Completing the hadronic Higgs boson decay at order α_s^4

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Higgs boson decay to hadrons

Hadronic decays: $\sim 70\%$ of total Higgs decay width.

- $\Gamma(H \to \bar{b}b)$, $\Gamma(H \to gg)$ form almost all of this.

All branching ratios are affected: want to know as precisely as possible.

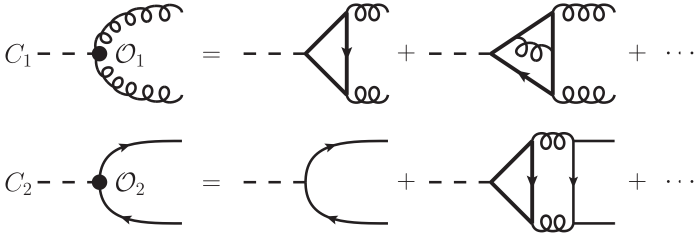
$$\mathrm{BR}_i = \frac{\Gamma_i}{\sum_j \Gamma_j}$$

Important for the determination of Higgs couplings.

Formalism

We work in an **effective theory**, in order to study top quark–mass effects.

$$\mathcal{L}_{\text{effective}} = \mathcal{L}_{\text{QCD}}^{(n_l=5)} - \frac{H}{v}\left(C_1\mathcal{O}_1 + C_2\mathcal{O}_2\right), \quad \mathcal{O}_1 = G^a_{\mu\nu}G^{a\,\mu\nu}, \quad \mathcal{O}_2 = m_b\bar{b}b$$



Coefficient functions C_1, C_2 known to 5 loops. [Schroder,Steinhauser '06]

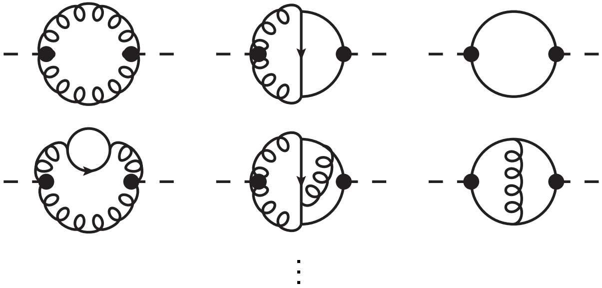
[Chetyrkin,Kühn,Sturm '06][Liu,Steinhauser '15]

Here, we require C_1, C_2 only to 4 loops.

Formalism

Optical theorem: Higgs decay related to imaginary part of correlators,

$$\Pi_{ij}(q^2 = M_H^2) = i \int \mathrm{d}x \, e^{iqx} \langle 0 | T[\mathcal{O}_i, \mathcal{O}_j] | 0 \rangle.$$



Then define:

$$\Delta_{11} \propto \mathrm{Im}\Pi_{11}, \quad \Delta_{12} \propto \mathrm{Im}\left(\Pi_{12} + \Pi_{21}\right), \quad \Delta_{22} \propto \mathrm{Im}\Pi_{22},$$

$$\sim 1 + m_b^2 + \mathcal{O}(m_b^4), \quad \sim m_b^2 + \mathcal{O}(m_b^4), \quad \sim m_b^2 + \mathcal{O}(m_b^4).$$

What is known already?

Decay rate:

$$\Gamma\left(H \to \bar{b}b, gg\right) = A_{gg}(C_1^2)\Delta_{11} + A_{b\bar{b}}\left[C_2^2(1+\Delta_{22}) + C_1 C_2 \Delta_{12}\right].$$

Corrections $\propto m_b^4$, $1/m_t^2$, EW corrections are small. Neglected here.

Already known:

- Δ_{11}: m_b^0 terms to 4 loops ($\alpha_s^5\,\Gamma$) [Baikov,Chetyrkin '06]
- Δ_{12}: m_b^2 terms to 3 loops ($\alpha_s^3\,\Gamma$) [Chetyrkin,Steinhauser '97]
- Δ_{22}: m_b^2 terms to 5 loops ($\alpha_s^4\,\Gamma$) [Baikov,Chetyrkin,Kühn '05]

Here we compute:

- Δ_{11}: m_b^2 terms to 3 loops ($\alpha_s^4\,\Gamma$)
- Δ_{12}: m_b^2 terms at 4 loops ($\alpha_s^4\,\Gamma$) (Note: $C_1 = \mathcal{O}(\alpha_s)$, $C_2 = \mathcal{O}(1)$)

How large are these corrections?

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Define the combinations

$$\Delta_{\text{light}} = \Delta_{22}, \qquad \Delta_{gg}^{m_b^0} = \# C_1^2 \Delta_{11}^{m_b^0},$$

$$\Delta_{\text{top}} = (C_2^2 - 1)(1 + \Delta_{22}) + C_1 C_2 \Delta_{12} + \# C_1^2 \Delta_{11}^{m_b^2},$$

such that

$$\Gamma\left(H \to \bar{b}b, gg\right) = A_{b\bar{b}}\left[1 + \Delta_{\text{light}} + \Delta_{\text{top}} + \Delta_{gg}^{m_b^0}\right].$$

Correction	α_s^1	α_s^2	α_s^3	α_s^4	α_s^5
Δ_{light}	0.2033	0.03752	0.001928	-0.001368	
Δ_{top}		0.00457	0.002562	**??**	
$\Delta_{gg}^{m_b^0}$		0.09699	0.06235	0.01911	0.001759

$[\alpha_s(M_H) = 0.1127,\ m_b(M_H) = 2.773 GeV,\ M_H = 125.09 GeV,\ M_t = 173.21 GeV]$

Motivates computation of α_s^4 corrections to Δ_{top}.

Renormalization

Strong coupling, bottom quark mass,

$$\alpha_s^{bare} = Z_{\alpha_s}\alpha_s,$$
$$m_b^{bare} = Z_m m_b.$$

The effective operators **mix** under renormalization,

$$\mathcal{O}_1 = \overbrace{\left(1 + \alpha_s \frac{\partial}{\partial \alpha_s} \log Z_{\alpha_s}\right)}^{Z_1} \mathcal{O}_1^{bare} \overbrace{- \left(4\alpha_s \frac{\partial}{\partial \alpha_s} \log Z_m\right)}^{Z_2} \mathcal{O}_2^{bare},$$
$$\mathcal{O}_2 = \mathcal{O}_2^{bare},$$

and so for Δ_{ij},

$$\Delta_{11} = Z_1^2 \Delta_{11}^{bare} + 2Z_1 Z_2 \Delta_{12}^{bare} + Z_2^2 \Delta_{22}^{bare},$$
$$\Delta_{12} = Z_1 \Delta_{12}^{bare} + Z_2 \Delta_{22}^{bare},$$
$$\Delta_{22} = \Delta_{22}^{bare}.$$

Software

Diagram generation	qgraf	[Nogueira '93]
Topology mapping	q2e/exp	[Harlander,Seidelsticker,Steinhauser '97]
Physics	TFORM	[Kuipers,Ueda,Vermaseren,Vollinga '13]
Scalar integrals: 3-L $\Delta_{11}^{m_b^2}$	MINCER	[Larin,Tkachov,Vermaseren '91]
4-L Δ_{12}	FIRE 5.2	[Smirnov '14]
	TSORT	[Pak '11]

4-loop case:

- 22K scalar integrals
- reduce to 28 4-loop master integrals (known) [Baikov,Chetyrkin '10]

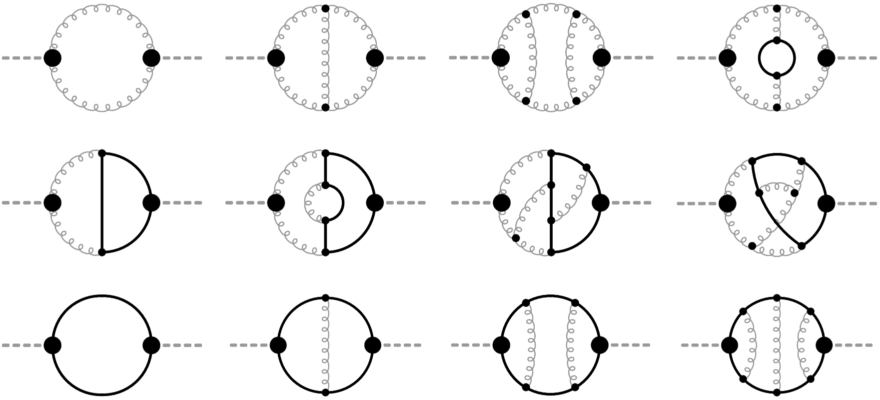
Verification of setup, re-calculate:

- $\Delta_{11}^{m_b^0}$ to 3 loops
- Δ_{22} to 4 loops

Arbitrary gauge parameter ξ, drops out after reduction to master integrals.

Sample Diagrams

Some diagrams contributing to Π_{11}, Π_{12} and Π_{22} ...



Results

$$
\begin{aligned}
\Delta_{12} = &+ \left(\frac{\alpha_s}{\pi}\right) \left[-\frac{92}{3} - 8\,L_H \right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{15073}{18} + 76\,\zeta_2 + 156\,\zeta_3 - \frac{1028}{3}\,L_H - 38\,L_H^2 \right. \\
&\qquad \left. + n_l \left(\frac{283}{9} - \frac{8}{3}\,\zeta_2 - \frac{16}{3}\,\zeta_3 + \frac{112}{9}\,L_H + \frac{4}{3}\,L_H^2 \right) \right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[-\frac{8957453}{432} + 4150\,\zeta_2 + \frac{131389}{18}\,\zeta_3 - 815\,\zeta_5 \right. \\
&\qquad - \left[\frac{65267}{6} - 855\,\zeta_2 - 1755\,\zeta_3 \right] L_H - 2075\,L_H^2 - \frac{285}{2}\,L_H^3 \\
&\qquad + n_l \left(\frac{279451}{162} - \frac{1003}{3}\,\zeta_2 - 446\,\zeta_3 + 10\,\zeta_4 + \frac{100}{3}\,\zeta_5 \right. \\
&\qquad\qquad \left. + \left[\frac{15973}{18} - 68\,\zeta_2 - 118\,\zeta_3 \right] L_H + \frac{1003}{6}\,L_H^2 + \frac{34}{3}\,L_H^3 \right) \\
&\qquad + n_l^2 \left(-\frac{25627}{972} + \frac{56}{9}\,\zeta_2 + \frac{20}{3}\,\zeta_3 - \left[\frac{407}{27} - \frac{4}{3}\,\zeta_2 - \frac{8}{3}\,\zeta_3 \right] L_H \right. \\
&\qquad\qquad \left. \left. - \frac{28}{9}\,L_H^2 - \frac{2}{9}\,L_H^3 \right) \right] + \mathcal{O}\left(\alpha_s^4\right)
\end{aligned}
$$

Scale dependence: $L_H = \log \frac{\mu^2}{M_H^2}$.

ζ_i: Riemann Zeta function.

Results

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$$\Gamma\left(H \to \bar{b}b, gg\right) = A_{b\bar{b}}\left[1 + \Delta_{\text{light}} + \Delta_{\text{top}} + \Delta_{gg}^{m_b^0}\right]$$

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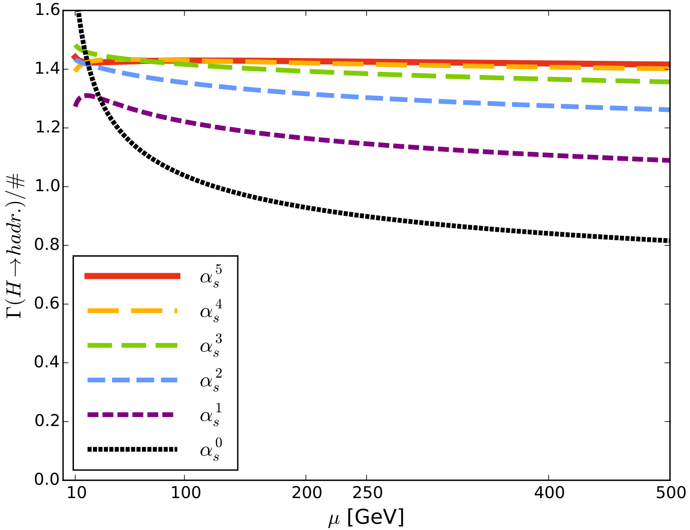
m_b^2 corrections:

- $1 + \Delta_{\text{light}} + \Delta_{\text{top}} = 1 + 0.2033 + 0.04209 + 0.00449 - 0.000709$
- $\Rightarrow$ Born $\xrightarrow{\text{1-loop}}$ 20.3% $\xrightarrow{\text{2-loop}}$ 3.50% $\xrightarrow{\text{3-loop}}$ 0.361% $\xrightarrow{\text{4-loop}}$ 0.0567%

Series is **well converging**.

Scale Dependence



Conclusions

We have computed the missing α_s^4 contributions to $\Gamma\left(H \to \bar{b}b, gg\right)$.

- Numerically small
- QCD series is well converging

Preprint: `arXiv:1703.02988`.

Computer-readable results (`FORM`, `Mathematica`) available at

- `https://www.ttp.kit.edu/preprints/2017/ttp17-010/`