GENERALIZED TMDS IN THE EXCLUSIVE DOUBLE DRELL-YAN PROCESS

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OUTLINE

- **Background and Generalized TMDs (GTMDs)**
- **Quark GTMDs in the Exclusive Double Drell-Yan Process**

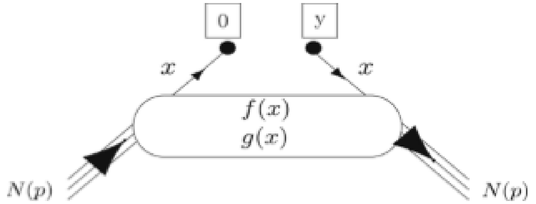
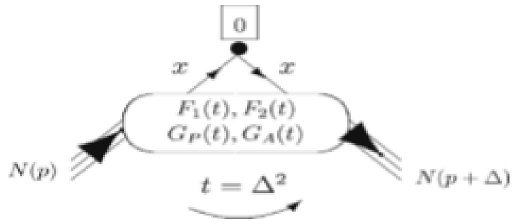
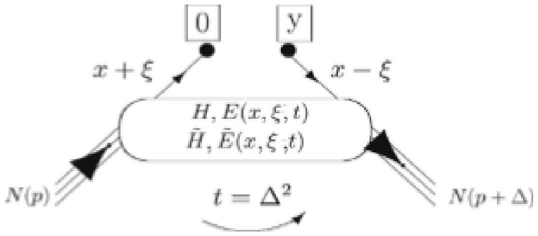
(S. Bhattacharya, A. Metz, J. Zhou / arXiv: 1702.04387v1)

1. **Define the Process**
2. **Leading-Order Diagrams**
3. **Kinematics**
4. **Scattering Amplitude**
5. **Quark GTMDs**
6. **Polarization Observables**

- **Summary**

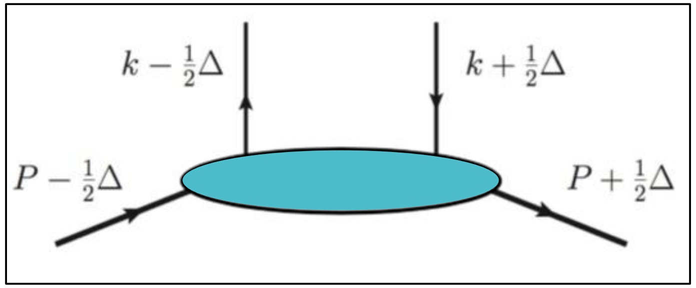
Background

- Still to resolve completely (to address few):
  1. How are quarks distributed spatially and in the momentum space inside the nucleon?
  2. How do they contribute to the properties of nucleon- spin?
- Scattering experiments: Inclusive, Exclusive, Semi-Inclusive
- QCD factorization: Hard part (point-like, perturbatively calculable), Soft part (non-perturbative QCD)
- Investigation of QCD matrix elements:

QCD MATRIX ELEMENT (Diagram)	FEATURES OF THE ELEMENT	PROCESS	NON-PERT. FUNCTIONS
	$\langle p \lvert \bar{\psi}_q(0)\mathcal{O}\psi_q(y) \rvert p \rangle$ • Non-local • Forward	DIS	PDFs $f_1(x)$ $g_1(x)$
	$\langle p' \lvert \bar{\psi}_q(0)\mathcal{O}\psi_q(0) \rvert p \rangle$ • Local • Off-forward	ELASTIC SCATTERING	FFs $F_1(t)$ $F_2(t)$ $G_P(t)$ $G_A(t)$
	$\langle p' \lvert \bar{\psi}_q(0)\mathcal{O}\psi_q(y) \rvert p \rangle$ • Non-local • Off-forward	EXCLUSIVE SCATTERING	GPDs $H(x, \xi, t)$ $\tilde{H}(x, \xi, t)$ $E(x, \xi, t)$ $\tilde{E}(x, \xi, t)$

Generalized Transverse Momentum Dependent distributions

- **GTMD matrix element graphical description:**



$$P = \frac{p+p'}{2} \qquad \Delta = p' - p \qquad t = \Delta^2$$

- **GTMD matrix element definition:** $\mathbf{W^{q[\Gamma]}_{\lambda,\lambda'}(P, \Delta, x, \vec{k}_t)}$

$$= \int \frac{dz^- d^2\vec{z}_t}{2(2\pi)^3} e^{ik.z} \left\langle p',\lambda' \middle| \bar{q}\left(-\frac{z}{2}\right) \Gamma\, q\left(\frac{z}{2}\right) \middle| p,\lambda \right\rangle |_{z^+=0}$$

- **Matrix elements parameterized through GTMDs:** $\mathbf{X^q(x, \xi, \vec{k}_t, \vec{\Delta}_t)}$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad \vec{\Delta}_t = \vec{p}'_t - \vec{p}_t$$

GTMDs as 'mother distributions'

GTMD $(x, \vec{k}_t, \Delta)$

$\int d^2\vec{k}_t$ → **GPD (x, Δ)**; $\Delta = 0$ → **TMD $(x, \vec{k}_t)$**

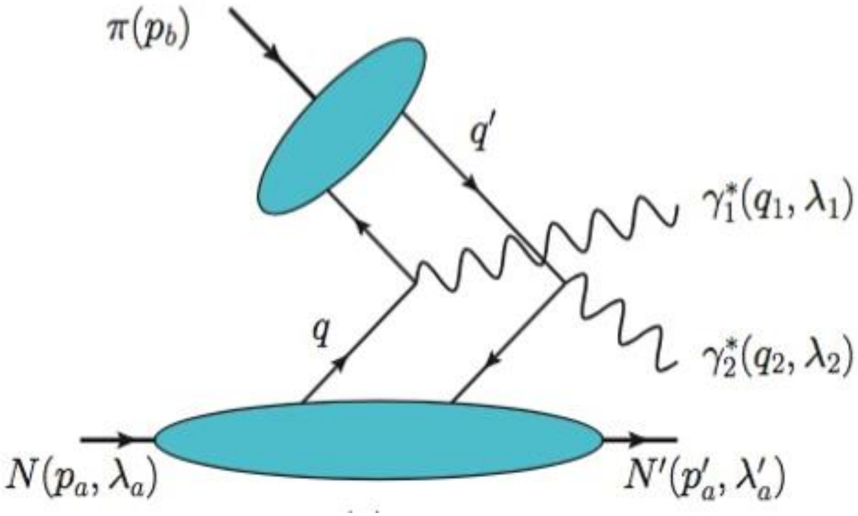
GPD (x, Δ): $\int x$ → **FFs (Δ)**; $\Delta = 0$ → **PDFs (x)**

In particular, $F_{1,4}$ and $G_{1,1}$ have a direct role in understanding spin-structure of hadrons
(Lorcé, Pasquini, 2011 / Hatta, 2011 / Lorcé, 2014).
But, how would we measure them?

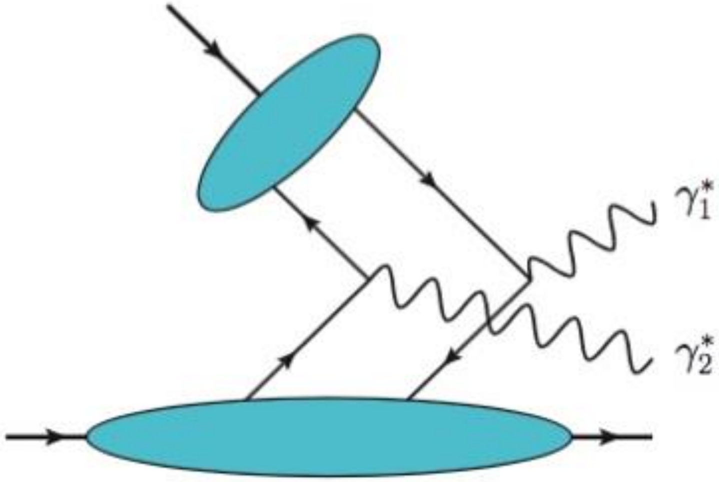
Exclusive Double Drell-Yan Process

(SB, Metz, Zhou / arXiv: 1702.04387v1)

1. **Process**: $\Pi(p_b)N(p_a,\lambda_a) \to (l_1^+ l_1^-)(l_2^+ l_2^-)N'(p'_a,\lambda'_a)$
2. **Leading-order diagrams**:



Graph (a)



Graph (b)

3. **Kinematics**:

Region: TMD type

$$s = (p_a + p_b)^2 = \text{large}$$

$$q_1^2, q_2^2 = \text{large}$$

$$q_i^2 \gg |\vec{q}_{it}|^2$$

Kinematical variables:

$$x_a = \frac{k_a^+}{P_a^+} \qquad \Delta_a = p'_a - p_a$$

$$x_b = \frac{k_b^-}{p_b^-} \qquad \xi_a = -\frac{\Delta_a^+}{2P_a^+} = \frac{q_1^+ + q_2^+}{2P_a^+}$$

4. Scattering Amplitude:

$$T^{\lambda_1\lambda_2}_{\lambda_a\lambda_a'} = T^{\mu\nu}_{\lambda_a\lambda_a'}\,\varepsilon^*_\mu(\lambda_1)\varepsilon^*_\nu(\lambda_2)$$

$$T^{\mu\nu}_{\lambda_a\lambda_a'} = i\sum_{qq'} e_q\, e_q' e^2 \frac{1}{N_c}\int d^2\vec{k}_{at}\int d^2\vec{k}_{bt}\,\delta^{(2)}\left(\frac{\Delta\vec{q}_t}{2}-\vec{k}_{at}-\vec{k}_{bt}\right)\phi_\pi^{q'q}(x_b,\vec{k}^2_{bt})$$
$$\left[-i\varepsilon_\perp^{\mu\nu}\left(W^{qq'[\gamma^+]}_{\lambda_a\lambda_{a'}}(x_a,\vec{k}_{at})-W^{qq'[\gamma^+]}_{\lambda_a\lambda_{a'}}(-x_a,-\vec{k}_{at})\right)-g_\perp^{\mu\nu}\left(W^{qq'[\gamma^+\gamma^5]}_{\lambda_a\lambda_{a'}}(x_a,\vec{k}_{at})+W^{qq'[\gamma^+\gamma^5]}_{\lambda_a\lambda_{a'}}(-x_a,-\vec{k}_{at})\right)\right]$$

Graph (a)

Longitudinal parton momenta fixed as:

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+} \qquad -\xi_a \le x_a \le \xi_a \quad \text{ERBL region}$$

$$x_b = 1 - \frac{q_1^-}{p_b^-} = \frac{q_2^-}{p_b^-}$$

- $\Delta\vec{q}_t = \vec{q}_{1t} - \vec{q}_{2t}$
- $\vec{\Delta}_{at} = -(\vec{q}_{1t} + \vec{q}_{2t})$

Graph (b)

Longitudinal parton momenta fixed as:

$$x_a = -\frac{(q_1^+ - q_2^+)}{2P_a^+} \qquad x_a(\text{Graph (b)}) = -x_a(\text{Graph (a)})$$

$$x_b = \frac{q_1^-}{p_b^-} \qquad x_b(\text{Graph (b)}) = 1 - x_b(\text{Graph (a)})$$

Symmetry considerations:

- $\phi_\pi^{q'q}(x_b,\vec{k}_{bt}) = \phi_\pi^{q'q}(x_b,-\vec{k}_{bt})$
- $\phi_\pi^{q'q}(x_b,\vec{k}^2_{bt}) = \phi_\pi^{q'q}(1-x_b,\vec{k}^2_{bt})$

5. **<u>Quark GTMDs</u>** (Meissner, Metz, Schlegel, 2009):

- $$\mathbf{W}^{q[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\left[F^q_{1,1} + \frac{i\sigma^{i+}k^i_t}{P^+}F^q_{1,2} + \frac{i\sigma^{i+}\Delta^i_t}{P^+}F^q_{1,3} + \frac{i\sigma^{ij}k^i_t\Delta^j_t}{M^2}F^q_{1,4}\right]u(p,\lambda)$$

$$= \frac{1}{M\sqrt{1-\xi^2}}\left\{\left[M\delta_{\lambda,\lambda'} - \frac{1}{2}(\lambda\Delta^1_t + i\Delta^2_t)\delta_{\lambda,-\lambda'}\right]F^q_{1,1}\right.$$
$$\left. + \frac{i\varepsilon^{ij}_t k^i_t \Delta^j_t}{M^2}\left[\lambda M\delta_{\lambda,\lambda'} - \frac{\xi}{2}(\Delta^1_t + i\lambda\Delta^2_t)\delta_{\lambda,-\lambda'}\right]F^q_{1,4} + \text{more helicity} - \text{flip terms}\right\}$$

- $$\mathbf{W}^{q[\gamma^+\gamma^5]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\left[-\frac{i\varepsilon^{ij}_t k^i_t\Delta^j_t}{M^2}G^q_{1,1} + \frac{i\sigma^{i+}\gamma^5 k^i_t}{P^+}G^q_{1,2} + \frac{i\sigma^{i+}\gamma^5\Delta^i_t}{P^+}G^q_{1,3} + i\sigma^{+-}\gamma^5 G^q_{1,4}\right]u(p,\lambda)$$

$$= \frac{1}{M\sqrt{1-\xi^2}}\left\{\left[-\frac{i\varepsilon^{ij}_t k^i_t\Delta^j_t}{M^2}\left(M\delta_{\lambda,\lambda'} - \frac{1}{2}(\lambda\Delta^1_t + i\Delta^2_t)\delta_{\lambda,-\lambda'}\right)\right]G^q_{1,1}\right.$$
$$\left. + \left[\lambda M\delta_{\lambda,\lambda'} - \frac{\xi}{2}(\Delta^1_t + i\lambda\Delta^2_t)\delta_{\lambda,-\lambda'}\right]G^q_{1,4} + \text{more helicity} - \text{flip terms}\right\}$$

JUST A MOMENT! **What do we have?**

- ✓ Scatt. Amp.($T_{\lambda,\lambda'}$)
- ✓ $\sigma \sim |T|^2$

What do we want?

$\sim F_{1,4}^2$ $\sim G_{1,1}^2$ $\sim F_{1,4}F_{1,1}$ $\sim G_{1,1}G_{1,4}$

Key point is to realize that if we ~

- **Define 'Polarization Observables' in a certain fashion**
- **Take proper linear combination of those observables**

'Single out' what you want!

6. Polarization Observables:

- **Unpolarized:** $\tau_{UU} = \frac{1}{2}\sum_{\lambda,\lambda'} |T_{\lambda,\lambda'}|^2$
- **SSA:** $\tau_{LU} = \frac{1}{2}\sum_{\lambda,\lambda'} (|T_{+,\lambda'}|^2 - |T_{-,\lambda'}|^2)$ Similarly, define τ_{XU}, τ_{YU} etc.
- **DSA:** $\tau_{LL} = \frac{1}{2}((|T_{+,+}|^2 - |T_{+,-}|^2) - (|T_{-,+}|^2 - |T_{-,-}|^2))$ Similarly, define τ_{XX}, τ_{YY} τ_{XY}, τ_{YX}

Remarks:

- Summation over photon polarization (λ_1, λ_2) is implied.
- If photon polarization (λ_1, λ_2) is summed over, there is no interference between the objects $W_{\lambda,\lambda'}^{q[\gamma^+]}$ and $W_{\lambda,\lambda'}^{q[\gamma^+\gamma^5]}$.

- **Choice 1~**

$$\frac{1}{4}(\tau_{UU}+\tau_{LL}-\tau_{XX}-\tau_{YY}) = \frac{2}{M^4}(\varepsilon_t^{ij}\Delta q_t^i \Delta_{at}^j)^2 C^{(+)}[\vec{\beta}_t.\vec{k}_{at}F_{1,4}\phi_\pi]C^{(+)}[\vec{\beta}_t.\vec{k}_{at}F_{1,4}^*\phi_\pi^*] + 2C^{(+)}[G_{1,4}\phi_\pi]C^{(+)}[G_{1,4}^*\phi_\pi^*]$$

$$C^{\pm}[w(\vec{k}_{at},\vec{k}_{bt})X\phi_\pi] = \frac{e^2}{\sqrt{1-\xi_a^2}}\frac{1}{N_c}\sum_{q,q'} e_q e_q' \int d^2\vec{k}_{at}\int d^2\vec{k}_{bt}\delta^{(2)}\left(\frac{\Delta\vec{q}_t}{2}-\vec{k}_{at}-\vec{k}_{bt}\right)w(\vec{k}_{at},\vec{k}_{bt})[X^{qq'}(x_a,\vec{k}_{at}) \pm X^{qq'}(-x_a,-\vec{k}_{at})]\phi_\pi^{q'q}(x_b,\vec{k}_{bt}^2)$$

$$\vec{\beta}_t = \frac{\vec{\Delta}_{at}^2\Delta\vec{q}_t - (\vec{\Delta}_{at}.\Delta\vec{q}_t)\vec{\Delta}_{at}}{\vec{\Delta}_{at}^2\Delta\vec{q}_t^2 - (\vec{\Delta}_{at}.\Delta\vec{q}_t)^2}$$

Recall: $\Delta\vec{q}_t = \vec{q}_{1t} - \vec{q}_{2t}$ & $\vec{\Delta}_{at} = -(\vec{q}_{1t} + \vec{q}_{2t})$

$\therefore\ \varepsilon_t^{ij}\Delta q_t^i \Delta_{at}^j$ **implies that the contribution from the first term is maximum when the photons come out perpendicular to each other.**

Remarks:

- If you consider specific polarization states of photons, you have the option to switch off the contribution from $G_{1,4}$.
- For extracting $G_{1,1}$, the relevant observable is $\frac{1}{4}(\tau_{UU}+\tau_{LL}+\tau_{XX}+\tau_{YY})$.

  The replacements in the above expression would be $F_{1,4} \rightarrow G_{1,1}$, $G_{1,4} \rightarrow F_{1,1}$, $C^{(+)} \rightarrow C^{(-)}$.
- The observables may be challenging.

- **Choice 2~**

$$\frac{1}{2}(\boldsymbol{\tau}_{LU}+\boldsymbol{\tau}_{UL}) = \frac{4}{M^2}\varepsilon_t^{ij}\Delta q_t^i \Delta_{at}^j \ \mathbf{Im}\left\{\mathbf{C}^{(-)}[\mathbf{F}_{1,1}\phi_\pi]\mathbf{C}^{(+)}[\vec{\beta}_t.\vec{k}_{at}\mathbf{F}_{1,4}^*\phi_\pi^*] - 4\mathbf{C}^{(+)}[\mathbf{G}_{1,4}\phi_\pi]\mathbf{C}^{(-)}[\vec{\beta}_t.\vec{k}_{at}\mathbf{G}_{1,1}^*\phi_\pi^*]\right\}$$

Remark:
This observable is sensitive to **Im** $\mathbf{F}_{1,4}$ and **Im** $\mathbf{G}_{1,1}$.

- **Choice 3~**

$$\frac{1}{2}(\boldsymbol{\tau}_{XY}-\boldsymbol{\tau}_{YX}) = \frac{4}{M^2}\varepsilon_t^{ij}\Delta q_t^i \Delta_{at}^j \ \mathbf{Re}\left\{\mathbf{C}^{(-)}[\mathbf{F}_{1,1}\phi_\pi]\mathbf{C}^{(+)}[\vec{\beta}_t.\vec{k}_{at}\mathbf{F}_{1,4}^*\phi_\pi^*] - 4\mathbf{C}^{(+)}[\mathbf{G}_{1,4}\phi_\pi]\mathbf{C}^{(-)}[\vec{\beta}_t.\vec{k}_{at}\mathbf{G}_{1,1}^*\phi_\pi^*]\right\}$$

Remark:
This observable is sensitive to **Re** $\mathbf{F}_{1,4}$ and **Re** $\mathbf{G}_{1,1}$ (sensitive to **O**rbital **A**ngular **M**omentum, spin-orbit correlation).

Summary

- Generalized Transverse Momentum Dependent distributions are interesting-
  - they are 'mother' distributions
  - their direct link to the orbital angular momentum (**$F_{1,4}$** and **$G_{1,1}$**)
- **Proposal:** A process that is sensitive to relevant quark **GTMDs** is the **Exclusive Double Drell-Yan process**

  1) Quark **GTMDs** can be accessed in the ERBL region

  2) **GTMDs** can be accessed through **P**olarization **O**bservables
- **What else can be done?**

  Calculation can be extended to processes like $\boldsymbol{pp \rightarrow \eta_c\, \eta_c\, pp}$ (work in progress) etc.