# Precision contraints on the top EFT at future lepton colliders

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# Introduction

# The NP EFT (often called SM EFT)

provides a systematic parametrization of the theory space
in direct vicinity of the SM

- in a low-energy limit
- through a proper QFT
- consistent when global

EFT analysis recipe:

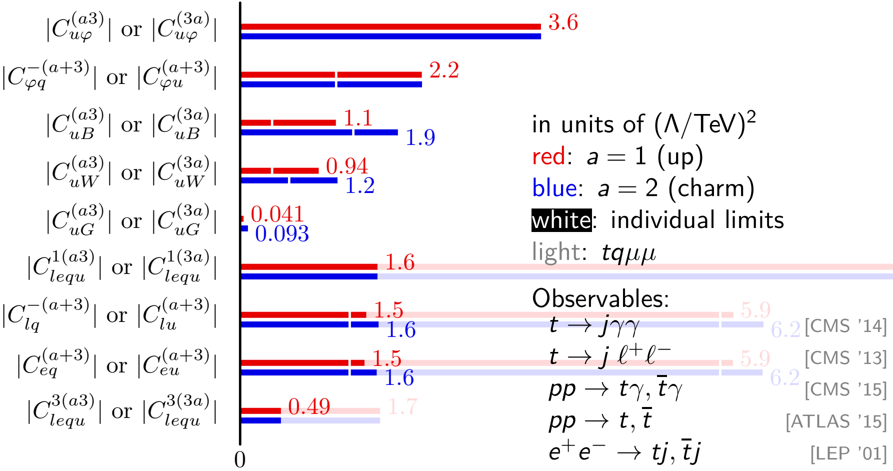
1. Go global!
2. Combine observables!
3. Offer yourself NLO QCD!

| | |
|---|---|
| FCNCs | [Degrande et al, '14] |
| top pair production | [Franzosi et al. '15] |
| single top production | [Zhang '16] |
| $t\bar{t}Z$, $t\bar{t}\gamma$ | [Bylund et al. '16] |
| $t\bar{t}h$ | [Maltoni et al. '16] |
| four-fermion operators | [$\bar{l}l\bar{q}q$ OK, $\bar{q}q\bar{q}q$ ongoing] |

# Global top FCNC constraints

Showcase example:
- (all) 52 complex coefficients
- full NLO QCD in the EFT

[GD-Maltoni-Zhang '14]



in units of $(\Lambda/\text{TeV})^2$

red: $a = 1$ (up)

blue: $a = 2$ (charm)

white: individual limits

light: $tq\mu\mu$

Observables:

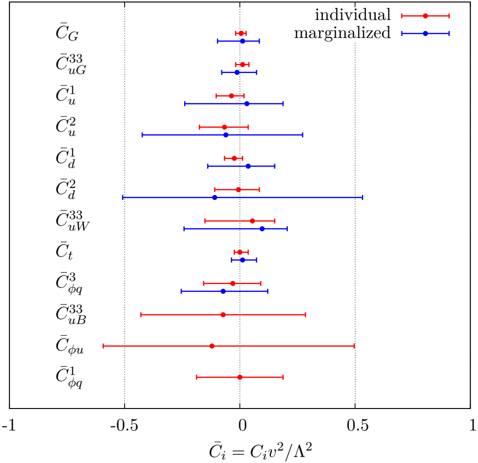
| | |
|---|---|
| $t \to j\gamma\gamma$ | [CMS '14] |
| $t \to j\,\ell^+\ell^-$ | [CMS '13] |
| $pp \to t\gamma, \bar{t}\gamma$ | [CMS '15] |
| $pp \to t, \bar{t}$ | [ATLAS '15] |
| $e^+e^- \to tj, \bar{t}j$ | [LEP '01] |

# Global top EFT constraints from LHC

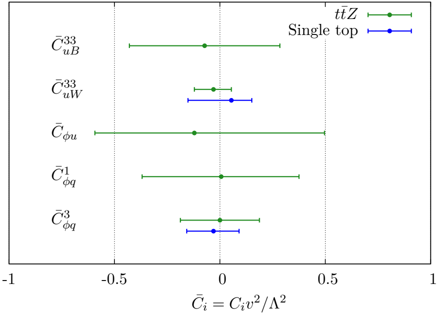
- 12 operators, 195 observables (174 from differential distributions)
- mainly from $t\bar{t}$, then single top, charge asymmetries, associated production, $W$ helicity fraction in decay
- standard-model-only (N)NLO k-factors in each bin

[Buckley *et al.* Jun. 15]

[Buckley *et al.* Dec. 15]



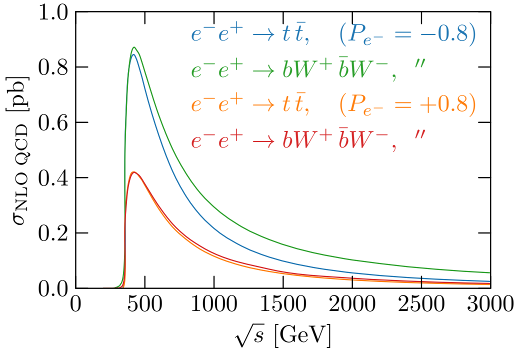
individual 95% CL limits from $t\bar{t}Z$ and $t\bar{t}\gamma$:

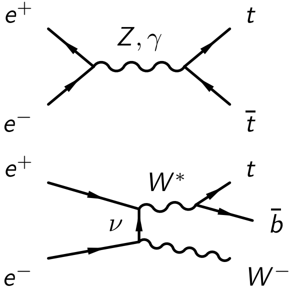


# Top EFT at lepton colliders

# Aiming at a global EFT analysis

- Including four-fermion operators, notably
- Examining the impact of
  - NLO QCD corrections
  - off-shell top effect
- Studying the sensitivity of various observables
  - at various center-of-mass energies
  - for various beam polarization





# Up-sector EFT

[Grzadkowski et al '10]

Two-quark operators:

Scalar: $O_{u\varphi} \equiv \bar{q} u \, \tilde{\varphi} \quad \varphi^\dagger \varphi,$

Vector: $O^1_{\varphi q} \equiv \bar{q} \gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \quad \equiv O^+_{\varphi q} + O^V_{\varphi q} - O^A_{\varphi q},$

$O^3_{\varphi q} \equiv \bar{q} \gamma^\mu \tau^I q \quad \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi \quad \equiv O^+_{\varphi q} - O^V_{\varphi q} + O^A_{\varphi q}$ (CC also)

$O_{\varphi u} \equiv \bar{u} \gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \quad \equiv O^V_{\varphi q} + O^A_{\varphi q}$

$O_{\varphi ud} \equiv \bar{u} \gamma^\mu d \quad \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi,$ (CC only, $m_b$ int.)

Tensor: $O_{uB} \equiv \bar{q} \sigma^{\mu\nu} u \, \tilde{\varphi} \quad B_{\mu\nu}, \quad \equiv O_{uA} - \tan\theta_W O_{uZ}$

$O_{uW} \equiv \bar{q} \sigma^{\mu\nu} \tau^I u \, \tilde{\varphi} \quad W^I_{\mu\nu}, \quad \equiv O_{uA} + \text{cotan}\,\theta_W O_{uZ}$ (CC also)

$O_{dW} \equiv \bar{q} \sigma^{\mu\nu} \tau^I d \, \tilde{\varphi} \quad W^I_{\mu\nu},$ (CC only, $m_b$ int.)

$O_{uG} \equiv \bar{q} \sigma^{\mu\nu} T^A u \, \tilde{\varphi} \quad G^A_{\mu\nu}.$

Two-quark–two-lepton operators:

Scalar: $O^S_{lequ} \equiv \bar{l} e \quad \varepsilon \quad \bar{q} u,$ (CC also, $m_e$ int.)

$O_{ledq} \equiv \bar{l} e \quad \bar{d} q,$ (CC only, $m_e$ int.)

Vector: $O^1_{lq} \equiv \bar{l} \gamma_\mu l \quad \bar{q} \gamma^\mu q \quad \equiv O^+_{lq} + O^V_{lq} - O^A_{lq},$

$O^3_{lq} \equiv \bar{l} \gamma_\mu \tau^I l \quad \bar{q} \gamma^\mu \tau^I q \quad \equiv O^+_{lq} - O^V_{lq} + O^A_{lq},$ (CC also)

$O_{lu} \equiv \bar{l} \gamma_\mu l \quad \bar{u} \gamma^\mu u \quad \equiv O^V_{lq} + O^A_{lq},$

$O_{eq} \equiv \bar{e} \gamma^\mu e \quad \bar{q} \gamma_\mu q \quad \equiv O^V_{eq} - O^A_{eq},$

$O_{eu} \equiv \bar{e} \gamma_\mu e \quad \bar{u} \gamma^\mu u \quad \equiv O^V_{eq} + O^A_{eq},$

Tensor: $O^T_{lequ} \equiv \bar{l} \sigma_{\mu\nu} e \quad \varepsilon \quad \bar{q} \sigma^{\mu\nu} u.$ (CC also, $m_e$ int.)

# Anomalous vertices

$$t\bar{t}\gamma: \quad \gamma_\mu \overbrace{\left(F_{1V}^{\gamma} + \gamma_5 F_{1A}^{\gamma}\right)}^{\sim\emptyset} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{\gamma} + i\gamma_5 F_{2A}^{\gamma}\right)}^{\sim\text{Re,Im}\{C_{uA}\}}$$

$$t\bar{t}Z: \quad \gamma_\mu \overbrace{\left(F_{1V}^{Z} + \gamma_5 F_{1A}^{Z}\right)}^{\sim C_{\varphi u}^{V}, C_{\varphi u}^{A}} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{Z} + i\gamma_5 F_{2A}^{Z}\right)}^{\sim\text{Re,Im}\{C_{uZ}\}}$$

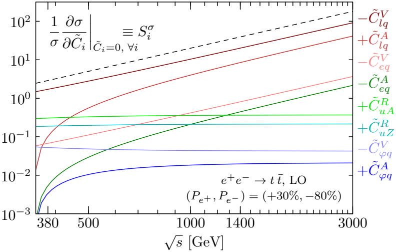
$$t\bar{b}W: \quad \gamma_\mu \overbrace{\left(F_{1V}^{W} + \gamma_5 F_{1A}^{W}\right)}^{\sim C_{\varphi ud}, C_{\varphi q}^{+} - \frac{1}{2}(C_{\varphi q}^{V} - C_{\varphi q}^{A})} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{W} + i\gamma_5 F_{2A}^{W}\right)}^{\sim\text{Re,Im}\{s_W^2 C_{uA} + s_W c_W C_{uZ}\}}$$

Insufficiencies:

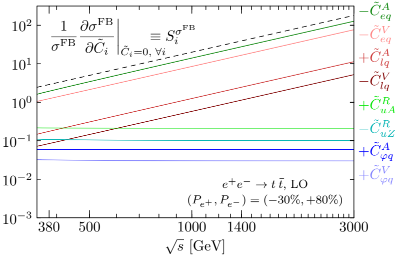
- Miss four-fermion operators
- Conflict with gauge invariance
  Do not allow for radiative corrections to be computed
- Complex couplings where the tree-level EFT prescribes real ones
- Hide correlations induced by gauge invariance
  Preclude the combination of measurements in various sectors

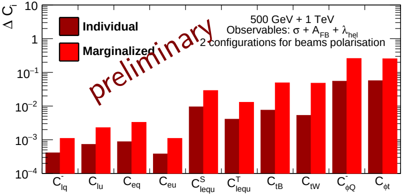
# Operator sensitivities in $e^+e^- \to t\,\bar{t}$

Total cross section (left pol.):



FB-integrated cross section (right pol.):





## Few features:

- quadratic energy growth for four-fermion operators
- no growth for two-fermion operators
  Some *azimuthal* $bW^+\bar{b}W^-$ observables have linearly growing sensitivity to $C_{uZ}, C_{uA}$.
- $p$-wave $\beta = \sqrt{1 - m_t^2/s}$ suppression of axial vectors at threshold
- enhanced sensitivity of axial vector operators in $\sigma^{\mathrm{FB}}$
- sensitivity sign flip for $C_{\varphi q}^V$ and $C_{uZ}^R$ when polarization is reversed
- etc.

# Helicity amplitude decomposition in $bW^+\bar{b}W^-$

[Jacob,Wick '59]

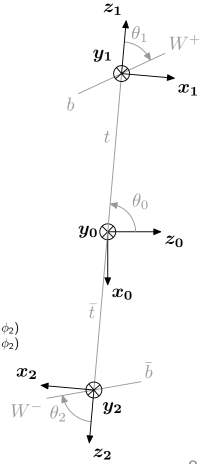
Production amplitudes:

[Schmidt '95]

$$
\begin{aligned}
++ &: A_1 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D - \beta\tilde{D}) \\
-- &: A_2 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D + \beta\tilde{D}) \\
+- &: A_3 \sim (V + \beta A) + 2m_t D \\
-+ &: A_4 \sim (V - \beta A) + 2m_t D
\end{aligned}
$$

In terms of $\Omega = \{\theta_0, \theta_1, \phi_1, \theta_2, \phi_2\}$ helicity angles:

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} =
\begin{array}{ccccccccc}
+3/4 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & (1+\cos^2\theta_0) & & & \\
+3/4 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & (1+\cos^2\theta_0) & & \cos\theta_2 & \\
+3/4 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & (1+\cos^2\theta_0) & \cos\theta_1 & & \\
+3/4 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & (1+\cos^2\theta_0) & \cos\theta_1 & \cos\theta_2 & \\
-3/2 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & \cos\theta_0 & & & \\
-3/2 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & \cos\theta_0 & & \cos\theta_2 & \\
-3/2 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & \cos\theta_0 & \cos\theta_1 & & \\
-3/2 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \cos\theta_0 & \cos\theta_1 & \cos\theta_2 & \\
+3/2 & & (|A_1|^2 + |A_2|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & \sin^2\theta_0 & & & \\
-3/2 & & (|A_1|^2 - |A_2|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & & \cos\theta_2 & \\
+3/2 & & (|A_1|^2 - |A_2|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & \sin^2\theta_0 & \cos\theta_1 & & \\
-3/2 & & (|A_1|^2 + |A_2|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \cos\theta_1 & \cos\theta_2 & \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \cos\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_4\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_4\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_3\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_3\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3 & & \mathrm{Re}\{A_1^* A_2\} & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \sin\theta_1 & \sin\theta_2 & \cos(\phi_1+\phi_2) \\
-3/2 & & \mathrm{Re}\{A_3^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \sin\theta_1 & \sin\theta_2 & \cos(\phi_1-\phi_2) \\
 & & & & & & & & \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \sin\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \sin\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \sin\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \sin\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_4\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \sin\phi_2 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_4\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \sin\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_3\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \sin\phi_2
\end{array}
$$



# NLO in QCD for $e^+e^- \to bW^+\bar{b}W^-$

For various beam polarizations and center-of-mass energies:

| pol | $\sqrt{s}$ [GeV] | $\sigma_{SM}$ [fb] | $\sigma_i$ [fb] | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\vert^A_{lq}$ | $\vert^A_{eq}$ | $\vert^A_{\phi q}$ | $\vert^V_{lq}$ | $\vert^V_{eq}$ | $\vert^V_{\phi q}$ | $\vert^R_{uZ}$ | $\vert^R_{uA}$ | $\vert^I_{uZ}$ | $\vert^I_{uA}$ | $\vert^R_{uG}$ |
| 00 | 300 | $\underset{1.15}{2.92}\pm0.3\%^{+1\%}_{-1\%}$ | $\underset{1.15}{0.353}\pm0.2\%^{+1\%}_{-1\%}$ | $\underset{1.27}{-0.0856}\pm0.4\%^{-2\%}_{+2\%}$ | $\underset{1.2}{0.14}\pm0.1\%^{+2\%}_{-1\%}$ | $\underset{1.34}{-0.621}\pm0.2\%^{-2\%}_{+3\%}$ | $\underset{1.51}{-0.303}\pm0.4\%^{-3\%}_{+4\%}$ | $\underset{1.21}{-0.136}\pm0.1\%^{-2\%}_{+2\%}$ | $\underset{1.21}{0.349}\pm0.3\%^{+2\%}_{-2\%}$ | $\underset{1.33}{0.32}\pm0.1\%^{+3\%}_{-2\%}$ | $\underset{—}{-0.000225}\pm90\%^{-7\%}_{+9\%}$ | $\underset{—}{-0.000125}\pm90\%^{-9\%}_{+10\%}$ | $\underset{—}{0.000214}\pm6\%^{+10\%}_{-9\%}$ |
| 00 | 380 | $\underset{1.18}{825}\pm0.5\%^{+2\%}_{-1\%}$ | $\underset{1.34}{77.1}\pm3\%^{+3\%}_{-2\%}$ | $\underset{1.32}{-55.6}\pm20\%^{-2\%}_{+3\%}$ | $\underset{1.2}{53.1}\pm0.1\%^{+2\%}_{-1\%}$ | $\underset{1.19}{-993}\pm2\%^{-1\%}_{+2\%}$ | $\underset{1.16}{-635}\pm1\%^{-1\%}_{+1\%}$ | $\underset{1.19}{-62.9}\pm0.6\%^{-1\%}_{+2\%}$ | $\underset{1.14}{118}\pm2\%^{+1\%}_{-1\%}$ | $\underset{1.19}{323}\pm2\%^{+2\%}_{-1\%}$ | $\underset{-1.36}{0.107}\pm300\%^{+20\%}_{-20\%}$ | $\underset{—}{-0.434}\pm80\%^{-10\%}_{+10\%}$ | $\underset{—}{0.25}\pm10\%^{+10\%}_{-9\%}$ |
| 00 | 500 | $\underset{0.952}{669}\pm0.5\%^{+0.4\%}_{-0.5\%}$ | $\underset{0.936}{258}\pm7\%^{+0.6\%}_{-0.7\%}$ | $\underset{1.04}{-233}\pm2\%^{-0.3\%}_{+0.4\%}$ | $\underset{0.99}{49.2}\pm1\%^{+0.08\%}_{-0.1\%}$ | $\underset{0.929}{-1230}\pm2\%^{-0.8\%}_{+0.7\%}$ | $\underset{0.872}{-750}\pm4\%^{-2\%}_{+1\%}$ | $\underset{0.909}{-45.2}\pm5\%^{-1\%}_{+0.9\%}$ | $\underset{0.97}{102}\pm5\%^{+0.3\%}_{-0.3\%}$ | $\underset{0.929}{263}\pm0.7\%^{+0.6\%}_{-0.8\%}$ | $\underset{—}{-2.08}\pm50\%^{-6\%}_{+8\%}$ | $\underset{—}{1.78}\pm80\%^{+10\%}_{-8\%}$ | $\underset{—}{0.715}\pm30\%^{+10\%}_{-9\%}$ |
| 00 | 1000 | $\underset{0.897}{221}\pm0.5\%^{+1\%}_{-1\%}$ | $\underset{1.21}{756}\pm9\%^{+2\%}_{-1\%}$ | $\underset{0.983}{-475}\pm20\%^{-0.2\%}_{+0.1\%}$ | $\underset{0.844}{15.6}\pm4\%^{+2\%}_{-2\%}$ | $\underset{0.784}{-1070}\pm20\%^{-3\%}_{+2\%}$ | $\underset{0.95}{-940}\pm6\%^{-0.6\%}_{+0.5\%}$ | $\underset{0.914}{-15.5}\pm3\%^{-1\%}_{+0.8\%}$ | $\underset{0.995}{36.1}\pm5\%^{+0.04\%}_{-0.05\%}$ | $\underset{0.899}{87.3}\pm3\%^{+1\%}_{-1\%}$ | $\underset{—}{0.392}\pm200\%^{+10\%}_{-10\%}$ | $\underset{—}{-8.74}\pm40\%^{-9\%}_{+10\%}$ | $\underset{—}{0.907}\pm5\%^{+10\%}_{-9\%}$ |
| 00 | 1400 | $\underset{0.936}{132}\pm3\%^{+0.6\%}_{-0.7\%}$ | $\underset{0.555}{391}\pm30\%^{+7\%}_{-9\%}$ | $\underset{0.798}{-412}\pm30\%^{-3\%}_{+2\%}$ | $\underset{0.803}{8.29}\pm3\%^{+2\%}_{-3\%}$ | $\underset{0.926}{-1460}\pm5\%^{-0.9\%}_{+0.7\%}$ | $\underset{0.794}{-816}\pm10\%^{-3\%}_{+2\%}$ | $\underset{0.859}{-8}\pm4\%^{-2\%}_{+1\%}$ | $\underset{1.03}{21.6}\pm8\%^{+0.2\%}_{-0.1\%}$ | $\underset{1.08}{59.8}\pm6\%^{+0.5\%}_{-0.4\%}$ | $\underset{—}{1.8}\pm80\%^{+8\%}_{-6\%}$ | $\underset{-0.906}{0.257}\pm200\%^{+20\%}_{-20\%}$ | $\underset{—}{0.414}\pm100\%^{+10\%}_{-10\%}$ |
| 00 | 3000 | $\underset{1.13}{40.2}\pm10\%^{+1\%}_{-1\%}$ | $\underset{1.1}{1080}\pm20\%^{+1\%}_{-0.8\%}$ | $\underset{0.128}{-70.4}\pm200\%^{-70\%}_{+60\%}$ | $\underset{0.533}{1.28}\pm10\%^{+8\%}_{-9\%}$ | $\underset{0.981}{-1270}\pm20\%^{-0.3\%}_{+0.2\%}$ | $\underset{0.668}{-689}\pm50\%^{-5\%}_{+4\%}$ | $\underset{0.406}{-0.717}\pm30\%^{-20\%}_{+10\%}$ | 10[illegible] $\underset{1.38}{}\pm30\%^{+3\%}_{-2\%}$ | $\underset{1.05}{14.2}\pm30\%^{+0.5\%}_{-0.4\%}$ | $\underset{0.441}{1.85}\pm100\%^{+7\%}_{-9\%}$ | $\underset{0.594}{2.15}\pm200\%^{+1\%}_{-1\%}$ | $\underset{—}{-0.261}\pm200\%^{-7\%}_{+8\%}$ |
| +− | 300 | $\underset{1.14}{2.73}\pm0.3\%^{+1\%}_{-1\%}$ | $\underset{1.14}{0.351}\pm0.2\%^{+1\%}_{-1\%}$ | — | $\underset{1.19}{0.126}\pm0.2\%^{+2\%}_{-1\%}$ | $\underset{1.34}{-0.62}\pm0.2\%^{-2\%}_{+3\%}$ | — | $\underset{1.22}{-0.14}$ ±[illegible] $^{-2\%}_{+2\%}$ | $\underset{1.23}{0.376}$ [illegible]0.1% $^{[illegible]2\%}_{-2\%}$ | $\underset{1.28}{0.241}\pm0.2\%^{+2\%}_{-2\%}$ | $\underset{-0.119}{6.25e-06}\pm2000\%^{+70\%}_{-50\%}$ | $\underset{0.19}{2.7e-06}\pm2000\%^{+20\%}_{-30\%}$ | $\underset{—}{0.000197}\pm4\%^{+10\%}_{-9\%}$ |
| +− | 380 | $\underset{1.19}{579}\pm2\%^{+2\%}_{-1\%}$ | $\underset{1.35}{73}\pm1\%^{+3\%}_{-2\%}$ | — | $\underset{1.18}{36.1}\pm0.5\%^{+2\%}_{-1\%}$ | $\underset{1.16}{-968}\pm2\%^{-1\%}_{+1\%}$ | — | [illegible]64.5 [illegible]16 ±1% −1[illegible] +[illegible] | $\underset{1.18}{165}\pm0.4\%^{+2\%}_{-1\%}$ | $\underset{1.18}{198}\pm0.6\%^{+2\%}_{-1\%}$ | $\underset{—}{0.44}\pm70\%^{+10\%}_{-9\%}$ | $\underset{—}{-0.324}\pm100\%^{-10\%}_{+10\%}$ | $\underset{—}{0.185}\pm10\%^{+10\%}_{-9\%}$ |
| +− | 500 | $\underset{0.96}{469}\pm0.5\%^{+0.4\%}_{-0.4\%}$ | $\underset{1.03}{287}\pm5\%^{+0.4\%}_{-0.3\%}$ | — | $\underset{0.953}{31.7}\pm2\%^{+0.4\%}_{-0.5\%}$ | $\underset{0.972}{-1270}\pm2\%^{-0.3\%}_{+0.2\%}$ | — | [illegible]1.1 $\underset{0.982}{}\pm3\%^{-0.2\%}_{+0.2\%}$ | $\underset{0.918}{130}\pm1\%^{+0.8\%}_{-0.9\%}$ | $\underset{0.943}{164}\pm0.5\%^{+0.5\%}_{-0.6\%}$ | $\underset{—}{1.35}\pm50\%^{+10\%}_{-9\%}$ | $\underset{—}{-0.442}\pm200\%^{-6\%}_{+8\%}$ | $\underset{—}{0.554}\pm10\%^{+10\%}_{-8\%}$ |
| +− | 1000 | $\underset{0.902}{160}\pm1\%^{+0.9\%}_{-1\%}$ | $\underset{0.742}{470}\pm8\%^{+3\%}_{-4\%}$ | — | $\underset{0.931}{11.8}\pm4\%^{+0.6\%}_{-0.8\%}$ | $\underset{0.926}{-1450}\pm4\%^{-0.8\%}_{+0.7\%}$ | — | $\underset{0.93}{-15.5}\pm2\%^{-0.8\%}_{+0.7\%}$ | $\underset{0.916}{44.9}\pm3\%^{+0.8\%}_{-1\%}$ | $\underset{0.883}{52.6}\pm6\%^{+1\%}_{-1\%}$ | $\underset{—}{-0.663}\pm200\%^{-7\%}_{+9\%}$ | $\underset{—}{5.09}\pm40\%^{+10\%}_{-8\%}$ | $\underset{—}{0.587}\pm4\%^{+10\%}_{-9\%}$ |
| +− | 1400 | $\underset{0.817}{84.9}\pm10\%^{+2\%}_{-2\%}$ | $\underset{0.715}{507}\pm20\%^{+3\%}_{-4\%}$ | — | $\underset{1.07}{7.57}\pm6\%^{+0.7\%}_{-0.6\%}$ | $\underset{0.772}{-1230}$ ±[illegible] −3[illegible] +3[illegible] | — | $\underset{0.835}{-7.76}\pm6\%^{-2\%}_{+2\%}$ | $\underset{0.806}{22.2}\pm8\%^{+2\%}_{-3\%}$ | $\underset{0.905}{29.5}\pm9\%^{+0.9\%}_{-1\%}$ | $\underset{—}{-1.22}\pm300\%^{-8\%}_{+10\%}$ | $\underset{—}{-2.38}\pm90\%^{-10\%}_{+10\%}$ | $\underset{—}{0.281}\pm100\%^{+10\%}_{-10\%}$ |
| +− | 3000 | $\underset{0.787}{23.8}\pm7\%^{+2\%}_{-3\%}$ | $\underset{0.414}{356}\pm30\%^{+10\%}_{-10\%}$ | — | $\underset{0.338}{0.574}\pm30\%^{+20\%}_{-20\%}$ | −14[illegible]0 $\underset{0.853}{}\pm10\%^{-2\%}_{+1\%}$ | — | $\underset{0.543}{-1.08}\pm30\%^{-9\%}_{+7\%}$ | $\underset{1.25}{6.28}\pm30\%^{+2\%}_{-1\%}$ | — | $\underset{—}{1.93}\pm90\%^{+10\%}_{-10\%}$ | $\underset{—}{8.36}\pm30\%^{+7\%}_{-5\%}$ | $\underset{—}{0.197}\pm70\%^{+10\%}_{-10\%}$ |
| −+ | 300 | $\underset{1.37}{0.218}\pm0.4\%^{-2\%}_{+0\%}$ | — | $\underset{1.27}{-0.0855}\pm0.1\%^{-2\%}_{+2\%}$ | $\underset{1.42}{0.0147}\pm0.1\%^{+3\%}_{-3\%}$ | — | $\underset{1.51}{-0.302}\pm0.2\%^{-3\%}_{+4\%}$ | $\underset{1.59}{0.00343}\pm0.1\%^{+4\%}_{-3\%}$ | $\underset{1.5}{-0.0259}\pm0.1\%^{-3\%}_{+4\%}$ | $\underset{1.5}{0.0799}\pm0.2\%^{+4\%}_{-3\%}$ | $\underset{—}{3.38e-06}\pm300\%^{+6\%}_{-5\%}$ | $\underset{1.3}{-7.78e-06}\pm200\%^{-0.1\%}_{+0.07\%}$ | $\underset{—}{1.84e-05}\pm4\%^{+10\%}_{-9\%}$ |
| −+ | 380 | $\underset{1.19}{249}\pm0.9\%^{-1\%}_{+0\%}$ | — | $\underset{1.18}{-51.6}\pm4\%^{-1\%}_{+2\%}$ | $\underset{1.18}{16.2}\pm0.6\%^{+2\%}_{-1\%}$ | — | $\underset{1.18}{-649}\pm0.2\%^{-1\%}_{+2\%}$ | $\underset{1.16}{3.31}\pm1\%^{+1\%}_{-1\%}$ | $\underset{1.15}{-41.8}\pm0.8\%^{-1\%}_{+1\%}$ | $\underset{1.19}{124}\pm0.4\%^{+2\%}_{-1\%}$ | $\underset{—}{0.0946}\pm200\%^{+10\%}_{-9\%}$ | $\underset{—}{-0.0633}\pm200\%^{-10\%}_{+20\%}$ | $\underset{-0.0614}{0.000205}\pm9000\%^{+200\%}_{-200\%}$ |
| −+ | 500 | $\underset{0.948}{203}\pm1\%^{+0.5\%}_{+0\%}$ | — | $\underset{0.958}{-213}\pm1\%^{-0.5\%}_{+0.4\%}$ | $\underset{0.975}{15.8}\pm0.9\%^{+0.2\%}_{-0.3\%}$ | — | $\underset{0.933}{-810}\pm0.7\%^{-0.8\%}_{+0.6\%}$ | $\underset{0.783}{0.767}\pm4\%^{+2\%}_{-3\%}$ | $\underset{0.923}{-34.2}\pm5\%^{-0.9\%}_{+0.7\%}$ | $\underset{0.909}{99.7}\pm0.5\%^{+0.9\%}_{-1\%}$ | $\underset{—}{0.316}\pm40\%^{+10\%}_{-8\%}$ | $\underset{—}{0.187}\pm200\%^{+10\%}_{-8\%}$ | $\underset{—}{0.255}\pm20\%^{+10\%}_{-9\%}$ |
| −+ | 1000 | $\underset{0.911}{63.4}\pm5\%^{+0.9\%}_{+0\%}$ | — | $\underset{0.7}{-327}\pm10\%^{-4\%}_{+4\%}$ | $\underset{0.828}{4.88}\pm3\%^{+2\%}_{-2\%}$ | — | $\underset{0.82}{-810}\pm5\%^{-2\%}_{+2\%}$ | $\underset{0.907}{-0.24}\pm10\%^{-1\%}_{+0.8\%}$ | $\underset{0.812}{-10.4}\pm6\%^{-2\%}_{+2\%}$ | $\underset{0.906}{34.1}\pm5\%^{+0.9\%}_{-1\%}$ | $\underset{—}{-0.832}\pm70\%^{-8\%}_{+10\%}$ | $\underset{1.26}{0.255}\pm300\%^{+3\%}_{-2\%}$ | $\underset{—}{0.39}\pm10\%^{+10\%}_{-8\%}$ |
| −+ | 1400 | $\underset{0.917}{33.8}\pm6\%^{+0.8\%}_{+0\%}$ | — | $\underset{0.93}{-493}\pm10\%^{-0.4\%}_{+0.3\%}$ | $\underset{0.897}{2.86}\pm4\%^{+1\%}_{-1\%}$ | — | $\underset{0.842}{-850}\pm4\%^{-2\%}_{+2\%}$ | $\underset{1.06}{-0.208}\pm10\%^{-0.4\%}_{+0.5\%}$ | $\underset{0.666}{-4.75}\pm20\%^{-5\%}_{+4\%}$ | $\underset{0.504}{10.2}\pm0\%^{+0\%}_{+0\%}$ | $\underset{1.23}{0.448}\pm300\%^{+2\%}_{-2\%}$ | $\underset{-0.982}{0.475}\pm400\%^{+20\%}_{-20\%}$ | $\underset{—}{0.258}\pm30\%^{+10\%}_{-9\%}$ |
| −+ | 3000 | — | — | $\underset{1.09}{-424}\pm60\%^{-0.7\%}_{+0.8\%}$ | $\underset{0.4}{0.226}\pm40\%^{+10\%}_{-20\%}$ | — | — | $\underset{—}{0.146}\pm0\%^{+0\%}_{+0\%}$ | $\underset{0.651}{-1.5}\pm70\%^{-6\%}_{+5\%}$ | $\underset{—}{-497}\pm0\%^{+0\%}_{+0\%}$ | $\underset{—}{2.52}\pm40\%^{+6\%}_{-5\%}$ | $\underset{—}{2110}\pm0\%^{+0\%}_{+0\%}$ | $\underset{1}{-0.0453}\pm200\%^{-0.04\%}_{+0.04\%}$ |

preliminary

(MG5_aMC@NLO, complex mass scheme, $m_b \to 0$,
EFT dependence of the total width not included)

# A couple of additional ideas

# Statistically optimal observables

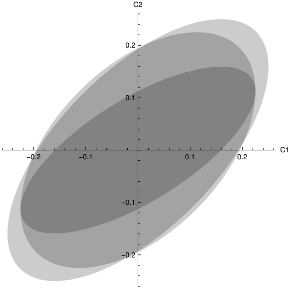
[Atwood,Soni '92]

[Diehl,Nachtmann '94]

minimize the one-sigma ellipsoid in EFT parameter space.

(*joint efficient* set of estimators, saturating the Rao-Cramér-Fréchet bound: $V^{-1} = I$)

For small $C_i$, with a phase-space distribution $\sigma(\phi) = \sigma_0(\phi) + \sum_i C_i\ \sigma_i(\phi)$,
the statistically optimal set of observables is: $O_i(\phi) = \sigma_i(\phi)/\sigma_0(\phi)$.



e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$

1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$
2. moments: $O_i \sim \sin(i\phi)$
3. statistically optimal: $O_i \sim \dfrac{\sin(i\phi)}{1 + \cos\phi}$

$\Longrightarrow$ area ratios 1.9 : 1.7 : 1
(total rate not used)

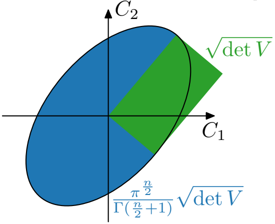
# Global determinant parameter (GDP)

[GD, Grojean, Gu, Wang]

In a $n$-dimensional Gaussian fit,
with covariance matrix $V$,

$$\mathrm{GDP} \equiv \sqrt[2n]{\det V}$$

provides a geometric average
of the constraints strengths.

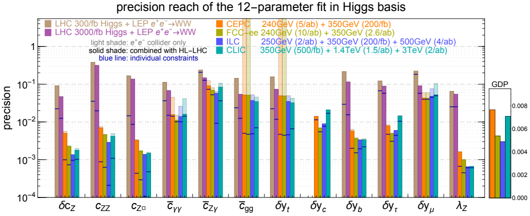


Interestingly, GDP ratios are operator-basis independent!

- as the volume scales linearly with coupling normalization
- as the volume is invariant under rotations

$\Longrightarrow$ conveniently assess constraint strengthening.

e.g. Global Higgs analysis
at lepton colliders:



# Summary

The EFT parametrizes systematically the parameter space in direct vicinity of the standard model.

A global analysis of future-lepton-collider constraints on the top EFT is ongoing.

Statistically optimal observables are surprisingly unexploited.

*Global determinant parameter* ratios assess the strengthening of global constraints, basis independently.

# Backup

# Independent coefficients

## Two-quark operators: 10 real degrees of freedom

Scalar: $C_{u\varphi}^{(33)},$

Vector:

$$\begin{array}{llll}
C_{\varphi q}^{+(33)} = C_{\varphi q}^{+(33)*}, & & & (\text{down-}Z,\ tbW)\\
C_{\varphi q}^{V(33)} = C_{\varphi q}^{V(33)*} & \equiv C_{\varphi u}^{(33)} + C_{\varphi q}^{-(33)}, & & (\text{up-}Z,\ tbW)\\
C_{\varphi q}^{A(33)} = C_{\varphi q}^{A(33)*} & \equiv C_{\varphi u}^{(33)} - C_{\varphi q}^{-(33)}, & & (\text{up-}Z,\ tbW)\\
C_{\varphi ud}^{(33)} & & &
\end{array}$$

Tensor:

$$\begin{array}{ll}
C_{uA}^{(33)} & \equiv C_{uW}^{(33)} + C_{uB}^{(33)},\\
C_{uZ}^{(33)} & \equiv \text{cotan}\,\theta_W\, C_{uW}^{(33)} - \tan\theta_W\, C_{uB}^{(33)},\\
C_{uG}^{(33)} &
\end{array}$$

## Two-quark–two-lepton operators: $9 \times 3^2$ real degrees of freedom

Scalar: $C_{lequ}^{S(33)},$

Vector:

$$\begin{array}{lll}
C_{lq}^{+(33)} = C_{lq}^{+(33)*}, & & (\text{up–}\nu,\ \text{down–}\ell)\\
C_{lq}^{V(33)} = C_{lq}^{V(33)*} & \equiv C_{lu}^{(33)} + C_{lq}^{-(33)}, & (\text{up–}\ell)\\
C_{lq}^{A(33)} = C_{lq}^{A(33)*} & \equiv C_{lu}^{(33)} - C_{lq}^{-(33)}, & (\text{up–}\ell)\\
C_{eq}^{V(33)} = C_{eq}^{V(33)*} & \equiv C_{eu}^{(33)} + C_{eq}^{(33)}, & (\text{up–}\ell,\ \text{down–}\ell)\\
C_{eq}^{A(33)} = C_{eq}^{A(33)*} & \equiv C_{eu}^{(33)} - C_{eq}^{(33)}, &
\end{array}$$

Tensor: $C_{lequ}^{T(33)}.$

# LO interferences

$$\sigma_{e^+e^-\to t\bar{t}}^{\sqrt{s}=500\text{ GeV}}\ [\text{fb}] = +568 + \left(\frac{1\text{ TeV}}{\Lambda}\right)^2 \begin{pmatrix} C_{lq}^A \\ C_{eq}^A \\ C_{\phi q}^A \\ C_{lq}^V \\ C_{eq}^V \\ C_{\phi q}^V \\ C_{uZ}^R \\ C_{uA}^R \\ C_{uZ}^I \\ C_{uA}^I \end{pmatrix}^T \begin{pmatrix} +221 \\ -194 \\ +7.01 \\ -1110 \\ -737 \\ -8.24 \\ +33.8 \\ +209 \\ \cdot \\ \cdot \end{pmatrix} + \left(\frac{1\text{ TeV}}{\Lambda}\right)^4 \begin{pmatrix} C_{lq}^A \\ C_{eq}^A \\ C_{\phi q}^A \\ C_{lq}^V \\ C_{eq}^V \\ C_{\phi q}^V \\ C_{uZ}^R \\ C_{uA}^R \\ C_{uZ}^I \\ C_{uA}^I \end{pmatrix}^T \begin{pmatrix} +367 & \cdot & +13.2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & +367 & -11.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & +0.209 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & +868 & \cdot & +31.1 & -128 & -197 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & +868 & -27.3 & +112 & -197 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & +0.493 & -4.05 & -0.432 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +9.36 & +2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +25.2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +2.51 & +0.536 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & +6.75 \end{pmatrix} \begin{pmatrix} C_{lq}^A \\ C_{eq}^A \\ C_{\phi q}^A \\ C_{lq}^V \\ C_{eq}^V \\ C_{\phi q}^V \\ C_{uZ}^R \\ C_{uA}^R \\ C_{uZ}^I \\ C_{uA}^I \end{pmatrix}$$

$$+ \left(\frac{1\text{ TeV}}{\Lambda}\right)^4 \left\{ + 1600 \left|C_{lequ}^S\right|^2 + 13900 \left|C_{lequ}^T\right|^2 \right\}$$