Two aspects of Regge limit in QCD:
Double Logs in Exclusive Observables
& Infrared Effects in Cross Sections

Agustín Sabio Vera

Universidad Autónoma de Madrid, Instituto de Física Teórica UAM/CSIC

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Two aspects of Regge limit in QCD - Plan

- Regge limit in QCD
- Double Logs in Exclusive Observables (+ a comment on the Odderon)
- Infrared Effects in Cross Sections
- Conclusions
Two aspects of Regge limit in QCD - Plan

♣ Regge limit in QCD
♣ Double Logs in Exclusive Observables
♣ Infrared Effects in Cross Sections
♣ Conclusions
Regge limit in QCD

Regge theory preludes QCD.
Microscopic description of Pomeron in terms of quarks & gluons?

We need a large scale $Q > \Lambda_{\text{QCD}}$ to use perturbation theory in $\alpha_s(Q) \ll 1$.

In the limit $s \gg t, Q^2$ we have $\alpha_s(Q) \log \left( \frac{s}{t} \right) \sim O(1)$.
These dominate the amplitudes and must be resummed to all orders.

Kinematic origin:

$$\sigma_{\text{tot}}(s = e^{y_A - y_B}) = \sum_{n=0}^{\infty} \frac{1}{s} \text{MUL I - REGGE KINEMATICS}$$

where $y_A - y_B$ is the difference in rapidity of particles A and B.
Regge limit in QCD

When $s \to \infty$ we should resum

$$\alpha_s^n \log^n (s) \sim \alpha_s^n (y_A - y_B)^n.$$  

$$\sigma_{\text{tot}}^{\text{LL}} = \sum_{n=0}^{\infty} C_n^{\text{LL}} (k_i) \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \ldots \int_{y_B}^{y_{n-1}} dy_n$$

$$= \sum_{n=0}^{\infty} \frac{C_n^{\text{LL}} (k_i)}{n!} \alpha_s^n (y_A - y_B)^n$$

LL BFKL formalism allows us to calculate the coefficients $C_n^{\text{LL}} (k_i)$.

NLL is more complicated, sensitive to the running & choice of energy scale:

$$\sigma_{\text{tot}} = \sum_{n=1}^{\infty} \frac{C_n^{\text{LL}} (k_i)}{n!} (\alpha_s - A \alpha_s^2)^n (y_A - y_B - B)^n$$

$$= \sigma_{\text{tot}}^{\text{LL}} - \sum_{n=1}^{\infty} \left( B C_n^{\text{LL}} (k_i) + (n - 1) A C_{n-1}^{\text{LL}} (k_i) \right) \frac{\alpha_s^n (y_A - y_B)^{n-1}}{(n - 1)!}$$

besides, quarks enter the game ...
Regge limit in QCD

Effective Feynman rules:
Simplest case, minijet production at LL.

Gluon Regge trajectory:
\[ \omega(\vec{q}) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2} \]

Modified propagators in the \( t \)-channel:
\[ \left( \frac{s_i}{s_0} \right)^{\omega(t_i)} = e^{\omega(t_i)(y_i-y_{i+1})} \]

\[
\left( \frac{\alpha_s N_c}{\pi} \right)^2 \int d^2 \vec{k}_1 \frac{\theta \left( k_1^2 - \lambda^2 \right)}{\pi k_1^2} \int d^2 \vec{k}_2 \frac{\theta \left( k_2^2 - \lambda^2 \right)}{\pi k_2^2} \delta(2) \left( \vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B \right) \\
\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega(\vec{k}_A)(Y-y_1)} e^{\omega(\vec{k}_A+\vec{k}_1)(y_1-y_2)} e^{\omega(\vec{k}_A+\vec{k}_1+\vec{k}_2)y_2}
\]
Regge limit in QCD

\[ \sigma(Q_1, Q_2, Y) = \int d^2 k_A d^2 k_B \phi_A(Q_1, k_A) \phi_B(Q_2, k_B) f(k_A, k_B, Y) \]

\[ f(k_A, k_B, Y) = \sum_n \delta^{(2)}(k_A - k_B) + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\alpha_s N_c}{\pi} \int d^2 k_i \frac{\theta(k_i^2 - \lambda^2)}{\pi k_i^2} \]

\[ \times \int_0^{y_i-1} dy_i e^{\omega(k_A + \sum_{l=1}^{i} k_l) - \omega(k_A + \sum_{l=1}^{i-1} k_l))y_i} \delta^{(2)} \left( k_A + \sum_{l=1}^{n} k_l - k_B \right) \]
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In collaboration with Grigorios CHACHAMIS (Madrid)

Monte Carlo implementation BFKLext

1511.03548
1512.03603
Double Logs in Exclusive Observables

Number of emissions?

Forward LO BFKL Singlet Green function ($q = 0 \text{ GeV}, c_R = 1$)

Non-forward LO BFKL Singlet Green function ($q = 5 \text{ GeV}, c_R = 1$)

Two aspects of Regge limit in QCD

Agustín Sabio Vera (UAM, IFT)
Reggeized (virtual) gluon $|p_T|$ at a given rapidity?
Double Logs in Exclusive Observables

Growth with energy?

\[ f \left( \vec{k}_A, \vec{k}_B, Y \right) = \sum_n \cdots \]

Forward LO BFKL Singlet Green function (\( q = 0 \))

Different growth for different components in the azimuthal angle:

\[ f_n \left( |\vec{k}_A|, |\vec{k}_B|, Y \right) = \int_0^{2\pi} \frac{d\theta}{2\pi} f \left( \vec{k}_A, \vec{k}_B, Y \right) \cos (n\theta) \]
All CCFM projections grow with energy, not in BFKL

This is a distinct feature of BFKL
Double Logs in Exclusive Observables

We can extend the formalism to include collinear regions

\[
f = e^{\omega(\vec{k}_A)} Y \left\{ \delta^{(2)} (\vec{k}_A - \vec{k}_B) + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\alpha_s N_c}{\pi} \int d^2 \vec{k}_i \frac{\theta (k_i^2 - \lambda^2)}{\pi k_i^2} \right. \]
\[
\times \int_0^{y_i-1} dy_i e^{\left( \omega(\vec{k}_A + \sum_{i=1}^{n} \vec{k}_i) - \omega(\vec{k}_A + \sum_{i=1}^{n-1} \vec{k}_i) \right)} y_i \delta^{(2)} (\vec{k}_A + \sum_{l=1}^{n} \vec{k}_l - \vec{k}_B) \right\}
\]

Key at NLL: \( \theta (k_i^2 - \lambda^2) \rightarrow \theta (k_i^2 - \lambda^2) - \frac{\bar{\alpha_s}}{4} \ln^2 \left( \frac{k_i^2}{(\vec{k}_A + \vec{k}_i)^2} \right) \)

This resums collinear emissions for more general kinematics:

\[
\theta (k_i^2 - \lambda^2) \rightarrow \theta (k_i^2 - \lambda^2) + \sum_{n=1}^{\infty} \frac{(-\bar{\alpha}_s)^n}{2^n n!(n+1)!} \ln^{2n} \left( \frac{k_i^2}{(\vec{k}_A + \vec{k}_i)^2} \right)
\]
Double Logs in Exclusive Observables

\[ \sigma(Q_1, Q_2, Y) = \int d^2k_a d^2k_b \phi_A(Q_1, k_a) \phi_B(Q_2, k_b) f(k_a, k_b, Y) \]

This is very important to go beyond the MRK limit. For BFKL domain we need "\(\delta\)-like" impact factors \(\phi_{A,B}\) & \(Q_1 \sim Q_2\).
Double Logs in Exclusive Observables

Average transverse momentum of emitted mini-jets?

\[ \langle p_t \rangle = \frac{1}{N} \sum_{i=1}^{N} |k_i| \]

Similar \( \langle p_t \rangle_{\text{max}} \) for different energies.

Two aspects of Regge limit in QCD.
Average rapidity separation among emitted mini-jets?

\[
\langle R_y \rangle = \frac{1}{N + 1} \sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}} \approx 1 + \frac{\Delta}{Y} \ln \frac{\Delta}{Y} \text{ if } Y \approx N\Delta \text{ in MRK and } Y \gg \Delta
\]

Higher \( \langle R_y \rangle_{\text{max}} \) for higher energies: \( \Delta_{\text{LO}} \approx 0.62, \Delta_{\text{LO+DLs}} \approx 0.81 \)

Lower mini-jet multiplicity when including higher order corrections
A comment on the Odderon

We have obtained the exact ODDERON Green function:

\[ f(Y, \vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \vec{p}_5, \vec{p}_6, \vec{q}) \]

\[ q = (4, 0) \]
\[ p_1 = (10, 0) \]
\[ p_2 = (-20, 0) \]
\[ p_3 = (14, 0) \]
\[ p_4 = (20, 0) \]
\[ p_5 = (-25, 0) \]
\[ p_6 = (9, 0) \]

It contains both Bartels-Lipatov-Vacca and Janik-Wosiek solutions.
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In collaboration with Douglas ROSS (Southampton)

1605.00692
1605.08265
Infrared Effects in Cross Sections

BFKL equation at LO with running coupling $\bar{\alpha}(t) = 1/(\bar{\beta}t)$:

$$\frac{\partial}{\partial Y} G(Y, t_1, t_2) = \frac{1}{\sqrt{\bar{\beta}t_1}} \int dt K(t_1, t) \frac{1}{\sqrt{\bar{\beta}t}} G(Y, t, t_2)$$

Partial wave, with $z(t) = \left(\frac{\bar{\beta}\omega}{14\zeta(3)}\right)^{1/3} \left(t - \frac{4\ln 2}{\bar{\beta}\omega}\right)$, is

$$G_\omega(t_1, t_2) = \frac{\pi}{4} \frac{\sqrt{t_1 t_2}}{\omega^{1/3}} \left(\frac{\bar{\beta}}{14\zeta(3)}\right)^{2/3} Ai(z(t_1)) Bi(z(t_2)) \theta(t_1 - t_2) + t_1 \leftrightarrow t_2$$

Lipatov’s Trick: Homogeneous solution admits an extra piece:

$$Bi(z) \rightarrow Bi(z) + Ai(z) \cot \left(\eta - \frac{2}{3} \sqrt{\frac{\bar{\beta}\omega}{14\zeta(3)}} \left(\frac{4\ln 2}{\bar{\beta}\omega} - t_0\right)^{3/2}\right)$$

which introduces $\infty$ poles in $\omega$-plane

$t_0$ is a UV/IR matching scale and $\eta$ has a non-perturbative origin
Infrared Effects in Cross Sections

\[ G(\gamma, t_1, t_2) = \frac{1}{2\pi i} \int_C d\omega \ e^{\omega \gamma} G_\omega(t_1, t_2) \]

Integration Contours in \( \omega \)-plane

Numerical integration takes into account the cut and all Regge poles

Two aspects of Regge limit in QCD:
Growth with rapidity for the Green function

\[ \eta = 0, \eta = \frac{\pi}{4}, \eta = \frac{\pi}{2} \]
Infrared Effects in Cross Sections

Collinear behaviour for the Green function with $Y=10$

Two aspects of Regge limit in QCD

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This model of the infrared does not affect the diffusion picture for larger external scales.
For smaller scales the diffusion into the IR is suppressed.

\[ t=6 \quad t'=4 \quad Y=10 \quad \eta=0 \]

\[ t=6 \quad t'=4 \quad Y=10 \quad \eta=0.2 \]
The IR suppression is also present in DIS-like configurations.
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At high energies new degrees of freedom appear: Reggeons

This is deep: it happens in QCD, SUSY and Gravity ...

Associated symmetries imply that the range of applicability is limited

Once key observables have been identified (ratios of azimuthal angle correlations in dijet events tagged at large rapidity separation) we need to introduce more physics in the original formulation

- Collinear contributions
- Non-perturbative models
- Saturation
- Temperature
- ...

Collinear contributions
Non-perturbative models
Saturation
Temperature
...