Pion electromagnetic form factor at high $Q^2$ from lattice QCD

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Simplest hadron

Space like “q”:

\[ q^2 = (p_2 - p_1)^2 \leq 0 \]

\[ Q^2 = -q^2 \]

\[ \langle \pi^+ (p') | j^\mu | \pi^+ (p) \rangle = (p + p')^\mu F_\pi (Q^2) \]

(in units of ‘e’)

\[ 2 \]

\[ (\pi^+ (p') | j^\mu | \pi^+ (p)) \]
Interplay between hard and soft scales

Hard tail \( (Q^2 \to \infty) \) from pQCD:

\[
F_\pi(Q^2) \to \frac{16\alpha_s(Q^2)f_{\pi}^2}{Q^2}
\]

G. P. Lepage, S.J. Brodsky, Phys. Lett. 87B(1979)359

Soft part \( (Q^2 < 1 \text{ GeV}^2) \):
vector meson dominance with \( F_\pi(0) = 1 \), data fits well

Need better understanding of the transition to the asymptotic region
Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?

D. Gaskell, T. Horn and G. Huber

$F_\pi$ measurements at $Q^2 \sim 8 \text{ GeV}^2$: E12-06-101 at JLAB Hall C
Lattice recipe for meson correlators

- Expectation values of observables:
  \[ \int D U D \bar{\psi} \exp(-\int L_{QCD} d^4 x) \]
- 4-D space-time lattice
- Gauge configurations: gluons + sea quarks

- Discretise:
  \[ L_q \equiv \bar{\psi} (\gamma_\mu D^\mu + m) \psi \rightarrow \bar{\psi} (\gamma \cdot \Delta + m a) \psi \]
- Inversion of Dirac matrix: propagator
- 2-point, 3-point correlation functions: extract meson properties
- Corrections for lattice artefacts
Two-point correlator construction

\[ C_{ij}(t) = \langle 0 | O_i(t) O_j^\dagger(0) | 0 \rangle \]

- Basis of operators

\[ \mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi \]

- Optimized operator for state \(|n>\)

\[ \Omega_n^\dagger = \sum_i w_i^{(n)} O_i^\dagger \]

in a variational sense by solving generalized eigenvalue problem-

\[ C(t) \nu^{(n)} = \lambda_n(t) C(t_0) \nu^{(n)} \]

- Diagonalize the correlation matrix – eigenvalues

\[ \lambda_n(t) = \exp[-E_n(t - t_0)] \]
Correlator Construction: smearing of quark fields - ‘distillation’ with

\[
\Box \bar{x}y(t) = \sum_{n=1}^{N_D} \xi_{\bar{x}}^{(n)}(t) \xi_{y}^{(n)\dagger}(t)
\]

Low lying hadron states

Meson creation operator:

\[
\mathcal{O}^\dagger(p) = \bar{\psi}_\bar{x} \Box \bar{x}y e^{-i\vec{p}\cdot\vec{y}} \Gamma_{\bar{y}\bar{z}} \Box \bar{z}\bar{w} \psi_\bar{w}
\]

Paramambulators by inverting the Dirac matrix

\[
+ \quad \text{Operator construction with momentum projection}
\]
Meson Spectrum

Some tools well established for spectroscopy

Hadron Spectrum Collaboration

Form factor calculation

Need three-point correlator

\[ C_{f\mu i}(\Delta t, t) = \langle 0 | \mathcal{O}_f(\Delta t) j_\mu(t) \mathcal{O}_i^\dagger(0) | 0 \rangle \]

\[ Z_V < \pi^+ (p_2) | J^\mu_\pi(0) | \pi^+ (p_1) > = e (p_1 + p_2)^\mu F_\pi(q^2) \]

\( Z_V \) calculated using \( F_\pi(q^2 = 0) = 1 \)

Clover discretised fermion action
Pion electromagnetic form factor: up to $Q^2 = 1 \text{ GeV}^2$

$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$

$a_s = 0.12 \text{ fm}, \quad \frac{a_s}{a_t} = 3.44$

Amendolia et. al.
JLAB expt.
JLAB lattice ongoing
Towards higher $Q^2$

More difficult on lattice for higher momenta

Signal-to-noise ratio:

2-point correlators:

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

3-point correlators:

$$\exp[-(E_\pi(p_i) + E_\pi(p_f) - 2m_\pi)t/2]$$

Minimize energies for a given $Q^2$ to get better signal

in the middle of the plateau
Towards higher $Q^2$

Achieve maximum $Q^2$ by using Breit frame: $\vec{P}_f = -\vec{P}_i$

Dispersion relation:

$$E^2 = m^2 + p^2$$

$$\left(a_t E\right)^2 = \left(a_t m\right)^2 + \left(\frac{2\pi}{\bar{\xi}(L/a_s)}\right)^2 |\vec{n}_{\vec{p}}|^2$$

$\vec{n}_{\vec{p}} = [0, 0, 0], [0, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0, 2], \ldots$
Current status of our calculation

- Signal for up to 4.5 GeV\(^2\) – first time with these method and pion mass 390 MeV on \(N_f = 2+1\) gauge configurations

- Building data analysis tools

- Needs more statistics

- Need smaller lattice spacing, larger volume
Outlook

Immediate goals:

- Pion form factor at $Q^2 \geq 6 \text{ GeV}^2$
- Extend to more ensembles with lighter pion masses, multiple volumes, multiple lattice spacing
- Take care of lattice artefacts

Long term goals:

- Hadron structure program: pion – distribution amplitude, PFDs, Quasi PDFs
- Extend to kaons & nucleons – charges, moments
- Unstable vector meson form factor