Collinear and TMD densities from Parton Branching Method

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Motivation

Advantages of the parton branching method

To be discussed today:

a parton branching method in analogy to a Parton Shower (PS) but used to solve DGLAP evolution equation.

Parton branching solution reproduces exactly semi-analytical results for collinear PDFs.

Similar codes exist (use similar formalism):
example: evolution code EvolFMC by Cracow group
S. Jadach et al., Markovian Monte Carlo program EvolFMC v.2 for solving QCD evolution equations, Comput.Phys.Commun. 181 (2010) 393-412

Advantages of the parton branching method

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Advantages of the parton branching method

A possibility of:

- studying different orderings (Q_T-ordering, virtuality ordering, angular ordering) (this will be not discussed in detail here),
- extraction of TMDs (structure of the grid suitable for usage in xFitter) (this will be discussed).

Why Transverse Momentum Dependent PDFs?

Goal: **TMD PDFs for all flavours, all x, μ^2 and k_T**

What is Transverse Momentum Dependent (TMD) Parton Distribution Function (PDF)?

- TMD PDF is a generalization of a concept of the PDF.
- TMD: depends not only on x and μ^2 but also on k_T: $TMD(x, \mu^2, k_T)$

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TDMs are important in studies on:

- resummation of all orders in the QCD coupling to many observables in high-energy hadronic collisions,
- nonperturbative information on hadron structure at very low k_T,
- perturbative region where QCD evolution equations describe processes
- a proper and consistent simulation of parton showers,
- multi-scale problems in hadronic collisions,
- ...

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Introduction to the method

DGLAP evolution equation

DGLAP evolution equation for momentum weighted parton density $xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz P_{ab}\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) \tag{1}$$

a, b- quark ($2N_f$ flavours) or gluon, x- longitudinal momentum fraction of the proton carried by a parton a,
$z = \frac{x_i}{x_{i-1}}$- splitting variable, μ- evolution mass scale

splitting function:

$$P_{ab}\left(\alpha_s(\mu^2), z\right) = D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)_+} + R_{ab}\left(\alpha_s(\mu^2), z\right), \tag{2}$$

$\int_0^1 f(x)g(x)_+ dx = \int_0^1 (f(x) - f(1))g(x)dx$
$R_{ab}\left(\alpha_s(\mu^2), z\right)$ has no power divergences $(1-z)^{-n}$ for $z \to 1$.

As long as $P_{ab}\left(\alpha_s(\mu^2), z\right)$ has this structure, the formalism presented today can be applied (LO, NLO, NNLO).

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As long as $P_{ab}\left(\alpha_s(\mu^2), z\right)$ has this structure, the formalism presented today can be applied (LO, NLO, NNLO).

Two potential problems for numerical solution: (details in Backup!)

- presence of the delta function → solved by *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\alpha_s(\mu^2), z\right) = 0$,
- integrals separately divergent for $(z \to 1)$ → solved by a parameter z_M: $\int_x^1 \to \int_x^{z_M}$

Sudakov formalism

After some algebraic transformations:

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{dz}\, P_{ab}^R\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right) \sum_c \int_0^{z_M} \mathrm{dz}\, z P_{ca}^R\left(\alpha_s(\mu^2), z\right) \tag{3}$$

where $P_{ab}^R\left(\alpha_s(\mu^2), z\right) = R_{ab}\left(\alpha_s(\mu^2), z\right) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{1-z}$ - *real* part of the splitting function.

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where $P_{ab}^R\left(\alpha_s(\mu^2), z\right) = R_{ab}\left(\alpha_s(\mu^2), z\right) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{1-z}$ - *real* part of the splitting function.

Define the *Sudakov form factor*:

$$\Delta_a(\mu^2) = \exp\left(-\int_{\ln\mu_0^2}^{\ln\mu^2} d\left(\ln\mu'^2\right) \sum_b \int_0^{z_M} dzzP_{ba}^R\left(\alpha_s(\mu'^2), z\right)\right) \tag{4}$$

insert it in Eq.(3) and integrate

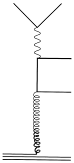
$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dzP_{ab}^R\left(\alpha_s(\mu'^2), z\right) \widetilde{f}_b\left(\frac{x}{z},\mu'^2\right). \tag{5}$$

Iterative solution

Example for a= gluon:

$$\widetilde{f}_a(x, \mu^2) = \widetilde{f}_a(x, \mu_0^2)\Delta_a(\mu^2)$$

Sudakov: probability of evolving from μ_0^2 to μ^2 without any resolvable branching.



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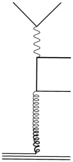
$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln \mu_0^2}^{\ln \mu^2} d\ln \mu'^2 \, \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P_{ab}^R\left(\alpha_s(\mu'^2), z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu_0^2\right) \Delta_b(\mu'^2)$$



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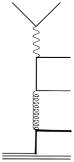

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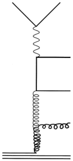
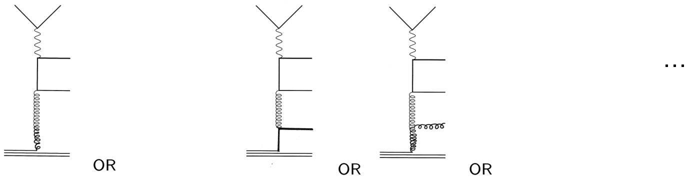


Iterative solution

Example for a= gluon:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu'^2 \, \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P_{ab}^R\left(\alpha_s(\mu'^2), z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu_0^2\right) \Delta_b(\mu'^2) + \ldots$$



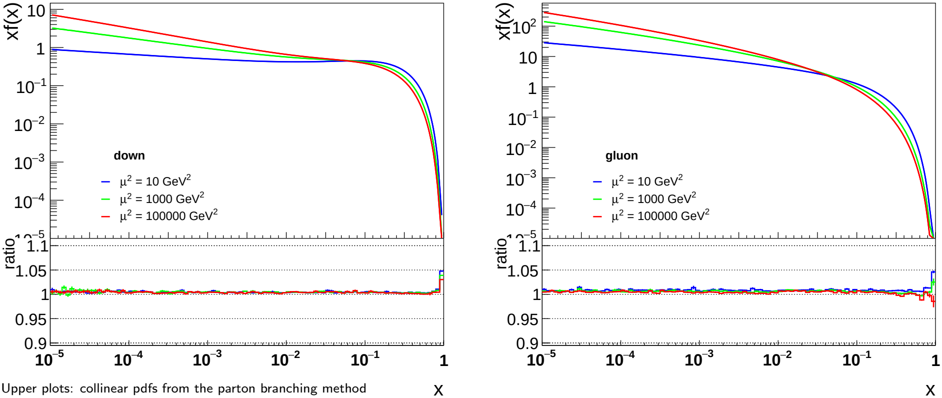
This problem has an iterative solution which can be easily implemented in the MC code.

Collinear PDFs from parton branching method

LO comparison with semi analytical methods

Initial distribution: $\widetilde{f}_{b_0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale μ^2.
Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)



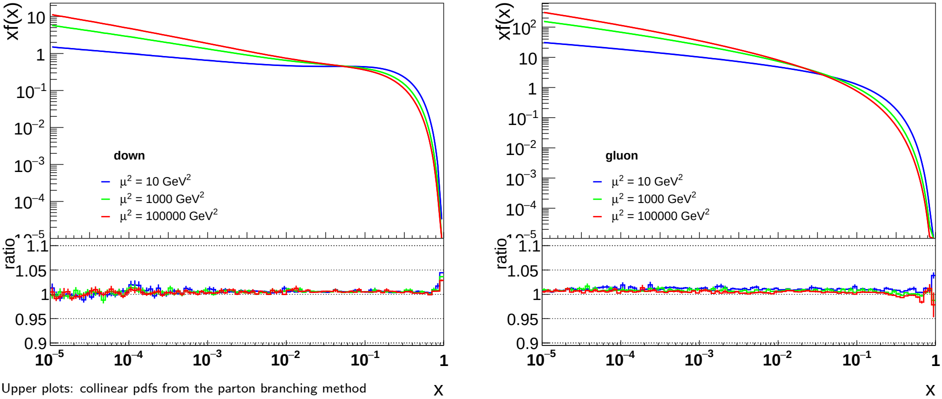
Upper plots: collinear pdfs from the parton branching method
Lower plots: ratios of the pdfs from a parton branching method and pdfs from QCDnum.

Very good agreement with the results coming from semi analytical methods (QCDnum).

NLO comparison with semi analytical methods

Initial distribution: $\widetilde{f}_{b_0}(x_0, \mu_0^2)$ - from QCDnum

The evolution performed with parton branching method up to a given scale μ^2.
Obtained distribution compared with a pdf calculated at the same scale by semi analytical method (QCDnum)

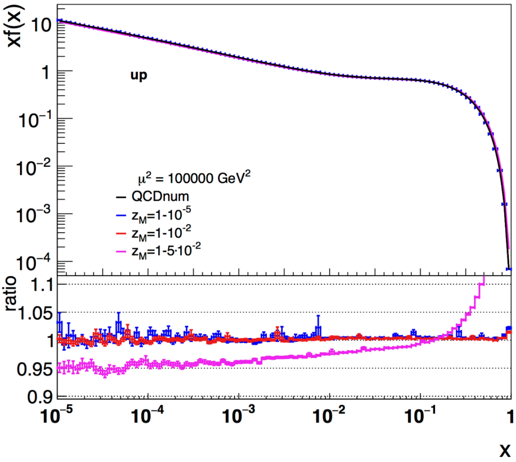


Upper plots: collinear pdfs from the parton branching method
Lower plots: ratios of the pdfs from a parton branching method and pdfs from QCDnum.

Very good agreement with the results coming from semi analytical methods (QCDnum).

Cross check for different z_M

Comparison of the results for different z_M values .



Upper plot: collinear pdfs from a MC method
Lower plot: ratios of the pdfs from a MC method and pdfs from QCDnum.

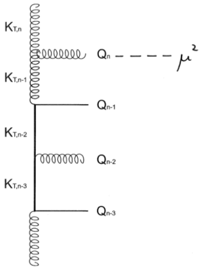
There is no dependence on z_M as long as z_M large enough.

Here results at NLO, at LO the same conclusion.

Results for TMDs

k_T dependence

Parton branching method: for every branching μ^2 is generated and available.



How to connect μ with Q_T of the emitted and k_T of the propagating parton?

- Q_T- ordering: $\vec{Q}^2_{T,n} = \mu^2$.
- virtuality ordering: $\vec{Q}^2_{T,n} = (1-z)\mu^2$

$$\vec{k}_{T,n} = \vec{k}_{T,n-1} - \vec{Q}_{T,n}$$

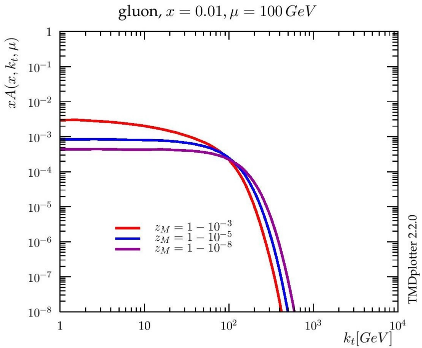
k_T contains the whole history of the evolution.

In this method k_T is treated properly from the beginning of the evolution- no extra reshuffling at the end is required.

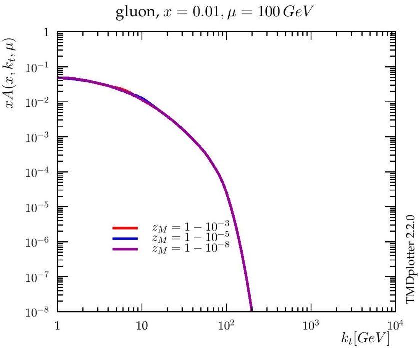
TMD PDFs from different k_T definition at LO

Reminder: for collinear PDFs there was no z_M dependence.

What about z_M dependence for TMDs?



Q_T- ordering: $\vec{Q}^2_{T,n} = \mu^2$



virtuality ordering: $\vec{Q}^2_{T,n} = (1 - z)\mu^2$

large z - soft gluons!

Q_T- ordering: for every z_M value we obtain different TMD

$\rightarrow$ not physical behaviour, Q_T- ordering shouldn't be used

Virtuality ordering: no z_M dependence (because of the $(1 - z)$ term soft gluons suppressed)

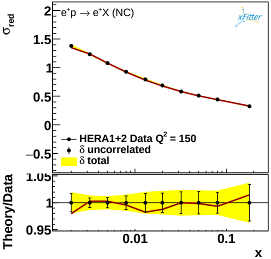
Fit of integrated TMDs for all flavours to HERA DIS data with xFitter

Procedure of the fit to the HERA 1+2 F_2 data

Goal: TMD PDF sets for all flavours, all x, Q^2 and k_T

- A kernel A_a^b is determined from the parton branching method from a toy starting distribution: $f_{0,b} = \delta(1-x)$.
- xFitter chooses a starting distribution $A_{0,b}$ and performs a convolution of the kernel A_a^b with the starting distribution $A_{0,b}$ to obtain a parton density

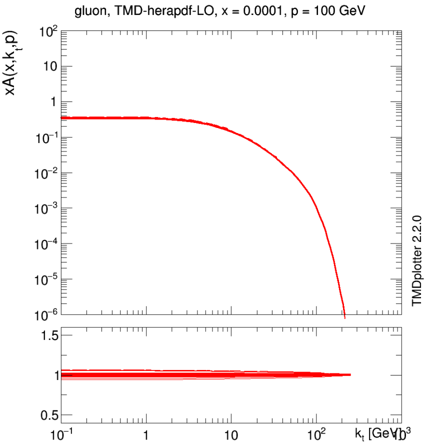
$$\widetilde{f}_a(x,\mu^2) = \int_0^\infty \frac{\mathrm{d}k_T^2}{k_T^2} \underbrace{\int \mathrm{d}x' A_{0,b}(x') \frac{x'}{x} A_a^b\left(\frac{x'}{x}, k_T^2, \mu^2\right)}_{\widetilde{f}_a(x,\mu^2,k_T^2)} \qquad (6)$$



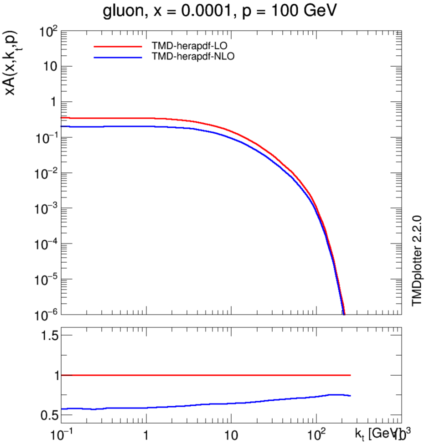
- Obtained parton density $\widetilde{f}_a(x,\mu^2)$ is fitted to the F_2 data and χ^2 is calculated.
  Data: arXiv:1506.06042v3, Abramowicz, H. and others.
- The procedure is repeated with the new starting distributions $A_{0,b}$ many times to minimize χ^2.

A very good $\chi^2/ndf \sim 1.2$ is obtained for $3.5 < Q^2 < 30000$ GeV^2 and $x > 4 \cdot 10^{-5}$.

TMDs from fits - comparison of LO and NLO TMDs



TMDs with experimental uncertainties.

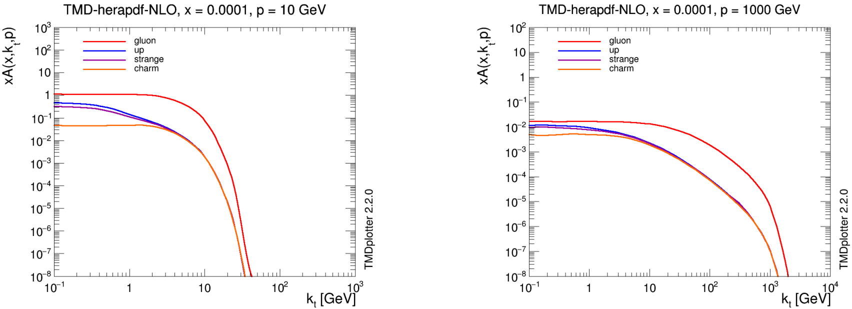


Comparison of the LO and NLO TMDs.

TMDs were fitted with experimental uncertainties at LO and NLO.

TMDs from the fit at NLO

From parton branching method we can obtain TMDs for all flavours



At small k_T (no branching or just a few branchings), the difference in the quark TMDs comes from initial distributions.
At large k_T (many branchings) TMDs for quarks the same.

TMDs sets available soon!

→ check on TMDplotter and TMDlib
http://tmdplotter.desy.de/
http://https://tmdlib.hepforge.org/doxy/html/index.html

Summary

Summary

New approach to solve coupled gluon and quark DGLAP evolution equation with a parton branching method was shown.

Advantages:

- it reproduces exactly semi-analytical solution for collinear PDFs (results consistent with QCDNum),
  moreover:
- extraction of TMD PDFs possible and done,
  fit to F2 Hera data at LO and NLO was performed within xFitter,
  TMDs sets for all flavours were obtained from the fit with experimental uncertainties,
- options to study different orderings for collinear and TMD PDFs available within this framework.

Prospects:

- TMD sets released soon,
- application in measurements,
- long term goal: direct usage in PS matched calculation.

Thank you!

Back up

DGLAP evolution equation

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a structure of a splitting function:

$$P_{ab}\left(\alpha_s(\mu^2), z\right) = D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)_+} + R_{ab}\left(\alpha_s(\mu^2), z\right), \tag{8}$$

$\int_0^1 f(x)g(x)_+ dx = \int_0^1 f(x)g(x)dx - \int_0^1 f(1)g(x)dx$
$D_{ab}\left(\alpha_s(\mu^2)\right) = \delta_{ab} d_a\left(\alpha_s(\mu^2)\right)$, $K_{ab}\left(\alpha_s(\mu^2)\right) = \delta_{ab} k_a\left(\alpha_s(\mu^2)\right)$,
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$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)} + R_{ab}\left(\alpha_s(\mu^2), z\right)\right)\widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \\ - \sum_b \widetilde{f}_b\left(x,\mu^2\right)\int_0^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)} - D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z)\right) \tag{9}$$

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$R_{ab}\left(\alpha_s(\mu^2), z\right)$ contains logarithmic terms in $\ln(1-z)$ and has no power divergences $(1-z)^{-n}$ for $z \to 1$.

$$\begin{aligned}\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = {} & \sum_b \int_x^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)} + R_{ab}\left(\alpha_s(\mu^2), z\right)\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \\ & - \sum_b \widetilde{f}_b\left(x,\mu^2\right) \int_0^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)} - D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z)\right)\end{aligned} \tag{9}$$

Two potential problems for numerical solution:

- presence of the delta function,
- integrals separately divergent for $z \to 1$.

Momentum sum rule

To get rid of the delta function:

We use *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\alpha_s(\mu^2), z\right) = 0$:

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$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{(1-z)} + R_{ab}\left(\alpha_s(\mu^2), z\right)\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$
$$- \sum_b \widetilde{f}_b\left(x,\mu^2\right) \int_0^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{(1-z)} - D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z)\right) +$$
$$- \widetilde{f}_a(x,\mu^2) \sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\alpha_s(\mu^2), z\right) =$$
$$= \sum_b \int_x^1 \mathrm{d}z \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right)\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$
$$- \widetilde{f}_a\left(x,\mu^2\right) \sum_c \int_0^1 \mathrm{d}z\, z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right)\right) \tag{10}$$

Momentum sum rule

To get rid of the delta function:
We use *momentum sum rule* $\sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\alpha_s(\mu^2), z\right) = 0$:

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{(1-z)} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$

$$- \sum_b \widetilde{f}_b\left(x,\mu^2\right) \int_0^1 dz \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{(1-z)} - D_{ab}\left(\alpha_s(\mu^2)\right) \delta(1-z) \right) +$$

$$- \widetilde{f}_a(x,\mu^2) \sum_c \int_0^1 \mathrm{d}z z P_{ca}\left(\alpha_s(\mu^2), z\right) =$$

$$= \sum_b \int_x^1 \mathrm{d}z \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) +$$

$$- \widetilde{f}_a\left(x,\mu^2\right) \sum_c \int_0^1 \mathrm{d}z\, z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right) \right) \quad (10)$$

We got rid of the delta function,
both pieces of the equation written in the same way.
Virtual and non-resorvable pieces still included.

Divergence for $z \to 1$

To avoid divergence when $z \to 1$ a cut off must be introduced:

$$\begin{aligned}
&\sum_b \int_x^{1} \mathrm{dz} \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \\
&\quad - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{1} \mathrm{dz}\, z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right) \right) \\
&\to \sum_b \int_x^{z_M} \mathrm{dz} \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \\
&\quad - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{z_M} \mathrm{dz}\, z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right) \right)
\end{aligned} \tag{11}$$

It can be shown that terms $\int_{zmax}^{1}$ skipped in the integral in eq. (11) are of order $\mathcal{O}(1 - z_{max})$.

Divergence for $z \to 1$

To avoid divergence when $z \to 1$ a cut off must be introduced:

$$
\begin{aligned}
&\sum_b \int_x^{1} \mathrm{dz} \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \\
&\qquad - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{1} \mathrm{dz}\; z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right) \right) \\
&\to \sum_b \int_x^{z_M} \mathrm{dz} \left(K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ab}\left(\alpha_s(\mu^2), z\right) \right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \\
&\qquad - \widetilde{f}_a\left(x, \mu^2\right) \sum_c \int_0^{z_M} \mathrm{dz}\; z \left(K_{ca}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z} + R_{ca}\left(\alpha_s(\mu^2), z\right) \right)
\end{aligned}
\tag{11}
$$

It can be shown that terms $\int_{zmax}^{1}$ skipped in the integral in eq. (11) are of order $\mathcal{O}(1 - z_{max})$.

Different choices of z_{max}:

- z_{max} - fixed
- z_{max} - can change dynamically with the scale:
  angular ordering: $z_{max} = 1 - \left(\frac{Q_0}{Q}\right)$
  or virtuality ordering: $z_{max} = 1 - \left(\frac{Q_0}{Q}\right)^2$

In this presentation: results from fixed z_{max}.

Sudakov form factor

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{d}z \; P^R_{ab}\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right) \sum_c \int_0^{z_M} \mathrm{d}z \; z P^R_{ca}\left(\alpha_s(\mu^2), z\right) \tag{12}$$

where $P^R_{ab}\left(\alpha_s(\mu^2), z\right) = R_{ab}\left(\alpha_s(\mu^2), z\right) + K_{ab}\left(\alpha_s(\mu^2)\right) \frac{1}{1-z}$ - *real* part of the splitting function.

Sudakov form factor

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} \mathrm{d}z\ P^R_{ab}\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right) \sum_c \int_0^{z_M} \mathrm{d}z\ zP^R_{ca}\left(\alpha_s(\mu^2), z\right) \tag{12}$$

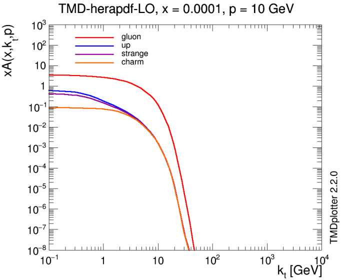
where $P^R_{ab}\left(\alpha_s(\mu^2), z\right) = R_{ab}\left(\alpha_s(\mu^2), z\right) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{1-z}$ - *real* part of the splitting function.
Defining the *Sudakov form factor*:

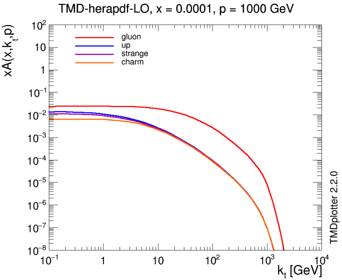
$$\Delta_a(\mu^2) = \exp\left(-\int_{\ln\mu_0^2}^{\ln\mu^2} d\left(\ln\mu'^2\right) \sum_b \int_0^{z_M} dzzP^R_{ba}\left(\alpha_s(\mu'^2), z\right)\right) \tag{13}$$

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^{z_M} dzP^R_{ab}\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \widetilde{f}_a\left(x,\mu^2\right) \frac{1}{\Delta_a(\mu^2)} \frac{d\Delta_a(\mu^2)}{d\ln\mu^2}, \tag{14}$$

TMDs from the fit at LO

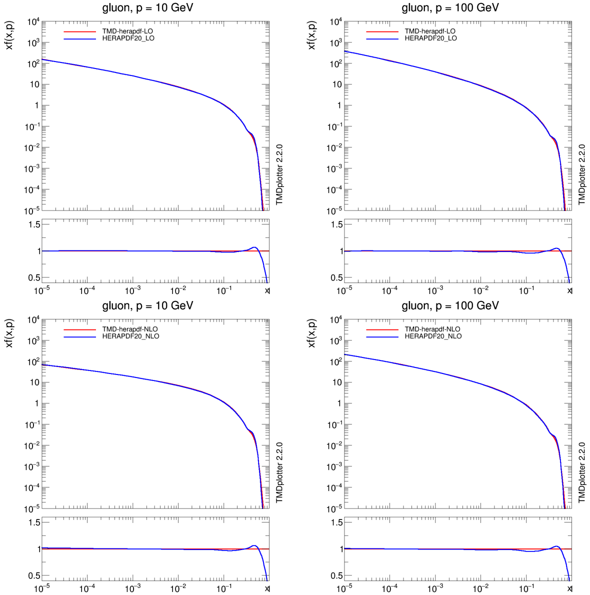
From parton branching method we can obtain TMDs for all flavours





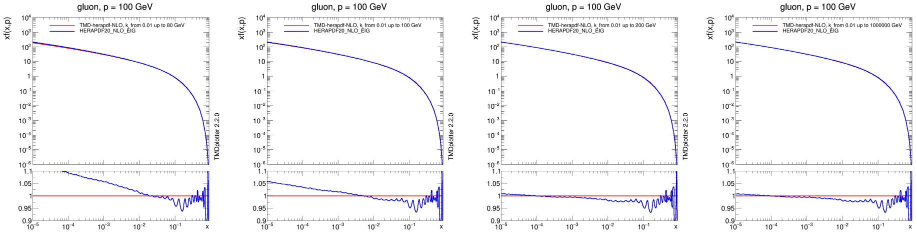
At small k_T (no branching or just a few branchings), the difference in the quark TMDs comes from initial distributions.
At large k_T (many branchings) TMDs for quarks the same.

integrated TMD from parton branching method and HERA pdf



Role of the limit in k_T integration

Comparison of int TMDs integrated up to a diffent k_T values



The integral over k_T has to be performed up to a value higher than the evolution scale to obtain collinear PDF which agrees well with the HERA pdf.