Collinear and TMD densities from Parton Branching Method

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To be discussed today:

a parton branching method in analogy to a Parton Shower (PS) but used to solve DGLAP evolution equation.

Parton branching solution reproduces exactly semi-analytical results for collinear PDFs.

Similar codes exist (use similar formalism):
example: evolution code EvolFMC by Cracow group
Advantages of the parton branching method

To be discussed today:

a parton branching method in analogy to a Parton Shower (PS) but used to solve DGLAP evolution equation.

Parton branching solution reproduces exactly semi-analytical results for collinear PDFs.

Similar codes exist (use similar formalism):
example: evolution code EvolFMC by Cracow group

Advantages of the parton branching method

A possibility of:

- studying different orderings ($Q_T$-ordering, virtuality ordering, angular ordering) (this will be not discussed in detail here),
- extraction of TMDs (structure of the grid suitable for usage in xFitter) (this will be discussed).
Why Transverse Momentum Dependent PDFs?

Goal: **TMD PDFs for all flavours, all $x$, $\mu^2$ and $k_T$**

What is Transverse Momentum Dependent (TMD) Parton Distribution Function (PDF)?
- TMD PDF is a generalization of a concept of the PDF.
- TMD: depends not only on $x$ and $\mu^2$ but also on $k_T$: $TMD(x, \mu^2, k_T)$
Why Transverse Momentum Dependent PDFs?

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TDMs are important in studies on:

- resummation of all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- nonperturbative information on hadron structure at very low \( k_T \),
- perturbative region where QCD evolution equations describe processes
- a proper and consistent simulation of parton showers,
- multi-scale problems in hadronic collisions,
- ...

Introduction to the method
DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density \( xf(x, \mu^2) = \tilde{f}(x, \mu^2) \)

\[
\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz P_{ab} (\alpha_s(\mu^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right)
\]  \( (1) \)

\( a, b \)- quark (2\( N_f \) flavours) or gluon, \( x \)- longitudinal momentum fraction of the proton carried by a parton \( a \),

\( z = \frac{x_i}{x_{i-1}} \)- splitting variable, \( \mu \)- evolution mass scale

splitting function:

\[
P_{ab} (\alpha_s(\mu^2), z) = D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) + K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)_+} + R_{ab} (\alpha_s(\mu^2), z), \quad (2)
\]

\[
f_0^1 f(x)g(x)dx = \int_0^1 (f(x) - f(1))g(x)dx
\]

\( R_{ab} (\alpha_s(\mu^2), z) \) has no power divergences \( (1 - z)^{-n} \) for \( z \to 1 \).

As long as \( P_{ab} (\alpha_s(\mu^2), z) \) has this structure, the formalism presented today can be applied (LO, NLO, NNLO).
Collinear and TMD densities from Parton Branching Method

Introduction to the method

DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density $xf(x, \mu^2) = \tilde{f}(x, \mu^2)$

$$\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz P_{ab} (\alpha_s(\mu^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right)$$

(1)

$a, b$- quark ($2N_f$ flavours) or gluon, $x$- longitudinal momentum fraction of the proton carried by a parton $a$,

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splitting function:

$$P_{ab} (\alpha_s(\mu^2), z) = D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) + K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)_+} + R_{ab} (\alpha_s(\mu^2), z) ,$$

(2)

$$\int_0^1 f(x)g(x)dx = \int_0^1 (f(x) - f(1))g(x)dx$$

$R_{ab} (\alpha_s(\mu^2), z)$ has no power divergences $(1 - z)^{-n}$ for $z \to 1$.

As long as $P_{ab} (\alpha_s(\mu^2), z)$ has this structure, the formalism presented today can be applied (LO, NLO, NNLO).

Two potential problems for numerical solution: (details in Backup!)

- presence of the delta function $\rightarrow$ solved by momentum sum rule
  $$\sum_c \int_0^1 dz z P_{ca} (\alpha_s(\mu^2), z) = 0,$$

- integrals separately divergent for $(z \to 1)$ $\rightarrow$ solved by a parameter $z_M$: $\int_x^1 \rightarrow \int_x^{z_M}$
Collinear and TMD densities from Parton Branching Method

Introduction to the method

**Sudakov formalism**

After some algebraic transformations:

\[
\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^{Z_M} dz \, P_{ab}^R(\alpha_s(\mu^2), z) \, \tilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \tilde{f}_a(x, \mu^2) \sum_c \int_0^{Z_M} dz \, z P_{ca}^R(\alpha_s(\mu^2), z)
\]

(3)

where \( P_{ab}^R(\alpha_s(\mu^2), z) = R_{ab}(\alpha_s(\mu^2), z) + K_{ab}(\alpha_s(\mu^2)) \, \frac{1}{1-z} \) - real part of the splitting function.
Sudakov formalism

After some algebraic transformations:

$$\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^{Z_M} dz \, P_{ab}^R (\alpha_s(\mu^2), z) \, \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) - \tilde{f}_a (x, \mu^2) \sum_c \int_0^{Z_M} dz \, z P_{ca}^R (\alpha_s(\mu^2), z)$$

(3)

where $P_{ab}^R (\alpha_s(\mu^2), z) = R_{ab} (\alpha_s(\mu^2), z) + K_{ab} (\alpha_s(\mu^2)) \frac{1}{1-z}$ - real part of the splitting function.

Define the Sudakov form factor:

$$\Delta_a(\mu^2) = \exp \left( - \int_{\ln \mu^2_0}^{\ln \mu^2} d \left( \ln \mu'^2 \right) \sum_b \int_0^{Z_M} dz \, z P_{ba}^R (\alpha_s(\mu'^2), z) \right)$$

(4)

insert it in Eq.(3) and integrate

$$\tilde{f}_a(x, \mu^2) = \tilde{f}_a(x, \mu^2_0) \Delta_a(\mu^2) + \int_{\ln \mu^2_0}^{\ln \mu^2} d \ln \mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{Z_M} dz \, P_{ab}^R (\alpha_s(\mu'^2), z) \, \tilde{f}_b \left( \frac{x}{z}, \mu'^2 \right)$$

(5)
Iterative solution

Example for a = gluon:

\[ \tilde{f}_a(x, \mu^2) = \tilde{f}_a(x, \mu_0^2) \Delta_a(\mu^2) \]

Sudakov: probability of evolving from \( \mu_0^2 \) to \( \mu^2 \) without any resolvable branching.
Iterative solution

Example for $a = \text{gluon}$:

$$
\tilde{f}_a(x, \mu^2) = \tilde{f}_a(x, \mu_0^2) \Delta_a(\mu^2) + \int_{\ln \mu_0^2}^{\ln \mu^2} d \ln \mu' \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} d z P_{ab}^R (\alpha_s(\mu'^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu_0^2 \right) \Delta_b(\mu'^2)
$$
Iterative solution

Example for $a = \text{gluon}$:

$$
\tilde{f}_a(x, \mu^2) = \tilde{f}_a(x, \mu_0^2) \Delta_a(\mu^2) + \int_{\ln \mu_0^2}^{\ln \mu^2} d\ln \mu' \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'\mu^2)} \sum_b \int_x^{z_M} dz P_{ab}^R \left( \alpha_s(\mu'^2), z \right) \tilde{f}_b \left( \frac{x}{z}, \mu_0^2 \right) \Delta_b(\mu'^2) + \ldots
$$

This problem has an iterative solution which can be easily implemented in the MC code.
Collinear PDFs from parton branching method
LO comparison with semi analytical methods

Initial distribution: $\bar{f}_{b0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale $\mu^2$.

Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)

Upper plots: collinear pdfs from the parton branching method
Lower plots: ratios of the pdfs from a parton branching method and pdfs from QCDnum.

Very good agreement with the results coming from semi analytical methods (QCDnum).
NLO comparison with semi analytical methods

Initial distribution: $\tilde{f}_{b_0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale $\mu^2$.

Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)

Very good agreement with the results coming from semi analytical methods (QCDnum).
Cross check for different $z_M$

Comparison of the results for different $z_M$ values.

Upper plot: collinear pdfs from a MC method
Lower plot: ratios of the pdfs from a MC method and pdfs from QCDnum.

There is no dependence on $z_M$ as long as $z_M$ large enough.

Here results at NLO, at LO the same conclusion.
Results for TMDs
$k_T$ dependence

Parton branching method: for every branching $\mu^2$ is generated and available.

How to connect $\mu$ with $Q_T$ of the emitted and $k_T$ of the propagating parton?

- $Q_T$- ordering: $\vec{Q}^2_{T,n} = \mu^2$.
- virtuality ordering: $\vec{Q}^2_{T,n} = (1 - z)\mu^2$

$$\vec{k}_{T,n} = \vec{k}_{T,n-1} - \vec{Q}_{T,n}$$

$k_T$ contains the whole history of the evolution.

In this method $k_T$ is treated properly from the beginning of the evolution- no extra reshuffling at the end is required.
Collinear and TMD densities from Parton Branching Method

Results for TMDs

TMD PDFs from different \( k_T \) definition at LO

Reminder: for collinear PDFs there was no \( z_M \) dependence.

What about \( z_M \) dependence for TMDs?

\[ Q_T \]- ordering:

\[ \overline{Q}_T \cdot n = \mu^2 \]

large \( z \) - soft gluons!

\[ Q_T \]- ordering: for every \( z_M \) value we obtain different TMD

\[ \rightarrow \] not physical behaviour, \( Q_T \)- ordering shouldn’t be used

Virtuality ordering: no \( z_M \) dependence (because of the \( (1 - z) \mu^2 \) term soft gluons suppressed)
Fit of integrated TMDs for all flavours to HERA DIS data with xFitter
Procedure of the fit to the HERA 1+2 $F_2$ data

Goal: TMD PDF sets for all flavours, all $x$, $Q^2$ and $k_T$

- A kernel $A^b_a$ is determined from the parton branching method from a toy starting distribution: $f_{0,b} = \delta(1-x)$.
- xFitter chooses a starting distribution $A_{0,b}$ and performs a convolution of the kernel $A^b_a$ with the starting distribution $A_{0,b}$ to obtain a parton density

$$\tilde{f}_a(x, \mu^2) = \int_0^\infty \frac{dk_T^2}{k_T^2} \int dx' A_{0,b}(x') \frac{x'}{x} A^b_a \left( \frac{x'}{x}, k_T^2, \mu^2 \right) \quad (6)$$

- Obtained parton density $\tilde{f}_a(x, \mu^2)$ is fitted to the $F_2$ data and $\chi^2$ is calculated.

- The procedure is repeated with the new starting distributions $A_{0,b}$ many times to minimize $\chi^2$.

A very good $\chi^2/ndf \sim 1.2$ is obtained for $3.5 < Q^2 < 30000 \text{ GeV}^2$ and $x > 4 \cdot 10^{-5}$.
Collinear and TMD densities from Parton Branching Method

Fit of integrated TMDs for all flavours to HERA DIS data with xFitter

TMDs from fits - comparison of LO and NLO TMDs

TMDs were fitted with experimental uncertainties at LO and NLO.
Collinear and TMD densities from Parton Branching Method

Fit of integrated TMDs for all flavours to HERA DIS data with xFitter

TMDs from the fit at NLO

From parton branching method we can obtain TMDs for all flavours

At small $k_T$ (no branching or just a few branchings), the difference in the quark TMDs comes from initial distributions.

At large $k_T$ (many branchings) TMDs for quarks the same.

TMDs sets available soon!

→ check on TMDplotter and TMDlib
http://tmdplotter.desy.de/
Summary
Summary

New approach to solve coupled gluon and quark DGLAP evolution equation with a parton branching method was shown.

Advantages:

- it reproduces exactly semi-analytical solution for collinear PDFs (results consistent with QCDNum),
- extraction of TMD PDFs possible and done,
  fit to F2 Hera data at LO and NLO was performed within xFitter,
  TMDs sets for all flavours were obtained from the fit with experimental uncertainties,
- options to study different orderings for collinear and TMD PDFs available within this framework.

Prospects:

- TMD sets released soon,
- application in measurements,
- long term goal: direct usage in PS matched calculation.
Thank you!
DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density \( xf(x, \mu^2) = \tilde{f}(x, \mu^2) \)

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\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz P_{ab} (\alpha_s(\mu^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right)
\]

\( a, b \)- quark \((2N_c \text{ flavours})\) or gluon, \( x \)- longitudinal momentum fraction of the proton carried by a parton \( a \), \( z = \frac{x_i}{x_{i-1}} \)- splitting variable, \( \mu \)- evolution mass scale and

a structure of a splitting function:

\[
P_{ab} (\alpha_s(\mu^2), z) = D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) + K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)^+} + R_{ab} (\alpha_s(\mu^2), z) ,
\]

\[
\int_0^1 f(x)g(x)dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx
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\[
D_{ab} (\alpha_s(\mu^2)) = \delta_{ab} d_a (\alpha_s(\mu^2)), K_{ab} (\alpha_s(\mu^2)) = \delta_{ab} k_a (\alpha_s(\mu^2)),
\]

\( R_{ab} (\alpha_s(\mu^2), z) \) contains logarithmic terms in \( \ln(1 - z) \) and has no power divergences \((1 - z)^{-n} \) for \( z \to 1 \) .
**DGLAP evolution equation**

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\]

$a, b$ - quark ($2N_f$ flavours) or gluon, $x$ - longitudinal momentum fraction of the proton carried by a parton $a$, $z = \frac{x_i}{x_i - 1}$ - splitting variable, $\mu$ - evolution mass scale and a structure of a splitting function:

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\begin{align*}
P_{ab} (\alpha_s(\mu^2), z) &= D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) + K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)^+} + R_{ab} (\alpha_s(\mu^2), z), \tag{8}
\end{align*}
\]

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\int_0^1 f(x)g(x)\, dx = \int_0^1 f(x)g(x)\, dx - \int_0^1 f(1)g(x)\, dx
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D_{ab} (\alpha_s(\mu^2)) = \delta_{ab} d_a (\alpha_s(\mu^2)), K_{ab} (\alpha_s(\mu^2)) = \delta_{ab} k_a (\alpha_s(\mu^2)),
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$R_{ab} (\alpha_s(\mu^2), z)$ contains logarithmic terms in $\ln(1 - z)$ and has no power divergences $(1 - z)^{-n}$ for $z \to 1$.

\[
\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)^+} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + 
\]

\[
- \sum_b \tilde{f}_b (x, \mu^2) \int_x^1 dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)^+} - D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) \right) \tag{9}
\]
DGLAP evolution equation

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(7)

$a, b$: quark ($2N_f$ flavours) or gluon, $x$: longitudinal momentum fraction of the proton carried by a parton $a$,

$z = \frac{x_i}{x_i - 1}$: splitting variable, $\mu$: evolution mass scale and

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\[
\int_0^1 f(x)g(x)dx = \int_0^1 f(x)g(x)dx - \int_0^1 (1)g(x)dx
\]

\[
D_{ab} (\alpha_s(\mu^2)) = \delta_{ab} d_a (\alpha_s(\mu^2)), K_{ab} (\alpha_s(\mu^2)) = \delta_{ab} k_a (\alpha_s(\mu^2)),
\]

\[
R_{ab} (\alpha_s(\mu^2), z) \text{ contains logarithmic terms in } \ln(1 - z) \text{ and has no power divergences } (1 - z)^{-n} \text{ for } z \to 1.
\]

\[
\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)_+} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) +
\]

\[
- \sum_b \tilde{f}_b (x, \mu^2) \int_0^1 dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{(1 - z)} - D_{ab} (\alpha_s(\mu^2)) \delta(1 - z) \right)
\]

(9)

Two potential problems for numerical solution:

▶ presence of the delta function,

▶ integrals separately divergent for $z \to 1$. 

2 / 8
Momentum sum rule

To get rid of the delta function:

We use momentum sum rule \( \sum_c \int_0^1 dz z P_{ca} (\alpha_s(\mu^2), z) = 0 \):
Momentum sum rule

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\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz \left( K_{ab}(\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab}(\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + \\
- \sum_b \tilde{f}_b(x, \mu^2) \int_0^1 dz \left( K_{ab}(\alpha_s(\mu^2)) \frac{1}{1-z} - D_{ab}(\alpha_s(\mu^2)) \delta(1-z) \right) + \\
- \tilde{f}_a(x, \mu^2) \sum_c \int_0^1 dz z P_{ca}(\alpha_s(\mu^2), z) = \\
= \sum_b \int_x^1 dz \left( K_{ab}(\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab}(\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + \\
- \tilde{f}_a(x, \mu^2) \sum_c \int_0^1 dz \left( K_{ca}(\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ca}(\alpha_s(\mu^2), z) \right) \tag{10}
\]
Momentum sum rule

To get rid of the delta function:
We use momentum sum rule $\sum_c \int_0^1 dz z P_{ca} (\alpha_s (\mu^2), z) = 0$:

$$\frac{d\tilde{f}_a (x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz \left( K_{ab} (\alpha_s (\mu^2)) \frac{1}{1 - z} + R_{ab} (\alpha_s (\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) +$$

$$- \sum_b \tilde{f}_b (x, \mu^2) \int_0^1 dz \left( K_{ab} (\alpha_s (\mu^2)) \frac{1}{1 - z} - D_{ab} (\alpha_s (\mu^2)) \delta(1 - z) \right) +$$

$$- \tilde{f}_a (x, \mu^2) \sum_c \int_0^1 dz z P_{ca} (\alpha_s (\mu^2), z) =$$

$$= \sum_b \int_x^1 dz \left( K_{ab} (\alpha_s (\mu^2)) \frac{1}{1 - z} + R_{ab} (\alpha_s (\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) +$$

$$- \tilde{f}_a (x, \mu^2) \sum_c \int_0^1 dz z \left( K_{ca} (\alpha_s (\mu^2)) \frac{1}{1 - z} + R_{ca} (\alpha_s (\mu^2), z) \right)$$  

(10)

We got rid of the delta function,
both pieces of the equation written in the same way.
Virtual and non-resorvable pieces still included.
Divergence for $z \to 1$

To avoid divergence when $z \to 1$ a cut off must be introduced:

$$
\sum_{b} \int_{x}^{1} dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + 
$$

$$
- \tilde{f}_a (x, \mu^2) \sum_{c} \int_{0}^{1} dz \left( K_{ca} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ca} (\alpha_s(\mu^2), z) \right) 
$$

$$
\rightarrow \sum_{b} \int_{x}^{z_M} dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + 
$$

$$
- \tilde{f}_a (x, \mu^2) \sum_{c} \int_{0}^{z_M} dz \left( K_{ca} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ca} (\alpha_s(\mu^2), z) \right) 
$$

(11)

It can be shown that terms $\int_{z_{\text{max}}}^{1} dz$ skipped in the integral in eq. (11) are of order $O(1 - z_{\text{max}})$. 

Collinear and TMD densities from Parton Branching Method

Back up

DGLAP evolution equation

Divergence for \( z \to 1 \)

To avoid divergence when \( z \to 1 \) a cut off must be introduced:

\[
\sum_b \int_x^1 dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) +
\]

\[
- \tilde{f}_a (x, \mu^2) \sum_c \int_0^1 dz \left( K_{ca} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ca} (\alpha_s(\mu^2), z) \right)
\]

\[
\to \sum_b \int_x^{z_M} dz \left( K_{ab} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ab} (\alpha_s(\mu^2), z) \right) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) +
\]

\[
- \tilde{f}_a (x, \mu^2) \sum_c \int_0^{z_M} dz \left( K_{ca} (\alpha_s(\mu^2)) \frac{1}{1-z} + R_{ca} (\alpha_s(\mu^2), z) \right) \quad (11)
\]

It can be shown that terms \( \int_{z_{max}}^1 \) skipped in the integral in eq. (11) are of order \( O(1 - z_{max}) \).

Different choices of \( z_{max} \):

- \( z_{max} \) - fixed
- \( z_{max} \) - can change dynamically with the scale:
  - angular ordering: \( z_{max} = 1 - \left( \frac{Q_0}{Q} \right) \)
  - or virtuality ordering: \( z_{max} = 1 - \left( \frac{Q_0}{Q} \right)^2 \)

In this presentation: results from fixed \( z_{max} \).
Collinear and TMD densities from Parton Branching Method

Sudakov formalism

Sudakov form factor

\[
\frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^{\mu} d\frac{dz}{z} P_{ab}^R(\alpha_s(\mu^2), z) \tilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \tilde{f}_a (x, \mu^2) \sum_c \int_0^{\mu} dz zP_{ca}^R(\alpha_s(\mu^2), z)
\]

(12)

where \( P_{ab}^R(\alpha_s(\mu^2), z) = R_{ab}(\alpha_s(\mu^2), z) + K_{ab}(\alpha_s(\mu^2)) \frac{1}{1-z} \) - real part of the splitting function.
Sudakov form factor

\[ \frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^{Z_M} dz \ P_{ab}^R(\alpha_s(\mu^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) - \tilde{f}_a(x, \mu^2) \sum_c \int_0^{Z_M} dz \ z P_{ca}^R(\alpha_s(\mu^2), z) \]

where \( P_{ab}^R(\alpha_s(\mu^2), z) = R_{ab}(\alpha_s(\mu^2), z) + K_{ab}(\alpha_s(\mu^2)) \frac{1}{1-z} \) - real part of the splitting function.

Defining the Sudakov form factor:

\[ \Delta_a(\mu^2) = \exp \left( - \int_{\ln \mu_0^2}^{\ln \mu^2} d(\ln \mu') \sum_b \int_0^{Z_M} dz z P_{ba}^R(\alpha_s(\mu'^2), z) \right) \]

\[ \frac{d\tilde{f}_a(x, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^{Z_M} dz P_{ab}^R(\alpha_s(\mu^2), z) \tilde{f}_b \left( \frac{x}{z}, \mu^2 \right) + \tilde{f}_a(x, \mu^2) \frac{1}{\Delta_a(\mu^2)} \frac{d\Delta_a(\mu^2)}{d \ln \mu^2} \]

(12)
TMDs from the fit at LO

From parton branching method we can obtain TMDs for all flavours

At small $k_T$ (no branching or just a few branchings), the difference in the quark TMDs comes from initial distributions. At large $k_T$ (many branchings) TMDs for quarks the same.
Collinear and TMD densities from Parton Branching Method

- Back up
- TMDs from fits at LO

integrated TMD from parton branching method and HERA pdf
Role of the limit in $k_T$ integration

Comparison of int TMDs integrated up to a different $k_T$ values

The integral over $k_T$ has to be performed up to a value higher than the evolution scale to obtain collinear PDF which agrees well with the HERA pdf.