Jefferson Lab
Thomas Jefferson National Accelerator Facility

Pion electromagnetic form factor at high Q^2 from lattice QCD

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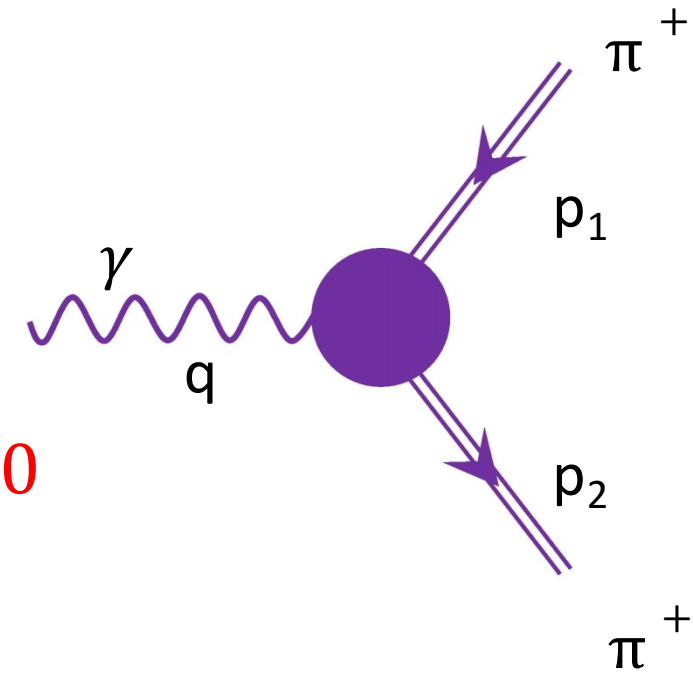
Definition

Simplest hadron

Space like "q":

$$q^2 = (p_2 - p_1)^2 \leq 0$$

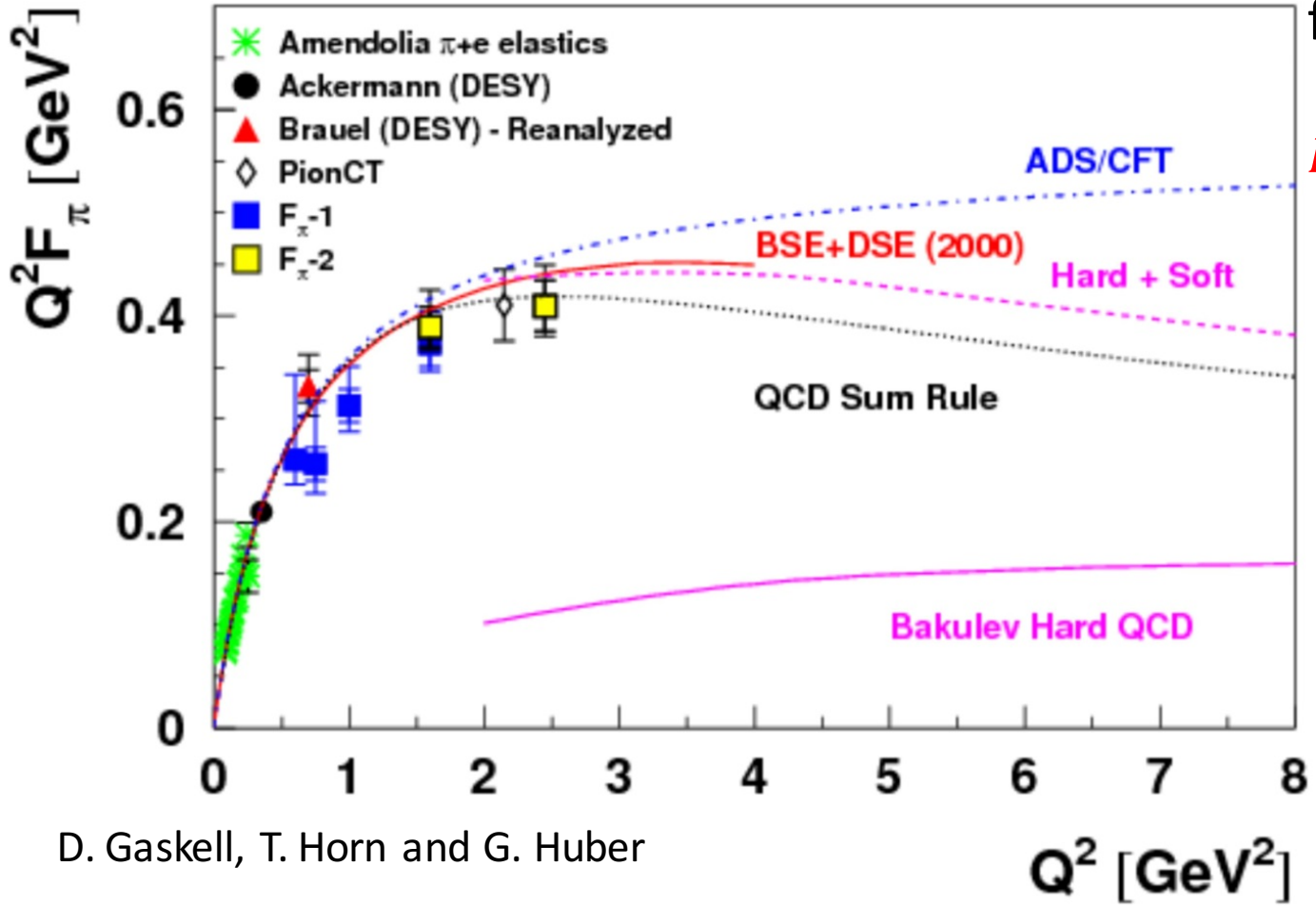
$$Q^2 = -q^2$$



$$\langle \pi^+(\vec{p}\,') | j^\mu | \pi^+(\vec{p}) \rangle = (p + p')^\mu \, F_\pi(Q^2)$$

(in units of 'e')

Interplay between hard and soft scales



D. Gaskell, T. Horn and G. Huber

Hard tail ($Q^2 \to \infty$) from pQCD:

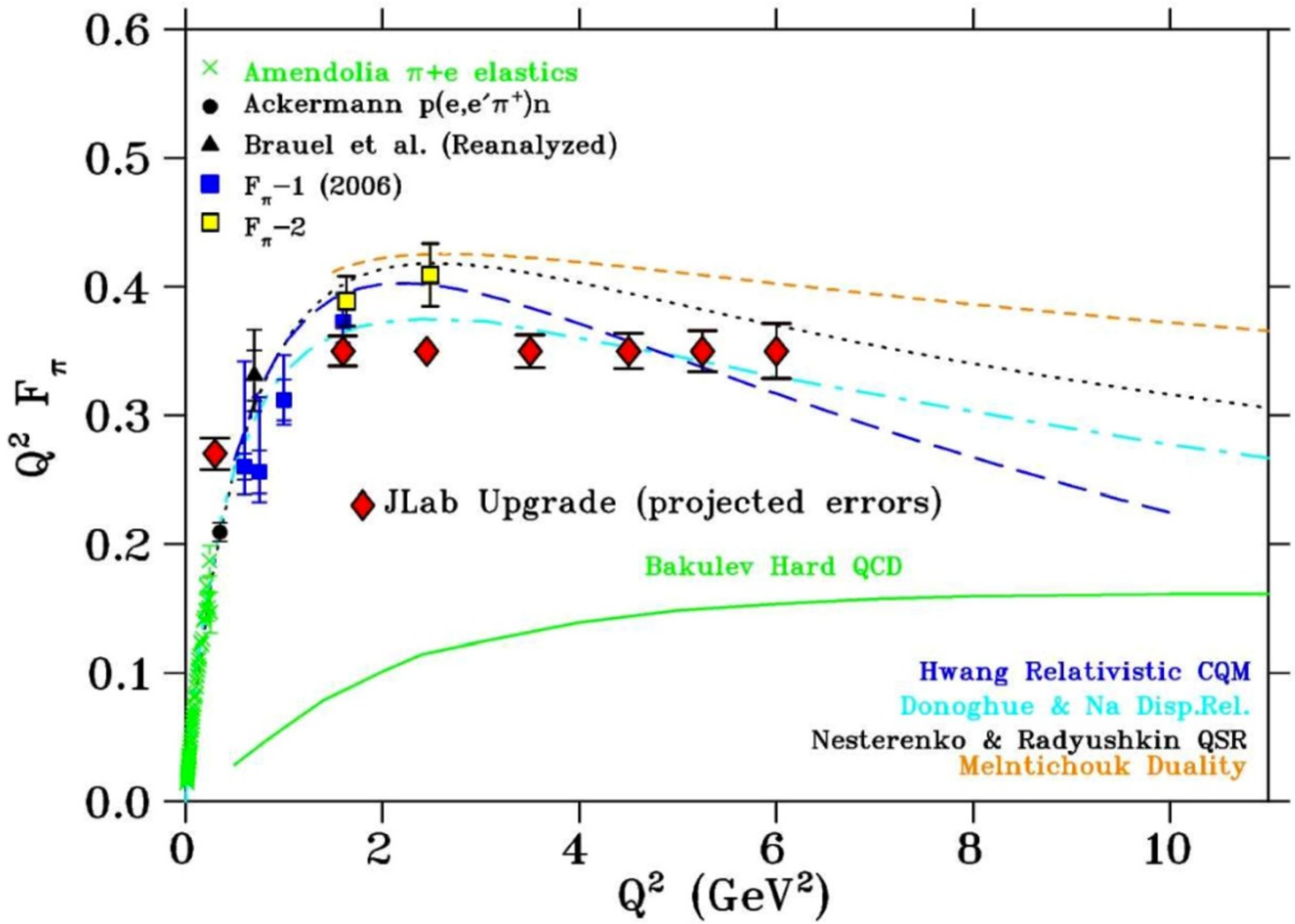
$$F_\pi(Q^2) \to \frac{16\pi\alpha_s(Q^2) f_\pi^{\,2}}{Q^2}$$

G. P. Lepage, S.J.Brodsky, Phys. Lett. 87B(1979)359

Soft part ($Q^2 < 1$ GeV²): vector meson dominance with $F_\pi(0) = 1$, data fits well

Need better understanding of the transition to the asymptotic region

JLAB 12 GeV upgrade



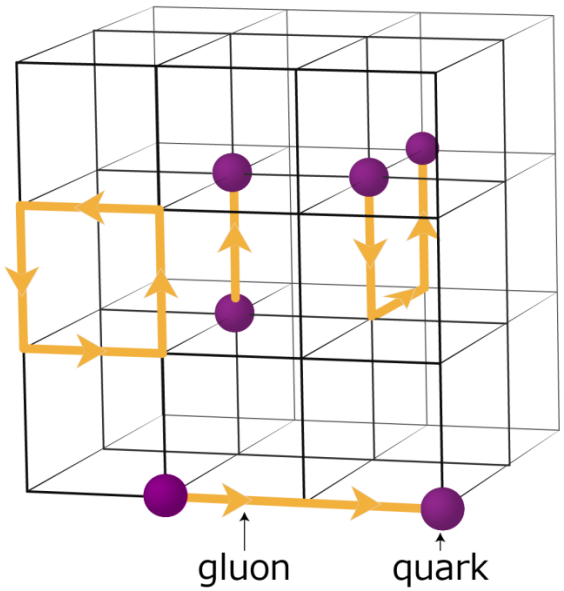
D. Gaskell, T. Horn and G. Huber

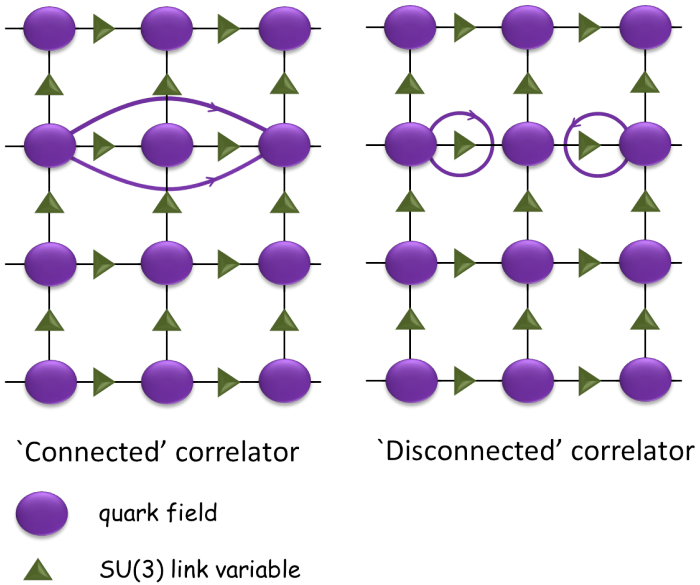
F_π measurements at $Q^2 \sim 8$ GeV^2: E12-06-101 at JLAB Hall C

Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?

Lattice recipe for meson correlators

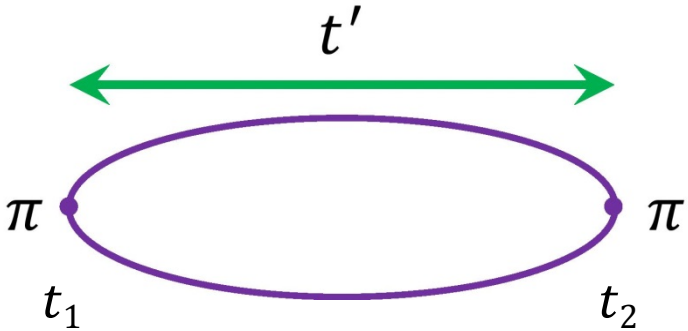
- Expectation values of observables : $\int DUD\psi D\bar{\psi} exp(-\int L_{QCD} d^4x)$
- 4-D space-time lattice
- Gauge configurations : gluons + sea quarks





- Discretise : $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma.\Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point, 3-point correlation functions : extract meson properties
- Corrections for lattice artefacts

Two-point correlator construction



$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi}\Gamma\overleftrightarrow{D}\cdots\overleftrightarrow{D}\psi$$

- Optimized operator for state $|n>$

$$\Omega_{\mathbf{n}}^\dagger = \sum_i w_i^{(\mathbf{n})}\mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

$$C(t)\,v^{(\mathbf{n})} = \lambda_{\mathbf{n}}(t)\,C(t_0)\,v^{(\mathbf{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-E_n(t-t_0)]$$

Two-point correlator construction

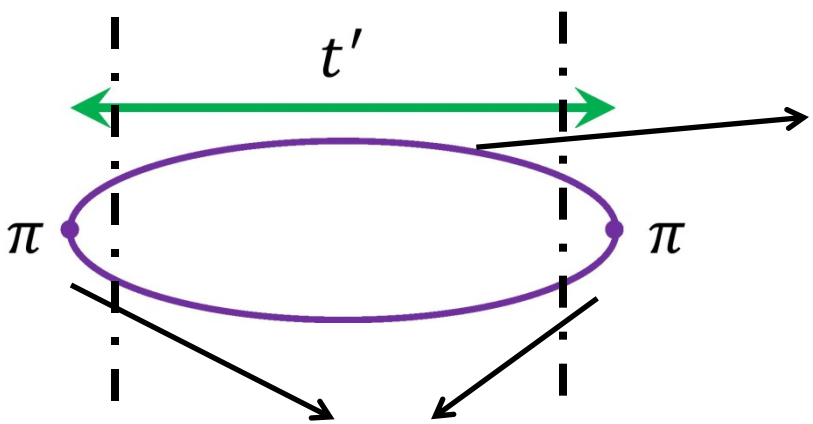
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi^{(n)}_{\vec{x}}(t)\, \xi^{(n)\dagger}_{\vec{y}}(t)$$

→ Low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}}\, \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



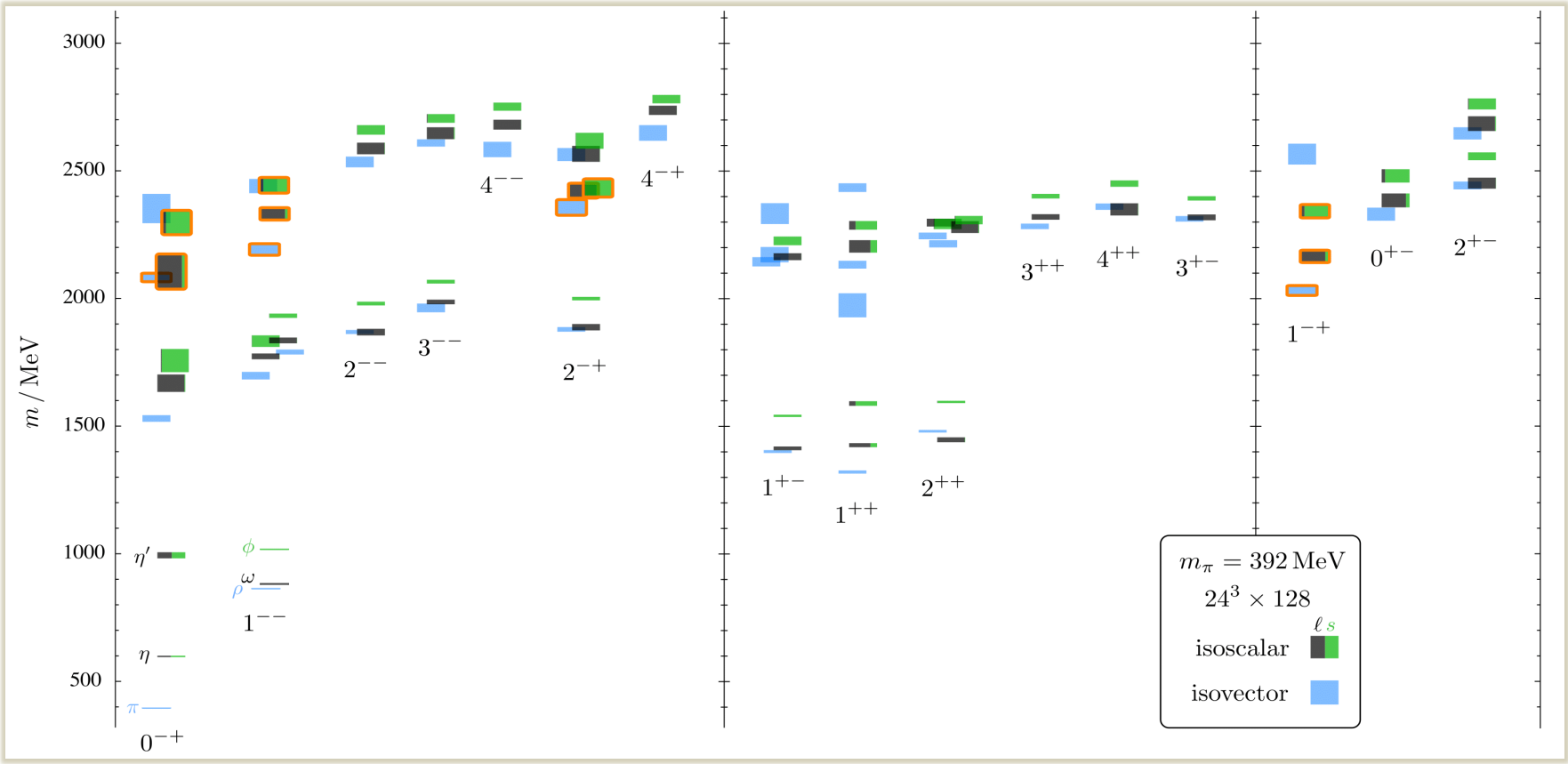
Parambulators by inverting the Dirac matrix

+

Operator construction with momentum projection

Meson Spectrum

Some tools well established for spectroscopy

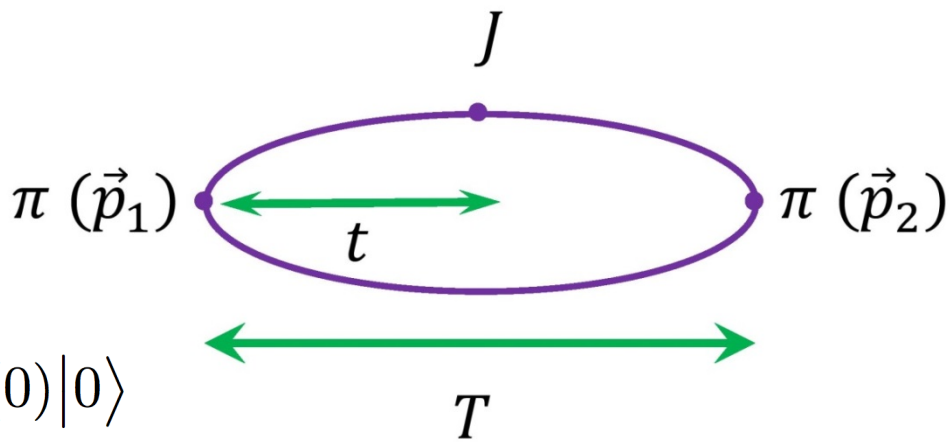


Hadron Spectrum Collaboration

Jozef J. Dudek *et. al*. Phys.Rev. D88 (2013)

Form factor calculation

Need three-point correlator



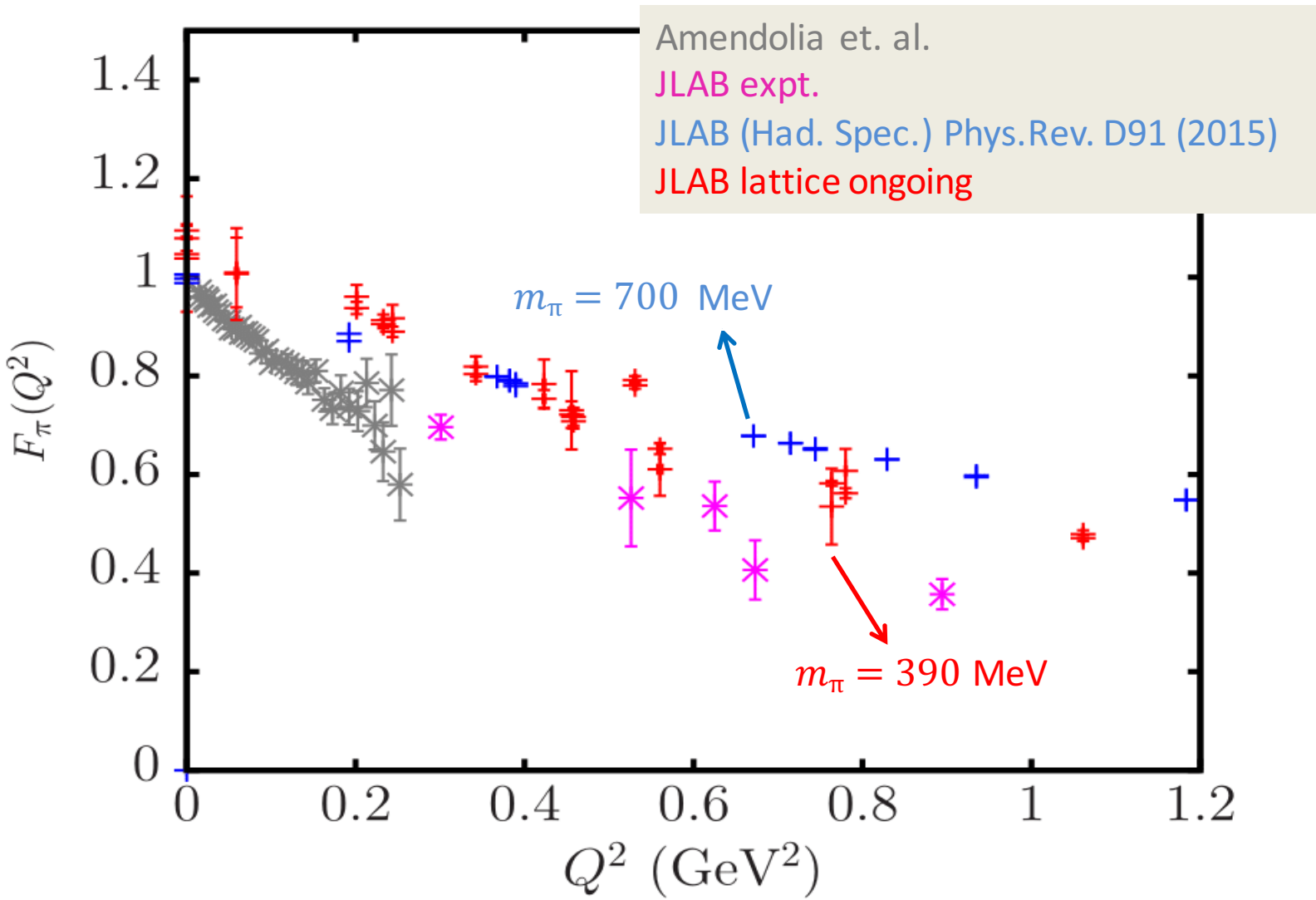
$$C_{\mathrm{f}\mu\mathrm{i}}(\Delta t, t) = \langle 0 | \mathcal{O}_{\mathrm{f}}(\Delta t)\, j_\mu(t)\, \mathcal{O}_{\mathrm{i}}^\dagger(0) | 0 \rangle$$

$$Z_V < \pi^+(p_2)|J^\mu{}_\pi(0)|\pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)$$

Clover discretised fermion action

Z_V calculated using $F_\pi(q^2 = 0) = 1$

Pion electromagnetic form factor: up to $Q^2 = 1$ GeV^2

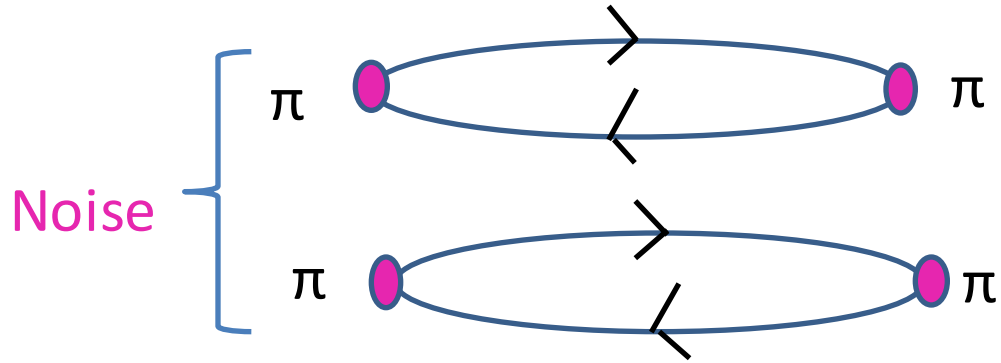


$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$ $\quad a_s = 0.12$ fm, $\quad \dfrac{a_s}{a_t} = 3.44$

Towards higher Q^2

More difficult on lattice for higher momenta

Signal-to-noise ratio:



2-point correlators :

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

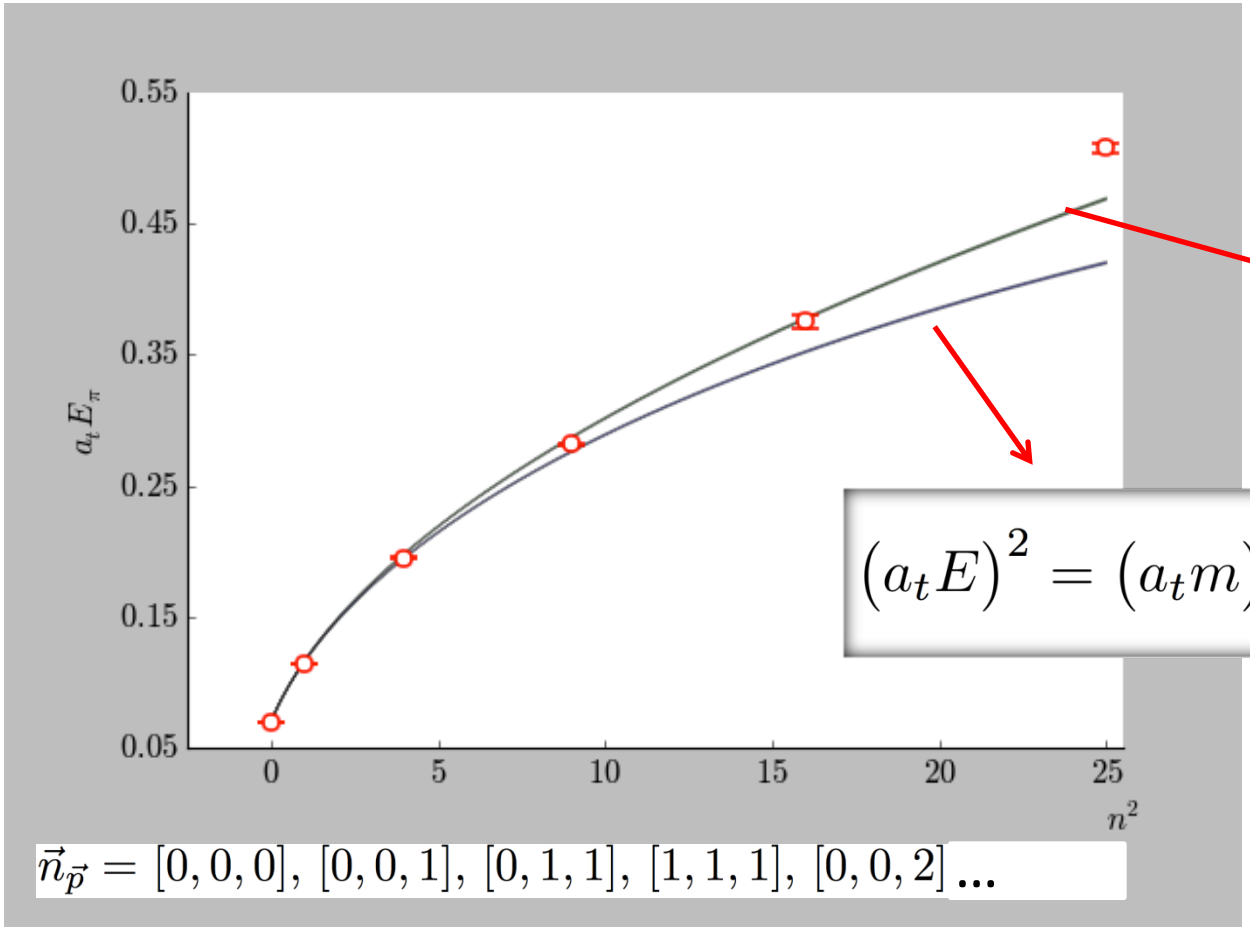
3-point correlators :

$$\exp[-(E_\pi(p_i) + E_\pi(p_f) - 2m_\pi)t/2]$$

in the middle of the plateau

Minimize energies
for a given Q^2
to get better signal

Towards higher Q^2



Dispersion relation:

$$E^2 = m^2 + p^2$$

$$(a_t E)^2 = (a_t m)^2 + \left(\frac{2\pi}{\xi(L/a_s)}\right)^2 |\vec{n}_{\vec{p}}|^2$$

Achieve maximum Q^2 by using Breit frame : $\overrightarrow{P_f} = -\overrightarrow{P_i}$

Current status of our calculation

- Signal for up to 4.5 GeV^2 – first time with these method and pion mass 390 MeV on N_f = 2+1 gauge configurations
- Building data analysis tools
- Needs more statistics
- Need smaller lattice spacing, larger volume

Outlook

Immediate goals:

- Pion form factor at $Q^2 \geq 6$ GeV^2
- Extend to more ensembles with lighter pion masses , multiple volumes, multiple lattice spacing

- Take care of lattice artefacts

Long term goals:

- Hadron structure program: pion – distribution amplitude, PFDs, Quasi PDFs

- Extend to kaons & nulceons – charges, moments

- Unstable vector meson form factor