

Parton shower and finite top mass effects in HH production



MAX-PLANCK-GESELLSCHAFT

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In collaboration with

Borowka, Greiner, Heinrich, Jones, Luisoni, Schlenk, Schubert, Vryonidou, Zirke

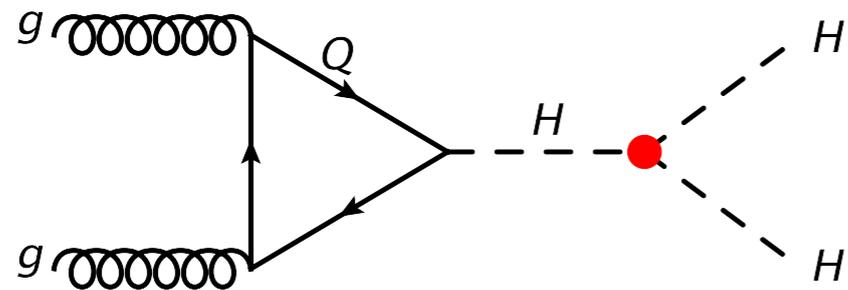
arXiv:1703.09252

JHEP 10 (2016) 107 [1608.04798]

PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

Motivation

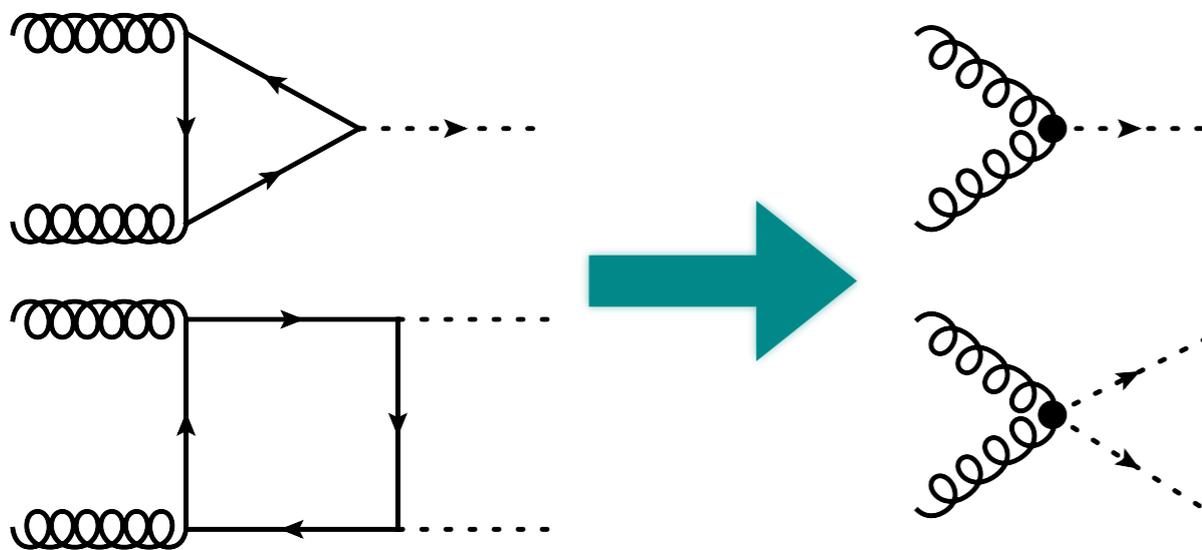
Dominant production mechanism for HH production is gluon fusion



Sensitive to HHH coupling

Test of Higgs potential & EW symmetry breaking

Most calculations are done in $m_t \rightarrow \infty$ limit (Higgs EFT)



HEFT valid for $\sqrt{s} \ll 2m_T$

Higgs pair production:

$$2m_H < \sqrt{s}$$

full top quark mass dependence required for accurate predictions

gg→HH calculations

1. LO, including full m_T dependence
Glover, van der Bij `88

2. NLO, (Born-improved) HEFT **K ≈ 2**
Dawson, Dittmaier, Spira `98

- including full m_T dependence in real radiation (FT approx.) **-10%**
Maltoni, Vryonidou, Zaro `14
- including $1/m_T$ expansion **±10%**
Grigo, Hoff, Melnikov, Steinhauser `13;
Grigo, Hoff, Steinhauser `15
Degrassi, Giardino, Gröber `16

3. NLO, including full m_T dependence
Borowka, Greiner, Heinrich, Jones, MK,
Schlenk, Schubert, Zirke `16

- NLO matched to parton shower
Heinrich, Jones, Luisoni, MK, Vryonidou `17
- transverse momentum NLL+NLO
Ferrera, Pires `16

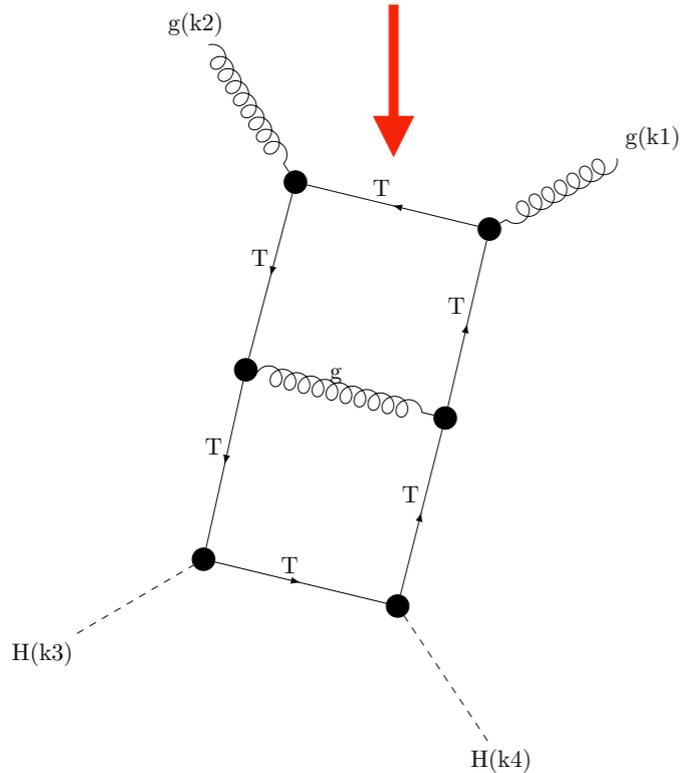
4. NNLO (HEFT) **+20%**
de Florian, Mazzitelli `13

- including all matching coefficients
Grigo, Melnikov, Steinhauser `14
- including $1/m_T$ expansion
Grigo, Hoff, Steinhauser `15
- NNLL soft gluon resummation
Shao, Li, Li, Wang `13
- NNLL + NNLO matching
de Florian, Mazzitelli `15
- fully differential
de Florian, Grazzini, Hanga, Kallweit,
Lindert, Maierhöfer, Mazzitelli, Rathlev `16

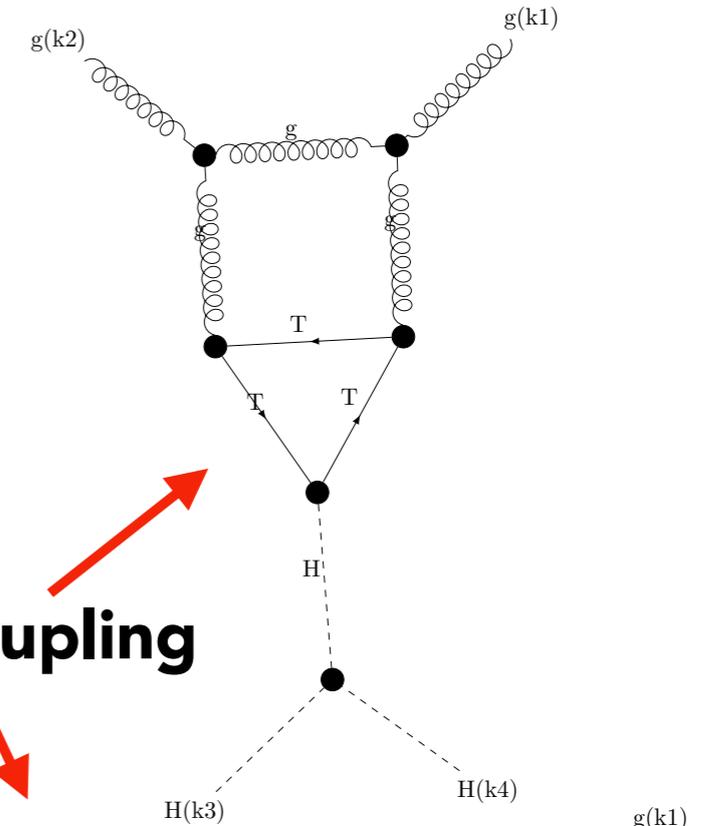
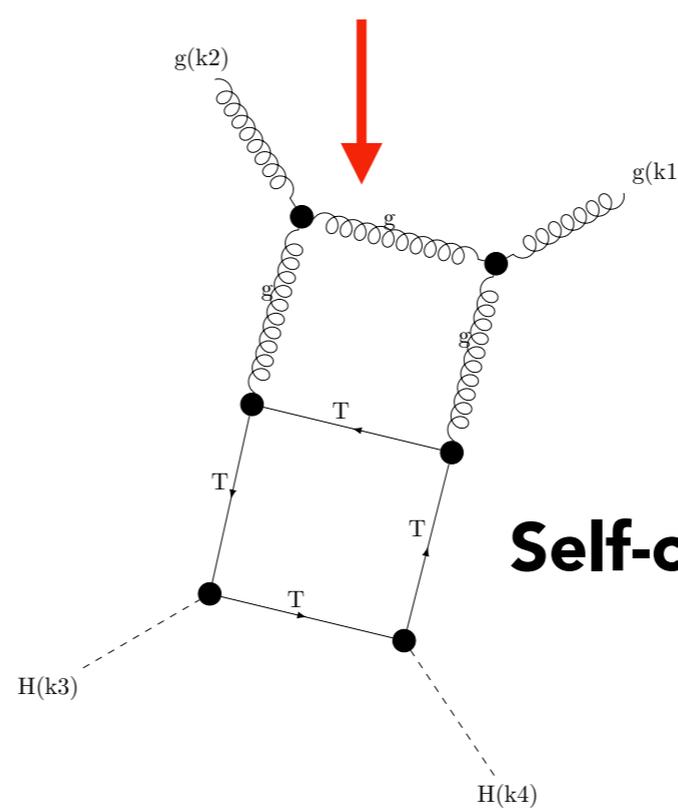
this talk

Two Loop Diagrams

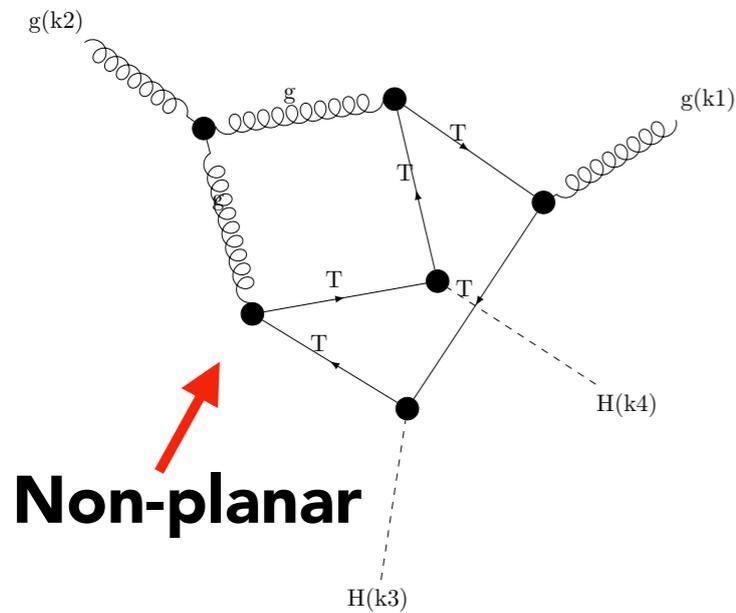
Massive Double Box



Massless/Massive Box

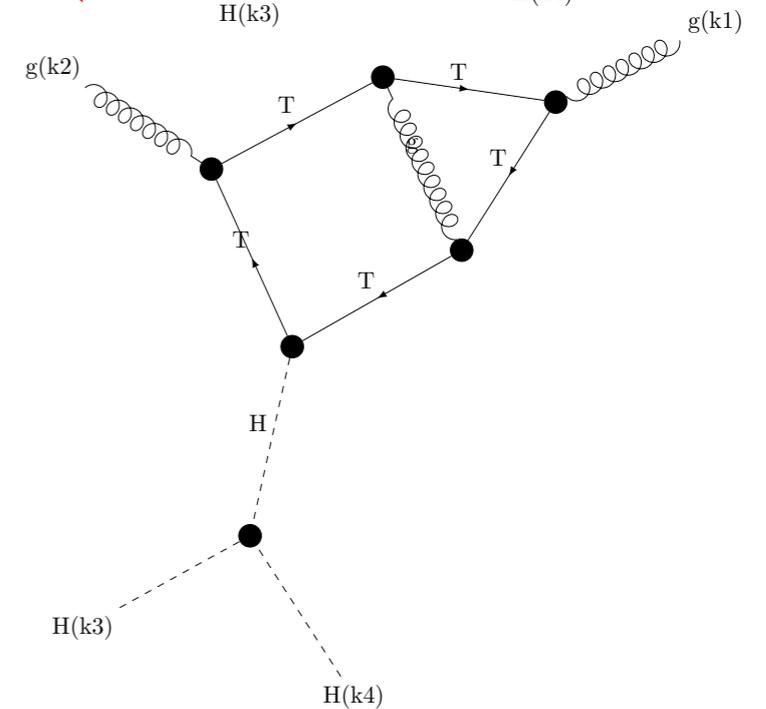
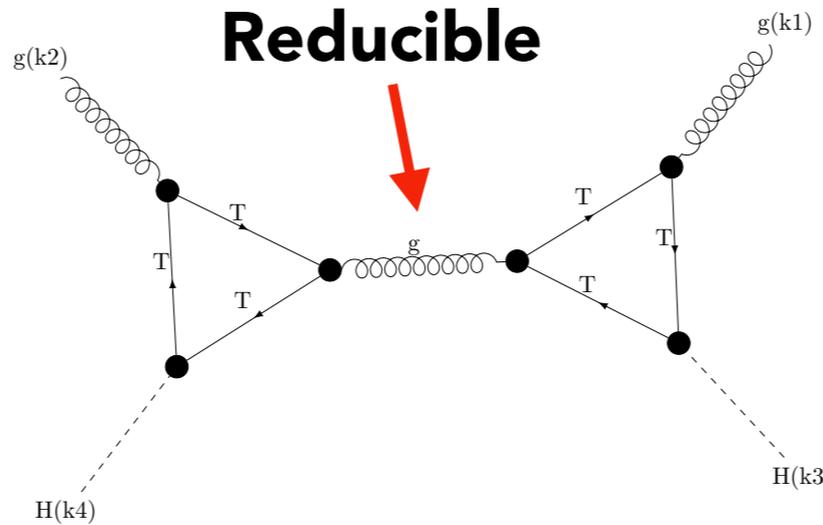


Self-coupling



Non-planar

Reducible

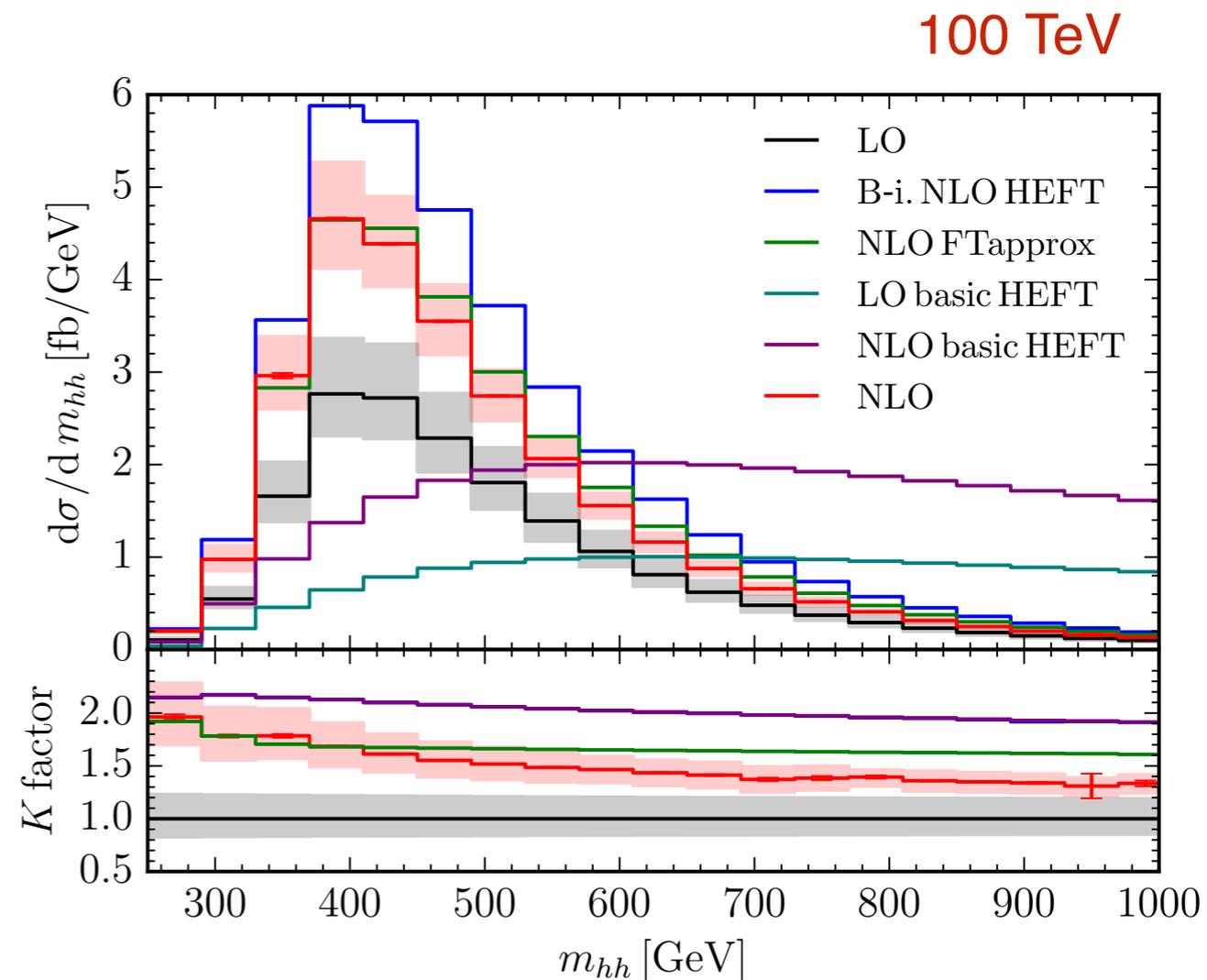
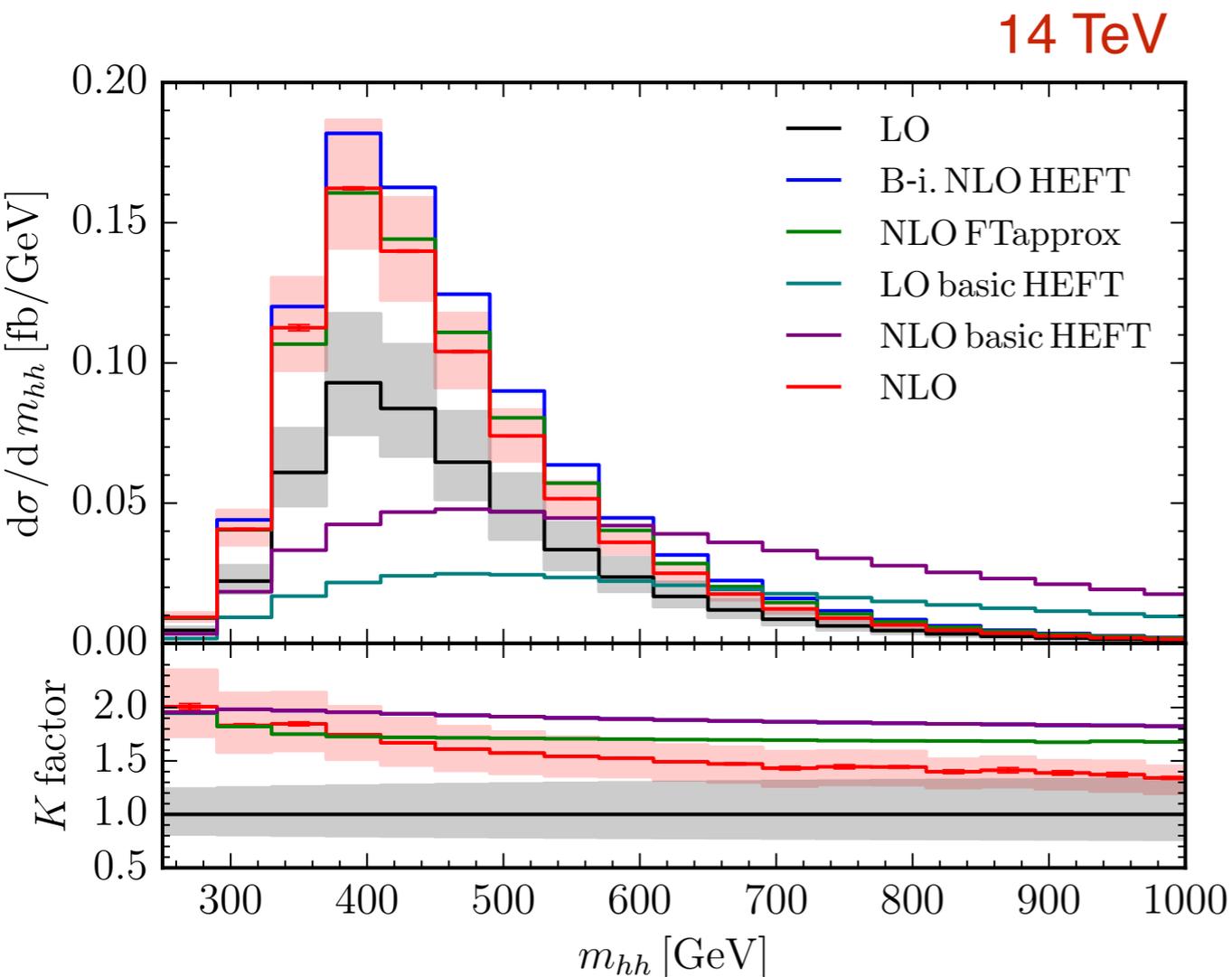


most complicated integrals not known analytically
 → numeric calculation required

NLO Calculation

1. Reduction to master integrals using Reduze [von Manteuffel, Studerus`12](#)
 - full reduction achieved only for planar integrals (non-planar integrals evaluated directly)
 - use finite basis [von Manteuffel, Panzer, Schabinger`15](#)
2. Numerical evaluation of 2-loop integrals using SecDec [Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke](#)
 - using Quasi-Monte-Carlo integration with $\mathcal{O}(n^{-1})$ scaling [Li, Wang, Yan, Zhao`15](#); Review: [Dick, Kuo, Sloan](#)
 - dynamically set number of sampling points for each integral
 - parallelization on gpu
3. Use unweighted events (based on LO) for phase space integration of virtuals
4. Real radiation amplitudes using GoSam [Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, von Soden-Fraunhofen, Tramontano](#)
5. Dipole subtraction [Catani Seymour](#) to deal with IR singularities

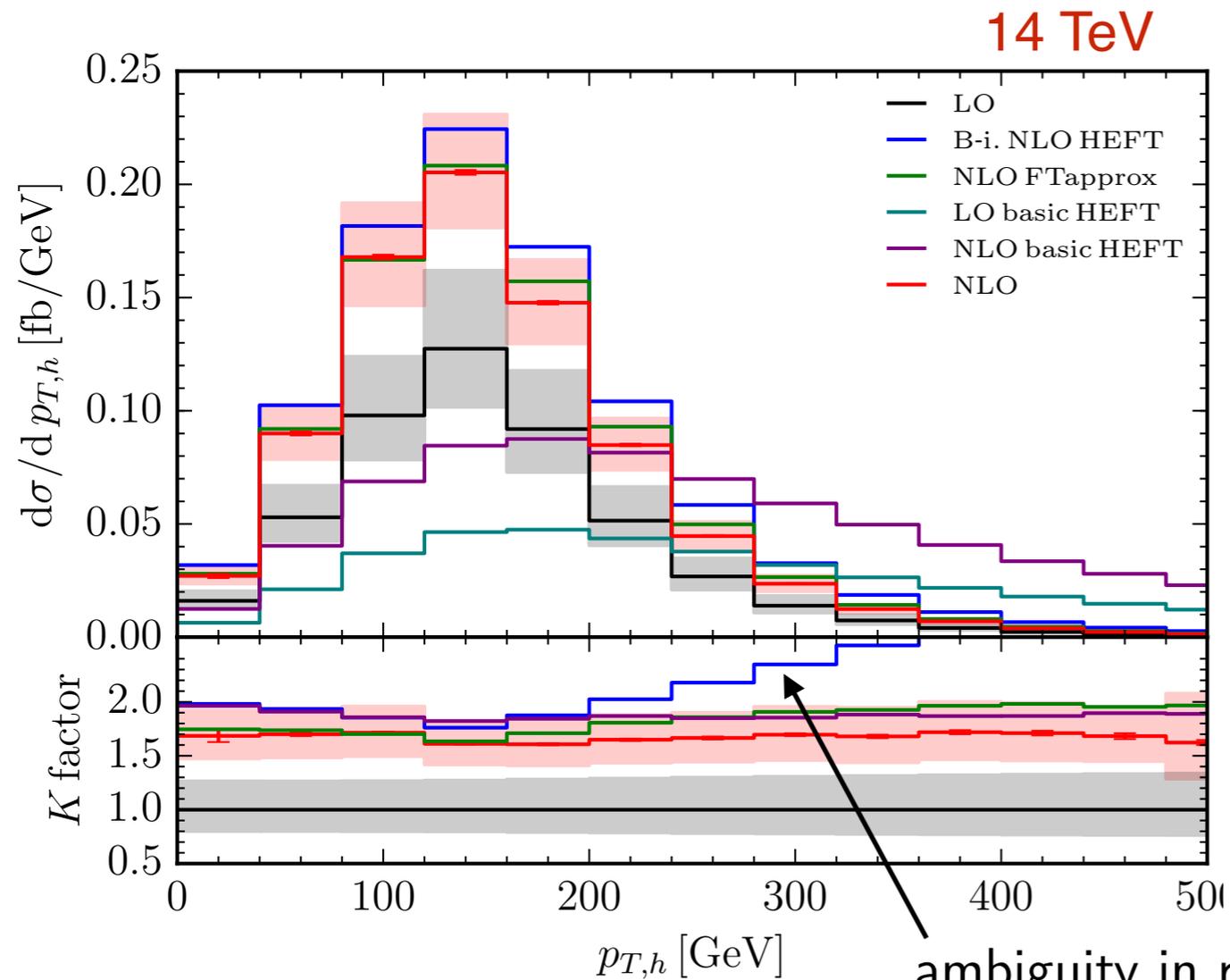
NLO Results — Invariant Mass



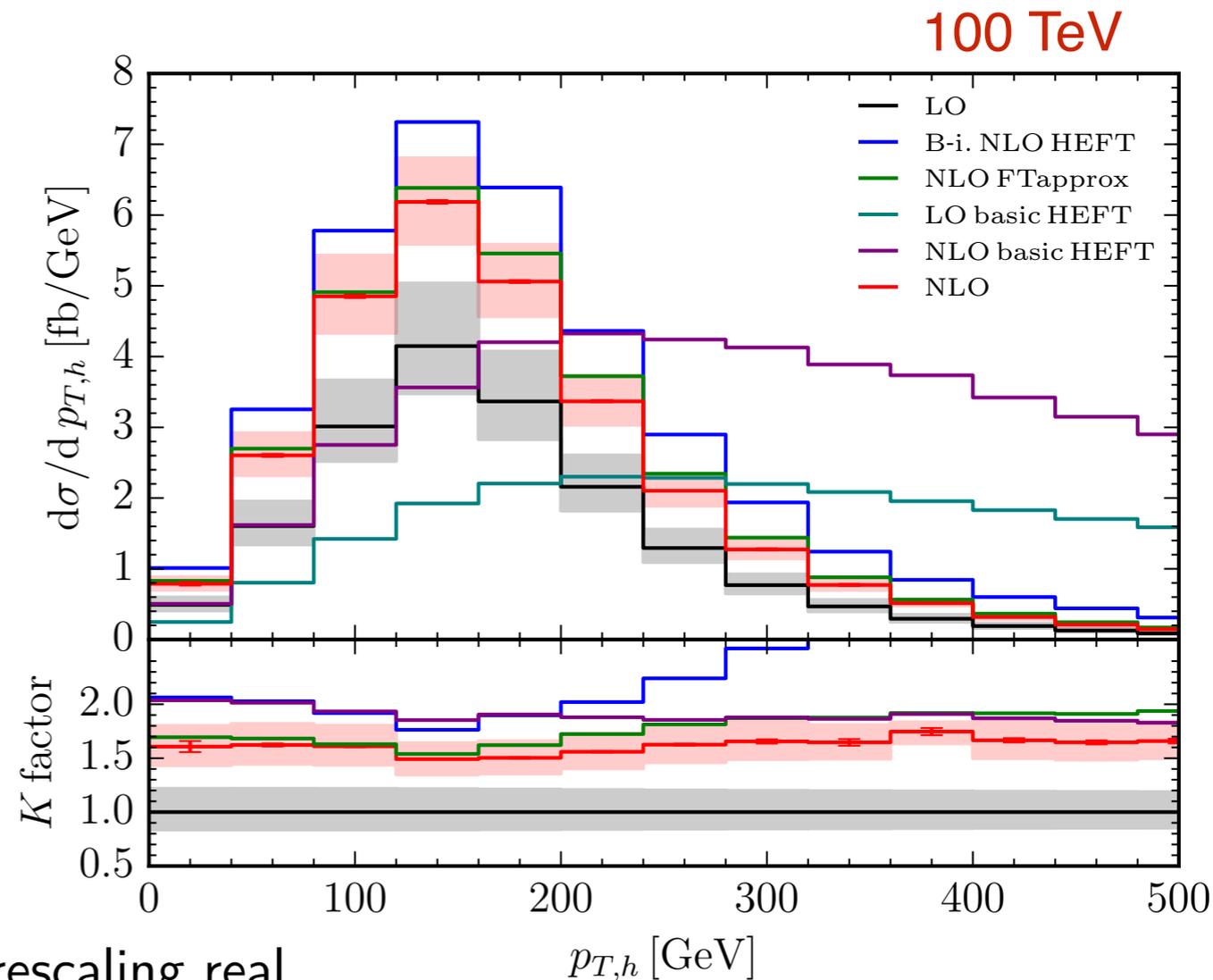
- basic HEFT leads to wrong shape
- B.I. HEFT overestimates by 16% / 30%
- FT approx closer to full result
(difference increasing with m_{hh})

	14 TeV	100 TeV
LO	$19.85^{+27.6\%}_{-20.5\%}$	$731.3^{+20.9\%}_{-15.9\%}$
B.i. HEFT	$38.32^{+18.1\%}_{-14.9\%}$	$1511^{+16.0\%}_{-13.0\%}$
FT approx	$34.26^{+14.7\%}_{-13.2\%}$	$1220^{+11.9\%}_{-10.7\%}$
NLO full	$32.91^{+13.6\%}_{-12.6\%}$	$1149^{+10.8\%}_{-10.0\%}$

NLO Results — Higgs Momentum



ambiguity in rescaling real radiation with full Born



top mass effects important, in particular at $\sqrt{s} = 100$ TeV

- basic HEFT leads to wrong shape
- B.I. HEFT overestimates by 16% / 30%
- FT approx closer to full result (difference increasing with $p_{T,h}$)

Parton Shower Interface

2-loop amplitude too slow (median 2h on gpu) for direct interface to PS

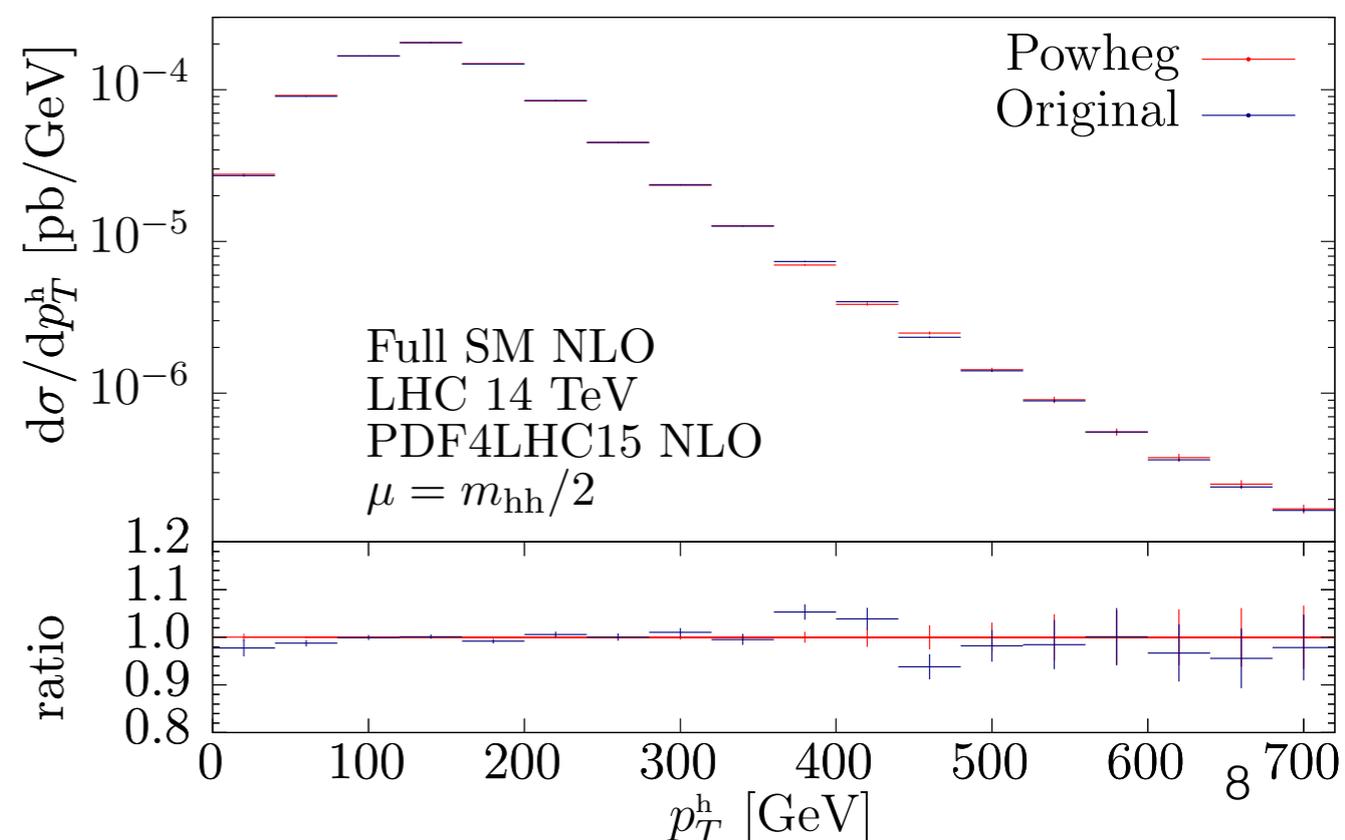
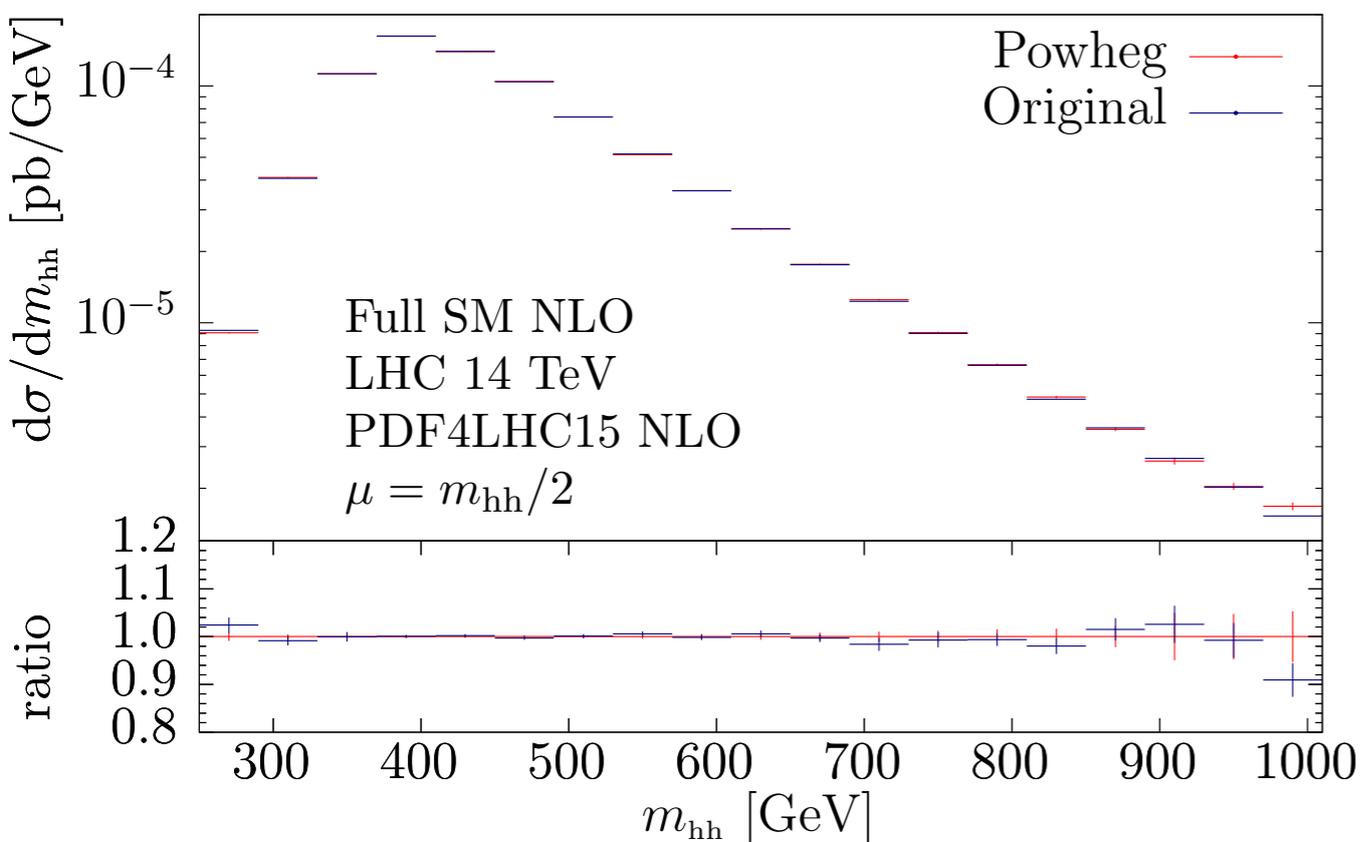
→ construct grid for interpolation of virtual amplitude (3741 2-loop results used)

input parameters (\hat{s}, \hat{t}) transformed to

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos \theta| = \left| \frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})} \right|, \quad \beta = \left(1 - \frac{4m_H^2}{\hat{s}} \right)^{\frac{1}{2}}$$

→ nearly uniform distribution of phase space points in $(x, c_\theta) \in [0, 1]^2$ if $f(\beta)$ chosen according to cumulative distribution of points in original calculation

Grid validation:



LHE Events in HEFT & comparison with NNLO

Les Houches Event Level:

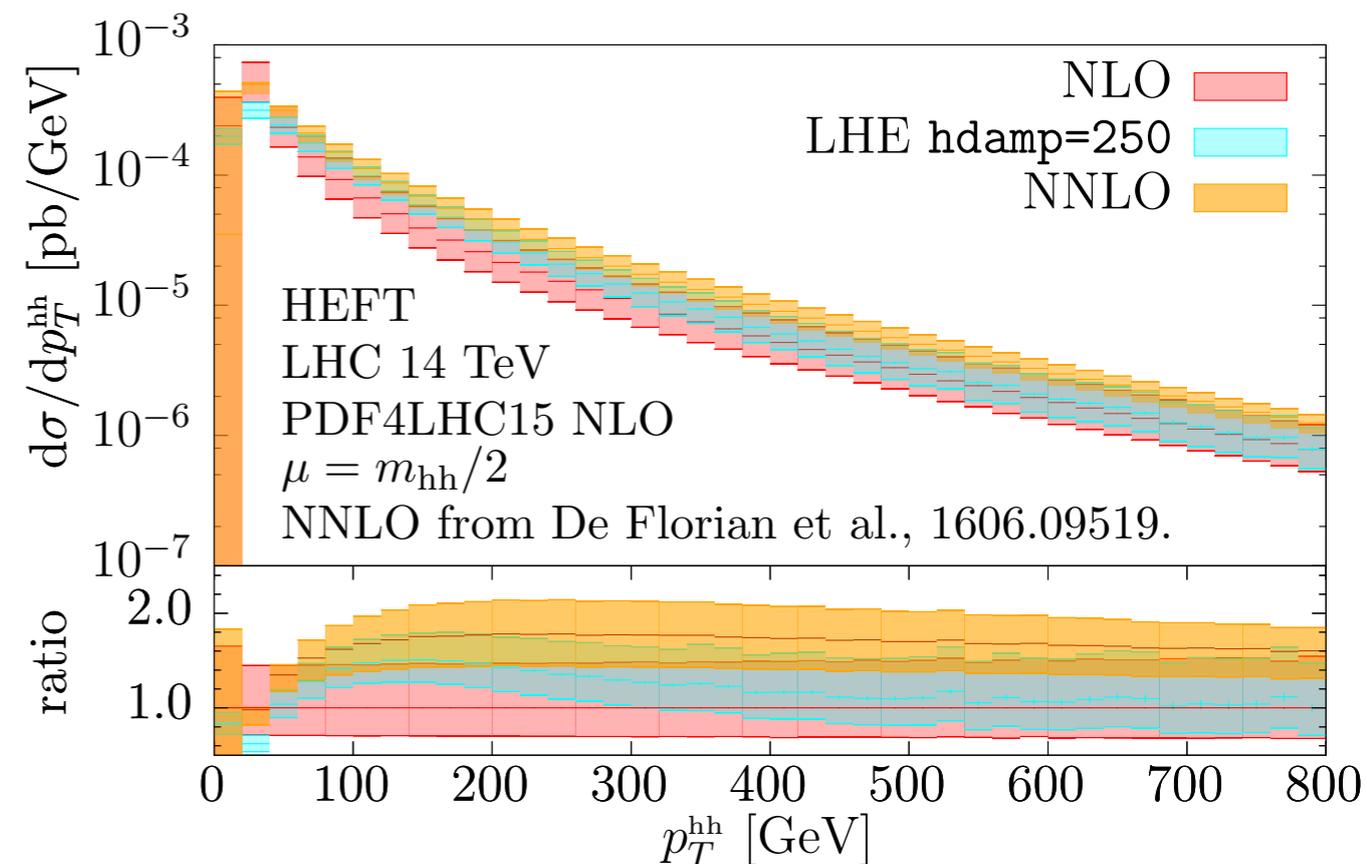
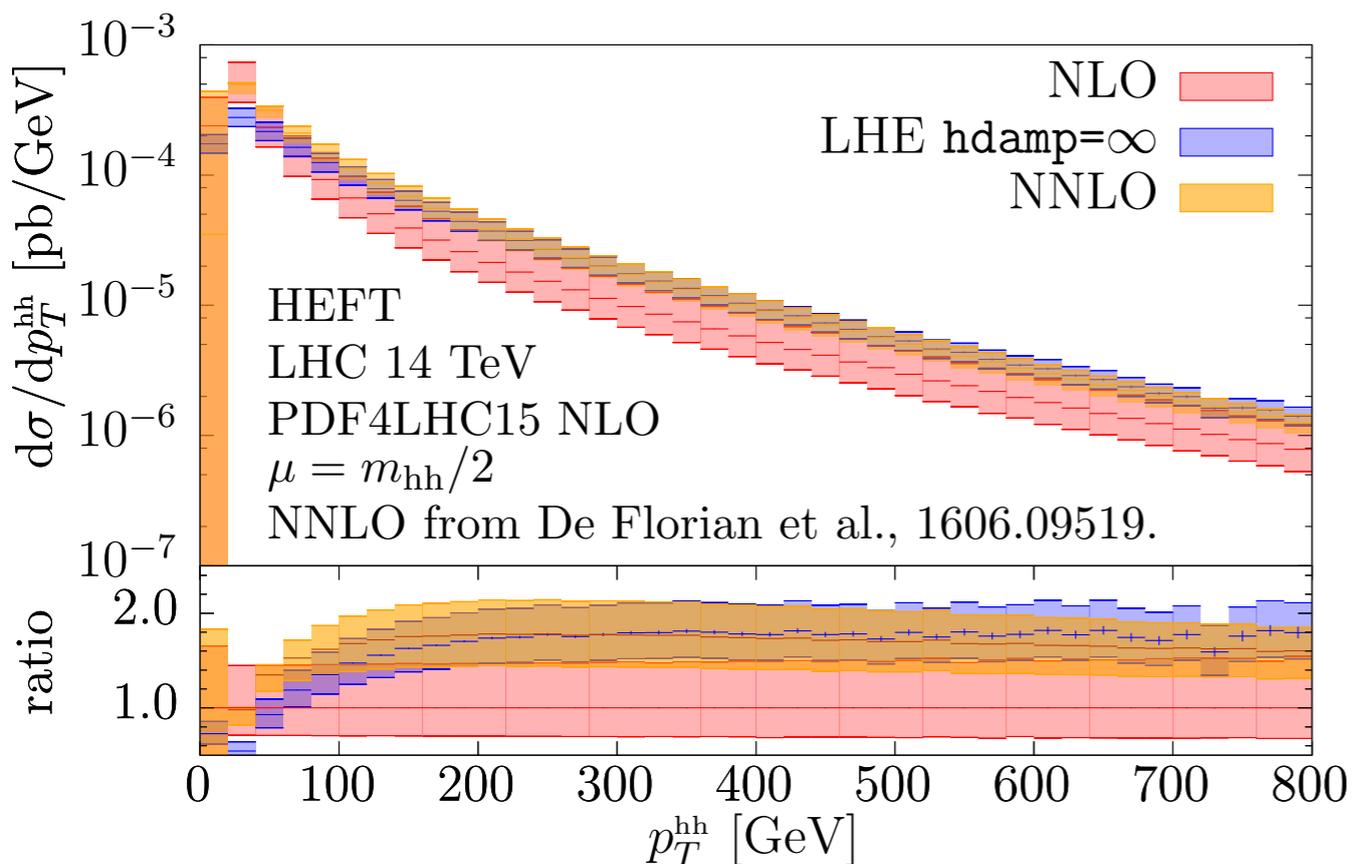
Sudakov factor included, but no parton shower

Powheg allows to split real radiation into (exponentiated) singular and regular part

$$R_{\text{sing}} = R \times F$$

$$R_{\text{reg}} = R \times (1 - F)$$

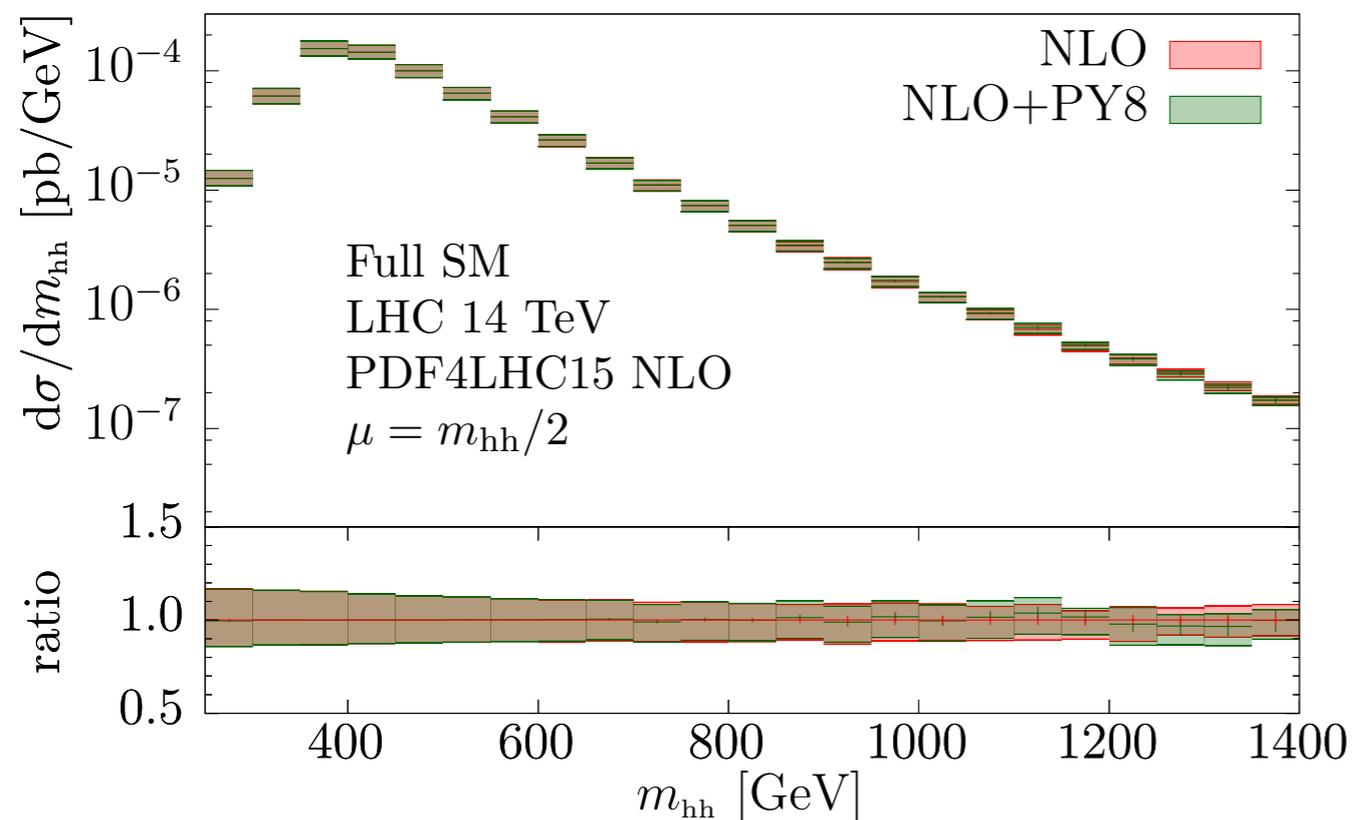
with
$$F = \frac{h^2}{(p_T^{\text{hh}})^2 + h^2}$$



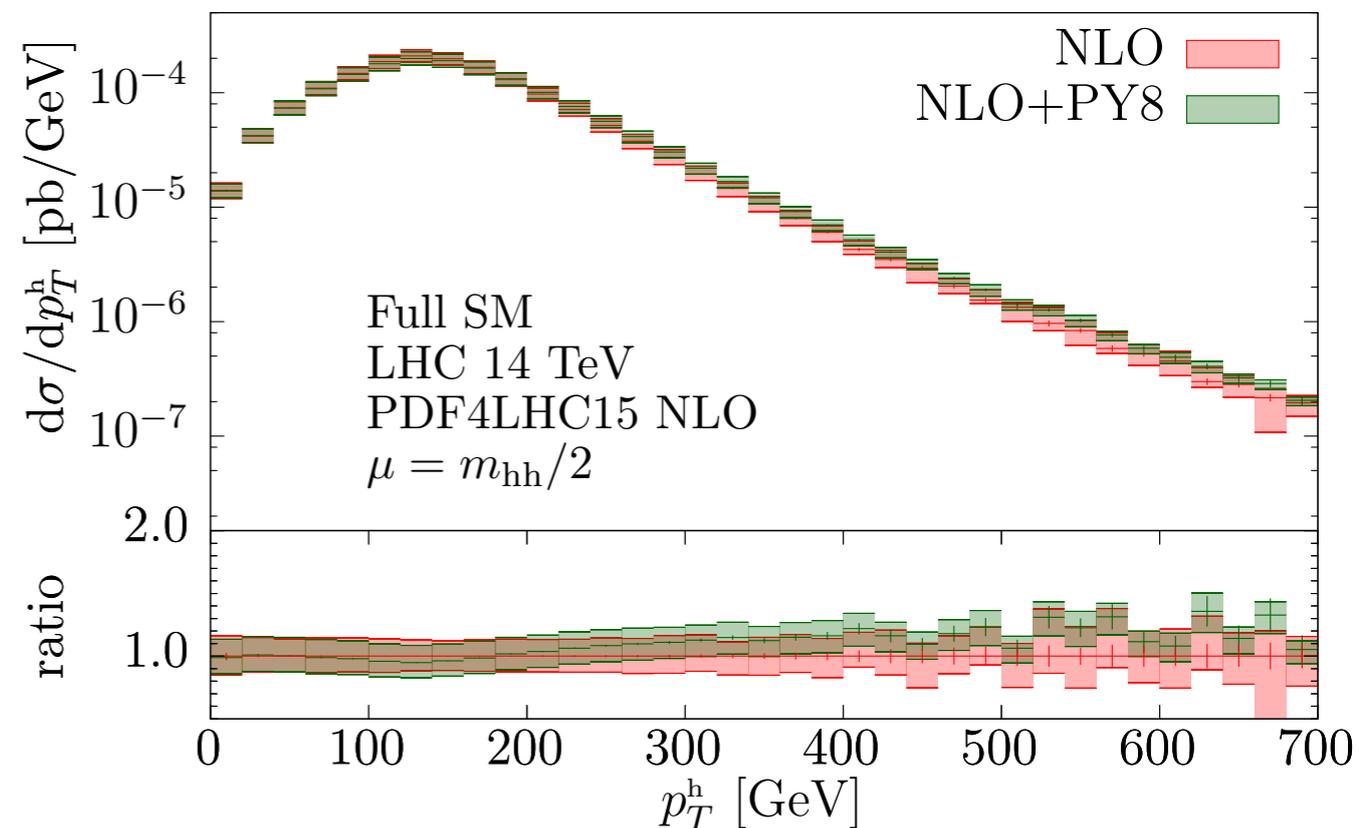
- $h = \infty$: LHE level results close to NNLO
- $h = 250$: LHE level approaches NLO in tail of p_T^{hh} distribution

Results including Parton Shower

Powheg + Pythia8



no effect on invariant mass



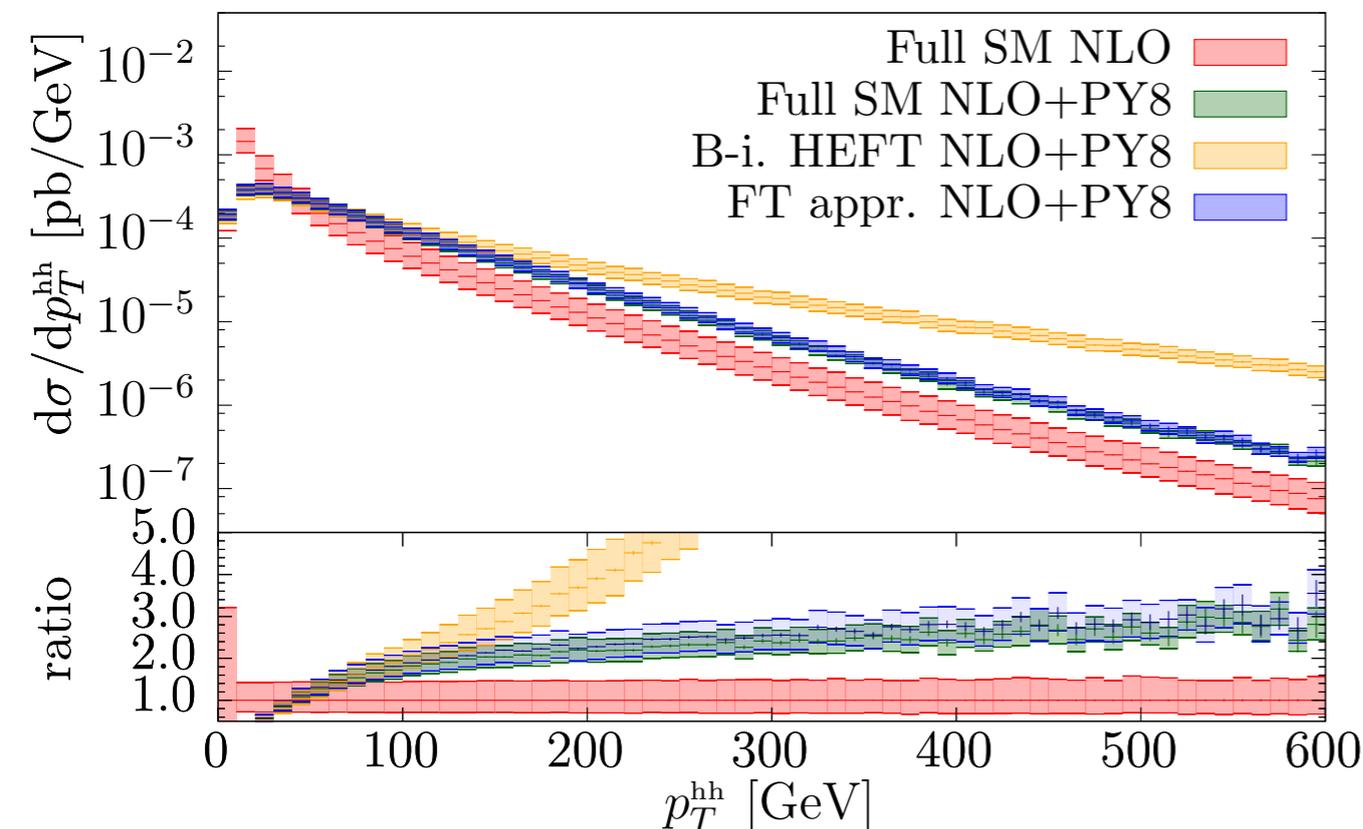
parton shower enhances tails of p_T distributions

only small parton shower effects on NLO accurate observables

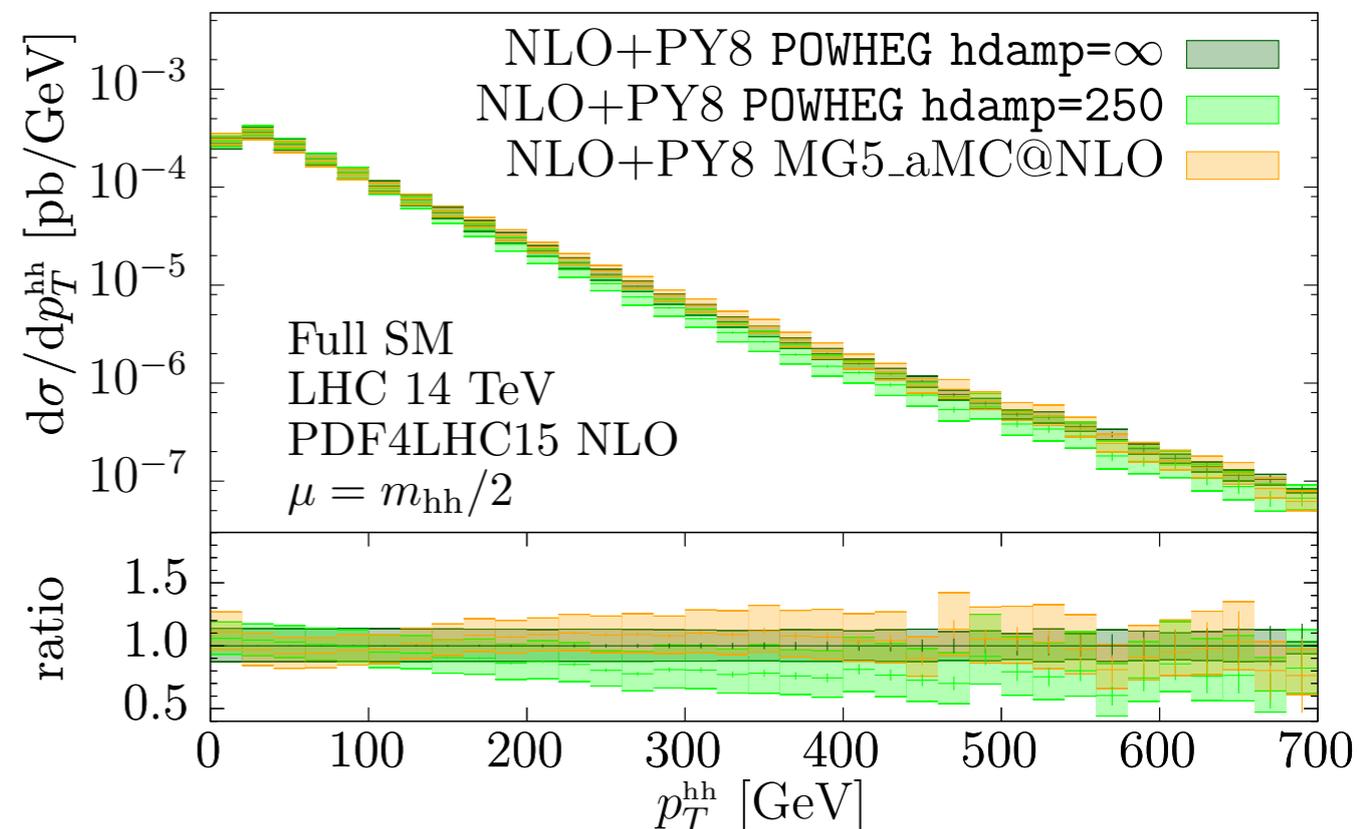
Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. p_T^{hh}

Powheg



MadGraph5_aMC@NLO

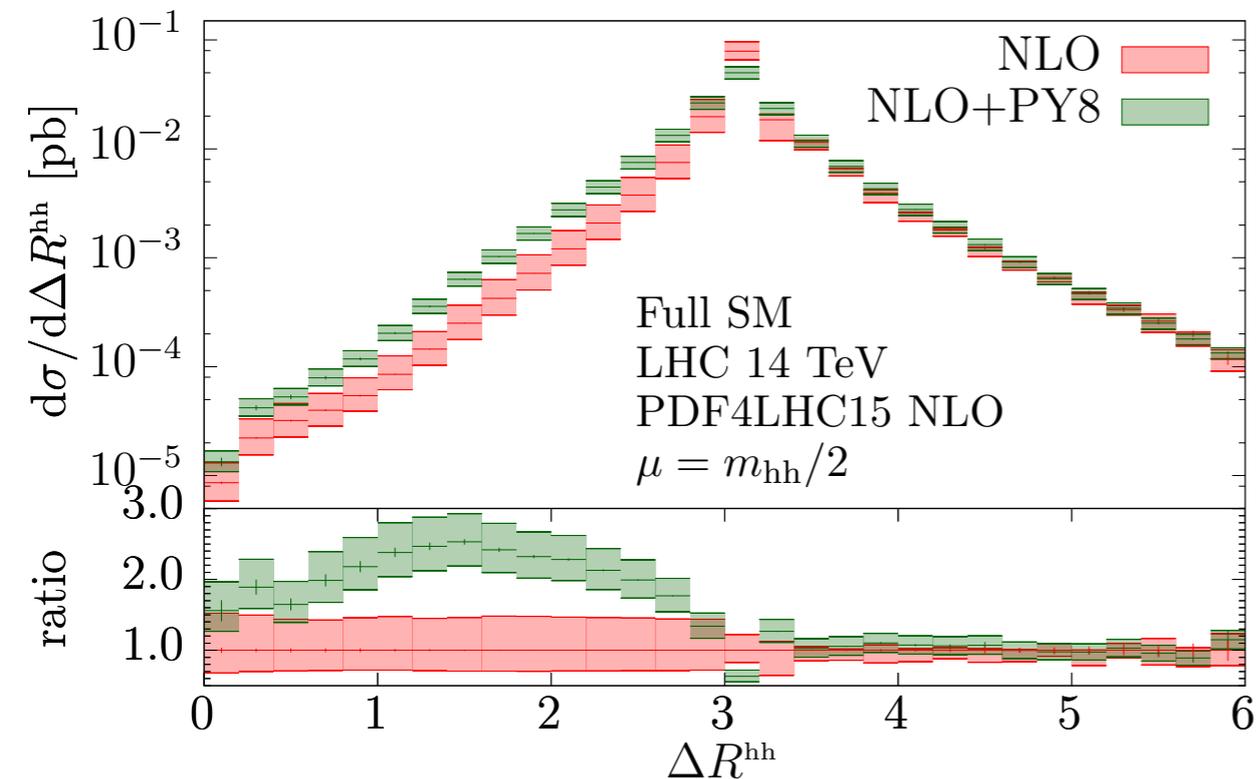


- parton shower enhances tail p_T^{hh} distribution by factor of ~ 2
- difference of matching schemes of $\sim 20\%$
- small difference between full NLO and FT approx. result

Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. ΔR^{hh}

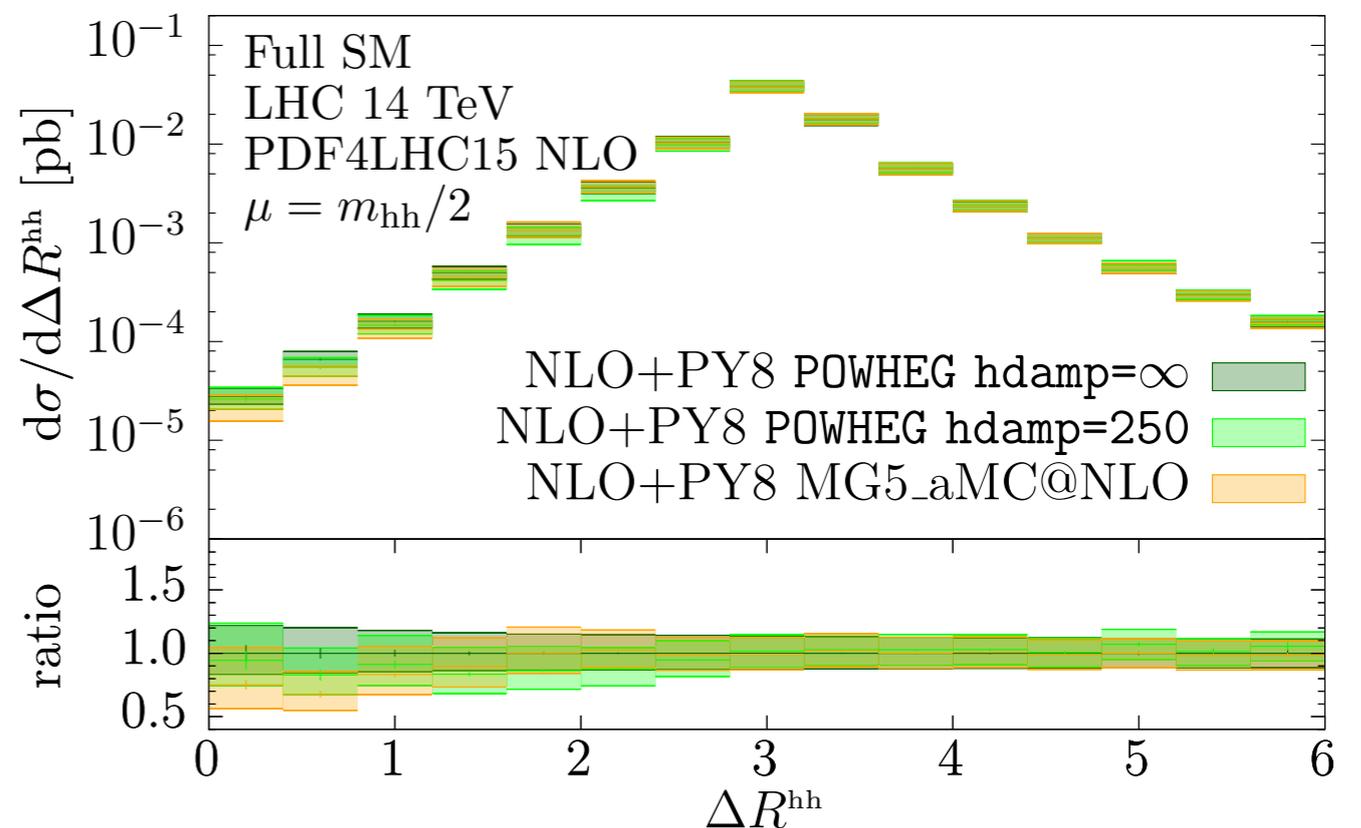
Powheg



$$\Delta R^{hh} < \pi$$

- filled by real radiation
- only LO accurate
- parton shower corrections up to factor of ~ 2.5
- differences due to matching method visible

MadGraph5_aMC@NLO



$$\Delta R^{hh} > \pi$$

- NLO accurate
- small dependence on parton shower / matching

Summary & Outlook

Higgs pair production at NLO

- retaining full m_t dependence
- numeric evaluation of 2-loop amplitudes
- reduces cross section by 14% compared to Born-improved HEFT
- corrections not uniform over phase space

Matching to Parton Showers

- up to ~20% differences for NLO accurate observables
only small dependence on matching method
- effects can be large for LO accurate observables

Outlook

- comparison with Herwig and Sherpa parton shower
- combination of NLO in full theory with NNLO HEFT

Backup

Two Loop Amplitude

- tensor structure Glover, van der Bij '88

$$\mathcal{M} = \epsilon_\mu(p_1, n_1) \epsilon_\nu(p_2, n_2) \mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = A_1(s, t, m_H^2, m_t^2, D) T_1^{\mu\nu} + A_2(s, t, m_H^2, m_t^2, D) T_2^{\mu\nu}$$

with

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2 (p_1 \cdot p_2)} \left\{ m_H^2 p_1^\nu p_2^\mu - 2 (p_1 \cdot p_3) p_3^\nu p_2^\mu - 2 (p_2 \cdot p_3) p_3^\nu p_1^\mu + 2 (p_1 \cdot p_2) p_3^\nu p_3^\mu \right\}$$

$$\begin{aligned} \mathcal{M}^{++} &= \mathcal{M}^{--} = -A_1 \\ \mathcal{M}^{+-} &= \mathcal{M}^{-+} = -A_2 \end{aligned}$$

triangle diagrams $gg \rightarrow H \rightarrow HH$
only contribute to A_1

- projectors

construct $P_i^{\mu\nu} = \sum_j c_{ij} T_j^{\mu\nu}$ such that

$$\begin{aligned} P_1^{\mu\nu} \mathcal{M}_{\mu\nu} &= A_1(s, t, m_H^2, m_t^2, D) \\ P_2^{\mu\nu} \mathcal{M}_{\mu\nu} &= A_2(s, t, m_H^2, m_t^2, D) \end{aligned}$$

Amplitude Structure

rewrite loop integrals with r propagators and s inverse propagators as

$$I_{r,s}(s, t, m_h^2, m_t^2) = (M^2)^{-L\epsilon} (M^2)^{2L-r+s} I_{r,s} \left(\frac{s}{M^2}, \frac{t}{M^2}, \frac{m_h^2}{M^2}, \frac{m_t^2}{M^2} \right)$$

arbitrary scale

and write renormalized form factors as

$$F^{\text{virt}} = aF^{(1)} + a^2 \left(\frac{n_g}{2} \delta Z_A + \delta Z_a \right) F^{(1)} + a^2 \delta m_t^2 F^{\text{ct},(1)} + a^2 F^{(2)} + \mathcal{O}(a^3)$$

$$F^{(1)} = \left(\frac{\mu_R^2}{M^2} \right)^\epsilon \left[b_0^{(1)} + b_1^{(1)} \epsilon + b_2^{(1)} \epsilon^2 + \mathcal{O}(\epsilon^3) \right], \quad \text{(1-loop)}$$

$$F^{\text{ct},(1)} = \left(\frac{\mu_R^2}{M^2} \right)^\epsilon \left[c_0^{(1)} + c_1^{(1)} \epsilon + \mathcal{O}(\epsilon^2) \right], \quad \text{(mass counter-term)}$$

$$F^{(2)} = \left(\frac{\mu_R^2}{M^2} \right)^{2\epsilon} \left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon) \right], \quad \text{(2-loop)}$$

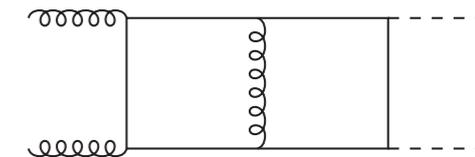
→ scale variations do not require re-computation of $b_i^{(n)}, c_i^{(n)}$

Amplitude Evaluation — Example

$$\sqrt{s} = 327.25 \text{ GeV}, \sqrt{-t} = 170.05 \text{ GeV}, M^2 = s/4$$

contributing integrals:

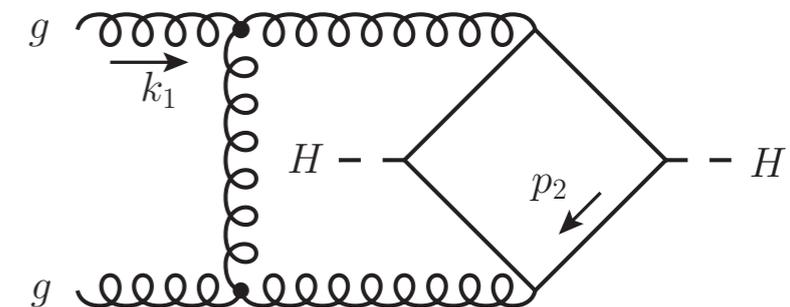
integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



≈ 700
integrals

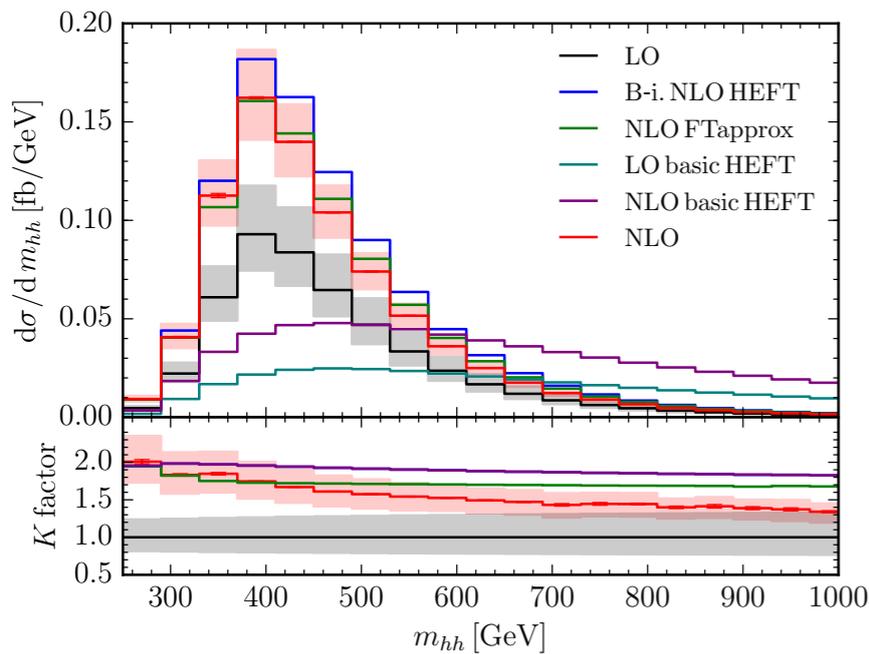
$$I(s, t, m_t^2, m_h^2) = - \left(\frac{\mu^2}{M^2} \right)^{2\epsilon} \Gamma(3 + 2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon) \right)$$

sector decomposition



sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

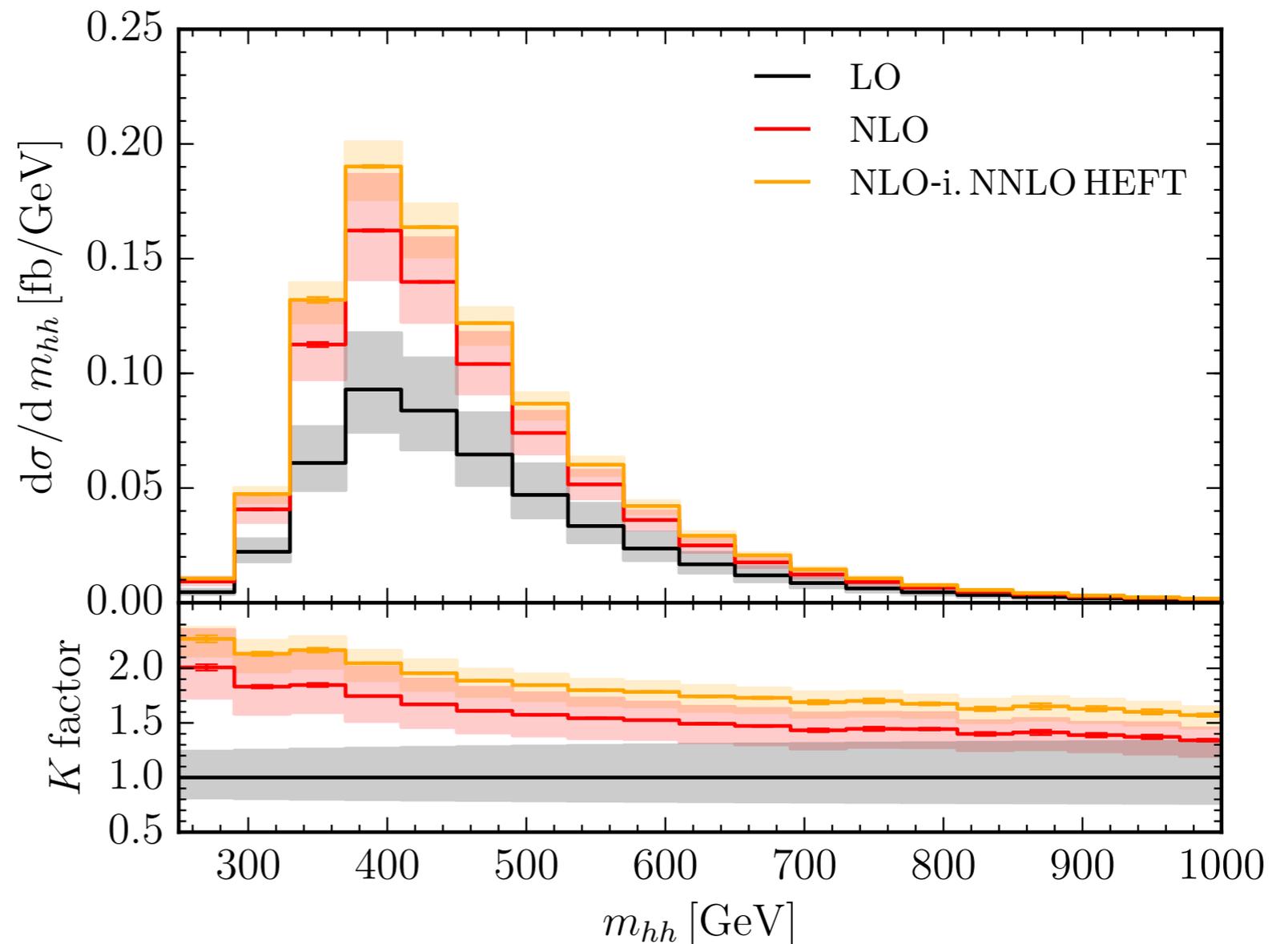
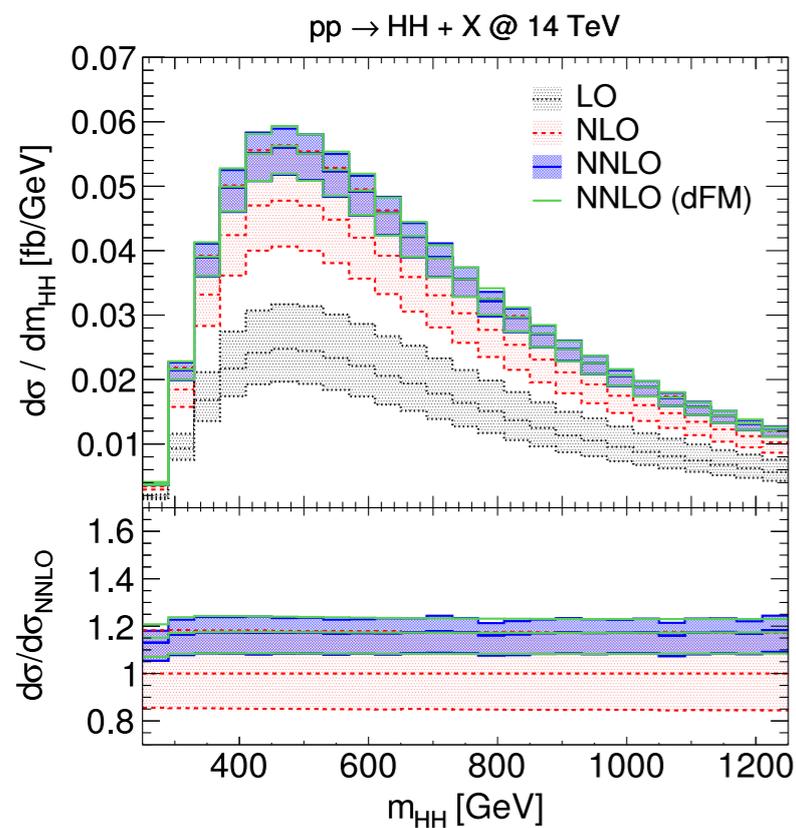
Results - Combination with NNLO_{HEFT}



first attempt to combine NLO_{full} with NNLO_{HEFT}

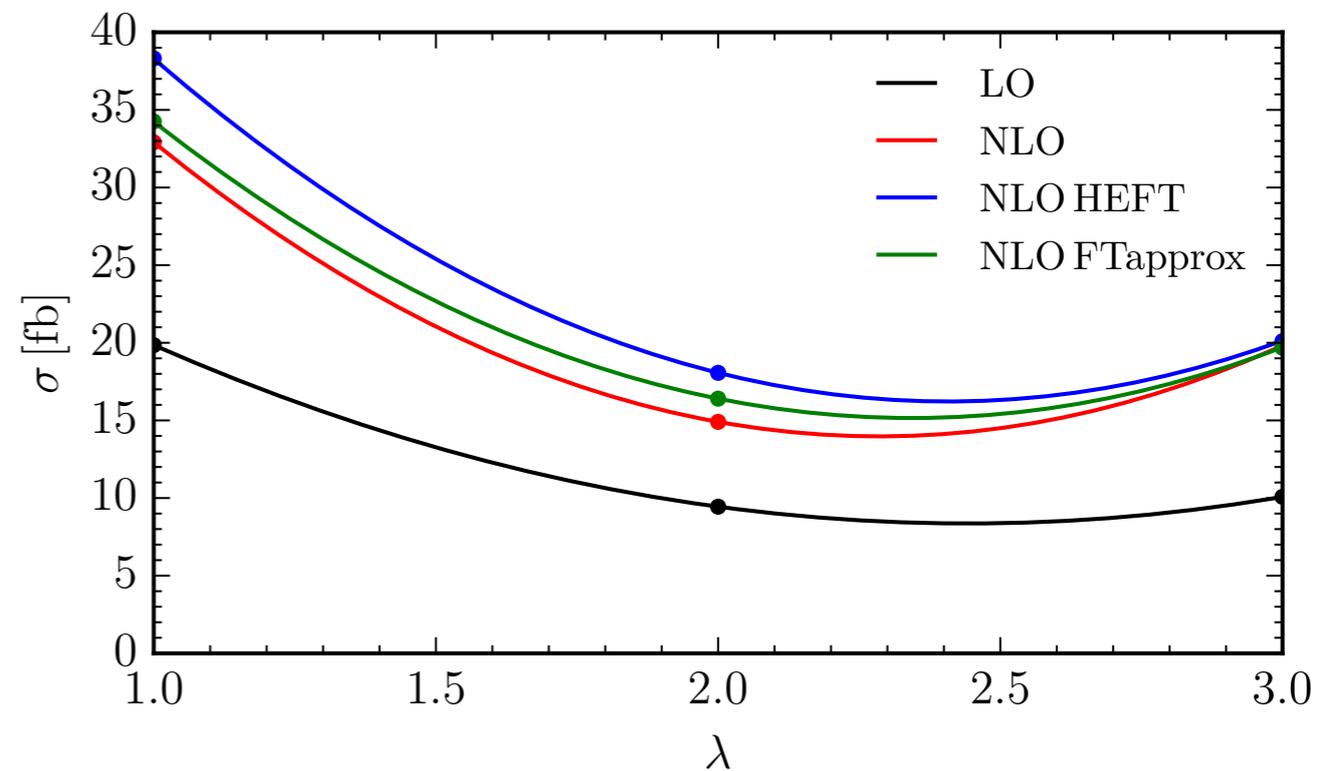
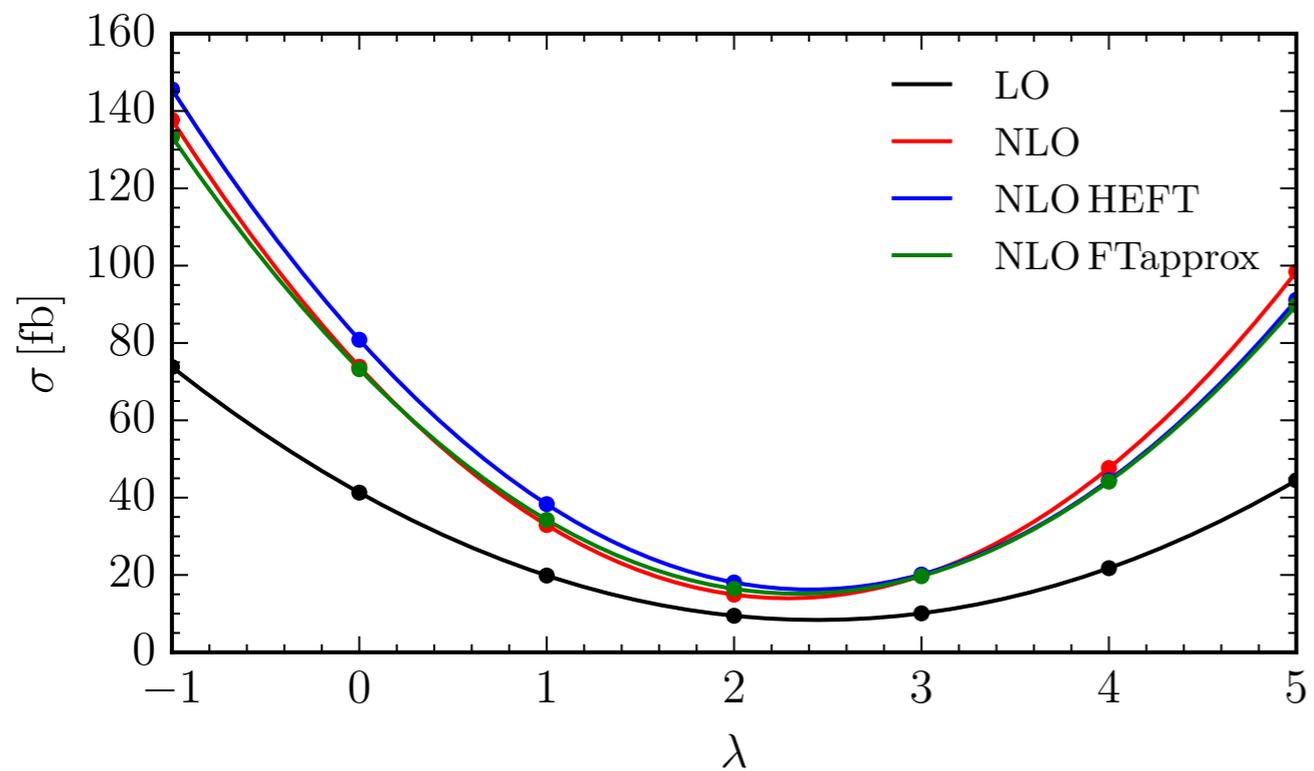
NLO-improved NNLO HEFT:

$$\frac{d\sigma_{\text{NLO}}^{\text{full}}}{dm_{hh}} \cdot \frac{d\sigma_{\text{NNLO}}^{\text{HEFT}}/dm_{hh}}{d\sigma_{\text{NLO}}^{\text{HEFT}}/dm_{hh}}$$

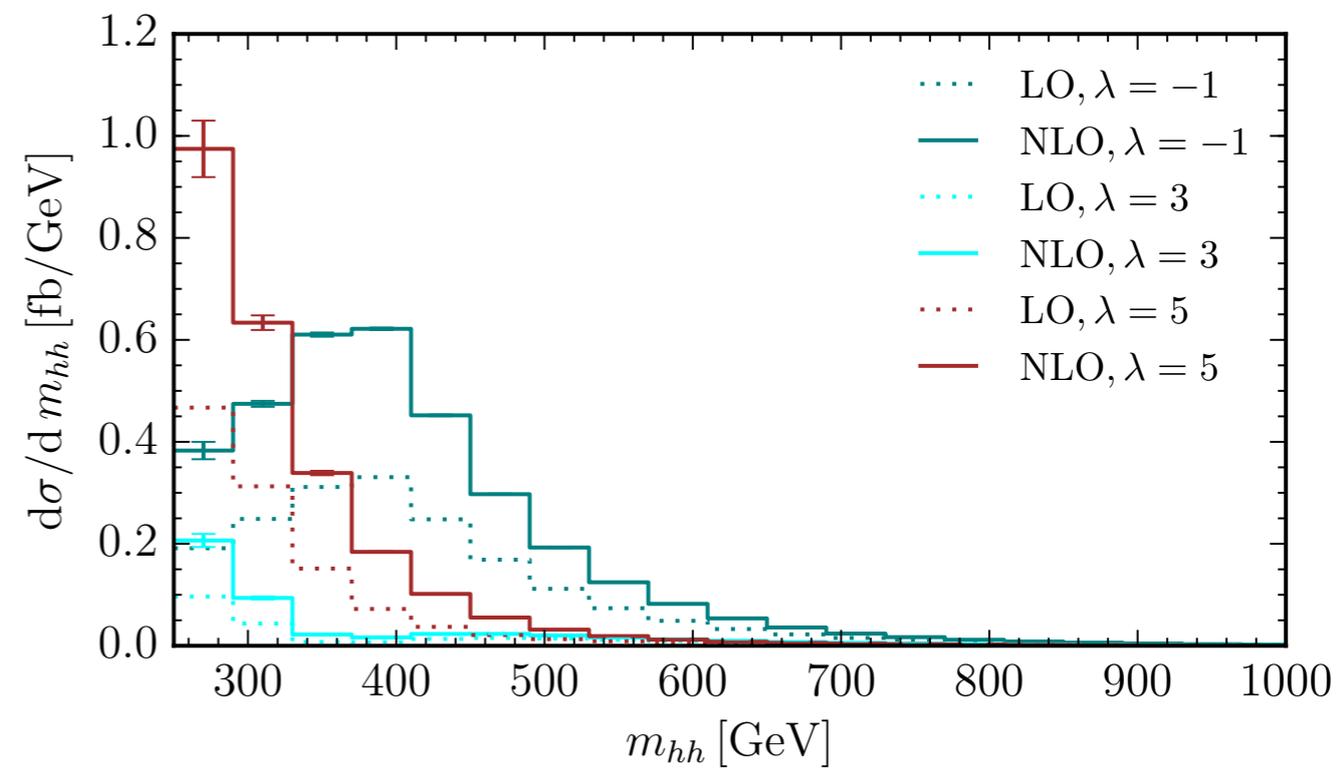
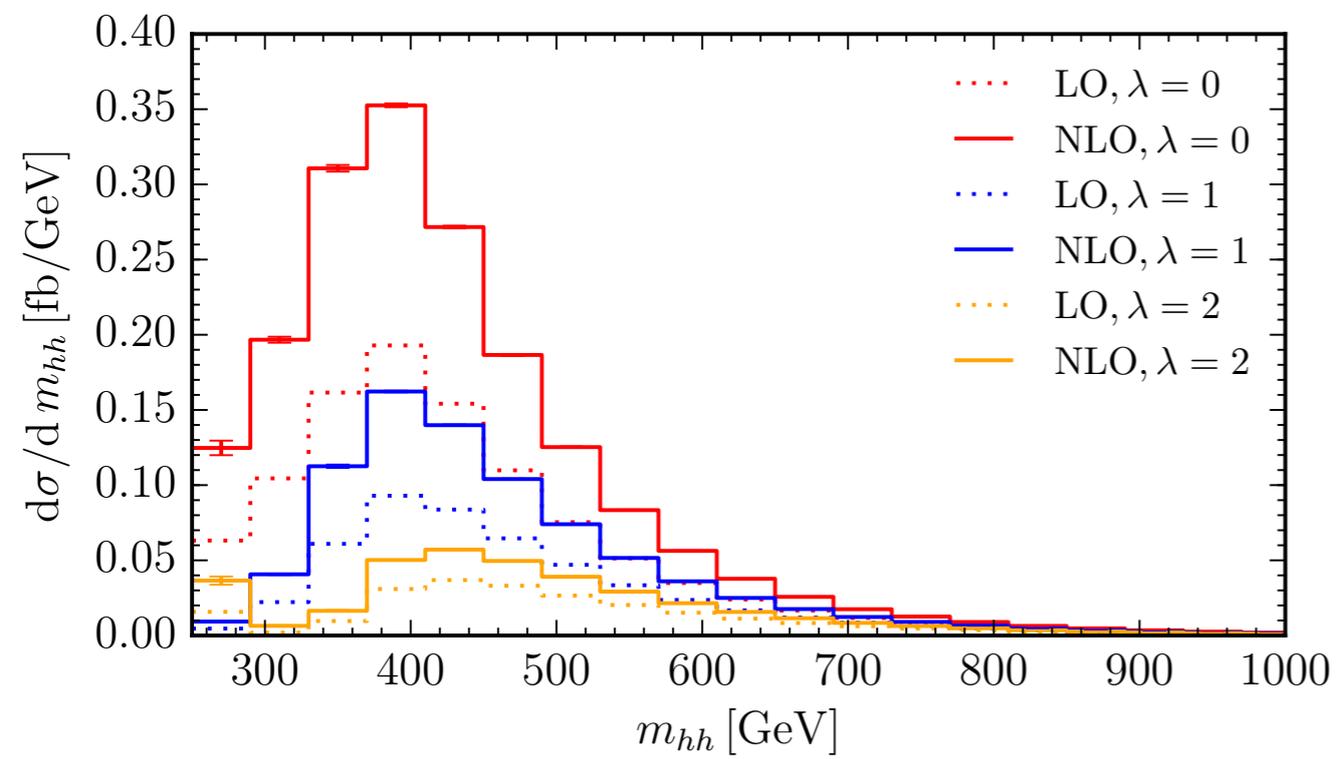


de Florian, Grazzini, Hanga,
Kallweit, Lindert, Maierhöfer,
Mazzitelli, Rathlev `16

modified Higgs self-interactions

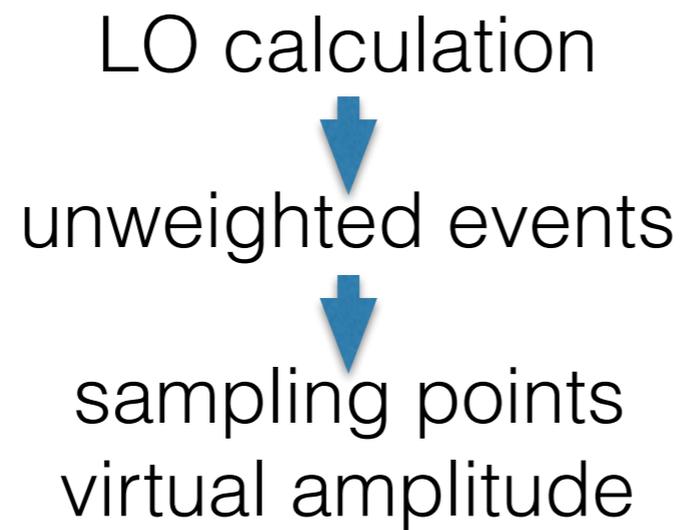


modified Higgs self-interactions



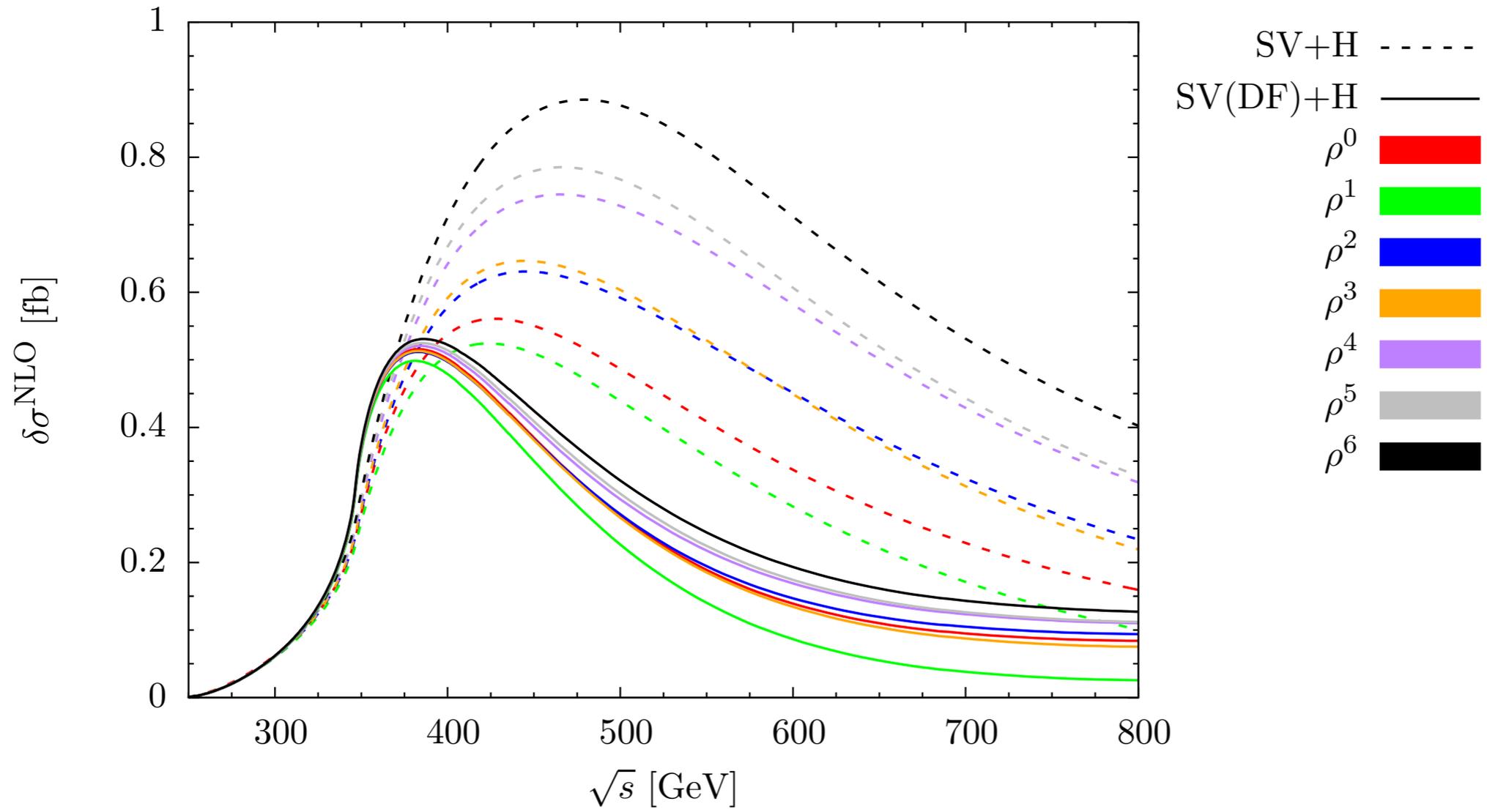
Calculation of σ^V

- Importance sampling:

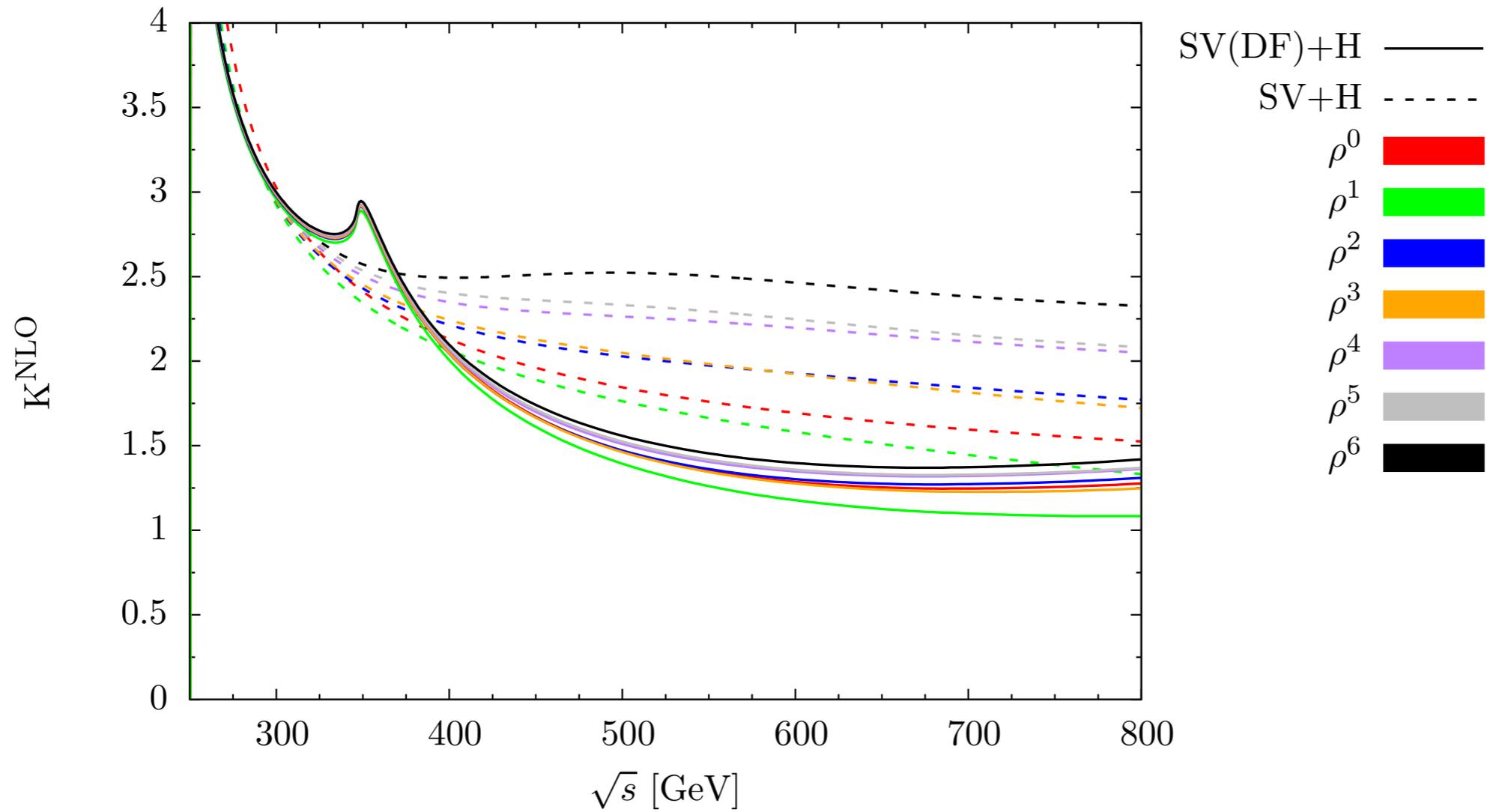


} σ^V with 2.5% accuracy
using
~1000 phase-space points

- Accuracy goal:
 - 3% for form factor F_1
 - 5-20% for form factor F_2 (depending on F_2/F_1)
- Run time: (gpu time)
 - 80 min - 2 d (\triangleq wall-clock limit)
 - median: 2h

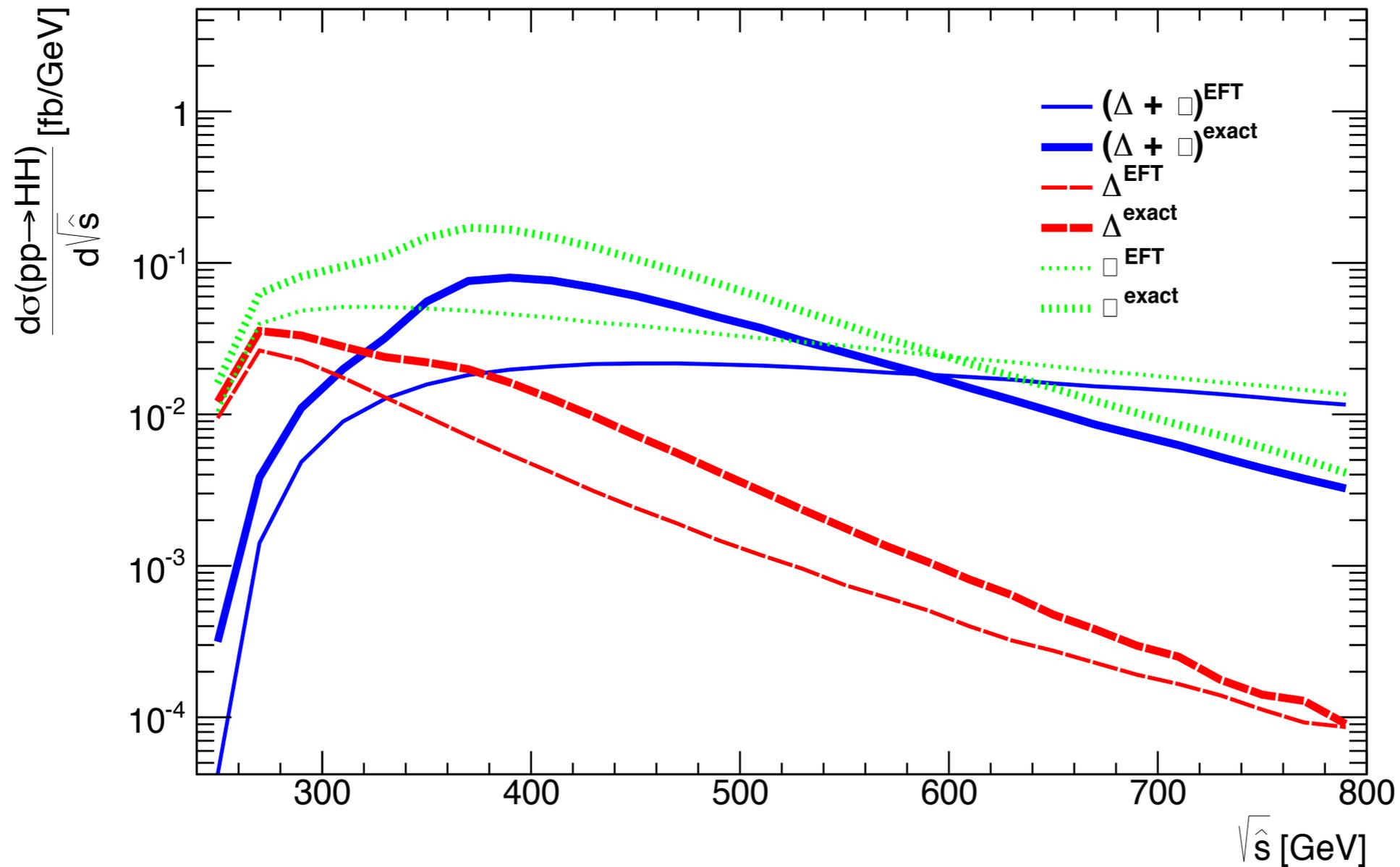


Grigo, Hoff, Steinhauser `15



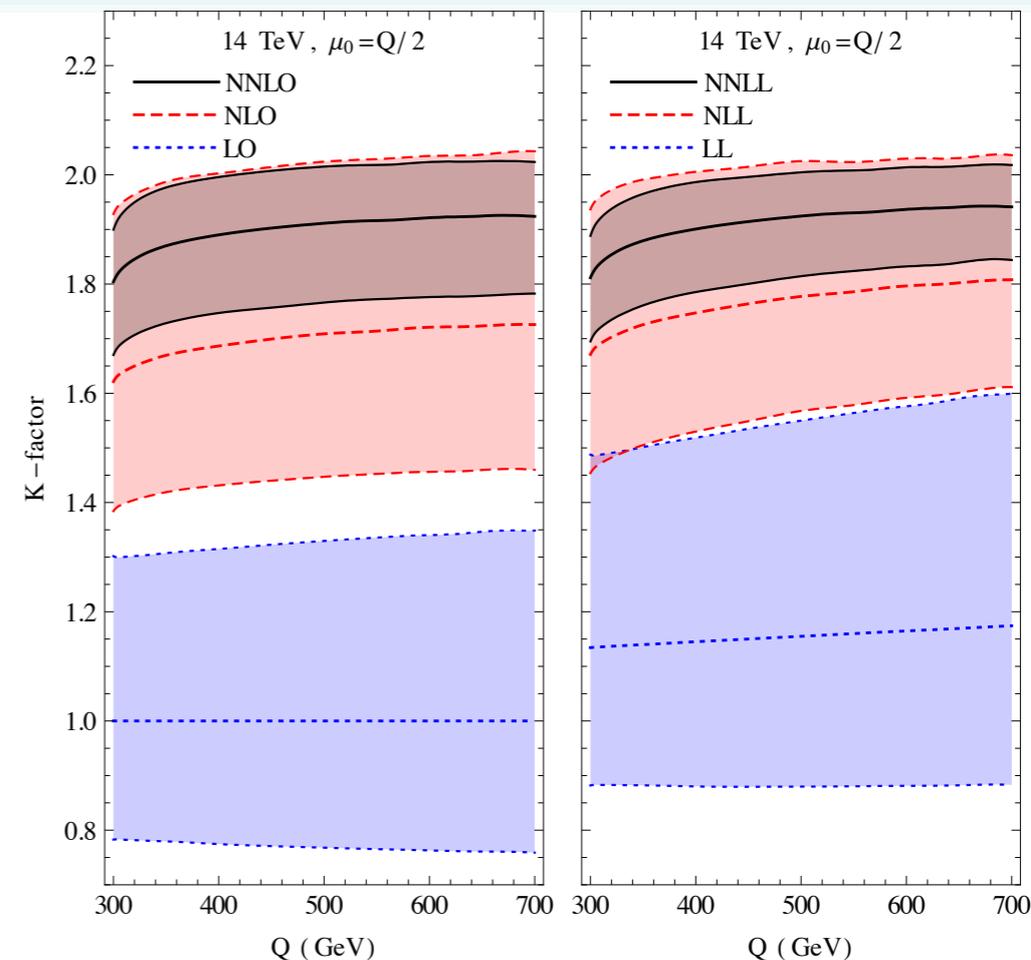
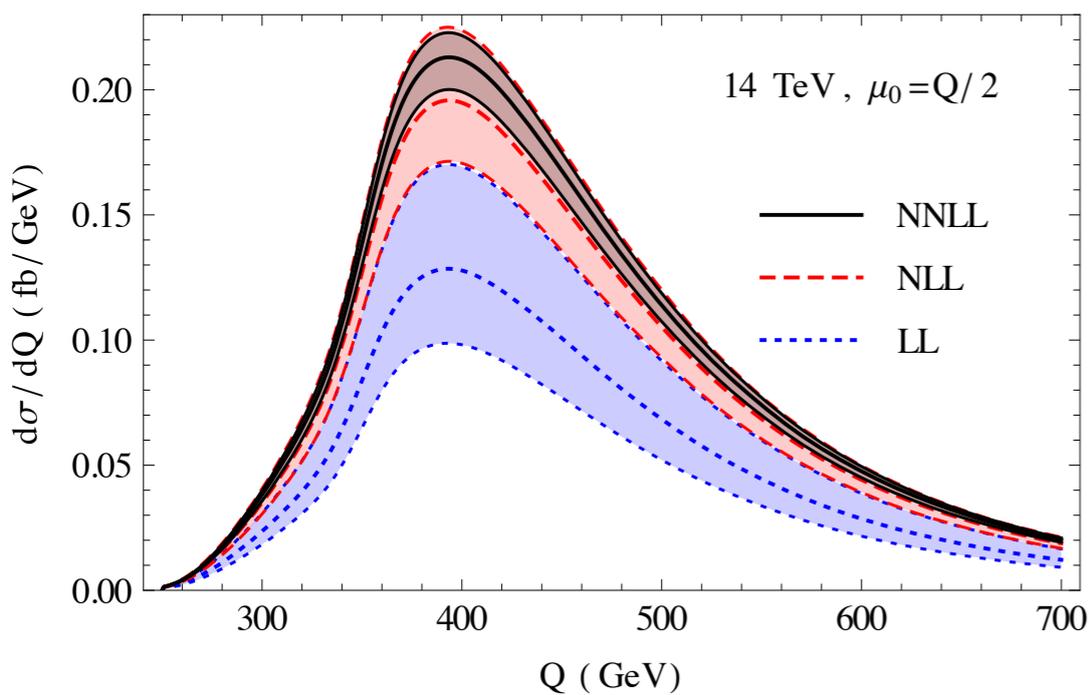
Grigo, Hoff, Steinhauser '15

Differential Cross Section



Slawinska, van den Wollenberg,
van Eijk, Bentvelsen '14

NNLO and NNLL results

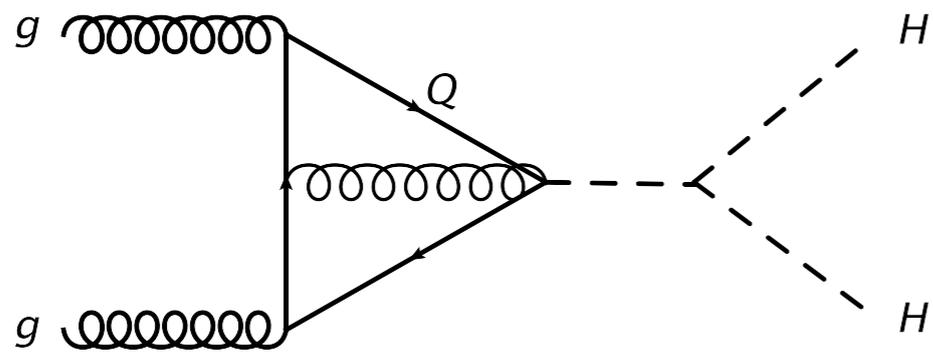


de Florian, Mazzitelli '15

$\mu_0 = Q$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+ α_S unc. (%)
8 TeV	9.92	+9.3 – 10	10.8	+5.4 – 5.9	+5.6 – 6.0	+9.3 – 9.2
13 TeV	34.3	+8.3 – 8.9	36.8	+5.1 – 6.0	+4.0 – 4.3	+7.7 – 7.5
14 TeV	40.9	+8.2 – 8.8	43.7	+5.1 – 6.0	+3.8 – 4.0	+7.5 – 7.3
33 TeV	247	+7.1 – 7.4	259	+5.0 – 6.1	+2.2 – 2.8	+6.1 – 6.1
100 TeV	1660	+6.8 – 7.1	1723	+5.2 – 6.1	+2.1 – 3.0	+5.7 – 5.8
$\mu_0 = Q/2$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+ α_S unc. (%)
8 TeV	10.8	+5.7 – 8.5	11.0	+4.0 – 5.6	+5.8 – 6.1	+9.6 – 9.3
13 TeV	37.2	+5.5 – 7.6	37.4	+4.2 – 5.8	+4.1 – 4.3	+7.8 – 7.6
14 TeV	44.2	+5.5 – 7.6	44.5	+4.2 – 5.9	+3.9 – 4.1	+7.6 – 7.4
33 TeV	264	+5.3 – 6.6	265	+4.6 – 6.1	+2.4 – 2.7	+6.3 – 6.1
100 TeV	1760	+5.3 – 6.7	1762	+4.9 – 6.4	+2.2 – 3.1	+6.2 – 7.0

Analytically known integrals

3-point, 1 off-shell leg



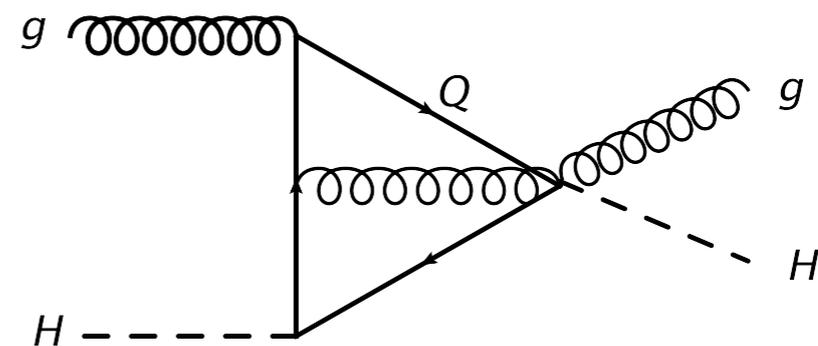
Spira, Djouadi et al. '93, '95

Bonciani, Mastrolia '03, '04

Anastasiou, Beerli et al. '06

→ HPLs

3-point, 2 off-shell leg



Gehrmann, Guns, Kara '15

→ generalized HPLs,
12 letters

Amplitude Structure

Form factors are sums of rational functions multiplied by integrals that depend on ratios of the scales s, t, m_h^2, m_t^2 and the arbitrary scale M^2

$$\begin{aligned} F^{(L)} &= \sum_i \left[\left(\sum_j C_{i,j}^{(L)} \epsilon^j \right) \cdot \left(\sum_k I_{i,k}^{(L)} \epsilon^k \right) \right] \\ &= \epsilon^{-2} \left[C_{1,-2}^{(L)} \cdot I_{1,0}^{(L)} + C_{1,-1}^{(L)} \cdot I_{1,-1}^{(L)} + \dots \right] \\ &\quad + \epsilon^{-1} \left[C_{1,-1}^{(L)} \cdot I_{1,0}^{(L)} + \dots \right] + \dots \end{aligned}$$

compute only once

Additionally, all L -loop form factors are computed simultaneously without re-evaluating common integrals

Note: $gg \rightarrow HH$ is a loop induced process, real subtraction and mass factorisation contained in $\mathbf{I}, \mathbf{P}, \mathbf{K}$ operators (not discussed here)

Catani, Seymour 96

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Phase-Space Sampling

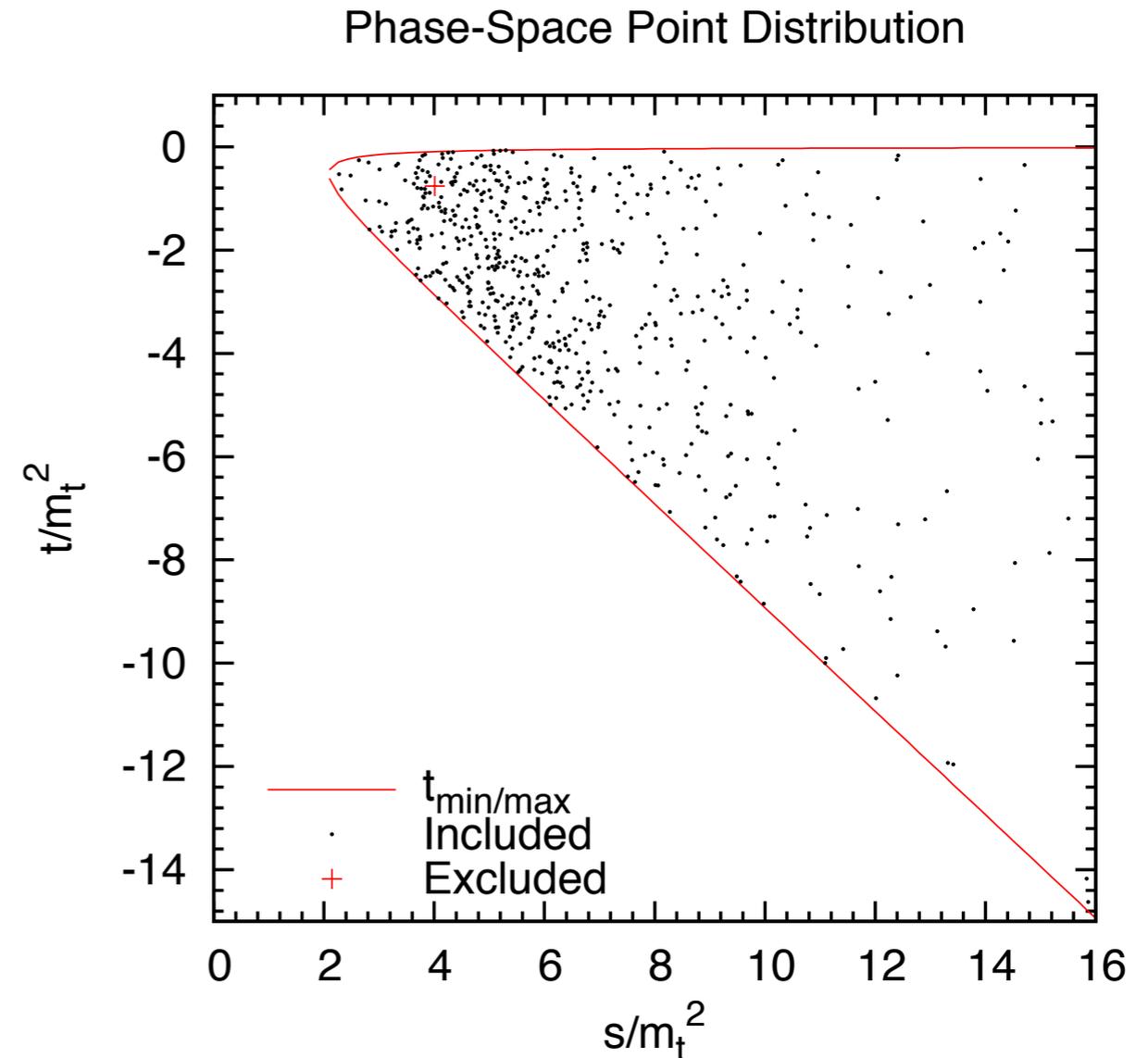
Phase-space implemented by hand

limited to 2-3 w/ 2 massive particles

Events for virtual:

- 1) VEGAS algorithm applied to LO matrix element $\mathcal{O}(100k)$ events computed
- 2) Using LO events unweighted events generated using accept/reject method $\mathcal{O}(30k)$ events remain
- 3) Randomly select 666 Events (woops), compute at NLO, exclude 1

Note: No grids used either for integrals or phase-space



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