Lipatov's effective action, color glass condensate and classical gluon field of relativistic color charge

Bondarenko Sergey

sergeyb@ariel.ac.il

Physics Department
Ariel University
Israel

In cooperation with L.Lipatov, work in progress.

Gluodynamic's Lagrangian with color field sources

- Action at some rapidity interval:

$$S_{eff} = -\int d^4 x \left(\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + v_- J^-(v_-) + v_+ J^+(v_+) \right),$$

with

$$\delta \left(v_\pm J^\pm(v_\pm) \right) = \delta v_\pm j^\pm(v_\pm)$$

and

$$D_\pm j^\pm_a(v_\pm) = 0 .$$

- Fixing current we request in LO (light-cone gauge): $v_+ = A_+$, $\partial_\mp A_\pm = 0$, with $A_\pm$ as some given (reggeon) fields. We can take $j^\pm$ as Lipatov's induced currents or we can fix the current's expression from the self-consistency condition of equations of motion (again obtaining Lipatov's induced current).

Fields ansatz

- Light-cone gauge

$$v_- = 0\,,\; v_\perp = \Lambda[g\,A_+,\,A_-] + g\,\Lambda_1[g\,A_+,\,A_-] + \cdots\,,\; v_+ = A_+ + g\Phi[g\,A_+,\,A_-] + \cdots\,,$$

- Complex, non-liner functionals of Reggeon fields in any perturbative order;
- No expansion of ordered exponentials is applied.

LO solution: first equation of motion

- the variation on $v_\perp^a$ field

$$2\left(D_+\,(\partial_- v_i)\,\right)_a - \left(\,D_i\,(\partial_- v_+)\,\right)_a + \left(D^j G_{ji}\right)_a = 0\,,$$

to LO (with $\partial_-\, v_+ = 0$):

$$\left(D_+\,(\partial_- v_i)\,\right)_a = 0$$

with $G_{ij} = 0$ in LO. The solution:

$$v_i^b = U^{bc}(v_+)\rho_{ci}\left(x^-, x_\perp\right)\,,\; D_+\,U^{bc}(v_+) = 0\,,$$

with still arbitrary form of $U^{bc}(v_+)$ and with some $\rho_{ci}\left(x^-, x_\perp\right)$ function (which is gluon density operator which determines a value of classical color charge in CGC approach).

LO solution: second equation of motion

- the variation on v_+^a field:

$$-\left(D_i\left(\partial_- v^i\right)\right)_a - \partial_-^2 v_{a+} = j_a^+(v_+),$$

that gives at LO:

$$-\partial_i \partial_- v_a^i = j_a^+ .$$

Self-consistency condition determines the form of the current as:

$$j_a^+ = -U^{ab}(v_+)\,\tilde{J}_b^+\left(x^-, x_\perp\right)$$

where

$$\partial_i \partial_- \rho_a^i = \tilde{J}_a^+\left(x^-, x_\perp\right),$$

which is CGC result.

- The form of $U^{ab}(v_+)$ still is not defined.

LO solution: third equation of motion

- The variation on v_-^a field:

$$\partial_\mu G_a^{\mu -} + g f_{abc} v_\mu^b G^{c\,\mu -} = j_a^-(v_- = 0)\,,$$

provides to LO

$$\partial_i \partial^i v_{a+} = j_a^-(v_- = 0) = \partial_i \partial^i A_{a+}$$

that gives

$$j_a^-(v_- = 0) = \partial_i \partial^i A_{a+}\,.$$

- The same can be achieved in any gauge in the limit $g \to 0$.

Lipatov's induced current

- Current's properties are

$$\delta\left(v_{\pm}\,J^{\pm}(v_{\pm})\right)\,=\,\delta\,v^{a}_{\pm}\,j^{\pm}_{a}(v_{\pm})\,,$$

$$j^{+}_{a}\,(v_{+})\,=\,-\,U^{ab}\,(v_{+})\,\partial_{i}\,\partial_{-}\,\rho^{i}_{b}\,,$$

$$D_{+}\,U^{ab}(v_{+})\,=\,0\,,$$

$$j^{-}_{a}\,(v_{-}\,=\,0\,)\,=\,\partial_{i}\,\partial^{i}\,A_{a\,+}\,.$$

- The same properties as of Lipatov's induced current;
- The same properties as classical current in equatons of motion in the CGC approach.

Current, Reggeon fields and CGC gluon density operator

- Induced current from Lipatov's effective action:

$$J^{\pm}(v_{\pm}) \,=\, O(v_{\pm})\, j^{\pm}_{reg}\,, \quad j^{\pm}_{reg\,a} \,=\, \frac{1}{N}\,\partial_i^2\, A^{\pm}\,,$$

with O as some operator which satisfies $D_+\, O \,=\, 0$ (will be discussed later). Induced current:

$$j^{+}_{ind}(v_+) \,=\, j^{+}(v_+) \,=\, \frac{1}{N}\, O(v_+)\, \left(\partial_i^2\, A^+\right)\, O^T(v_+)\,.$$

For the case of adjoint representation with f_a as $f_{a\,b\,c}$:

$$\partial_i\, \partial_-\, \rho^i_a \,=\, \frac{1}{N}\, \partial^2_{\perp}\, A^+_a\,, \quad U^{a\,b} \,=\, -\,tr\,[\,f_a\, O\, f_b\, O^T\,]\,.$$

The connection between CGC and Lipatov's effective action:

$$\rho^i_a \,=\, -\,\frac{1}{N}\,\partial_-^{-1}\, \left(\partial^i\, A^a_-\right)\,, \quad O \,=\, W_{-\infty,\, x^+} \,=\, P\, e^{g\int_{-\infty}^{x^+} dx'^+\, A^a_+\left(x'^+,\, x^-,\, x_\perp\right)\, f_{abc}}\,.$$

LO solution

- Lipatov's effective action LO solution:

$$v_{-}^{a} = 0\,,\;\; v_{+}^{a} = A_{+},\;\; v_{i}^{a} = -\frac{1}{N}U^{ab}\left(A_{+}\right)\left(\partial_{-}^{-1}\left(\partial_{i}\,A_{-}^{b}\right)\right)\,.$$

- CGC fields are reproduced in the limit $A_{+} \to 0$:

$$v_{-}^{a} = 0\,,\;\; v_{+}^{a} = 0,\;\; v_{i}^{a} = \rho_{i}^{a}\left(x^{-}, x_{\perp}\right)\,.$$

- The second Reggeon field A_{+} is present in the CGC approach as semi-hard fluctuation and further it is integrated out in calculations of the loop corrections without changing classical field solution of equations motion.

NLO classical solution

- The classical fields is not zero on any perturbativ order.
- For v_{+}^{a} field we have:

$$v_{+}^{a} = A_{+}^{a} + g\, v_{+1}^{a}(x_{\perp}, x^{-}, x^{+})\,,$$

$$v_{+1}^{a} = -\frac{2}{g}\,\Box^{-1}\left(\left(\partial_{+}\partial^{i}U^{ab}\right)\,\rho_{i}^{b}\right)\,.$$

For v_{i}^{a} field we have:

$$v_{i}^{a} = v_{i\,0}^{a} + g\, v_{i\,1}^{a}(x_{\perp}, x^{-}, x^{+})\,,$$

$$v_{i\,1}^{a}(x_{\perp}, x^{-}, x^{+}) = -\Box^{-1}\left(\partial^{j}\,F_{j\,i}^{a} + \frac{1}{g}\,\partial_{i}\left(\left(\partial^{j}U^{ab}\right)\rho_{j}^{b}\right) + \partial_{i}\partial_{-}^{-1}\,j_{a\,1}^{+}\right)\,,$$

$$g\,j_{a\,1}^{+} = g\,f_{abc}\left(U^{b\,b'}\,\rho^{ib'}\right)\left(U^{c\,c'}\left(\partial_{-}\rho_{i}^{c'}\right)\right)\,.$$

- This is correct result for the NLO CGC classical field as well.

$O(v_+)$ operators representation

- Different integral representation of $\frac{1}{\partial_+}$ integral operator will lead to different representations of O operator. In the simplest case:

$$\frac{1}{\partial_+} f(x^+) = \int_{-\infty}^{x^+} f(y^+)\, dy^+$$

we reproduce:

$$O_x = P\, e^{g \int_{-\infty}^{x^+} dx'^+ \, v_+(x'^+)} \, , \quad O_x^T = P\, e^{g \int_{x^+}^{\infty} dx'^+ \, v_+(x'^+)} \, .$$

- The induced current derivation:

$$\partial_{+x}\, \delta\, O_x = g\, O_x^T\, \delta v_{+x}\, O_x \, ,$$

regardless to particular form of $O(v_+)$ and $O^T(v_+)$ operators.

LO structure of effective action

- General representation of the action:

$$S_{eff} = -\int d^4 x \, (s_1[g, A_+, A_-] + g \, s_2[g, A_+, A_-] + \cdots) \, .$$

For LO classical solution in light-cone gauge:

$$S_{eff} = - \int d^4x \left(\left(\partial_i A_+^a\right) U^{a\,b}(A_+) \, (\partial_- \rho_{i\,b}(x_\perp)) - A_+^a \left(U^{a\,b}(A_+) - N \, \delta^{a\,b} \right) \cdot \cdot \left(\partial_i \partial_- \, \rho_a^i \right) \right) ,$$

or

$$S_{eff} = -\frac{1}{N} \int d^4x \left(\left(\partial_i A_+^a\right) U(A_+)^{a\,b} \left(\partial^i \, A_-^b\right) - A_+^a \left(U^{a\,b}(A_+) - N \, \delta^{a\,b} \right) \left(\partial_\perp^2 \, A_-^b \right) \right) ,$$

for any form of O operator.

Conclusion:

- Lipatov's effective action can be considered as directly derivable from the gluodynamics Lagrangian with external color sources of longitudinal fields added, where the form of the currents is determined by general requests and constraints on the LO form of the longitudinal fields;
- LO CGC classical fields can be obtained from the classical solution of Lipatov's effective action equations of motion taking $A_+ \to 0$ limit, there is a simple "translation" between the results in this order;
- In general the forms of the Lagrangians int two approaches are different. The possible consequence is that the LO currents are the same only for the special choice of O operator as Wilson line, whereas it is not clear if the form of CGC Lagrangian is invariant to the form of O operator, the Lipatov's effective action does;
- Lipatov's effective action classical field solution is a non-linear functional of $A_+\,,\;A_-$ Reggeon fields in any perturbative order, whereas in CGC approach the higher orders of the perturbative classical solution are not accounted (as far as I know);
- As a consequence, the different forms of classical solutions will result in the loop calculations (RFT construction).