GENERALIZED TMDS
IN THE EXCLUSIVE DOUBLE DRELL-YAN PROCESS

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OUTLINE

➢ Background and Generalized TMDs (GTMDs)
➢ Quark GTMDs in the Exclusive Double Drell-Yan Process
  (S. Bhattacharya, A. Metz, J. Zhou / arXiv: 1702.04387v1)

1. Define the Process
2. Leading-Order Diagrams
3. Kinematics
4. Scattering Amplitude
5. Quark GTMDs
6. Polarization Observables

➢ Summary
Still to resolve completely (to address few):
1. How are quarks distributed spatially and in the momentum space inside the nucleon?
2. How do they contribute to the properties of nucleon- spin?

Scattering experiments: Inclusive, Exclusive, Semi-Inclusive

QCD factorization: Hard part (point-like, perturbatively calculable), Soft part (non-perturbative QCD)

Investigation of QCD matrix elements:

<table>
<thead>
<tr>
<th>QCD MATRIX ELEMENT (Diagram)</th>
<th>FEATURES OF THE ELEMENT</th>
<th>PROCESS</th>
<th>NON-PERT. FUNCTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>( \langle p</td>
<td>\bar{\psi}_q(0) \mathcal{O} \psi_q(y)</td>
<td>p \rangle )</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>( \langle p'</td>
<td>\bar{\psi}_q(0) \mathcal{O} \psi_q(0)</td>
<td>p \rangle )</td>
</tr>
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<td><img src="image3.png" alt="Diagram" /></td>
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<td>\bar{\psi}_q(0) \mathcal{O} \psi_q(y)</td>
<td>p \rangle )</td>
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Generalized Transverse Momentum Dependent distributions

➢ GTMD matrix element graphical description:

\[ p = \frac{p + p'}{2}, \quad \Delta = p' - p, \quad t = \Delta^2 \]

➢ GTMD matrix element definition: \( W_{\lambda, \lambda'}^{q\left[\alpha\right]}(P, \Delta, x, \vec{k}_t) \)

\[
= \int \frac{dz^- d^2 \vec{z}_t}{2(2\pi)^3} e^{ikz} \left( p', \lambda' \left| q' \left( -\frac{z}{2} \right) \Gamma q \left( \frac{z}{2} \right) \right| p, \lambda \right) \bigg|_{z^+ = 0}
\]

➢ Matrix elements parameterized through GTMDs: \( X^q(x, \xi, \vec{k}_t, \Delta_t) \)

\[
x = \frac{k^+}{p^+}, \quad \xi = \frac{p^+ - p'}{p^+ + p'} = -\frac{\Delta^+}{2p^+}, \quad \Delta_t = p_t' - p_t
\]

GTMDs as ‘mother distributions’

\[
\begin{align*}
\text{GPD (x, } \Delta) & \quad \text{TMD (x, } \vec{k}_t) \\
\int \Delta = 0 & \quad \int x, \quad \Delta = 0
\end{align*}
\]

FFs (\( \Delta \))  PDFs (\( x \))

In particular, \( F_{1,4} \) and \( G_{1,1} \) have a direct role in understanding spin-structure of hadrons (Lorcé, Pasquini, 2011 / Hatta, 2011 / Lorcé, 2014).

But, how would we measure them?
1. **Process**: $\Pi(p_b)N(p_a, \lambda_a) \rightarrow (I_1^+I_1^-)(I_2^+I_2^-)N'(p'_a, \lambda'_a)$

2. **Leading-order diagrams**:

3. **Kinematics**:

   **Region: TMD type**
   
   - $s = (p_a + p_b)^2 = \text{large}$
   - $q_1^2, q_2^2 = \text{large}$
   - $q_i^2 \gg |\vec{q}_{it}|^2$

   **Kinematical variables**:
   
   $$x_a = \frac{k_a^+}{p_a^+}, \quad x_b = \frac{k_b^-}{p_b^-}$$
   
   $$\Delta_a = p'_a - p_a$$
   
   $$\xi_a = -\frac{\Delta_a^+}{2P_a^+} = \frac{q_1^++q_2^+}{2P_a^+}$$
4. Scattering Amplitude:

\[ T^{\lambda_1 \lambda_2}_{\lambda_a \lambda_a'} = T^{\mu \nu}_{\lambda_a \lambda_a'} \epsilon^*_\mu(\lambda_1) \epsilon^*_\nu(\lambda_2) \]

\[ T^{\mu \nu}_{\lambda_a \lambda_a'} = i \sum_{q q'} e_q e'_q e^2 \frac{1}{N_c} \int d^2 \vec{k}_a \int d^2 \vec{k}_{bt} \delta^{(2)} \left( \frac{\Delta \vec{q}_t}{2} - \vec{k}_a - \vec{k}_{bt} \right) \phi_{\pi}^{q'q}(x_b, \vec{k}_{bt}^2) \]

\[ -i \epsilon^{\mu \nu}_\perp \left( W_{\lambda_a \lambda_a'}^{qq'[y^+]}(x_a, \vec{k}_a) - W_{\lambda_a \lambda_a'}^{qq'[y^+]}(-x_a, -\vec{k}_a) \right) - g^{\mu \nu}_\perp \left( W_{\lambda_a \lambda_a'}^{qq'[y^+y^5]}(x_a, \vec{k}_a) + W_{\lambda_a \lambda_a'}^{qq'[y^+y^5]}(-x_a, -\vec{k}_a) \right) \]

Graph (a)

- **Longitudinal parton momenta fixed as:**
  \[ x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+} \]
  \[-\xi_a \leq x_a \leq \xi_a \quad \text{ERBL region} \]

\[ x_b = 1 - \frac{q_1^-}{P_b} = \frac{q_2^-}{P_b} \]

- \( \Delta \vec{q}_t = \vec{q}_{1t} - \vec{q}_{2t} \)
- \( \vec{\Delta}_{at} = -(\vec{q}_{1t} + \vec{q}_{2t}) \)

Graph (b)

- **Longitudinal parton momenta fixed as:**
  \[ x_a = -\frac{(q_1^+ - q_2^+)}{2P_a^+} \quad x_a(\text{Graph (b)}) = -x_a(\text{Graph (a)}) \]

\[ x_b = \frac{q_1^-}{P_b} \quad x_b(\text{Graph (b)}) = 1 - x_b(\text{Graph (a)}) \]

**Symmetry considerations:**

- \( \phi_{\pi}^{q'q}(x_b, \vec{k}_{bt}) = \phi_{\pi}^{q'q}(x_b, -\vec{k}_{bt}) \)
- \( \phi_{\pi}^{q'q}(x_b, \vec{k}_{bt}^2) = \phi_{\pi}^{q'q}(1 - x_b, \vec{k}_{bt}^2) \)
5. Quark GTMDs (Meissner, Metz, Schlegel, 2009):

\[ W^{q[\gamma^+]_{\lambda,\lambda'}} = \frac{1}{2M} \bar{u}(p', \lambda') \left( F^{q\lambda}_{1,1} + \frac{i \sigma^i + k^i}{p^+} F^{q\lambda}_{1,2} + \frac{i \sigma^i + \Delta^i_t}{M^2} F^{q\lambda}_{1,3} + \frac{i \sigma^{ij} k^i \Delta^j_t}{M^2} F^{q\lambda}_{1,4} \right) u(p, \lambda) \]

\[ = \frac{1}{M \sqrt{1 - \xi^2}} \left\{ M \delta_{\lambda,\lambda'} - \frac{1}{2} (\lambda \Delta^1_t + i \Delta^2_t) \delta_{\lambda,-\lambda'} \right\} F^{q\lambda}_{1,1} \]

\[ + \frac{i \varepsilon^i t^j k^i \Delta^j_t}{M^2} \left[ \lambda M \delta_{\lambda,\lambda'} - \frac{\xi}{2} (\Delta^1_t + i \lambda \Delta^2_t) \delta_{\lambda,-\lambda'} \right] F^{q\lambda}_{1,4} + \text{more helicity - flip terms} \]

\[ W^{q[\gamma^+\gamma^5]_{\lambda,\lambda'}} = \frac{1}{2M} \bar{u}(p', \lambda') \left( -\frac{i \varepsilon^i t^j k^i \Delta^j_t}{M^2} G^{q\lambda}_{1,1} + \frac{i \sigma^i + \gamma^5 k^i}{p^+} G^{q\lambda}_{1,2} + \frac{i \sigma^i + \gamma^5 \Delta^i_t}{M^2} G^{q\lambda}_{1,3} + i \sigma^+ - \gamma^5 G^{q\lambda}_{1,4} \right) u(p, \lambda) \]

\[ = \frac{1}{M \sqrt{1 - \xi^2}} \left\{ -\frac{i \varepsilon^i t^j k^i \Delta^j_t}{M^2} (M \delta_{\lambda,\lambda'} - \frac{1}{2} (\lambda \Delta^1_t + i \Delta^2_t) \delta_{\lambda,-\lambda'}) \right\} G^{q\lambda}_{1,1} \]

\[ + \left[ \lambda M \delta_{\lambda,\lambda'} - \frac{\xi}{2} (\Delta^1_t + i \lambda \Delta^2_t) \delta_{\lambda,-\lambda'} \right] G^{q\lambda}_{1,4} + \text{more helicity - flip terms} \]
JUST A MOMENT!  What do we have?  What do we want?

✓ Scatt. Amp. \((T_{\lambda,\lambda'})\)
✓ \(\sigma \sim |T|^2\)

\[ \tau_{UU} = \frac{1}{2} \sum_{\lambda,\lambda'} |T_{\lambda,\lambda'}|^2 \]

\[ \tau_{LU} = \frac{1}{2} \sum_{\lambda,\lambda'} (|T_{+,\lambda'}|^2 - |T_{-,\lambda'}|^2) \]

\[ \tau_{LL} = \frac{1}{2} (|T_{+,+}|^2 - |T_{+,+}|^2) - (|T_{-,+}|^2 - |T_{-,+}|^2)) \]

Similarly, define \(\tau_{XU}, \tau_{YU}\) etc.

Similarly, define \(\tau_{XX}, \tau_{YY}, \tau_{XY}, \tau_{YX}\)

Key point is to realize that if we –

• Define ‘Polarization Observables’ in a certain fashion
• Take proper linear combination of those observables

‘Single out’ what you want!

Remarks:

➢ Summation over photon polarization \((\lambda_1, \lambda_2)\) is implied.

➢ If photon polarization \((\lambda_1, \lambda_2)\) is summed over, there is no interference between the objects \(W^{q[y^+]}_{\lambda,\lambda'}\) and \(W^{q[y^+y^5]}_{\lambda,\lambda'}\).
\[
\frac{1}{4} (\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY}) = \frac{2}{M^2} (\varepsilon_{t}^{ij} \Delta q_{t}^{i} \Delta \Delta_{at})^2 C(+) [\vec{\beta}_{t} \cdot \vec{k}_{at} F_{1,4} \Phi_{\pi}] C(+) [\vec{\beta}_{t} \cdot \vec{k}_{at} F_{1,4}^{*} \Phi_{\pi}^{*}] + 2 C(+) [G_{1,4} \Phi_{\pi}] C(+) [G_{1,4}^{*} \Phi_{\pi}^{*}]
\]

\[
C^\pm [w(\vec{k}_{at}, \vec{k}_{bt}) X \Phi_{\pi}]
\]

\[
= \frac{e^2}{\sqrt{1 - \xi_a^2} N_c} \sum_{q,q'} e_q e'_{q'} \int d^2 \vec{k}_{at} \int d^2 \vec{k}_{bt} \delta(2) \left( \frac{\Delta q_{t}}{2} - \vec{k}_{at} - \vec{k}_{bt} \right) w(\vec{k}_{at}, \vec{k}_{bt}) [X^{qq'} (x_a, \vec{k}_{at})
\]

\[
\pm X^{qq'} (-x_a, -\vec{k}_{at})] \Phi_{\pi}^{q'q} (x_b, \vec{k}_{bt}^{*})
\]

\[
\vec{\beta}_{t} = \frac{\Delta_{at} \Delta q_{t} - (\Delta_{at} \cdot \Delta q_{t}) \Delta \Delta_{at}}{\Delta_{at}^2 \Delta q_{t}^2 - (\Delta_{at} \cdot \Delta q_{t})^2}
\]

\(\Delta q_{t} = \vec{q}_{1t} - \vec{q}_{2t} \) & \(\Delta \Delta_{at} = -(\vec{q}_{1t} + \vec{q}_{2t})\)

\(\varepsilon_{t}^{ij} \Delta q_{t}^{i} \Delta \Delta_{at}\) implies that the contribution from the first term is maximum when the photons come out perpendicular to each other.

**Remarks:**

- If you consider specific polarization states of photons, you have the option to switch off the contribution from \(G_{1,4}\).
- For extracting \(G_{1,1,1}\), the relevant observable is \(\frac{1}{4} (\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY})\).
  - The replacements in the above expression would be \(F_{1,4} \rightarrow G_{1,1}, G_{1,4} \rightarrow F_{1,1}, C(+) \rightarrow C(-)\).
- The observables may be challenging.
Choice 2~

\[
\frac{1}{2}(\tau_{LU} + \tau_{UL}) = \frac{4}{M^2} \varepsilon_t^{ij} \Delta q_t^i \Delta_a^j \text{Im} \left\{ C^(-) [F_{1,1} \Phi_\pi] C^{(+)} [\vec{\beta}_t \cdot \vec{k}_{at} F_{1,4}^* \Phi_\pi^*] - 4 C^{(+)} [G_{1,4} \Phi_\pi] C^(-) [\vec{\beta}_t \cdot \vec{k}_{at} G_{1,1}^* \Phi_\pi^*] \right\}
\]

Remark:
This observable is sensitive to \text{Im} F_{1,4} and \text{Im} G_{1,1}.

Choice 3~

\[
\frac{1}{2}(\tau_{XY} - \tau_{YX}) = \frac{4}{M^2} \varepsilon_t^{ij} \Delta q_t^i \Delta_a^j \text{Re} \left\{ C^(-) [F_{1,1} \Phi_\pi] C^{(+)} [\vec{\beta}_t \cdot \vec{k}_{at} F_{1,4}^* \Phi_\pi^*] - 4 C^{(+)} [G_{1,4} \Phi_\pi] C^(-) [\vec{\beta}_t \cdot \vec{k}_{at} G_{1,1}^* \Phi_\pi^*] \right\}
\]

Remark:
This observable is sensitive to \text{Re} F_{1,4} and \text{Re} G_{1,1} (sensitive to Orbital Angular Momentum, spin-orbit correlation).
Summary

➢ Generalized Transverse Momentum Dependent distributions are interesting-
   • they are ‘mother’ distributions
   • their direct link to the orbital angular momentum ($F_{1,4}$ and $G_{1,1}$)

➢ Proposal: A process that is sensitive to relevant quark GTMDs is the Exclusive Double Drell-Yan process

   1) Quark GTMDs can be accessed in the ERBL region

   2) GTMDs can be accessed through Polarization Observables

➢ What else can be done?
Calculation can be extended to processes like $pp \rightarrow \eta_c \eta_c pp$ (work in progress)
etc.