Limits on the effective quark radius from inclusive $e^\pm p$ scattering and contact interactions at HERA

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On behalf of ZEUS Collaboration

- Combined inclusive cross sections from HERA
- Beyond-the-Standard-Model analysis simultaneously with PDFs fit
- Simplified procedure for QCD+BSM fits

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HERA — world's only $e^\pm p$ collider

Operated during 1992 - 2007

$e^\pm$ energy 27.5 GeV;
$p$ energies 920, 820, 575 and 460 GeV.

Kinematics of the $e^\pm p$ collisions:

$$Q^2 = -(k - k')^2$$

$$x_{Bj} = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

H1 and ZEUS — two collider experiments at HERA:

~0.5 $\text{fb}^{-1}$ of luminosity recorded by each experiment.
**HERA inclusive data combination**

**NC e⁺p**

H1 and ZEUS

- 2927 data points combined to 1307
- up to 8 data points combined to 1

• impressive improvement of precision due to:
  - increased statistics
  - better understanding of systematics
  - cross-calibration of the data from two experiments

**σ_T,NC**

- HERA NC e⁺p 0.5 fb⁻¹
- √s = 318 GeV

$Q^2/\text{GeV}^2$

$\times_{Bj} = 0.002$

$\times_{Bj} = 0.008$

$\times_{Bj} = 0.032$

$\times_{Bj} = 0.08$

$\times_{Bj} = 0.25$
QCD analysis of the combined DIS data

Neutral Current:
\[
\frac{d^2\sigma_{NC}^{e^+p}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \cdot (Y_+ \cdot F_2 \pm Y_- \cdot x \cdot F_3 - y^2 \cdot F_L) \\
Y_\pm = 1 \pm (1 - y)^2
\]
\[F_L \sim \alpha_s g\]

At the Quark-Parton Model:
\[
F_2 = \frac{4}{9} (xU + x\bar{U}) + \frac{1}{9} (xD + x\bar{D})
\]
\[x \cdot F_3 \sim xu_v + xd_v\]

Charged Current:
\[
\frac{d^2\sigma_{CC}^{e^+p}}{dx dQ^2} = \frac{G_F^2}{4\pi x} \cdot \kappa^2 \cdot (Y_+ \cdot W_2^\mp + Y_- \cdot x \cdot W_3^\mp - y^2 \cdot W_L^\mp) \\
\kappa = \frac{M_w^2}{M_w^2 + Q^2}
\]

\[x \cdot W_2 = x (U + \bar{D}) \quad x \cdot W_2^+ = x (D + \bar{U})\]
\[x \cdot W_3 = x (U - \bar{D}) \quad x \cdot W_3^+ = x (D - \bar{U})\]

Parton Density Functions parametrization at starting scale \(Q_0^2 = 1.9 \text{ GeV}^2\): 
\[
x \cdot g(x) = A_g x^B g (1 - x)^{C_g} - A'_g x^{B'_g} (1 - x)^{C'_g}
\]
\[
x \cdot u_v(x) = A_{u_v} x^{B_u} (1 - x)^{C_u} (1 + D_{u_v} x + E_{u_v} x^2)
\]
\[
x \cdot d_v(x) = A_{d_v} x^{B_d} (1 - x)^{C_d}
\]
\[
x \cdot \bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x)
\]
\[
x \cdot \bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}}
\]

- fixed or calculated by sum-rules
- set equal

Evolve to any \(Q^2 > Q_0^2\) with DGLAP. Obtained PDFs are referred to as ZCIPDFs and have a good agreement with the HERAPDF 2.0.
How big is a quark?

One of the possible parameterisations of deviations from SM – spatial distribution or substructure of electrons and/or quarks.

In a semi-classical form factor approach cross sections are expected to **decrease** at high-$Q^2$:

\[
\frac{d\sigma}{dQ^2} = \frac{d\sigma^{SM}}{dQ^2} \left(1 - \frac{R_e^2}{6 Q^2}\right)^2 \left(1 - \frac{R_q^2}{6 Q^2}\right)^2
\]

$R_e$, $R_q$ – root mean square radii of the electroweak charge distributions in the electron and quark.

**Same dependence** expected for **NC** and **CC** $e^+p$ and $e^-p$.

Electrons were assumed to be point-like, $R^2_e = 0$, and both, positive and negative values of $R^2_q$ were considered.
Reason for the simultaneous fit procedure

- BSM signal in the data could affect the PDF fit and result in biased PDFs.

- Use of the biased PDFs in the BSM analysis would result in overestimated limits.

- This cannot be avoided for the analysis of HERA data by using another available PDF set, since all high-precision PDF fits include the DIS data from HERA (MMHT2014, NNPDF3.0, etc.).

- The proper procedure for a BSM analysis of the HERA data - global QCD analysis which includes a possible contribution from BSM processes.
Necessity of the simultaneous fit procedure

Pseudodata generated for values of $R^2_q = R^2_q^{\text{True}}$

- $R^2_q + \text{PDF fit}$
- $R^2_q$-only fit after SM PDF fit

Pseudodata generated for $R^2_q = 0$

- $R^2_q + \text{PDF } \chi^2$ scan
- $R^2_q$-only $\chi^2$ scan

$R^2_q + \text{PDF procedure provides unbiased values of } R^2_q^{\text{Fit}}$

$R^2_q$-only procedure results in too strong limits
Limit setting method

Limits are derived in a frequentist approach using the technique of Monte Carlo replicas (probability method). Two procedures were used:

- **$R_q$-only**
  - Monte Carlo replicas generated for $R_q^{\text{True}}$ using ZCIPDFs and $R_q$ parameter fitted with PDFs fixed to ZCIPDFs.

- **$R_q$+PDF**
  - Monte Carlo replicas generated for $R_q^{\text{True}}$ using ZCIPDFs and $R_q$ parameter fitted simultaneously with PDFs.

The $R_q$+$\text{PDF}$ probability method was a main analysis method.
Monte Carlo replicas

Monte Carlo replicas of cross-section measurements calculated with

Cross-section prediction from the ZCIPDF modified with $R_q^{\text{True}}$

$$\mu_i = [m_i^0 + \sqrt{\delta_{i,\text{stat}}^2 + \delta_{i,\text{uncor}}^2} \cdot \mu_0 \cdot r_i] \cdot (1 + \sum_j \gamma_j \cdot r_j)$$

Measured cross-section value

Correlated systematic uncertainties

Relative statistical and uncorrelated systematic uncertainties

Random numbers from a normal distribution

For $R_q^{\text{True}} = 0.48 \cdot 10^{-16}$ cm:

**$R_q$-only**

- $\chi^2_{\text{free}} = -0.3544 \times 10^6$ GeV$^{-2}$
- $\langle R_q \rangle_{\text{free}} = 6 \times 10^6$ GeV$^{-2}$

Fraction of $\langle R_q^{2,\text{free}} \rangle < \langle R_q \rangle_{\text{free}}$: 0.96 %

**$R_q + \text{PDF}$**

- $\chi^2_{\text{free}} = -0.4786 \times 10^6$ GeV$^{-2}$
- $\langle R_q \rangle_{\text{free}} = 6 \times 10^6$ GeV$^{-2}$

Fraction of $\langle R_q^{2,\text{free}} \rangle < \langle R_q \rangle_{\text{free}}$: 1.84 %
**$R_q$ limits with the MC replicas**

**$R_q$-only**

**ZEUS**

Fractions close to 5% fitted with:

$$f(x) = 5 \cdot \exp((-x - A) \cdot B)$$

$$[0.42 \times 10^{-16} \text{ cm}]^2 \leq R_q^2 \leq [0.40 \times 10^{-16} \text{ cm}]^2$$
$R_q$ limits with the MC replicas

Fractions close to 5% fitted with:

$$f(x) = 5 \cdot \exp((x - A) \cdot B)$$

$$- [0.47 \times 10^{-16} \text{ cm}]^2 \leq R_q^2 \leq [0.43 \times 10^{-16} \text{ cm}]^2$$
Comparison to Data

NC $e^\pm p$

- HERA NC $e^p$ 0.5 fb$^{-1}$
- HERA NC $e^p$ 0.4 fb$^{-1}$
- ZCIPDF total unc.

CC $e^\pm p$

- HERA CC $e^p$ 0.5 fb$^{-1}$
- HERA CC $e^p$ 0.4 fb$^{-1}$
- ZCIPDF total unc.

Quark Radius
95% CL Limits
$R_q^2 = (0.43 \times 10^{-16} \text{cm})^2$
$R_q^2 = -(0.47 \times 10^{-16} \text{cm})^2$
Simplified fit procedure

On average every Cl+PDF fit takes $\sim 1.5 \text{ hours}$ of cpu time. For final $R_q$ analysis 215000 replicas were fitted, taking $\sim 36.8 \text{ years}$ of cpu time.

To proceed with other BSM models a simplified fit procedure based on the approximation of the cross-section predictions with a Taylor expansion have been developed and implemented, reducing the average fit duration to $\sim 2 \text{ minutes}$ of cpu time.

For $R_{q,\text{True}} = 0.43 \cdot 10^{-16}$ cm:

**Graphs:**
- Scatter plot showing the comparison between full fit and simplified fit.
- Histogram showing CPU time for single replica fit for different $R_q$ values.
Contact interactions

Four-fermion $eeqq$ contact interactions provide a convenient method to search for possible effects due to the virtual exchange of new particles with mass much higher than center of mass energy.

\[ \mathcal{L}_{CI} = \sum_{i,j=L,R, \quad q=u,d} \eta_{ij}^{eq} (\bar{e}_i \gamma^\mu e_i) (\bar{q}_j \gamma_\mu q_j) \]

\[ \eta_{ij} = \epsilon_{ij} \frac{4\pi}{\Lambda^2} \]

\[ \epsilon_{ij} = \pm 1; 0 \]

Considered models:

<table>
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<tr>
<th>Model</th>
<th>$\eta_{LL}^{eq}$</th>
<th>$\eta_{LR}^{eq}$</th>
<th>$\eta_{RL}^{eq}$</th>
<th>$\eta_{RR}^{eq}$</th>
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<td>-$\eta$</td>
<td>-$\eta$</td>
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<tr>
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Following approach from the $R_q$ analysis:

**VV model**
(highest sensitivity)

Evaluated 95% C.L. limits:

$-5.8 \cdot 10^{-8} \text{ GeV}^{-2} < \eta < 13.9 \cdot 10^{-8} \text{ GeV}^{-2}$

$\Lambda^- > 14.7 \text{ Tev} \quad \Lambda^+ > 9.5 \text{ Tev}$
Contact interactions

Following approach from the $R_q$ analysis:

**AA model**
(deviation from SM 2.5 $\sigma$)

Evaluated 95% C.L. limits:

\[
11.6 \cdot 10^{-8} \text{ GeV}^{-2} < \eta < 53.1 \cdot 10^{-8} \text{ GeV}^{-2} \\
\Lambda^+ < 10.4 \text{ Tev} \quad \Lambda^+ > 4.8 \text{ Tev}
\]
Evaluated CI limits

HERA $e^+p$ 1994-2007 95% C.L.

<table>
<thead>
<tr>
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<th>Measured</th>
<th>Expected</th>
<th>$p_{SM}$ (%)</th>
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<td>$\Lambda^-$</td>
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<td>4.8 - 10.4</td>
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<tr>
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<td>—</td>
<td>3.6 - 10.1</td>
<td>4.1</td>
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<td>9.2</td>
<td>8.0</td>
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Summary

- Combined HERA inclusive DIS cross sections allow BSM searches up to TeV scales

- Limits on the quark form factor:

  \[ -[0.47 \times 10^{-16} \text{ cm}]^2 \leq R_q^2 \leq [0.43 \times 10^{-16} \text{ cm}]^2 \]

- Simultaneous fit procedure is necessary since limits obtained with fixed PDFs are too strong

- Some of the contact interactions models provide improved description of the data
Determination of ZCIPDFs

The QCD analysis done with the HERAFitter, ancestor of the xFitter. (available at www.xfitter.org/xFitter/).

The procedure established for HERAPDF 2.0 was closely followed:

- $Q^2_{\text{min}} = 3.5 \text{ GeV}^2$ → 1145 data points used
- Renormalisation and factorisation in the $\overline{\text{MS}}$ scheme, with $\mu^2_R = \mu^2_F = Q^2$
- NLO calculations and DGLAP evolution
- Heavy quarks evaluated in RTOPT scheme with $M_c = 1.47 \text{ GeV}$ and $M_b = 4.5 \text{ GeV}$
- Starting scale $Q^2_0 = 1.9 \text{ GeV}^2$
- $\alpha_s(M^2_Z) = 0.118$, $f_s = 0.4$

The $\chi^2$ definition for ZCIPDF was different from HERAPDF 2.0:

$$\chi^2(m, s) = \sum_i \left[ \frac{m^i - \sum_j y^i_j m^i s^j - \mu^i_0}{\delta^2_{i, \text{stat}} (\mu^i_0)^2 + \delta^2_{i, \text{uncorr}} (\mu^i_0)^2} \right]^2 + \sum_j s^2_j$$
Good agreement with HERAPDF 2.0
Simplified fit procedure

In simplified procedure cross-section predictions were approximated by first-order Taylor expansion in PDFs $\bar{p}$ and second-order expansion in BSM parameter $\eta$:

$$m(x_i, Q^2_i, \bar{p}, \eta) = m_0^i + \sum_k \Theta_{0,k}^i \Delta p^k + (m_1^i + \sum_k \Theta_{1,k}^i \Delta p^{k^*}) \cdot \eta + (m_2^i + \sum_k \Theta_{2,k}^i \Delta p^{k^{**}}) \cdot \eta^2$$

Comparing simplified and full fit results for $R^\text{True}_q = 0.43 \cdot 10^{-16} \text{ cm}$:
$R_q$ limits with simplified procedure

Very good agreement of the analyses results

[arXiv:1606.0667].
## Comparison to other experiments

<table>
<thead>
<tr>
<th>Measured 95% C.L. limits (×10^{-16} cm)</th>
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<tr>
<td>HERA combined</td>
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<tr>
<td>( R_{q}^- )</td>
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<table>
<thead>
<tr>
<th>Measured 95% C.L. limits (TeV)</th>
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<td>( \Lambda^- )</td>
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