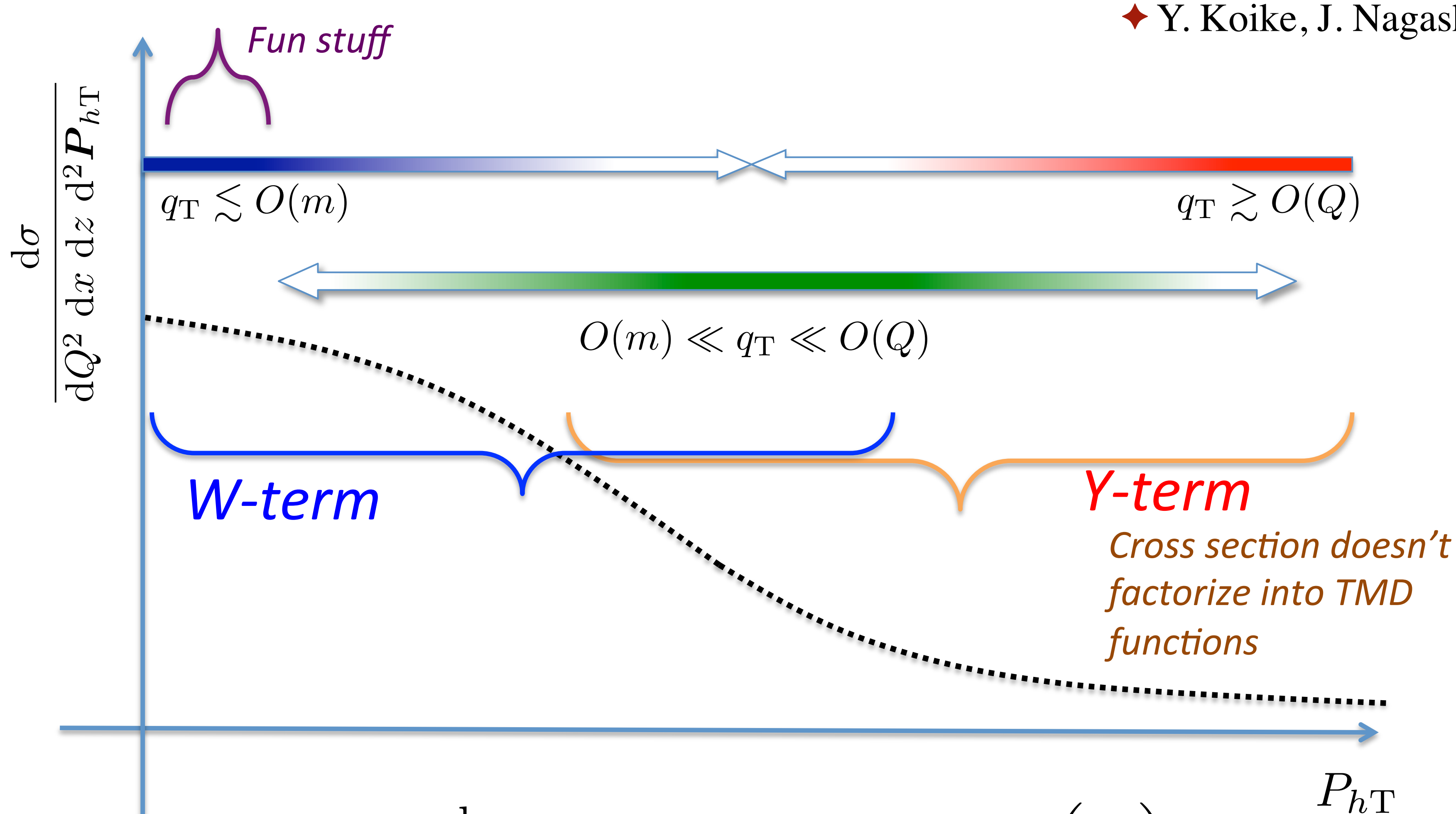
- This was *designed* with the aim to have a formalism that is valid to leading power in m/Q uniformly in q_T, where m is a typical hadronic mass scale
- and where there is a broad intermediate range of transverse momentum characterized by $m \ll q_T \ll Q$

Implementations/studies

✦ Nadolsky Stump C.P. Yuan PRD 1999 **HERA data**

✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **eRHIC**

From Ted Rogers **W + Y**



note $P_{hT} = z q_T$

$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

“Matching-1” $W + Y$ studies

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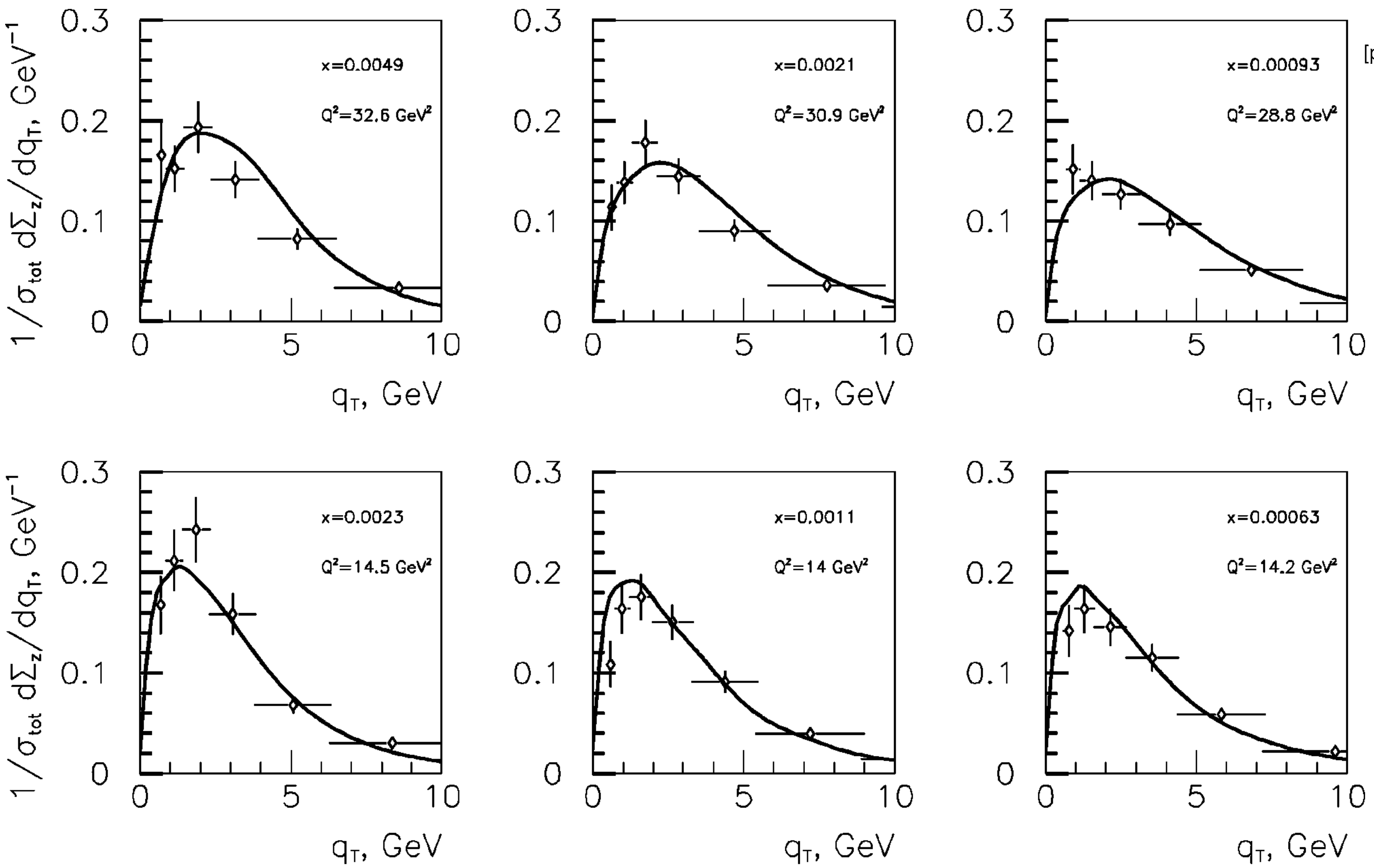
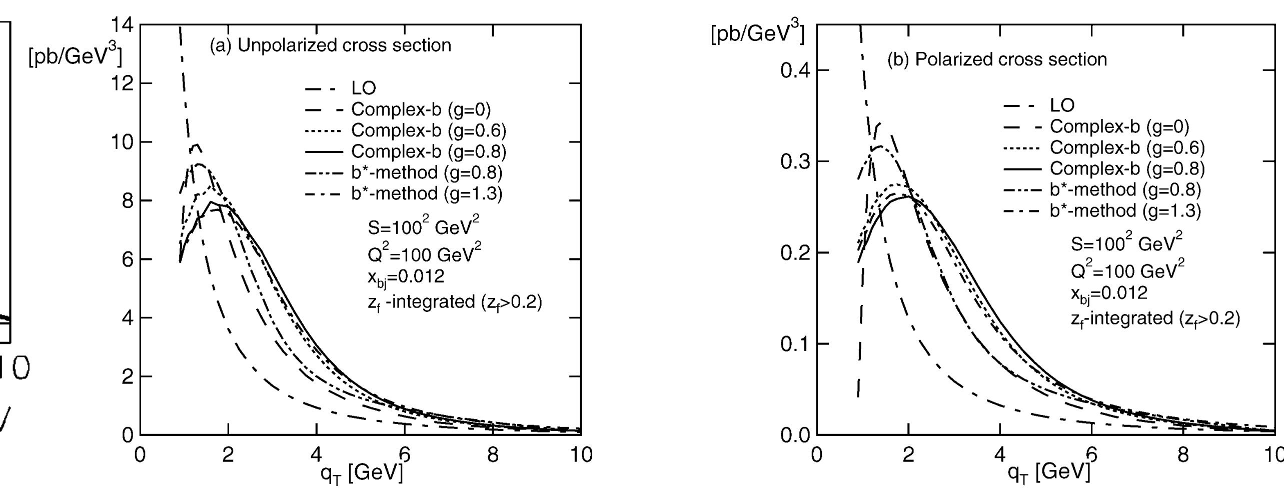
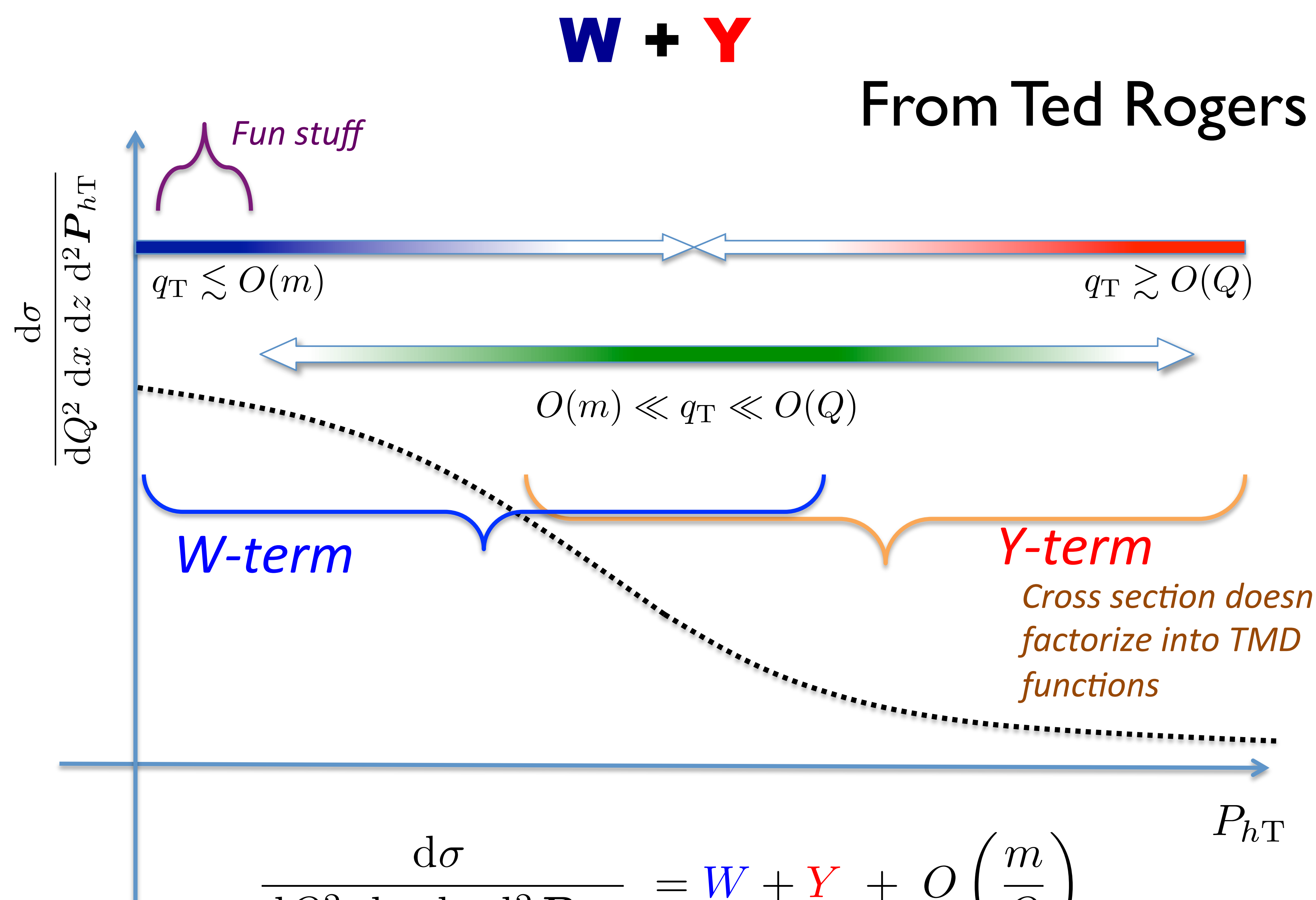
$$\frac{1}{d\sigma_{tot}/(dxdQ^2)}\frac{d\Sigma_z}{dxdQ^2dq_T}$$





"Matching-1" and $W + Y$-schematic

- However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic



note $P_{hT} = z q_T$

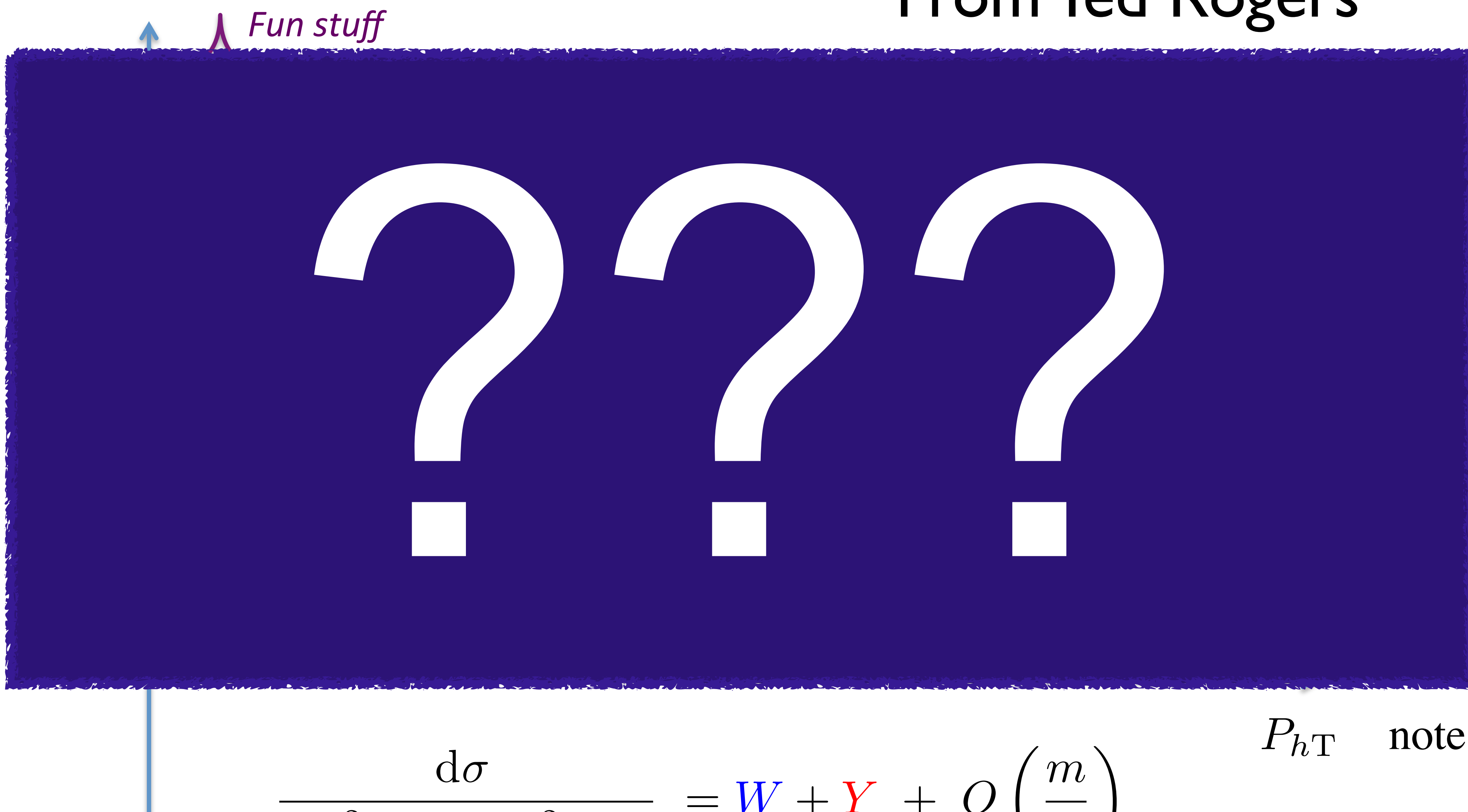
$$\frac{d\sigma}{dQ^2\ dx\ dz\ d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

Matching and $W + Y$-schematic

- However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

W + Y

From Ted Rogers



$P_{h\mathrm{T}}$ note $P_{hT} = zq_T$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\ \mathrm{d}x\ \mathrm{d}z\ \mathrm{d}^2\boldsymbol{P}_{h\mathrm{T}}} = W + Y + O\left(\frac{m}{Q}\right)$$

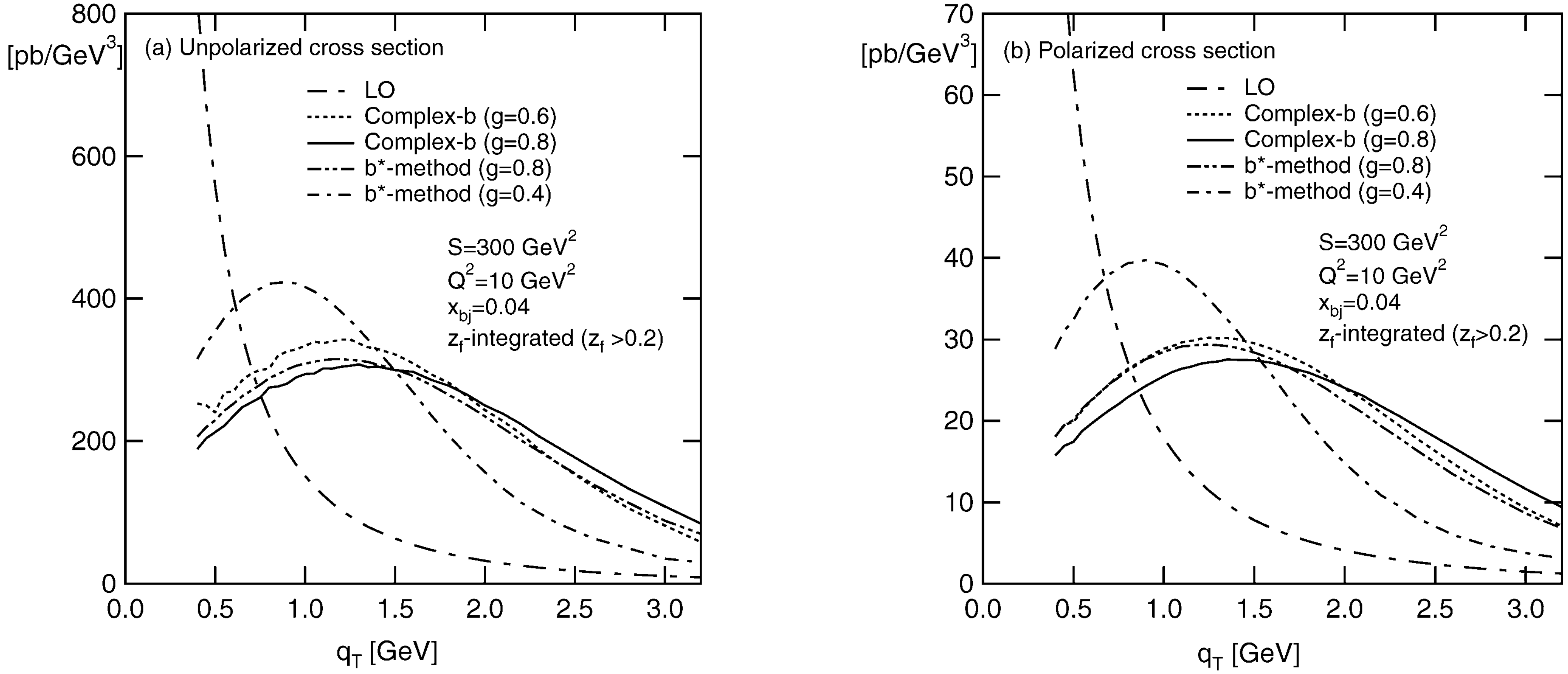
Matching and $W + Y$ -studies

This impacts studies of non-perturbative nucleon structure @ COMPASS & JLAB !!!

$$m \lesssim q_T \lesssim Q$$

Implementations

- ✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006). "…***COMPASS no data yet…***"



Matching and $W + Y$ -studies

Compass Example

Boglione Prokudin et al. JHEP 2015

- When q_T is above some small fraction of Q, W deviates a lot from $d\sigma(q_T, Q)$
- Then it becomes negative and "asymptotes" to $\frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$

  Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006)
- At large q_T $W+Y$ is then difference of large terms and *truncation errors* can be augmented

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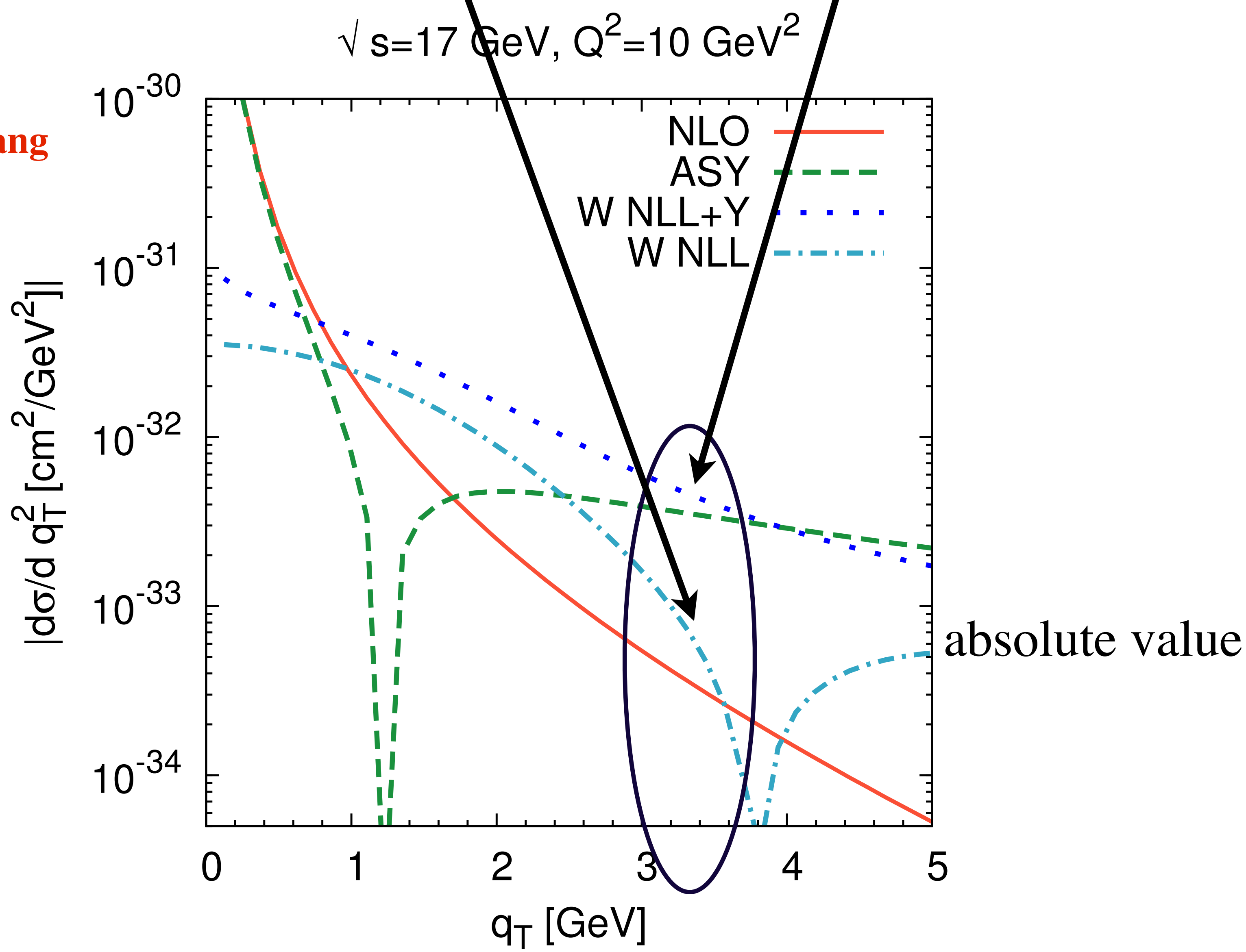
Matching becomes a problem COMPASS/Jlab like energies



- To get a sense of these *truncation errors* we further "unpack" *W+ Y* via their "*Approximators*" and its *construction in terms of* W, Y, FO, ASY <u>terms</u>

Review of Region Analysis "Construction"

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- CONSTRUCTION: one starts with smallest-size region which is in a neighborhood of $q_T = 0$, where T_{TMD} gives a very good approximation adding and subtracting the T_{TMD} approximation

$$d\sigma(q_T, Q) = T_{TMD}\, d\sigma(q_T, Q) + [d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)]$$

- The error in the bracket is order $(q_T/Q)^a$ and is only unsuppressed at $q_T >> m$

- **Now, *extend* the range of q_T ...**

e.g. Collins PRD 1998, Cambridge Press 2011

Review of Region Analysis "Construction"
W, Y, FO, ASY Definitions

- Extending q_T, one then applies T_{coll} to the bracket & uses the fixed order (FO) perturbative expansion

The Result is the combination

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx T_{TMD}\, d\sigma(q_T, Q) + T_{coll}\left[d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$q_T/Q << 1$

$q_T \sim Q$ or $m/q_T << 1$

Now we see the definition of the Y term via "approximators"

$$Y(q_T, Q) \equiv T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}\, d\sigma(q_T, Q)$$

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section

- ***N.B. At small q_T the FO and ASY are dominated by the same diverging terms***

$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$$

- ***Thus its expected that the Y term is small or zero leaving***

$$d\sigma(q_T \ll Q, Q) \approx W(q_T, Q)$$

The Asymptotic piece of the NLO cross section in detail

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

$$\left(\frac{d\sigma_{BA}}{dx\,dz\,dQ^2\,dq_T^2\,d\phi}\right)_{\text{asym}} = \frac{\sigma_0 F_l}{S_{eA}}\,\frac{\alpha_s}{\pi}\,\frac{1}{2q_T^2}\,\frac{A_1(\psi,\phi)}{2\pi}$$

$$\times\sum_j e_j^2 \Bigg[D_{B/j}(z,\mu)\{(P_{qq}\otimes f_{j/A})(x,\mu)+(P_{qg}\otimes f_{g/A})(x,\mu)\}$$

$$+\{(D_{B/j}\otimes P_{qq})(z,\mu)+(D_{B/g}\otimes P_{gq})(z,\mu)\}f_{j/A}(x,\mu)$$

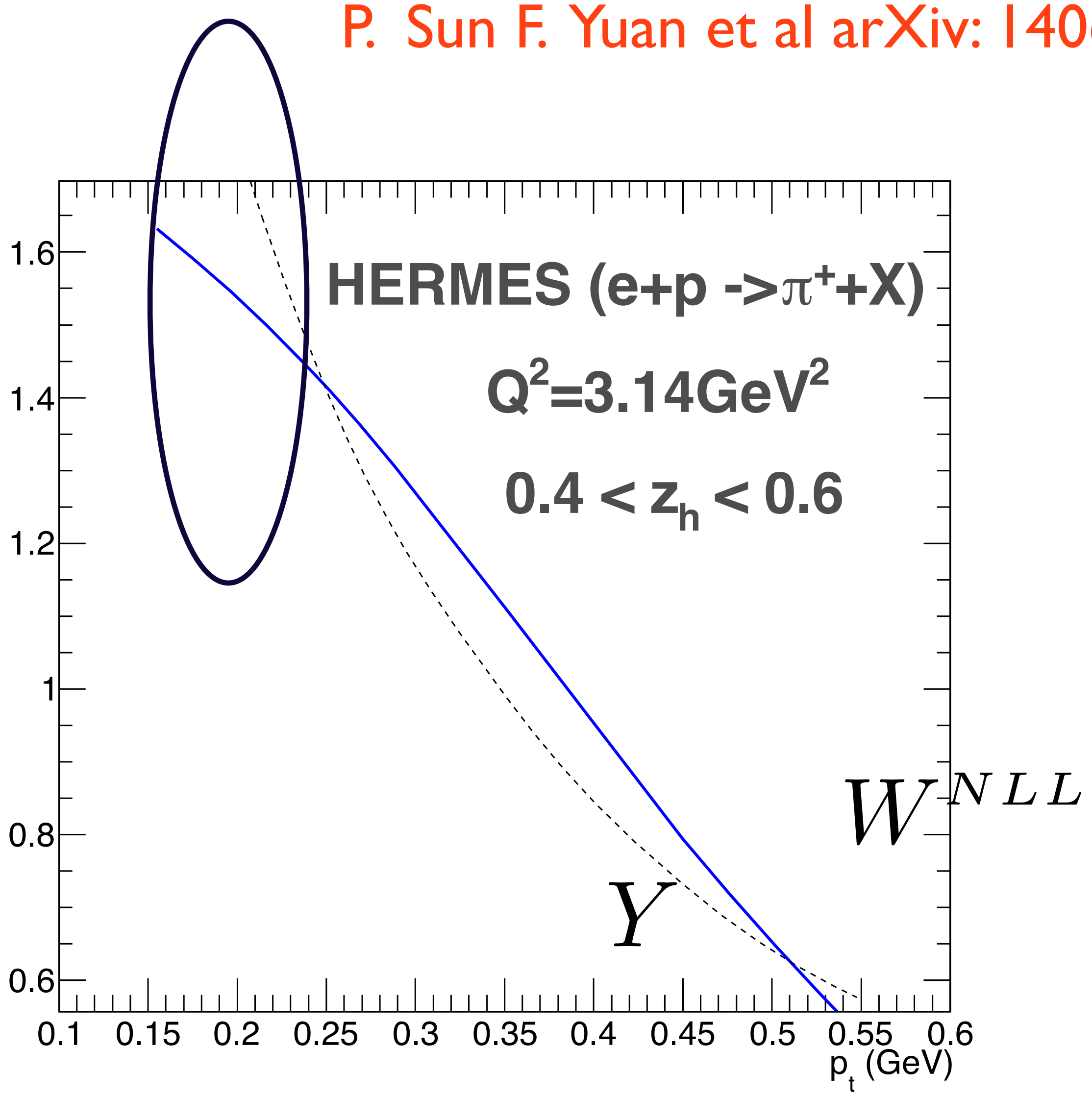
$$+2D_{B/j}(z,\mu)f_{j/A}(x,\mu)\left\{C_F\log\frac{Q^2}{q_T^2}-\frac{3}{2}C_F\right\}+\mathcal{O}\left(\frac{\alpha_s}{\pi},q_T^2\right)\Bigg].$$

- Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006)

Matching and $W + Y$ -studies

- At small q_T the Y term is in principle suppressed: it is the difference of the FO perturbative calculation of the cross section and the asymptotic contribution of W for small q_T
- But there can be a difference of of large terms and truncation errors are augmented: **Here the Y term is larger than W?!**

P. Sun F. Yuan et al arXiv: 1406.3073



$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

Matching and $W + Y$-schematic

- Thus the region *between* large and small q_T needs special treatment if errors are to be strictly power suppressed point-by-point in q_T

Matching and $W + Y$-schematic

- Thus the region *between* large and small q_T needs special treatment if errors are to be strictly power suppressed point-by-point in q_T

Address by extending formalism

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$$q_T \lesssim m \quad \text{and} \quad q_T \gtrsim Q$$

Extend/enhanced formalism

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \lesssim m$$

- For $q_T \lesssim m$ collinear factorization is not applicable for the differential cross section. But this region is actually where the *W-term* in has its highest validity. So one simply must ensure that the *Y -term* is sufficiently suppressed in Eq. (10) for $q_T \lesssim m$
- Modify *Y*

$$Y(q_T, Q) = \{FO(q_T, Q) - ASY(q_T, Q)\} X(q_T/\lambda)$$

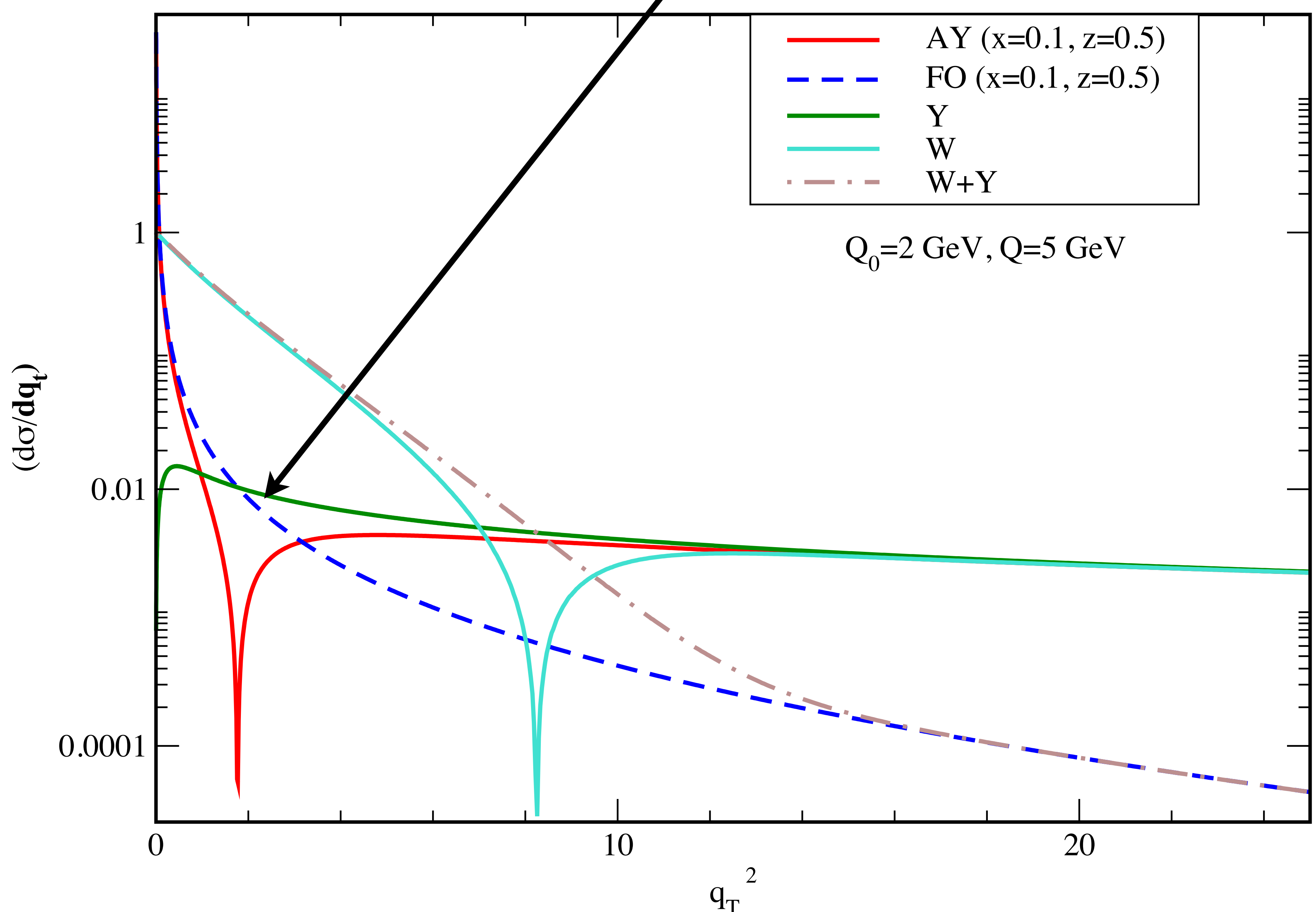
with small q_T cutoff

hinted in Collins Soper Sterman NPB 1985

$$X(q_\mathrm{T}/\lambda) = 1 - \exp\{-(q_\mathrm{T}/\lambda)^{a_X}\}$$

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$d\sigma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



Extend/enhanced formalism

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$$q_T \gtrsim Q$$

Modification of the cross section leaves the standard treatment of TMD factorization only slightly modified

In particular the op. definitions along with evolution properties are the same as in the usual formalism

We do this in two steps however now we need explicit expression for *W from JCC* formalism

see Collins Rogers PRD 2015

Summary of elements of TMD factorization

$$W(q_T,Q)=\int\frac{d^2b_T}{(2\pi)^2}e^{iq_T\cdot b_T}\tilde{W}(b_T,Q)$$

Collins 2011 QCD
Aybat Rogers PRD 2011

- Factorization and TMD evolution in b_T *space*
- Solve the CSS & RG evolution eqs. for W term in SIDIS with "boundary condition" to freeze b_T above some b_{max}

$$b_*(b_T)=\sqrt{\frac{b_T^2}{1+b_T^2/b_{max}}}$$

$$\tilde{W}(q_T,Q)=\int\frac{d^2b_T}{(2\pi)^2}e^{iq_T\cdot b_T}\tilde{W}^{OPE}(b_*(b_T),Q)\,\tilde{W}_{NP}(b_T,Q;b_{max})$$

$$\tilde{W}_i^{OPE}(b_*(b_T),Q)=H_i(Q)\;\tilde{C}^{pdf}_{i/i'}(x_A/\hat{x},b_*b_\star)\otimes\tilde{f}_{i'/A}(\hat{x},\mu_{b_\star})\;\tilde{C}^{ff}_{j'/i}(z_B/\hat{z},b_*)\otimes\tilde{d}_{B/i'}(\hat{z},\mu_b)e^{-S^{pert}(b_*,Q)}$$

Collinear pdfs

$$\tilde{W}_{NP}(b_T,Q;b_{max})=e^{-S_{NP}(b_T,Q;b_{max})}$$

Aidala, Field, Gamberg, Rogers PRD 2015

$$g_K(b_T;b_{\max})=\frac{g_2(b_{\max})b_{\mathrm{NP}}^2}{2}\ \ln\left(1+\frac{b_T^2}{b_{\mathrm{NP}}^2}\right)$$

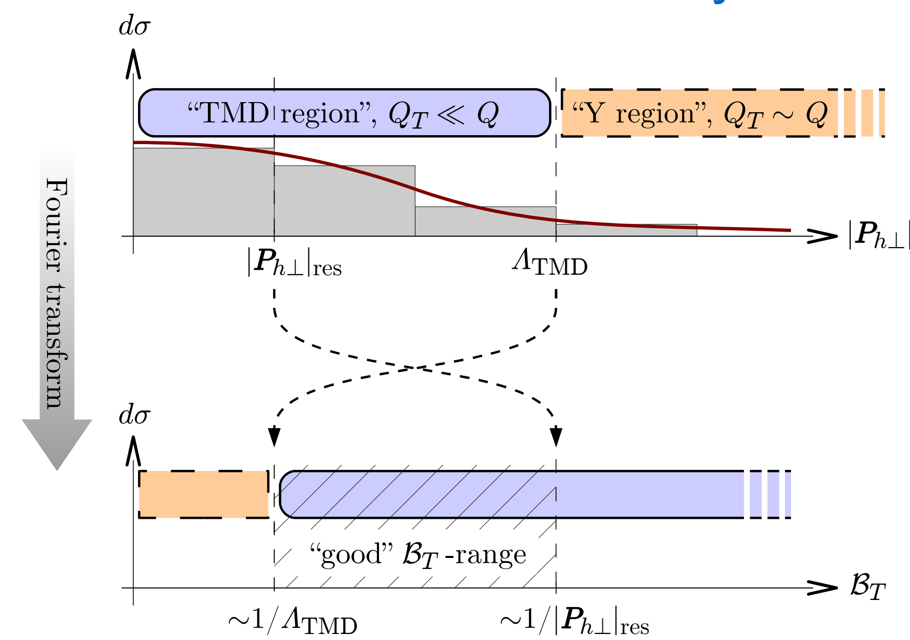
$$S_{NP}(b_T,Q;b_{max})=g_A(x_A,b_T;b_{max})+g_B(z_B,b_T;b_{max})-2g_K(b_T;b_{max})\ln\left(\frac{Q}{Q_0}\right)$$

Fourier Transforms of TMDs and universal soft function g_K

Two modifications

a) B.C. Introduce small b-cuttoff Similar to Catani et al. NPB 2006 &

Bessel Weighting
Boer LG Musch Prokudin JHEP 2011



$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

b) Introduce large q_T-cuttoff so that $W_{\rm New}$ vanishes at large $q_{\rm T}$ Similar to Nadolsky et al. PRD 1999

$$\Xi\left(\frac{q_T}{Q},\eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{New}(q_T,Q;\eta,C_5) = \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2b_T}{(2\pi)^2}e^{iq_T\cdot b_T}\,\tilde{W}^{OPE}\left(b_*(b_c(b_T)),Q\right)\tilde{W}_{NP}(b_c(b_T)),Q;b_{max})$$

Generalized B.C.

$$b_*(b_c(b_{\rm T})) \longrightarrow \begin{cases} b_{\rm min} & b_{\rm T} \ll b_{\rm min} \\ b_{\rm T} & b_{\rm min} \ll b_{\rm T} \ll b_{\rm max} \\ b_{\rm max} & b_{\rm T} \gg b_{\rm max}\,. \end{cases}$$

Now Y term is further modified

$$Y_{New}(q_T, Q) = [T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}^{New}\, d\sigma(q_T, Q)]\, X(q_T/\lambda)$$

$$= [FO(q_T, Q) - ASY_{New}(q_T, Q)]\, X(q_T/\lambda)$$

Putting all together

$$d\sigma(q_T,Q) \approx T_{TMD}^{New} d\sigma(q_T,Q) + T_{coll}\left[\, d\sigma(q_T,Q) - T_{TMD}^{New}\, d\sigma(q_T,Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

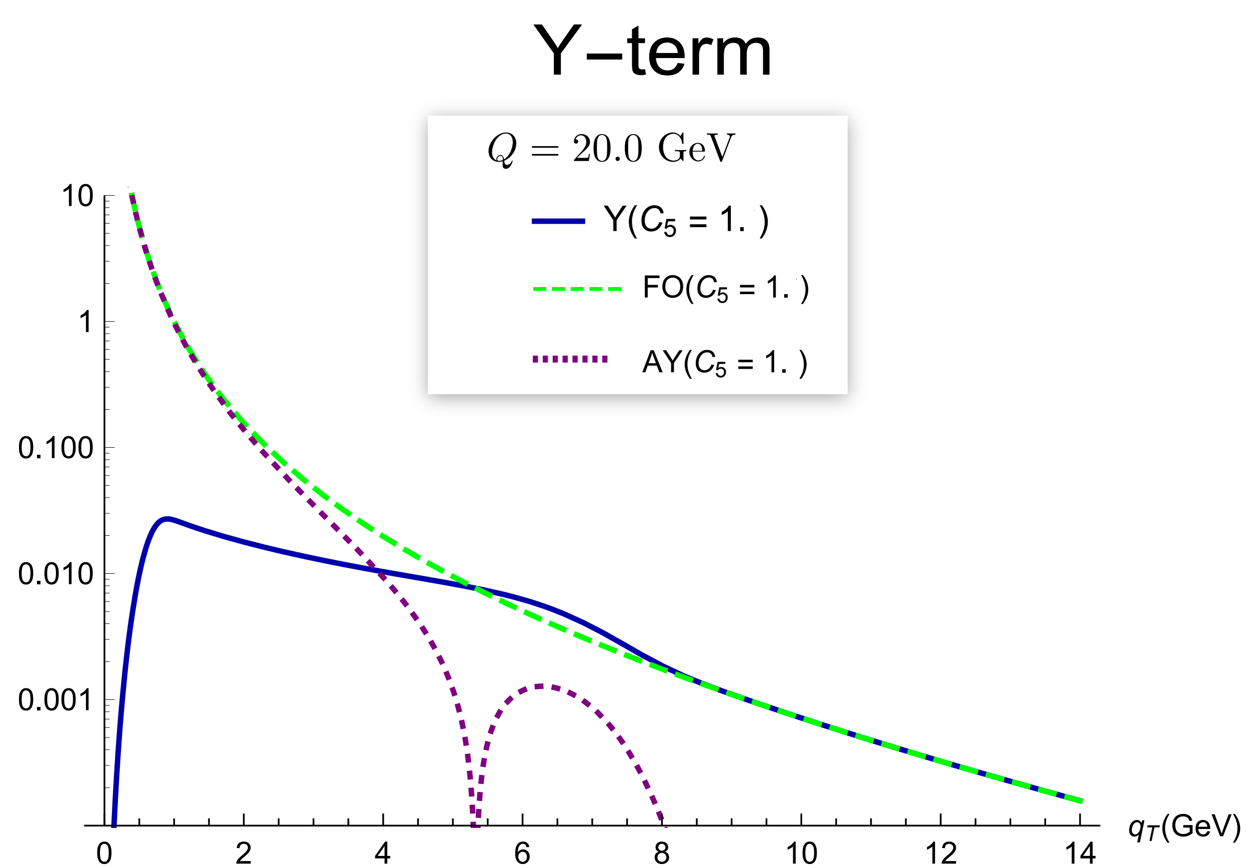
or

$$d\sigma(q_T,Q) \approx W_{New}(q_T,Q) + Y_{New}(q_T,Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

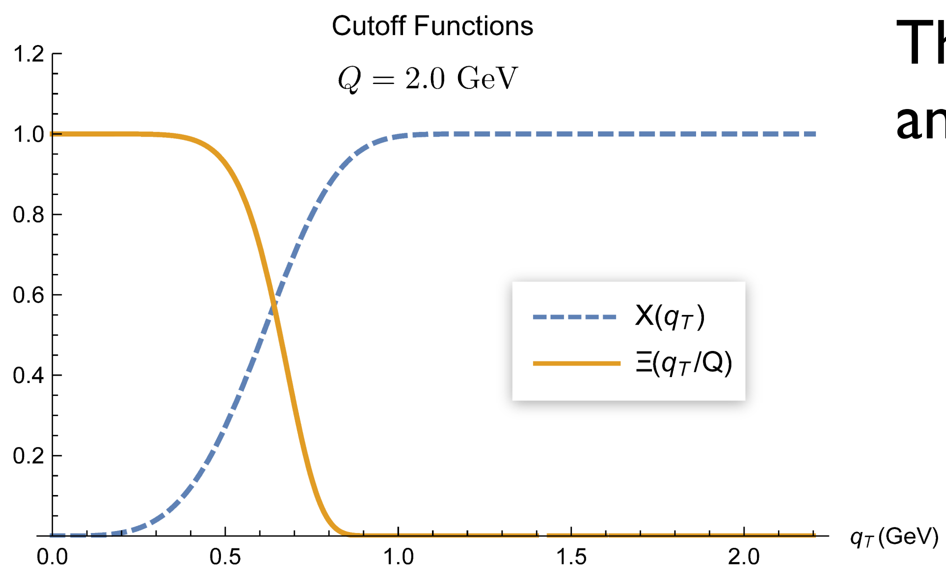
Putting all together demonstration

To illustrate the steps above, we have performed sample calculations of the Y-term using analytic approximations for the collinear pdfs and collinear ffs. We consider only the target up-quark gamma q to $q+g$ channel, and for the running alpha_s we use the two-loop beta function f = 3 since we are mainly interested in the transition to low Q.
Thus we use $\Lambda_{QCD} = 0.330$

To further simplify our calculations, we use analytic expressions for the collinear correlation functions, taken from appendix A1 of GRV ZPC 1992 for the up-quark pdf and from Eq. (A4) of KKP NPB 2001 for the up-quark-to-pion fragmentation function.

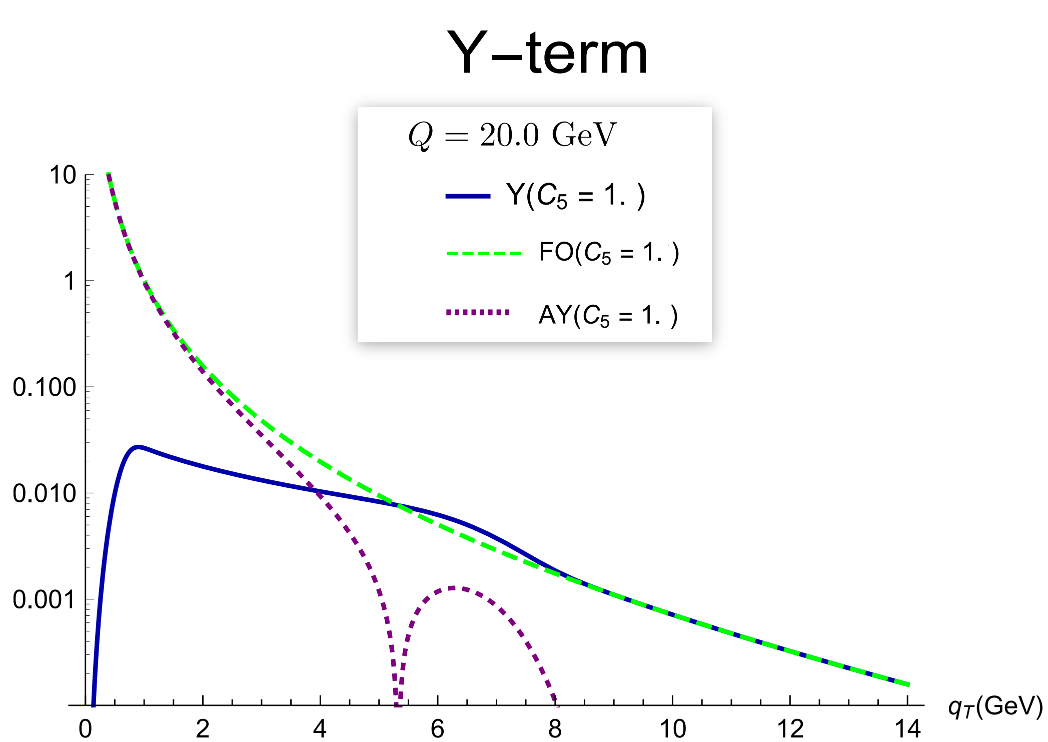


Putting all together demonstration

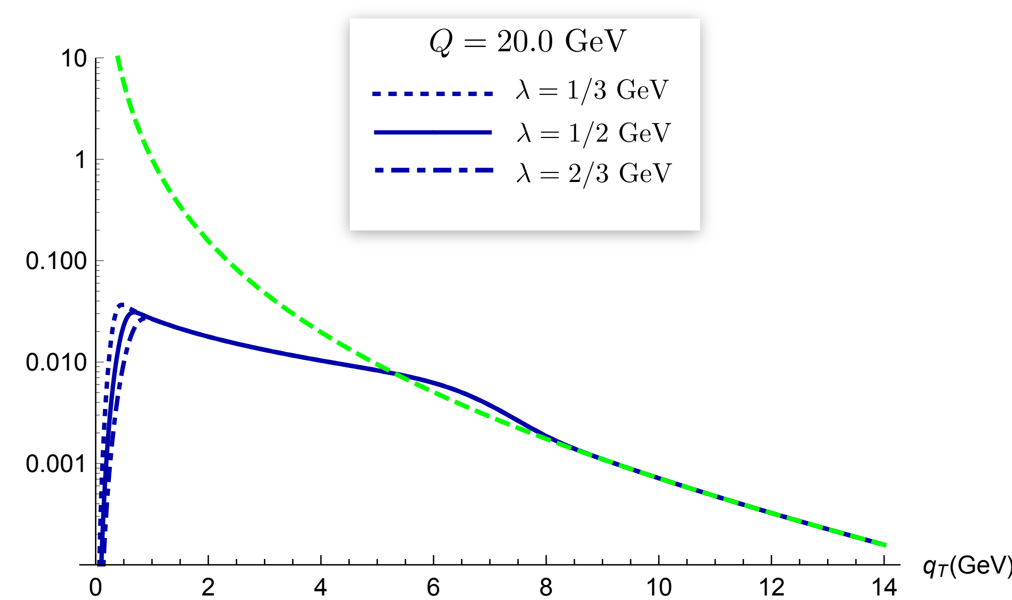


(a)

The cutoff functions in for low q_T/lambda (blue dashed line) and large qT/Q (brown solid line) for Q = 20.0 GeV



Insenstive to λ



Now Semi-inclusive to Colinear integrate over q_T

- Parton Model W-term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2 k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2 q_T \, W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building
w/ and w/o evolution using TMD and collinear
evolution approach   Anselmino et al. 2005-2016

- Standard CSS W-term

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

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See appendix for details Phys.Rev. D 94 (2016)
J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = \int \delta^2(b_T)\, b_T^a \times \text{logarithmic corrections}$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

For details Phys.Rev. D 94 (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang

$$W_{New}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{New}(b_T, Q)$$

$$\int d^2 q_T W_{New}(q_T, Q) = \tilde{W}(b_{c\ min}, Q)$$

$$\int d^2 q_T \, W_{New}(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \ + O(\alpha_s(Q))$$

Has a normal collinear factorization in terms of collinear pdfs

$$\int d^2 q_T \, W_{New}(q_T, Q) + Y(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \ + O(\alpha_s(Q))$$

$$+ \text{ terms dominated by large } q_T \text{ contribution to } Y \text{ term}$$

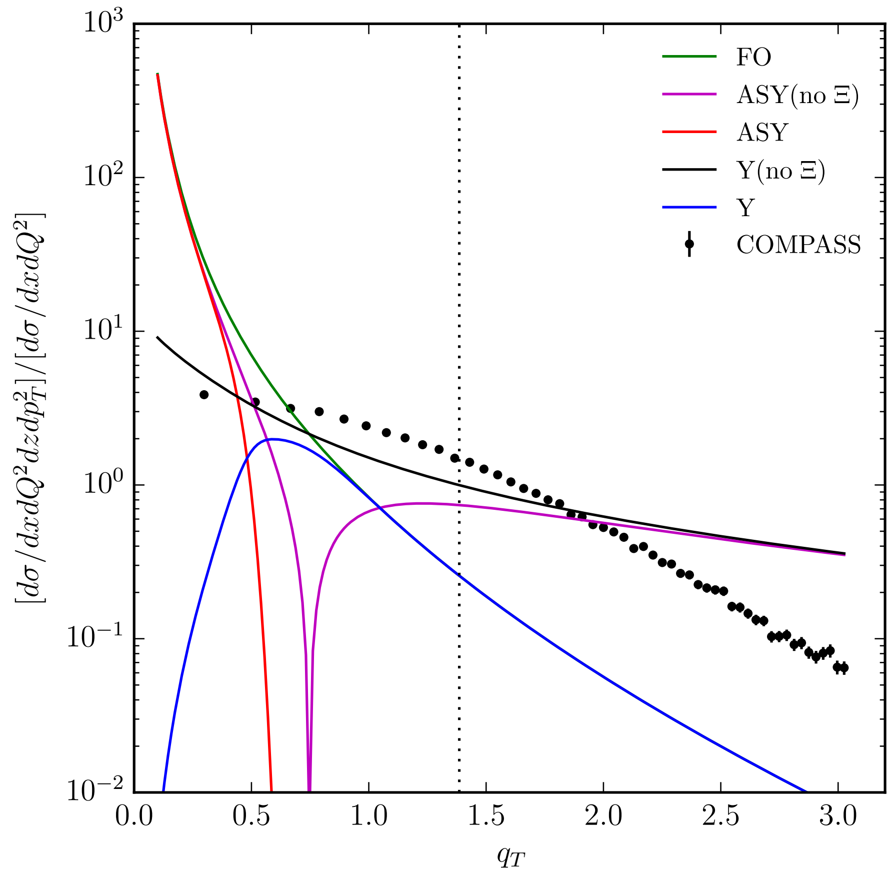
Has implications for modeling TMD and fitting

Comments

- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of 1/Q.
- Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used.
- This work has dealt only with unpolarized cross sections.
- We are studying the analogous topic applied to polarized phenomena.
- This is central to the EIC and studying the 3-D momentum and spatial structure of the nucleon and further exploring the connection between TMD and collinear factorization

Matching with fixed-order calculations

Collins et al., arXiv: 1605.00671



$Q^2 = 1.92\mathrm{GeV}^2, x = 0.0318, z = 0.375$

The collinear calculation (green line) is much smaller than data
Standard Y term is bigger than data (black line) ➞ modifications needed (blue line)

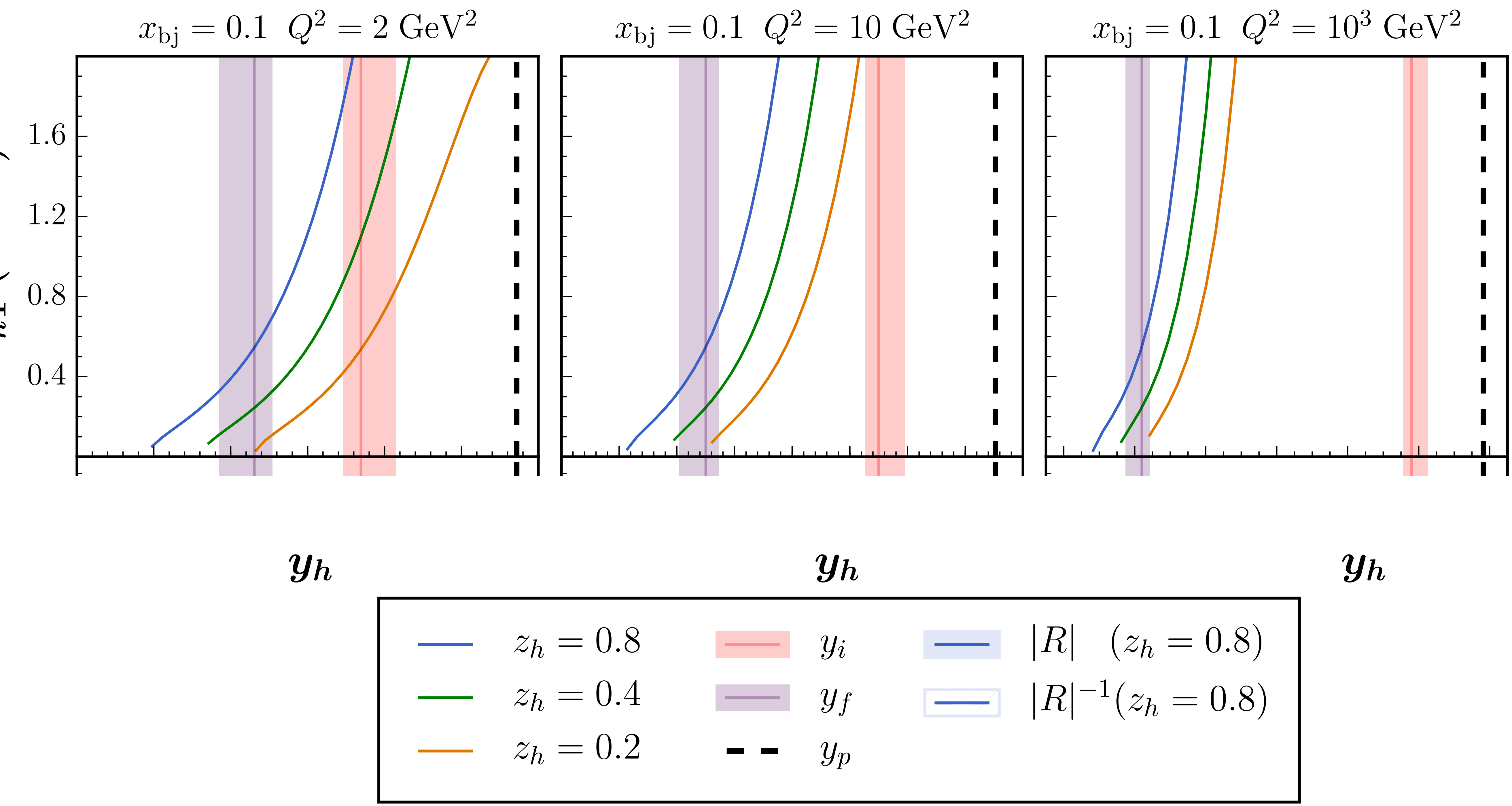
EXTRA Slides

The flow of the variable z

$$\frac{d\Sigma_z}{dx\,dQ^2\,dq_T^2} = \sum_B \int_{z_{min}}^{1} z \frac{d\sigma(e+A \to e+B+X)}{dx\,dz\,dQ^2\,dq_T^2} dz$$

$$= \frac{\sigma_0 F_l}{S_{eA}} \frac{A_1(\psi,\phi)}{2} \int \frac{d^{n-2}b}{(2\pi)^{n-2}} e^{i\vec{q}_T \cdot \vec{b}} W_z(b,x,Q) + Y_z$$

Kinematics of Current Region Fragmentation in Semi-Inclusive Deeply Inelastic Scattering
M. Boglione, Collins, Gamberg, Gonzalez-Hernandez, Rogers, Sato
To appear today/tomorrow ...



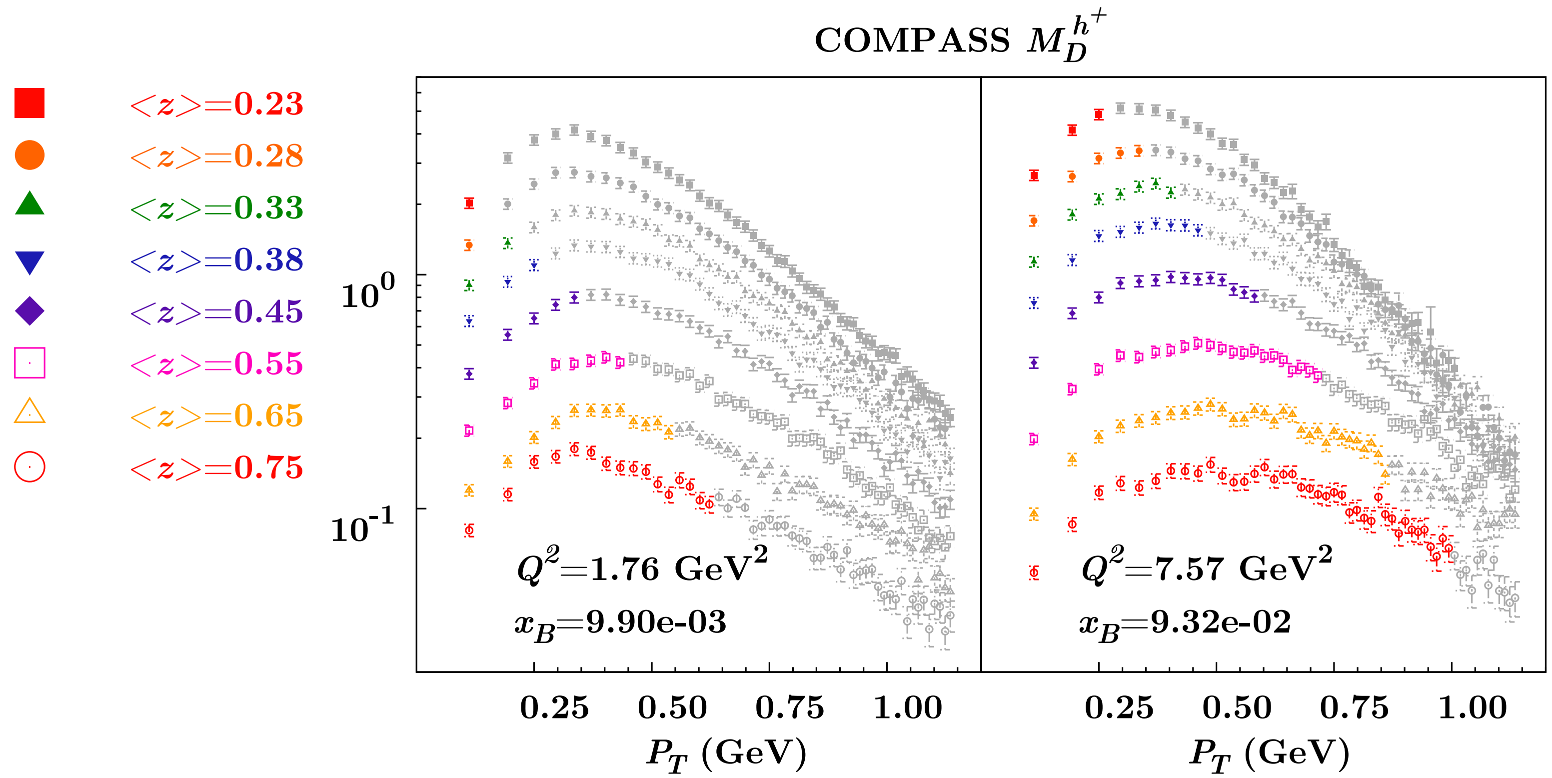


Figure 4: A selection of COMPASS data from [23]. The colored points correspond to the hadron moving with rapidity smaller than some maximum value, which has been chosen to be a quarter-way between the largest estimate of $y_{\rm f}$ and the value of $y_{\rm h}$ for which $R = 1$. This ensures that for $Q^2 \sim 10\,{\rm GeV}^2$, $R \lesssim 0.25$. Within our rough order of magnitude estimate, grey points are likely to receive important contributions from non-current regions. For detailed phenomenological calculations, it is important to improve the estimates of Eq. (26) by more precise constraints on $M_{\rm iT}$ and $M_{\rm fT}$, and also to use a range of rapidity cutoffs.

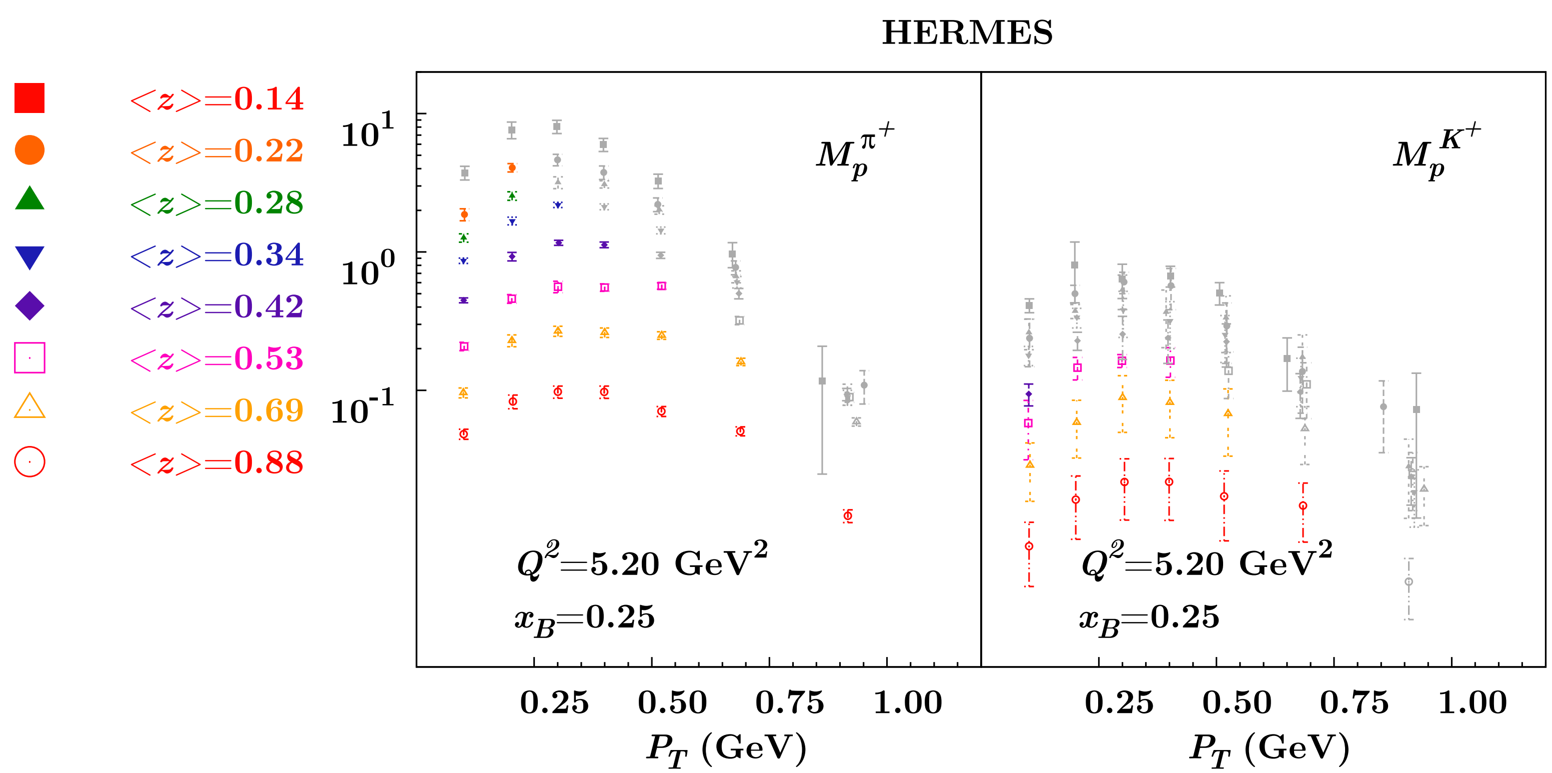
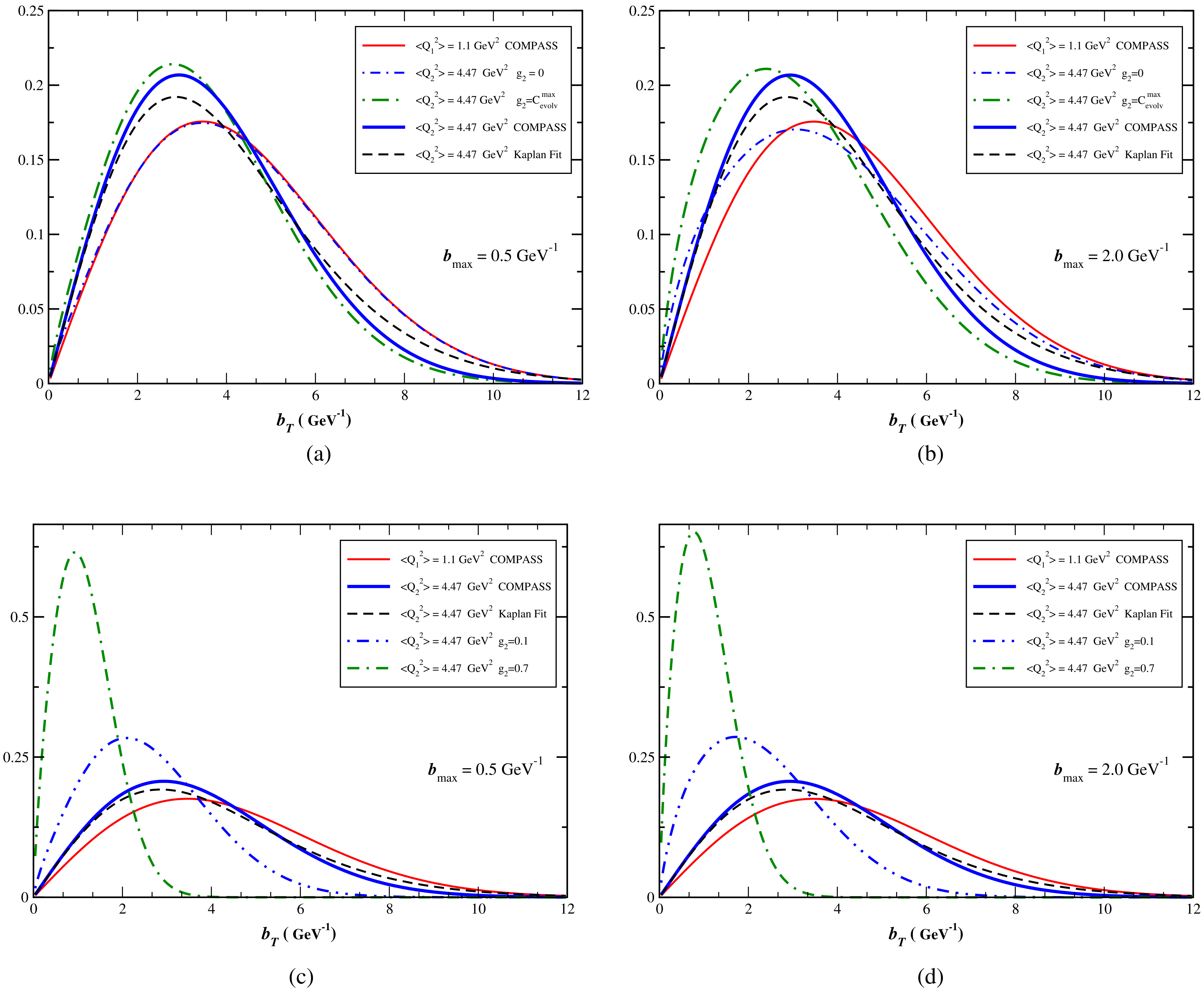


Figure 5: A selection of HERMES data from [24]. Points are as described in Fig. 4. The larger mass of the kaon results in a larger number of points that are likely to receive significant contributions from the non-current regions, within our rough order of magnitude estimate. For detailed phenomenological calculations, it is important to improve the estimates of Eq. (26) by more precise constraints on $M_{\rm iT}$ and $M_{\rm fT}$, and also to use a range of rapidity cutoffs.

TMD Evolution and COMPASS Data

Aidala, Field, Gamberg, Rogers PRD 2015

$$g_K(b_T; b_{\max}) = \frac{g_2(b_{\max}) b_{\rm NP}^2}{2} \ \ln\left(1 + \frac{b_T^2}{b_{\rm NP}^2}\right)$$



Expression for $W(b_c,Q)$

$$\tilde{W}(b_c(b_{\rm T}),Q) = H(\mu_Q,Q)\sum_{j'i'}\int_{x_A}^{1}\frac{d\hat{x}}{\hat{x}}\tilde{C}^{\rm pdf}_{j/j'}(x_A/\hat{x},b_*(b_c(b_{\rm T}));\bar{\mu}^2,\bar{\mu},\alpha_s(\bar{\mu}))f_{j'/A}(\hat{x};\bar{\mu})\times$$
$$\times\int_{z_B}^{1}\frac{d\hat{z}}{\hat{z}^3}\tilde{C}^{\rm ff}_{i'/j}(z_B/\hat{z},b_*(b_c(b_{\rm T}));\bar{\mu}^2,\bar{\mu},\alpha_s(\bar{\mu}))d_{B/i'}(\hat{z};\bar{\mu})\times$$
$$\times\exp\left\{\ln\frac{Q^2}{\bar{\mu}^2}\tilde{K}(b_*(b_c(b_{\rm T}));\bar{\mu})+\int_{\bar{\mu}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1)-\ln\frac{Q^2}{{\mu'}^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$
$$\times\exp\left\{-g_A(x_A,b_c(b_{\rm T});b_{\rm max})-g_B(z_B,b_c(b_{\rm T});b_{\rm max})-2g_K(b_c(b_{\rm T});b_{\rm max})\ln\left(\frac{Q}{Q_0}\right)\right\}$$

Boundary conditions

$$b_*(b_c(b_{\rm T}))\longrightarrow\begin{cases} b_{\rm min} & b_{\rm T}\ll b_{\rm min} \\ b_{\rm T} & b_{\rm min}\ll b_{\rm T}\ll b_{\rm max} \\ b_{\rm max} & b_{\rm T}\gg b_{\rm max}\,. \end{cases}$$

Review of Region Analysis "Approximators"
W, Y, FO, ASY <u>Definitions</u>

Original CSS definition of W is given by instruction to carryout an approximation of the *cross section* designed to be good in the region $q_T << Q$ up to powers of q_T/Q and m/Q

$$T_{TMD} d\sigma(q_T, Q) \approx d\sigma(q_T \ll Q, Q) + O\left(\frac{q_T}{Q}\right)^a d\sigma(q_T, Q) + O\left(\frac{m}{Q}\right)^{a'} d\sigma(q_T, Q)$$

$$W(q_T, Q) \equiv T_{TMD} d\sigma(q_T, Q)$$

Review of Region Analysis "Approximators"
W, Y, FO, ASY Definitions

Another approximator for the design "region" of $q_T \sim Q$ defines FO up to powers of m/q_T

$$T_{coll} d\sigma(q_T, Q) \approx d\sigma(q_T \gtrsim Q, Q) + O\left(\frac{m}{q_T}\right)^b d\sigma(q_T, Q)$$

$$FO(q_T, Q) \equiv T_{coll} d\sigma(q_T, Q)$$

Bacchetta's talk GHP 2017

Y term in Z boson production

Bozzi et al. arXiv:0812.2862