Two aspects of Regge limit in QCD:

Double Logs in Exclusive Observables & Infrared Effects in Cross Sections

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- Regge limit in QCD
- Double Logs in Exclusive Observables (+ a comment on the Odderon)
- Infrared Effects in Cross Sections
- Conclusions

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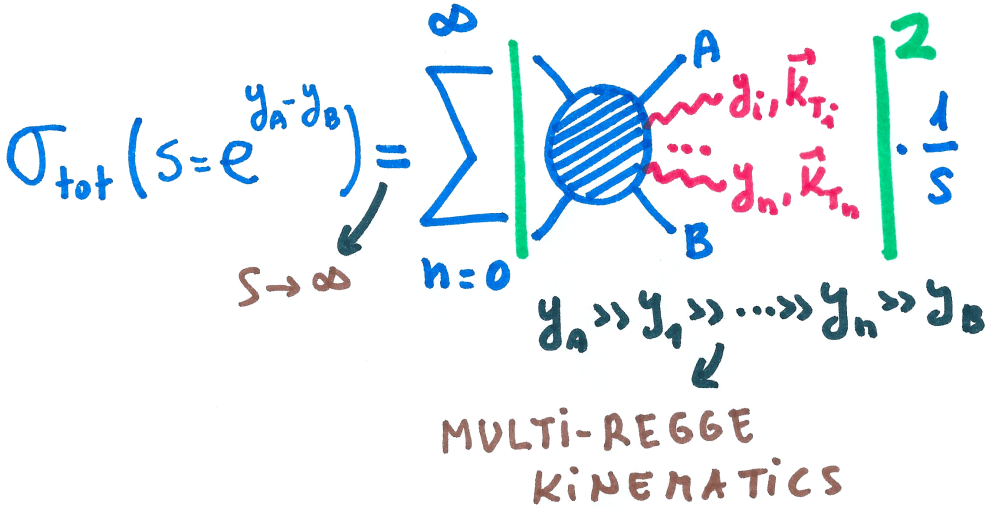
Regge theory preludes QCD.
Microscopic description of Pomeron in terms of quarks & gluons?

We need a large scale $Q > \Lambda_{\mathrm{QCD}}$ to use perturbation theory in $\alpha_s(Q) \ll 1$.

In the limit $s \gg t, Q^2$ we have $\alpha_s(Q) \log\left(\frac{s}{t}\right) \sim \mathcal{O}(1)$.
These dominate the amplitudes and must be resummed to all orders.

Kinematic origin:



where $y_A - y_B$ is the difference in rapidity of particles A and B.

When $s \to \infty$ we should resum $\alpha_s^n \log^n(s) \sim \alpha_s^n (y_A - y_B)^n$.

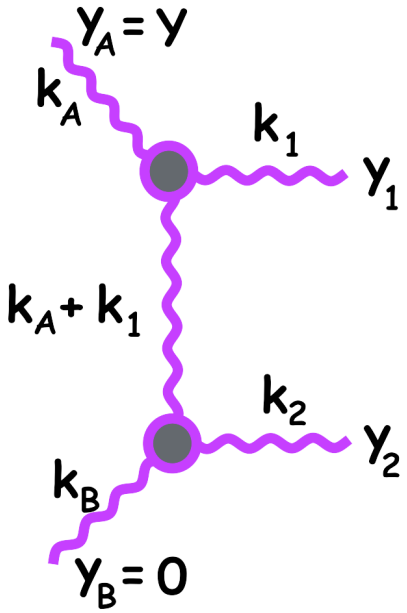
$$\begin{aligned}\sigma_{\rm tot}^{\rm LL} &= \sum_{n=0}^{\infty} \mathcal{C}_n^{\rm LL}(\mathbf{k}_i)\, \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \ldots \int_{y_B}^{y_{n-1}} dy_n \\ &= \sum_{n=0}^{\infty} \frac{\mathcal{C}_n^{\rm LL}(\mathbf{k}_i)}{n!} \underbrace{\alpha_s^n (y_A - y_B)^n}_{\rm LL}\end{aligned}$$

LL BFKL formalism allows us to calculate the coefficients $\mathcal{C}_n^{\rm LL}(\mathbf{k}_i)$.
NLL is more complicated, sensitive to the running & choice of energy scale:

$$\begin{aligned}\sigma_{\rm tot} &= \sum_{n=1}^{\infty} \frac{\mathcal{C}_n^{\rm LL}(\mathbf{k}_i)}{n!} \left(\alpha_s - \mathcal{A}\alpha_s^2\right)^n (y_A - y_B - \mathcal{B})^n \\ &= \sigma_{\rm tot}^{\rm LL} - \sum_{n=1}^{\infty} \frac{\left(\mathcal{B}\,\mathcal{C}_n^{\rm LL}(\mathbf{k}_i) + (n-1)\,\mathcal{A}\,\mathcal{C}_{n-1}^{\rm LL}(\mathbf{k}_i)\right)}{(n-1)!} \underbrace{\alpha_s^n (y_A - y_B)^{n-1}}_{\rm NLL}\end{aligned}$$

besides, quarks enter the game ...

Effective Feynman rules:
Simplest case, minijet production at LL.



Gluon Regge trajectory:

$$\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2}$$

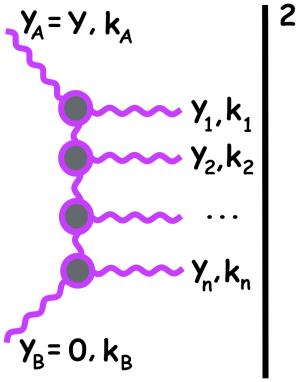
Modified propagators in the t-channel:

$$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$$

$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$
$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2\, e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right) y_2}$$

$$\sigma(Q_1, Q_2, Y) = \int d^2\vec{k}_A d^2\vec{k}_B \underbrace{\phi_A(Q_1, \vec{k}_A)\,\phi_B(Q_2, \vec{k}_B)}_{\text{PROCESS-DEPENDENT}} \underbrace{f(\vec{k}_A, \vec{k}_B, Y)}_{\text{UNIVERSAL}}$$

$$f\left(\vec{k}_A, \vec{k}_B, Y\right) = \sum_n$$



$$= e^{\omega(\vec{k}_A)Y}\left\{\delta^{(2)}\left(\vec{k}_A - \vec{k}_B\right) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i \frac{\theta\left(k_i^2 - \lambda^2\right)}{\pi k_i^2}\right.$$

$$\left.\times \int_0^{y_{i-1}} dy_i\, e^{\left(\omega\left(\vec{k}_A + \sum_{l=1}^{i}\vec{k}_l\right) - \omega\left(\vec{k}_A + \sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\,\delta^{(2)}\left(\vec{k}_A + \sum_{l=1}^{n}\vec{k}_l - \vec{k}_B\right)\right\}$$

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In collaboration with Grigorios CHACHAMIS (Madrid)

Monte Carlo implementation BFKLex

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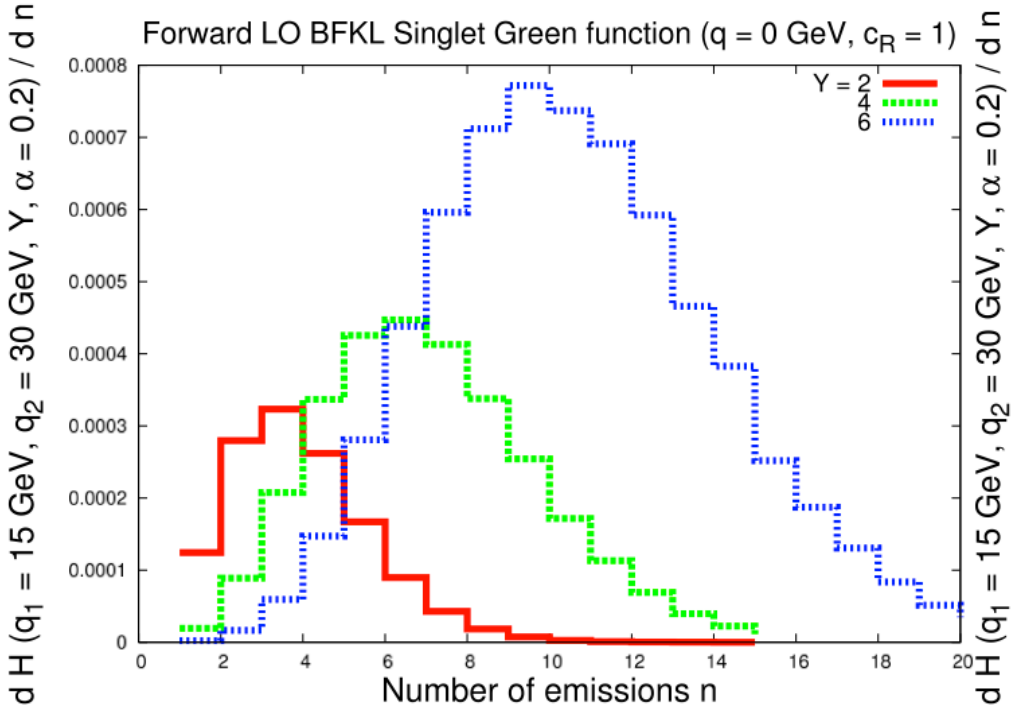
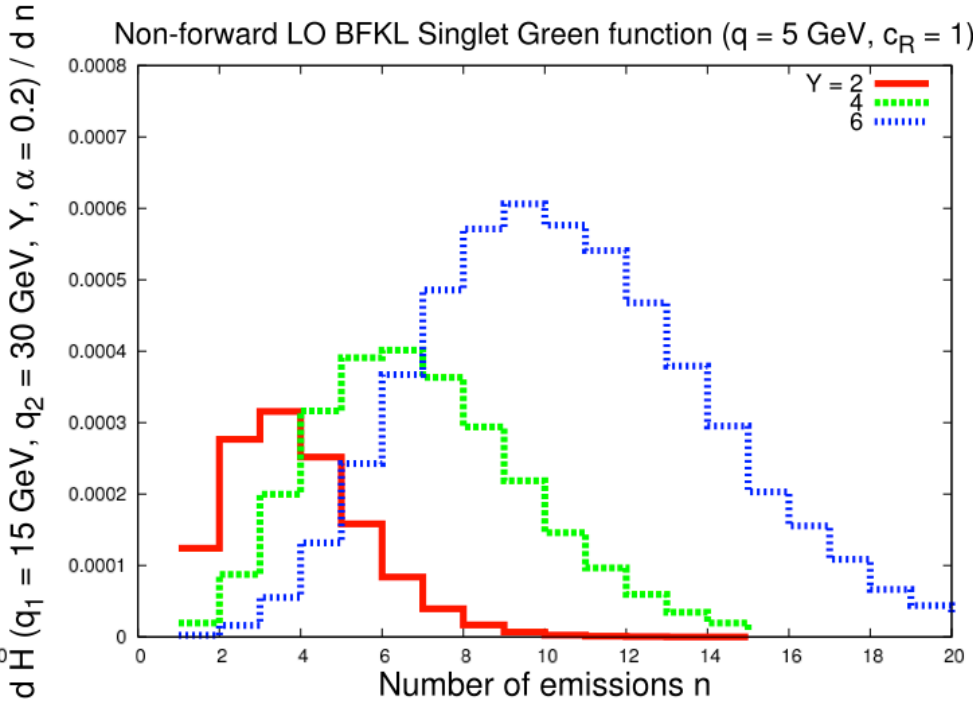
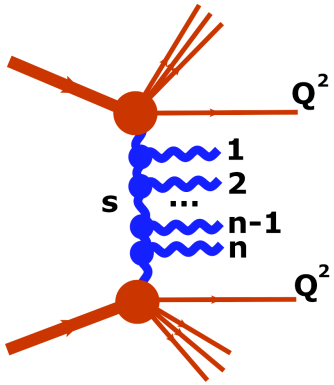
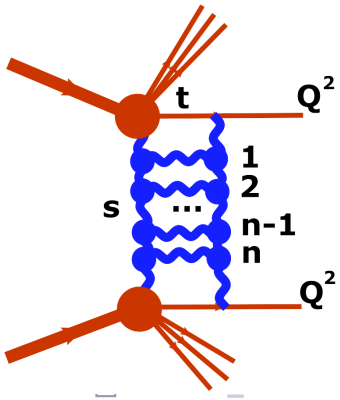
1512.03603

Number of emissions?

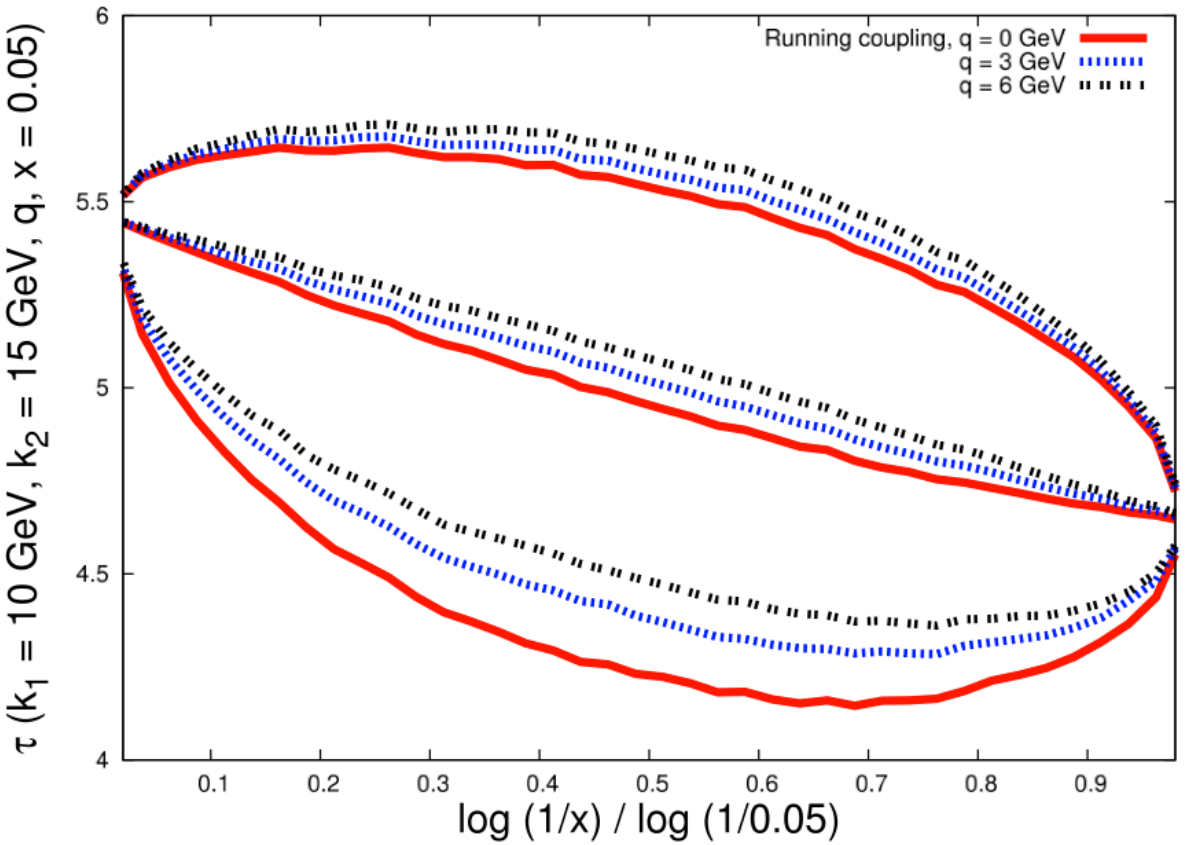


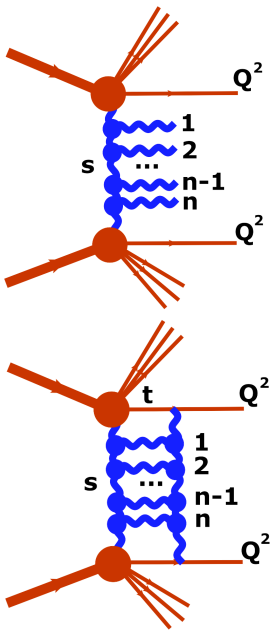






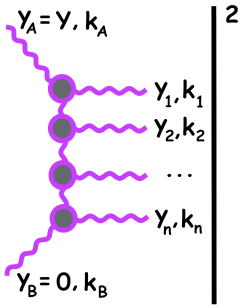
Reggeized (virtual) gluon $|p_T|$ at a given rapidity?

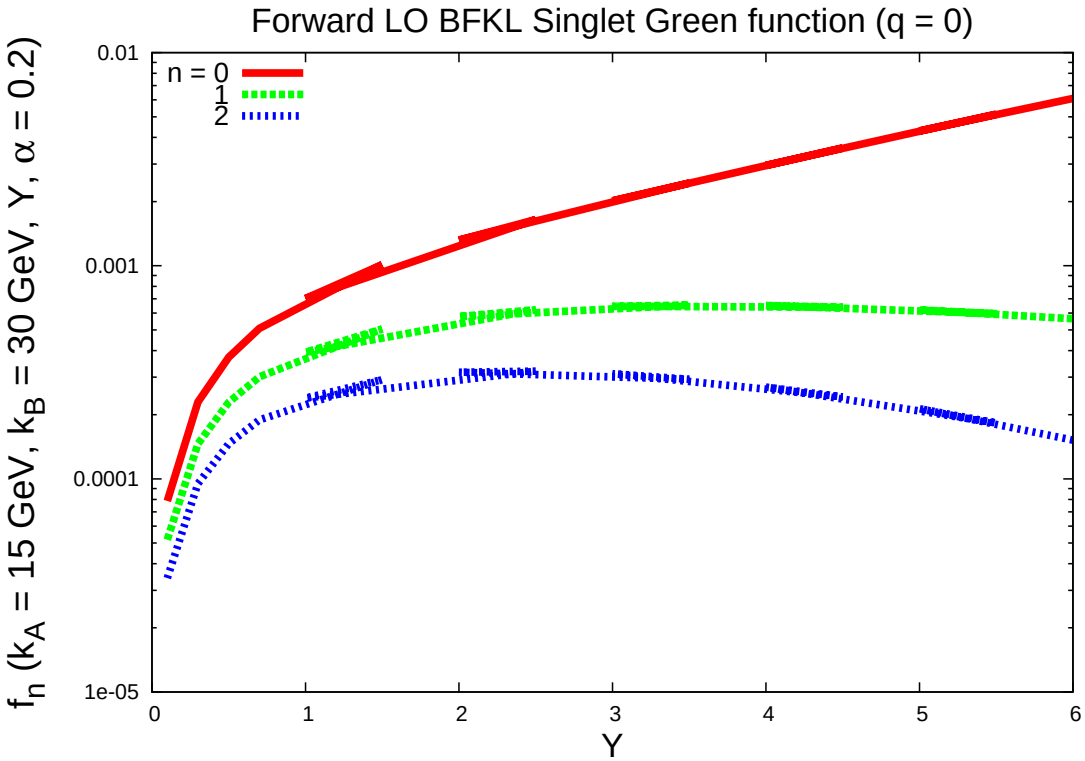




Growth with energy?

$$f\left(\vec{k}_A, \vec{k}_B, Y\right) = \sum_n$$

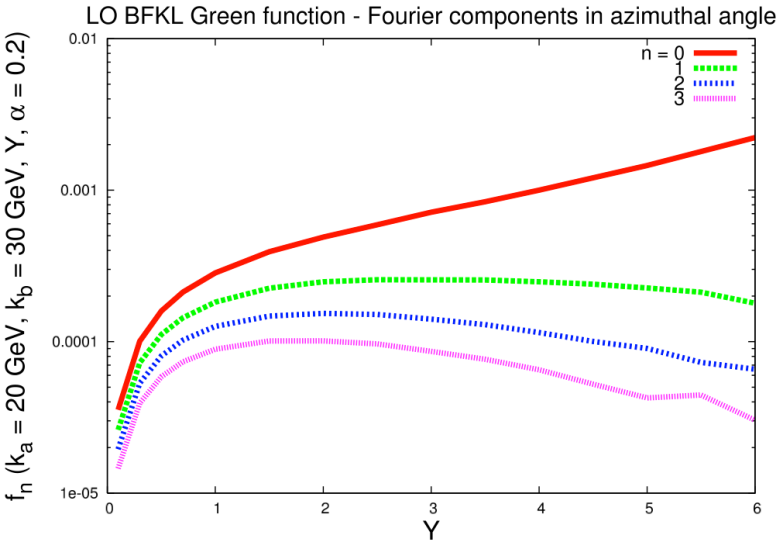




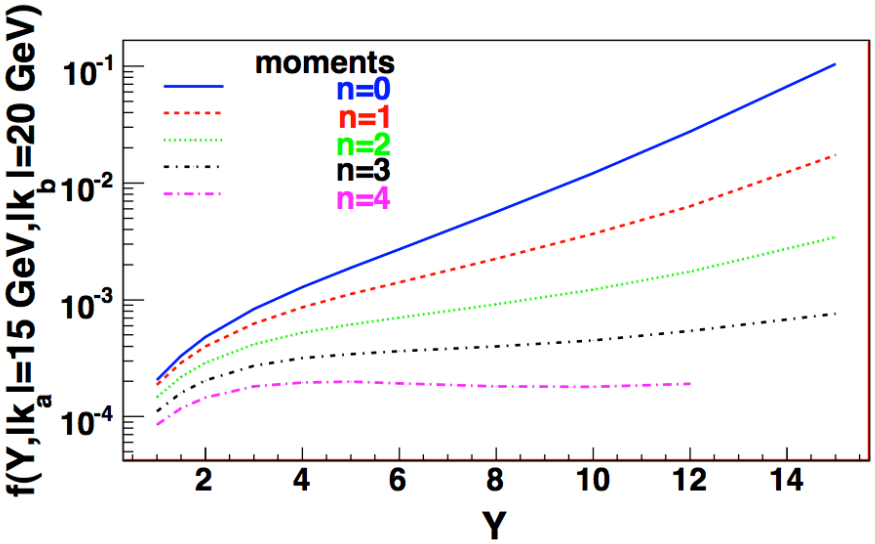
Different growth for different components in the azimuthal angle:

$$f_n\left(|\vec{k}_A|, |\vec{k}_B|, Y\right) = \int_0^{2\pi} \frac{d\theta}{2\pi} f\left(\vec{k}_A, \vec{k}_B, Y\right) \cos\left(n\theta\right)$$

BFKL



CCFM



All CCFM projections grow with energy, not in BFKL

This is a distinct feature of BFKL

1102.1890

We can extend the formalism to include collinear regions

$$f = e^{\omega\left(\vec{k}_A\right)Y}\left\{\delta^{(2)}\left(\vec{k}_A-\vec{k}_B\right)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i\frac{\theta\left(k_i^2-\lambda^2\right)}{\pi k_i^2}\right.$$
$$\left.\times\int_0^{y_{i-1}}dy_i\, e^{\left(\omega\left(\vec{k}_A+\sum_{l=1}^{i}\vec{k}_l\right)-\omega\left(\vec{k}_A+\sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A+\sum_{l=1}^{n}\vec{k}_l-\vec{k}_B\right)\right\}$$

Key at NLL: $\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)-\underbrace{\frac{\bar{\alpha}_s}{4}\ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}_{\text{NLL}}$

This resums collinear emissions for more general kinematics:

$$\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)+\sum_{n=1}^{\infty}\frac{\left(-\bar{\alpha}_s\right)^n}{2^n n!(n+1)!}\ln^{2n}\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)$$

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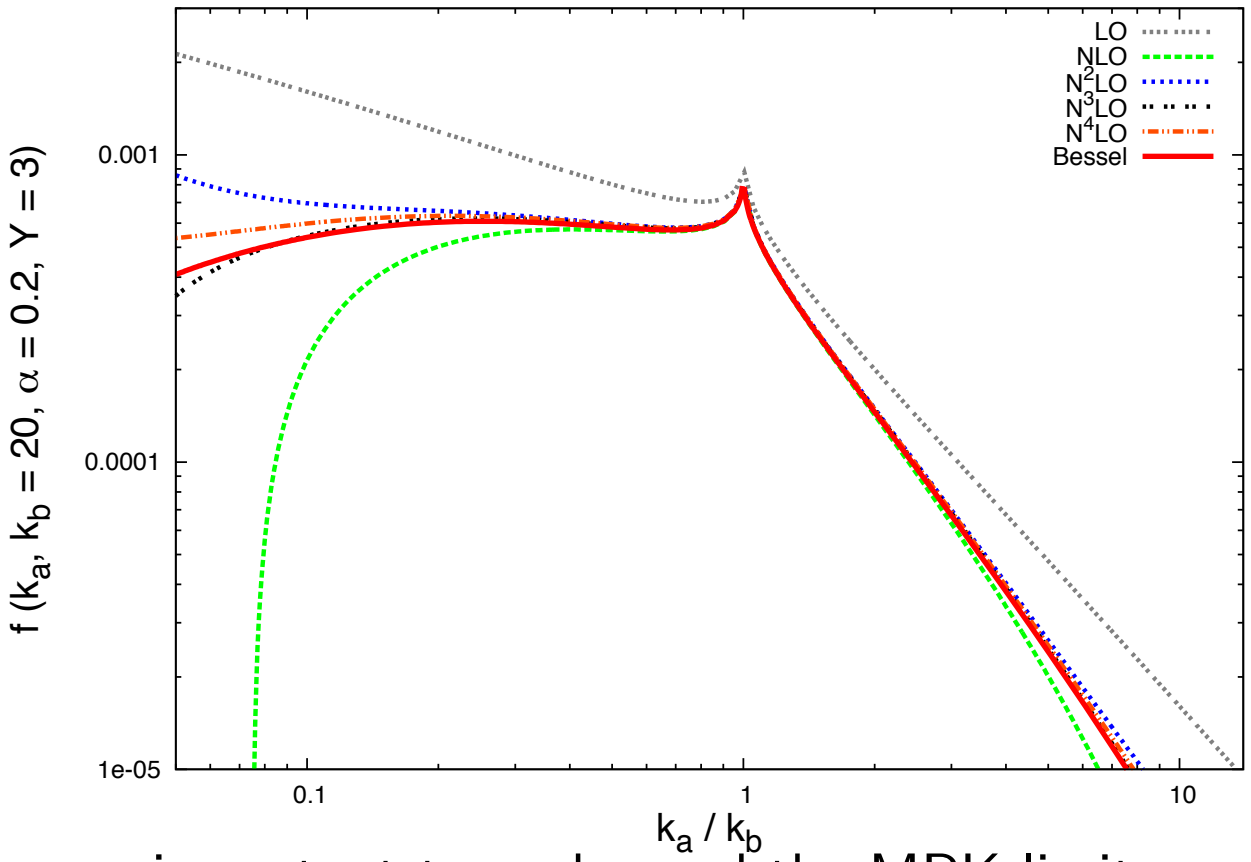
$$\sigma(Q_1, Q_2, Y) \;=\; \int d^2\mathbf{k}_a d^2\mathbf{k}_b\, \phi_A(Q_1, \mathbf{k}_a)\, \phi_B(Q_2, \mathbf{k}_b)\, f(\mathbf{k}_a, \mathbf{k}_b, Y)$$

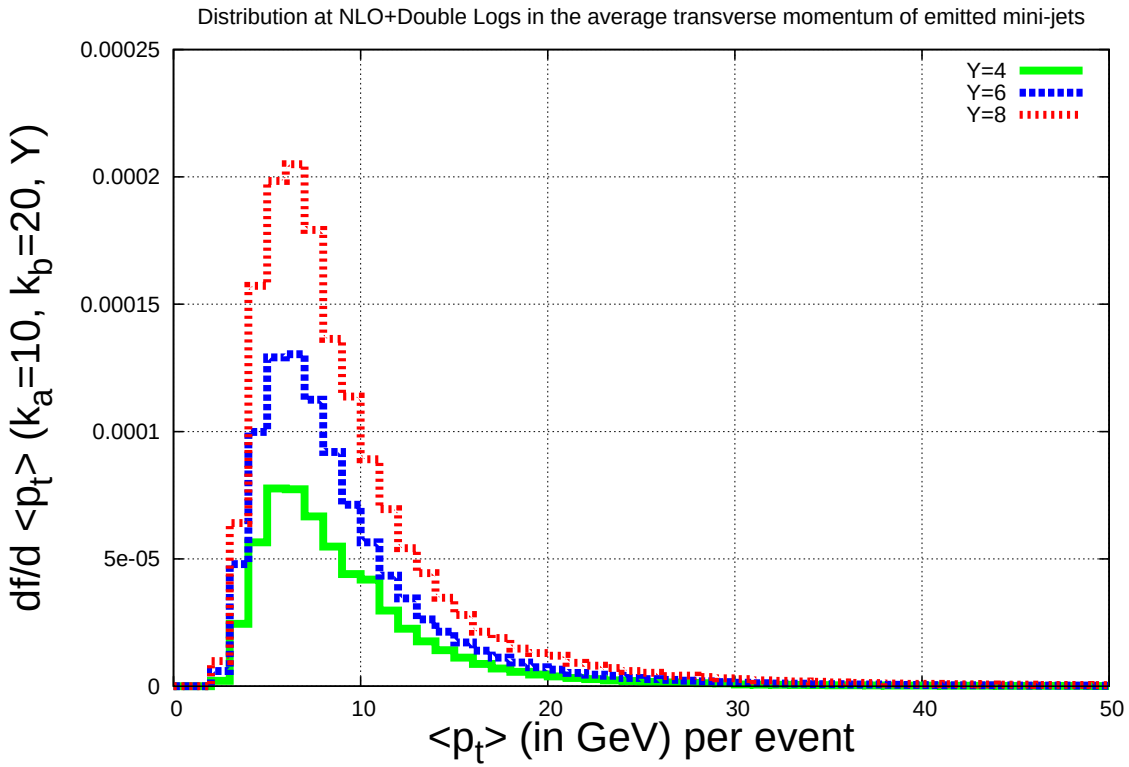


This is very important to go beyond the MRK limit.

For BFKL domain we need "δ-like" impact factors $\phi_{A,B}$ & $Q_1 \simeq Q_2$.

Average transverse momentum of emitted mini-jets?

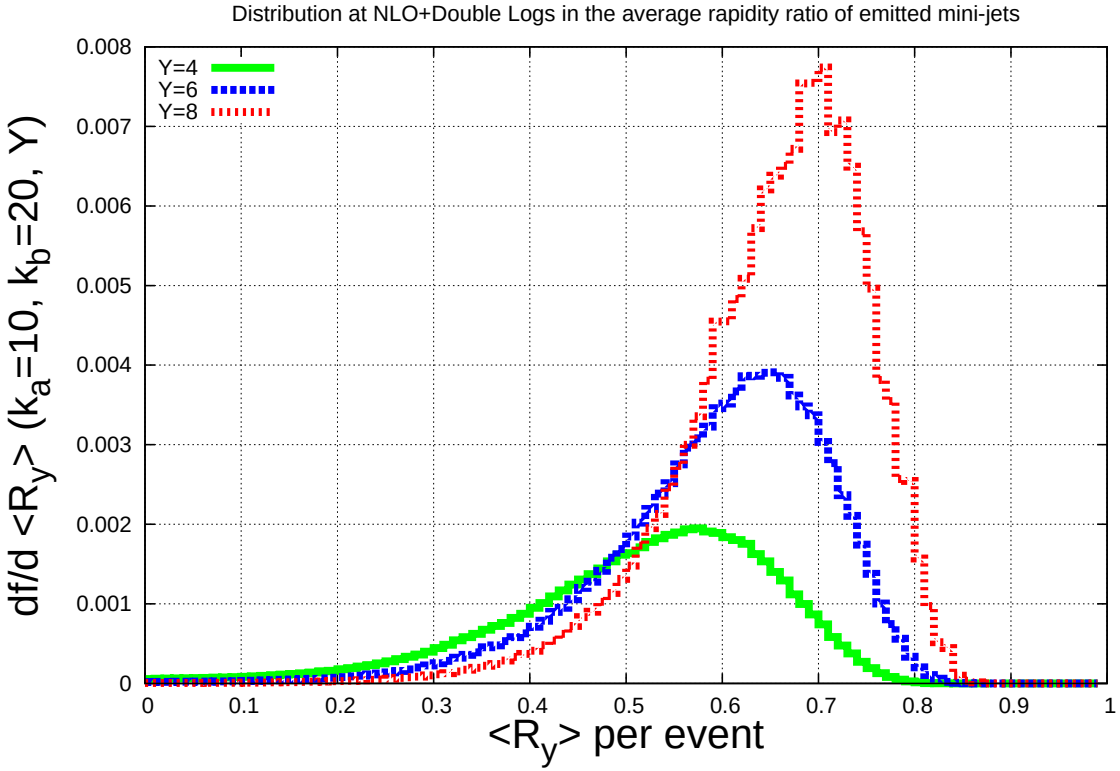
$$\langle p_t \rangle = \frac{1}{N} \sum_{i=1}^{N} |k_i|$$



Similar $\langle p_t \rangle_{\max}$ for different energies.

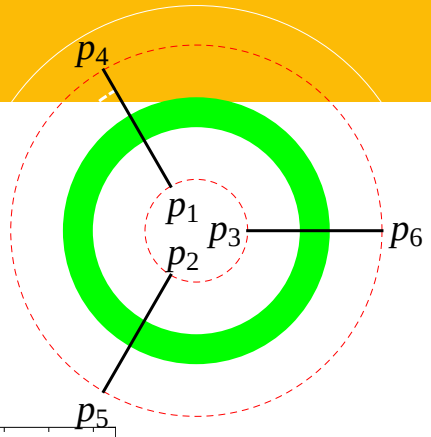
Average rapidity separation among emitted mini-jets?

$$\langle \mathcal{R}_y \rangle = \frac{1}{N+1} \sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}} \simeq 1 + \frac{\Delta}{Y} \ln \frac{\Delta}{Y} \text{ if } Y \simeq N\Delta \text{ in MRK and } Y \gg \Delta$$



Higher $\langle \mathcal{R}_y \rangle_{\max}$ for higher energies: $\Delta_{\rm LO} \simeq 0.62$, $\Delta_{\rm LO+DLs} \simeq 0.81$

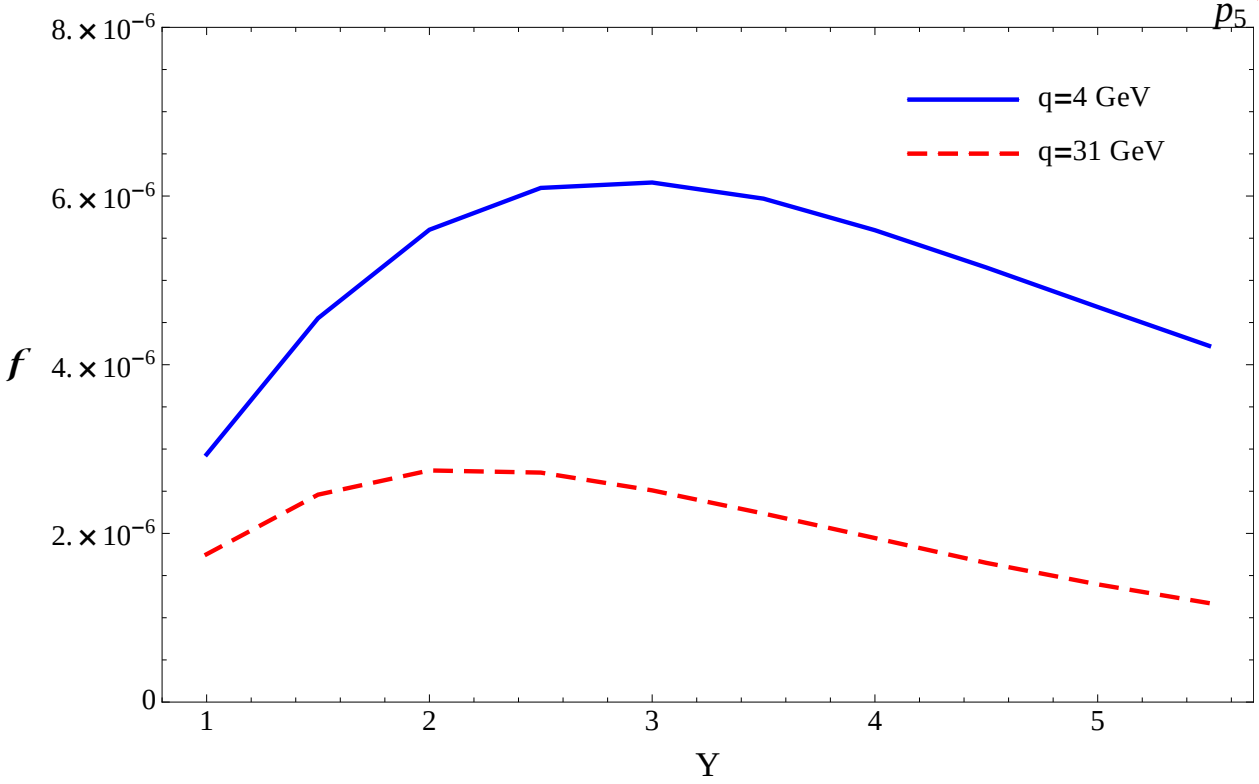
Lower mini-jet multiplicity when including higher order corrections



We have obtained the exact ODDERON Green function:

$$f\left(Y, \vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \vec{p}_5, \vec{p}_6, \vec{q}\right)$$

$\mathbf{q} = (4, 0)$
$\mathbf{p}_1 = (10, 0)$
$\mathbf{p}_2 = (-20, 0)$
$\mathbf{p}_3 = (14, 0)$
$\mathbf{p}_4 = (20, 0)$
$\mathbf{p}_5 = (-25, 0)$
$\mathbf{p}_6 = (9, 0)$



$\mathbf{q} = (31, 0)$
$\mathbf{p}_1 = (10, 0)$
$\mathbf{p}_2 = (-20, 0)$
$\mathbf{p}_3 = (41, 0)$
$\mathbf{p}_4 = (20, 0)$
$\mathbf{p}_5 = (-25, 0)$
$\mathbf{p}_6 = (36, 0)$

It contains both Bartels-Lipatov-Vacca and Janik-Wosiek solutions

1606.07349

- Regge limit in QCD
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In collaboration with Douglas ROSS (Southampton)

1605.00692
1605.08265

BFKL equation at LO with running coupling $\bar{\alpha}(t) \;=\; 1/(\bar{\beta}t)$:

$$\frac{\partial}{\partial Y}\mathcal{G}(Y,t_1,t_2) \;=\; \frac{1}{\sqrt{\bar{\beta}t_1}}\int dt\,\mathcal{K}(t_1,t)\frac{1}{\sqrt{\bar{\beta}t}}\,\mathcal{G}(Y,t,t_2)$$

Partial wave, with $z(t) \;\equiv \left(\frac{\bar{\beta}\omega}{14\zeta(3)}\right)^{1/3}\left(t-\frac{4\ln 2}{\bar{\beta}\omega}\right)$, is

$$\mathcal{G}_\omega\left(t_1,t_2\right) = \frac{\pi}{4}\frac{\sqrt{t_1 t_2}}{\omega^{1/3}}\left(\frac{\bar{\beta}}{14\zeta(3)}\right)^{2/3} Ai\left(z(t_1)\right) Bi\left(z(t_2)\right)\theta\left(t_1-t_2\right) + t_1 \leftrightarrow t_2$$

Lipatov's Trick: Homogeneous solution admits an extra piece:

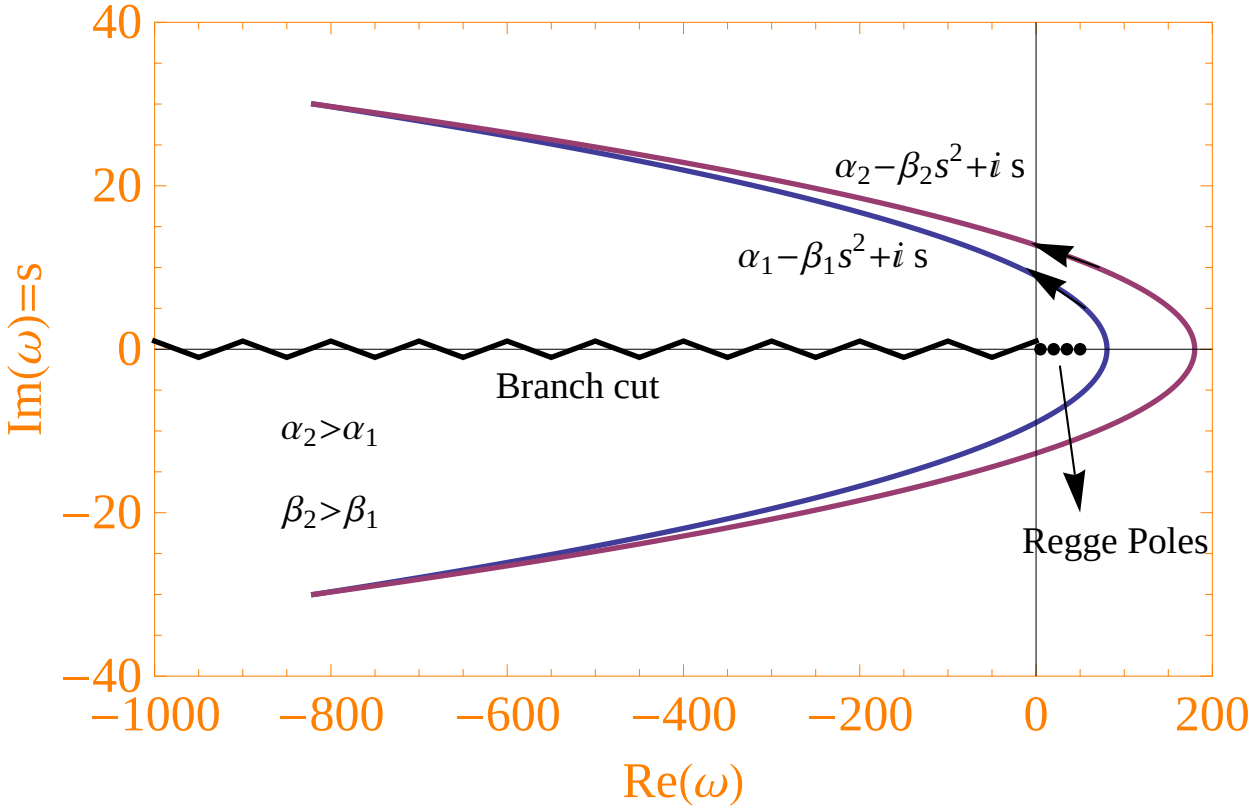
$$Bi(z) \to Bi(z) + Ai(z)\cot\left(\eta - \frac{2}{3}\sqrt{\frac{\bar{\beta}\omega}{14\zeta(3)}}\left(\frac{4\ln 2}{\bar{\beta}\omega} - t_0\right)^{3/2}\right)$$

which introduces ∞ poles in ω-plane

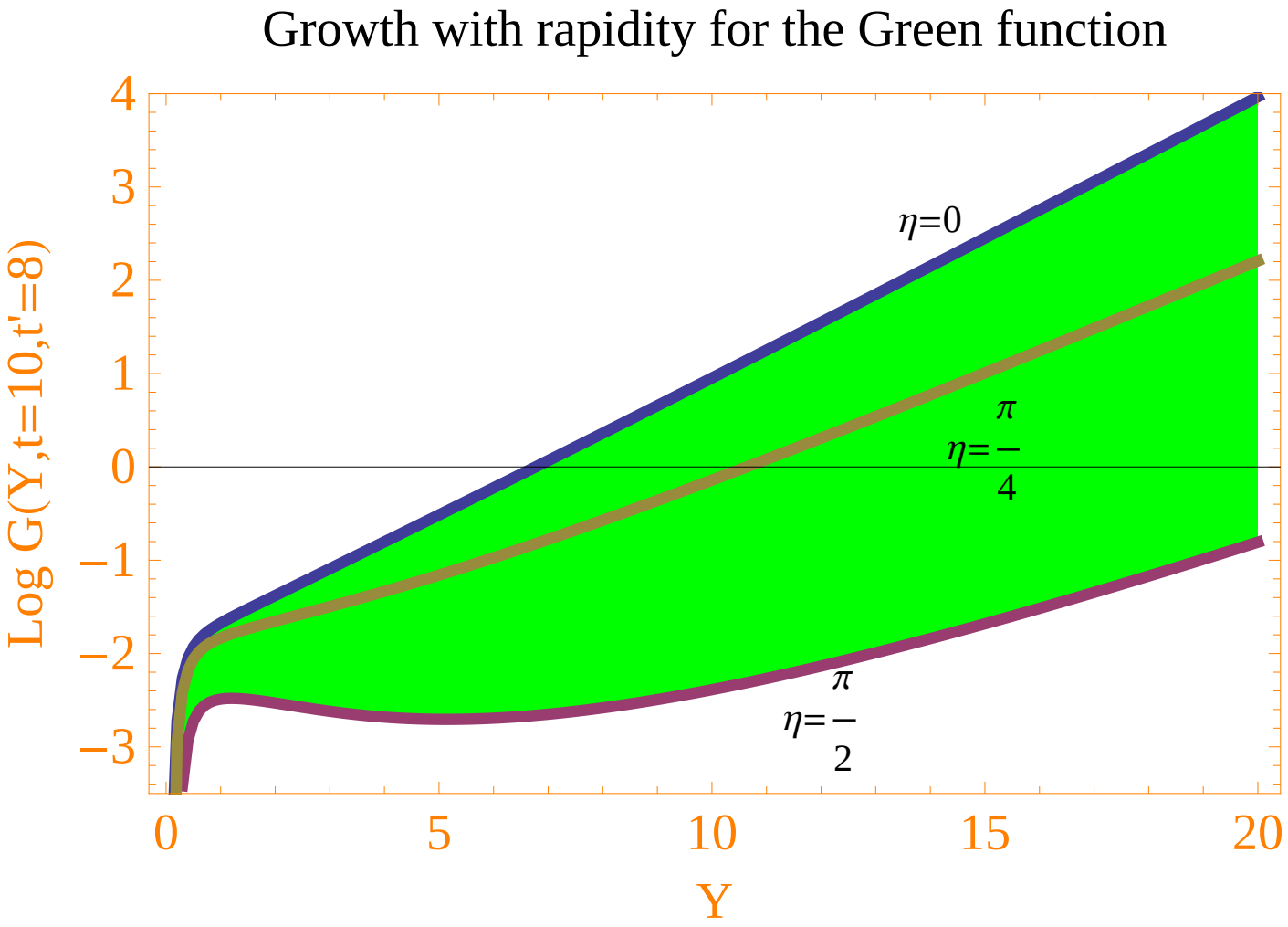
t_0 is a UV/IR matching scale and η has a non-perturbative origin

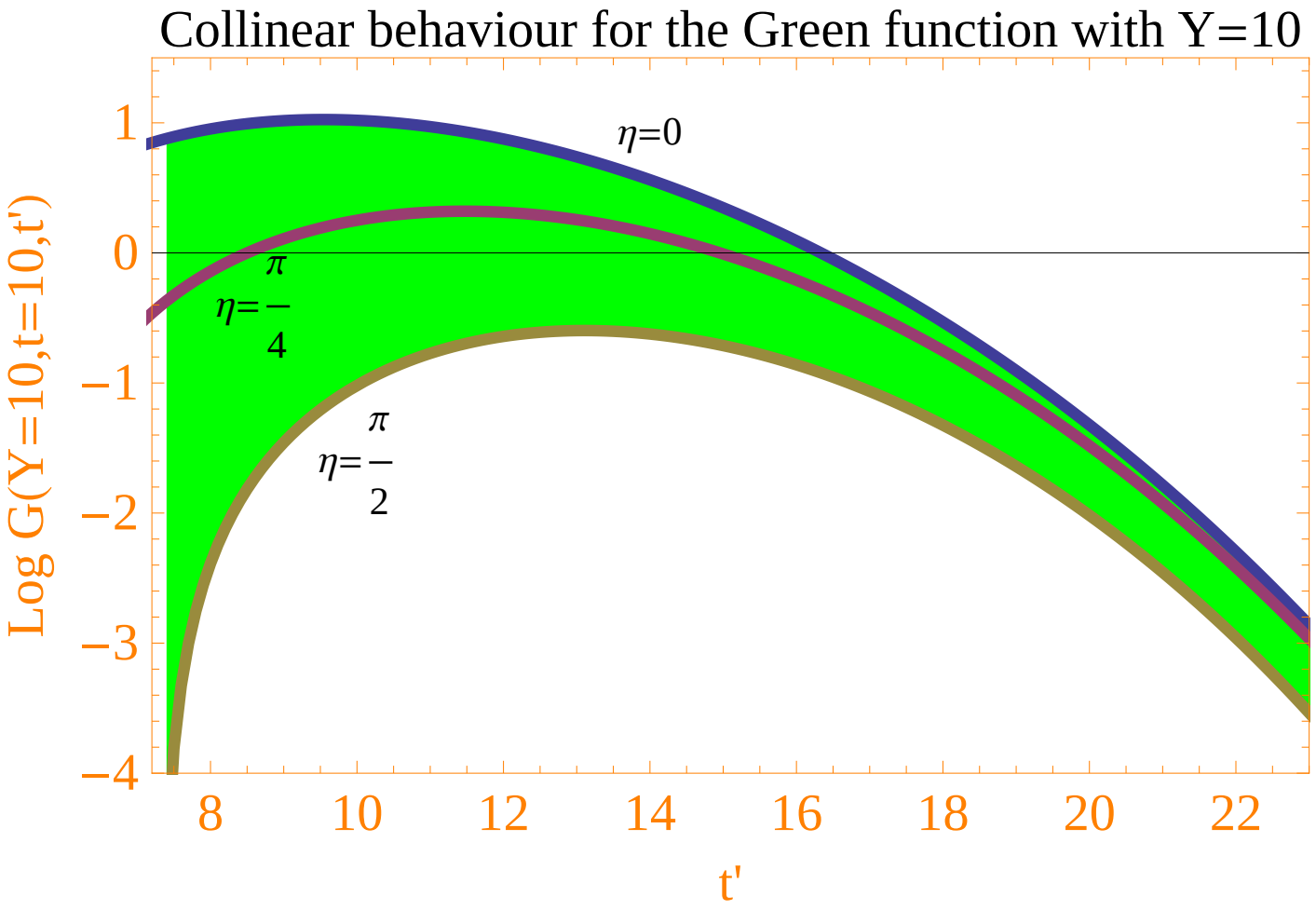
$$\mathcal{G}(Y, t_1, t_2) \;=\; \frac{1}{2\pi i}\int_{\mathcal{C}} d\omega\, e^{\omega Y}\mathcal{G}_\omega(t_1, t_2)$$

Integration Contours in ω–plane

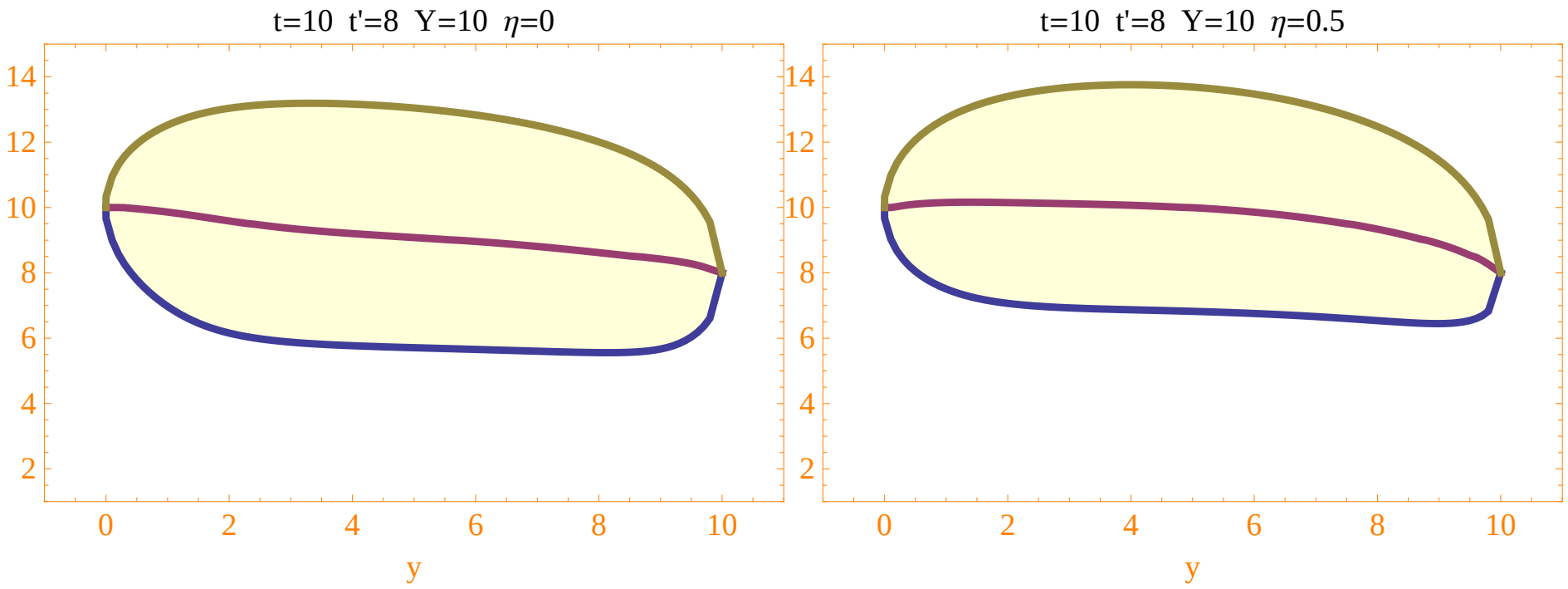


Numerical integration takes into account the cut and all Regge poles

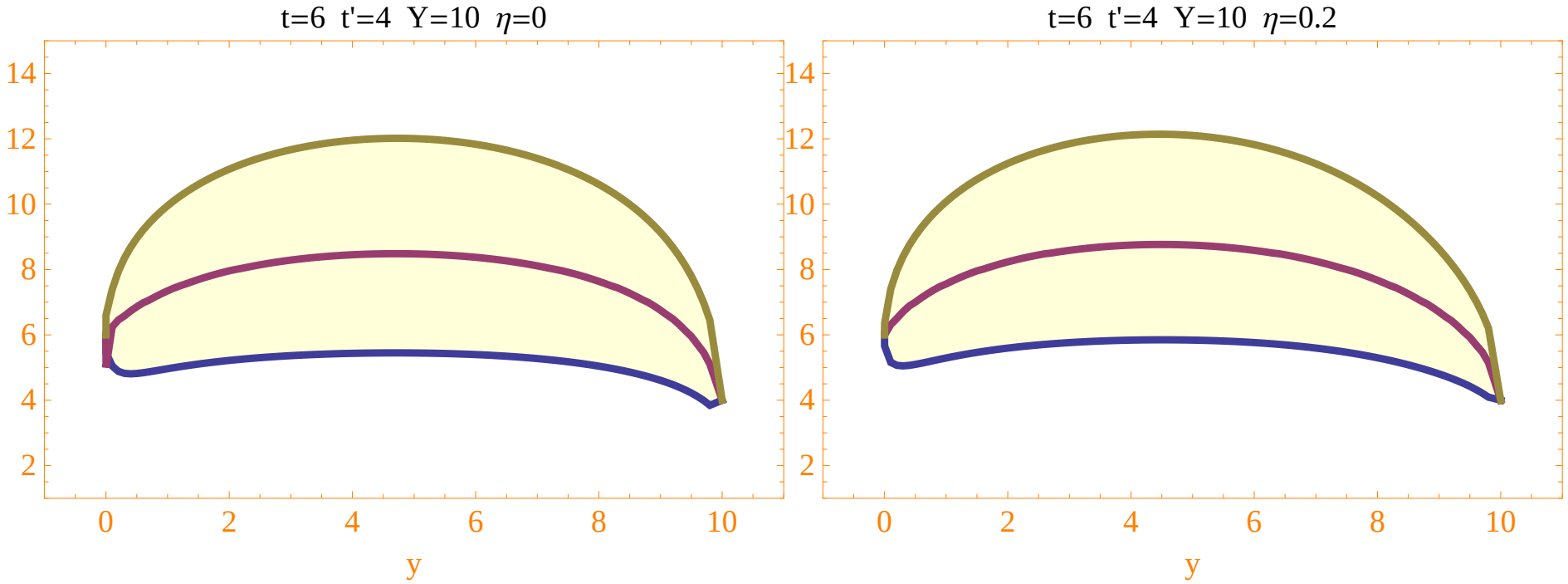

Growth with rapidity for the Green function
Log G(Y,t=10,t'=8)
Y
η=0
η=π/4
η=π/2
4
3
2
1
0
−1
−2
−3
0
5
10
15
20


Collinear behaviour for the Green function with Y=10
Log G(Y=10,t=10,t')
t'
η=0
η=π/4
η=π/2
1
0
−1
−2
−3
−4
8
10
12
14
16
18
20
22

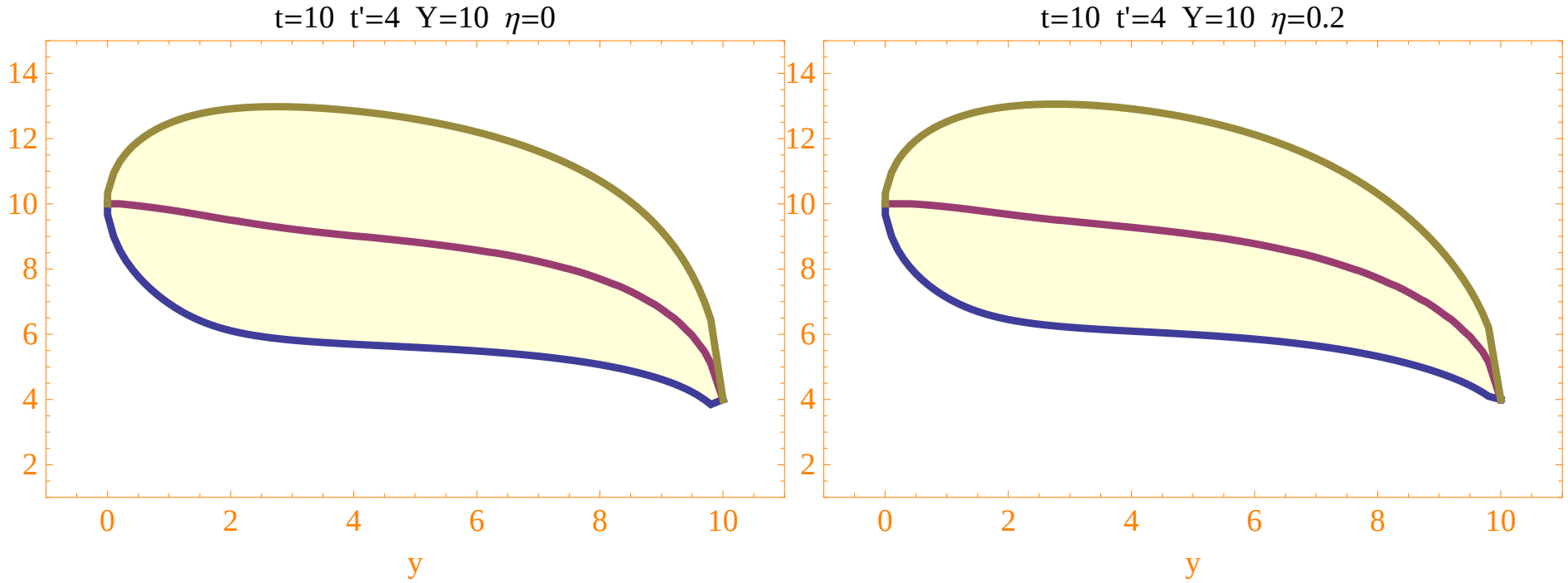
This model of the infrared does not affect the diffusion picture for larger external scales



For smaller scales the diffusion into the IR is suppressed



The IR suppression is also present in DIS-like configurations



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- At high energies new degrees of freedom appear: Reggeons
- This is deep: it happens in QCD, SUSY and Gravity ...
- Associated symmetries imply that the range of applicability is limited
- Once key observables have been identified (ratios of azimuthal angle correlations in dijet events tagged at large rapidity separation) we need to introduce more physics in the original formulation
  - Collinear contributions
  - Non-perturbative models
  - Saturation
  - Temperature
  - ...