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Azimuthal-angle Observables in Inclusive Three-jet Production

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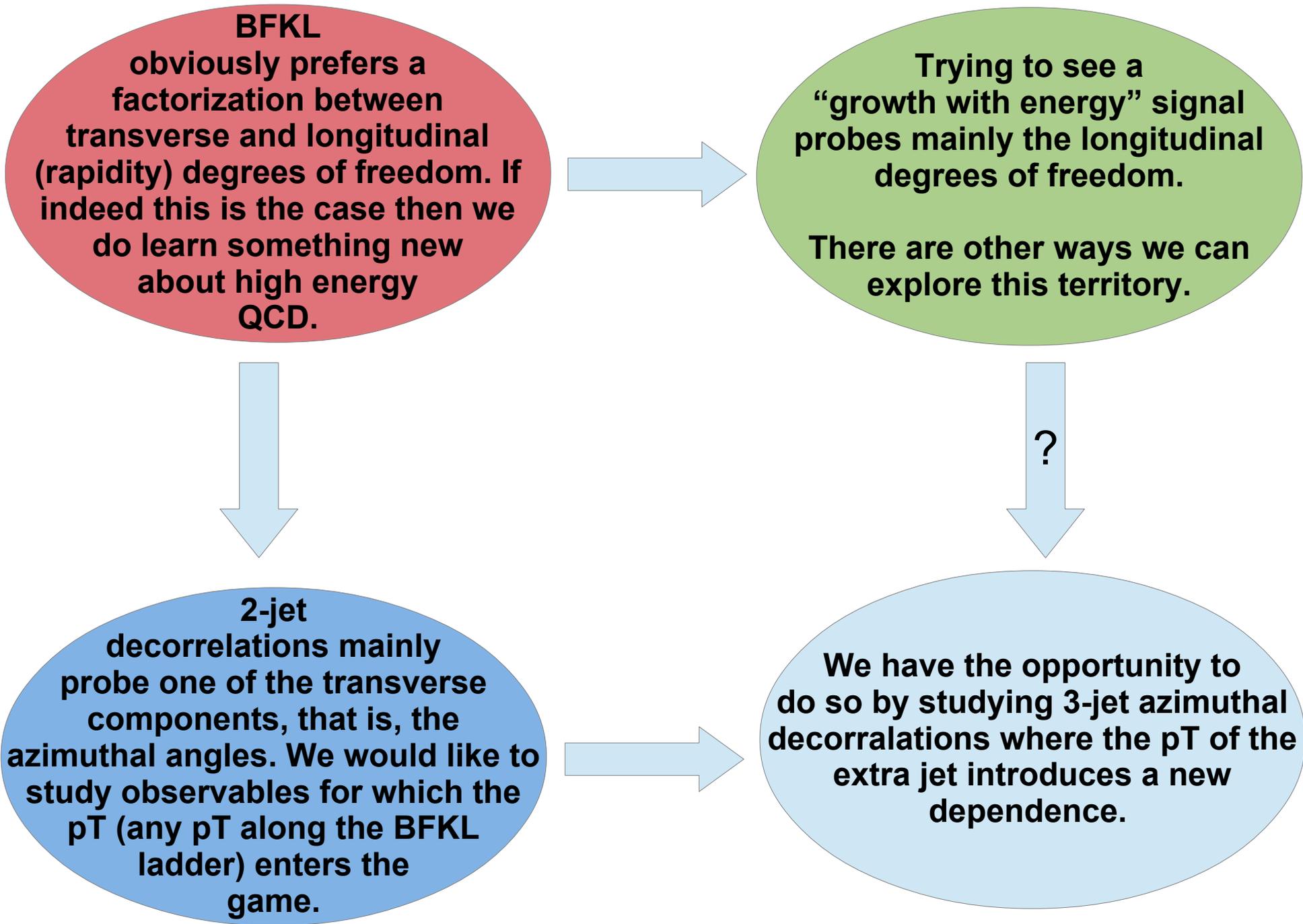
DIS 2017, 3-7 April 2017, Birmingham, UK

Outline

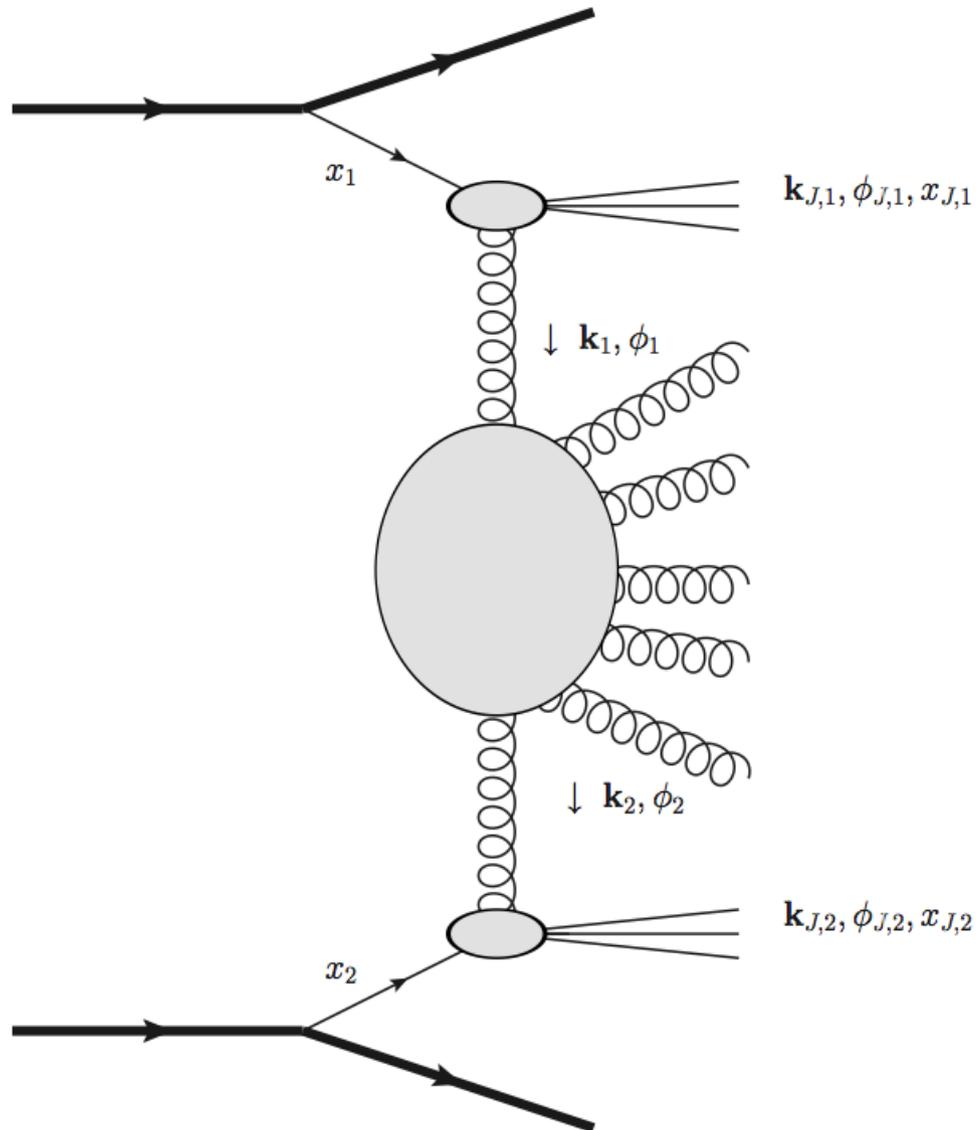
- Motivation
- Kinematics
- 3-jet production within the BFKL framework
- New observables relevant to 3-jet production
- Include higher order corrections and check stability
- Conclusions and outlook

BFKL phenomenology

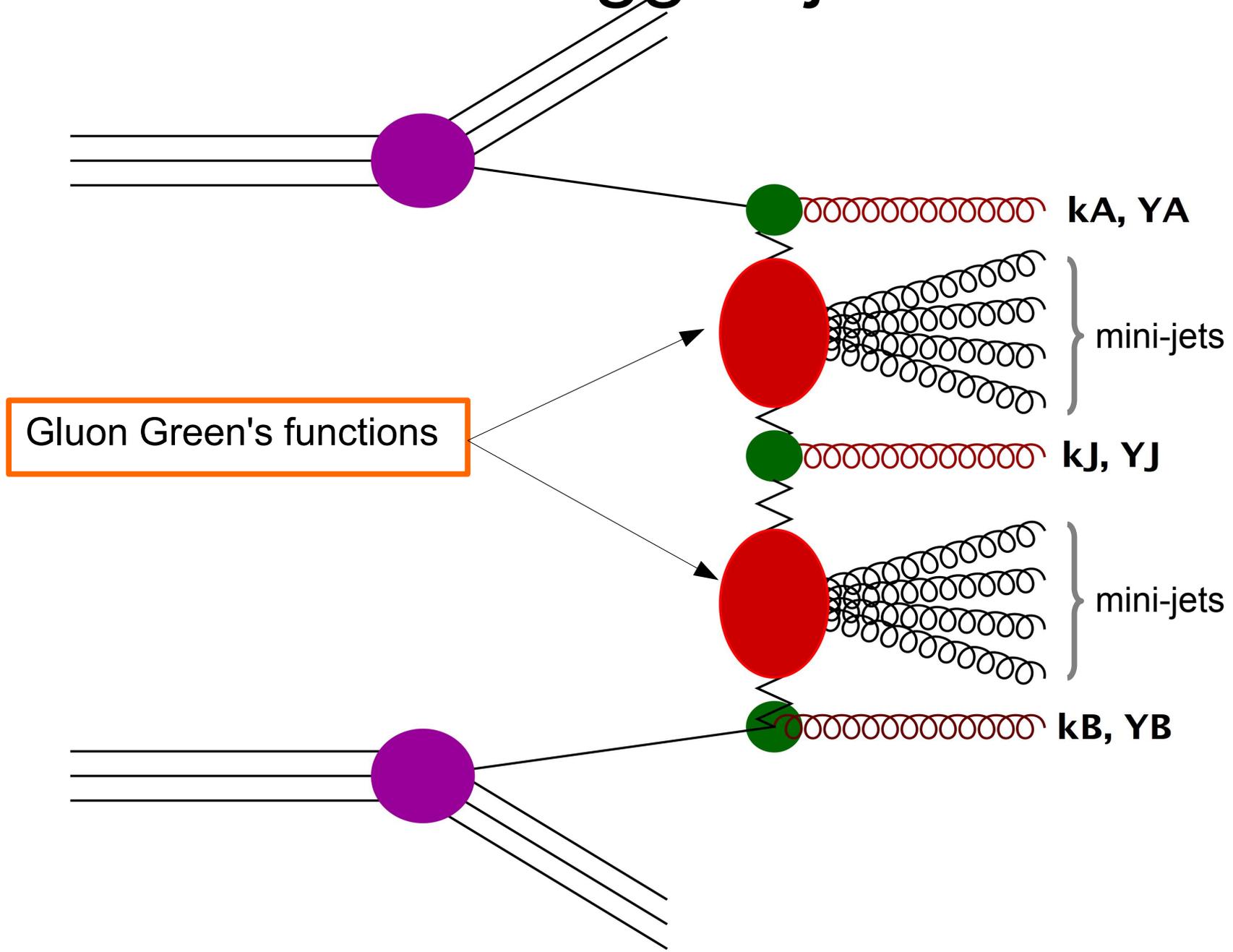
- LHC has produced and will further produce an abundance of data
- This is the best time to investigate the applicability of the BFKL resummation program within the context of a hadron collider
- In the last years: the big hit from the theory/experimental side was the study of Mueller-Navelet jets (dijets). We only touch here one subfield for which BFKL is relevant, small- x physics/forward physics is much richer of course: Diffraction, Saturation, DPI etc.

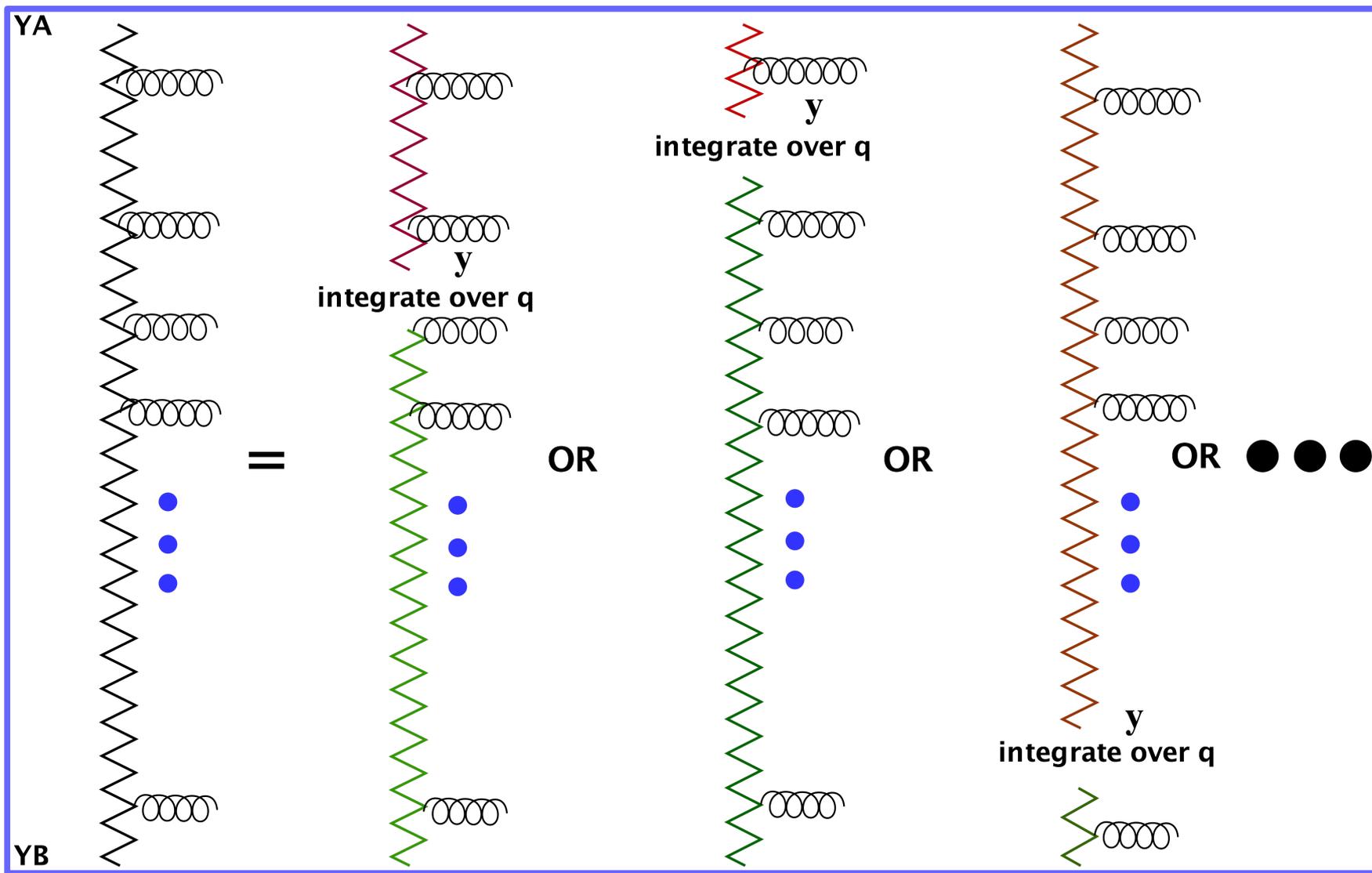


Mueller-Navelet jets



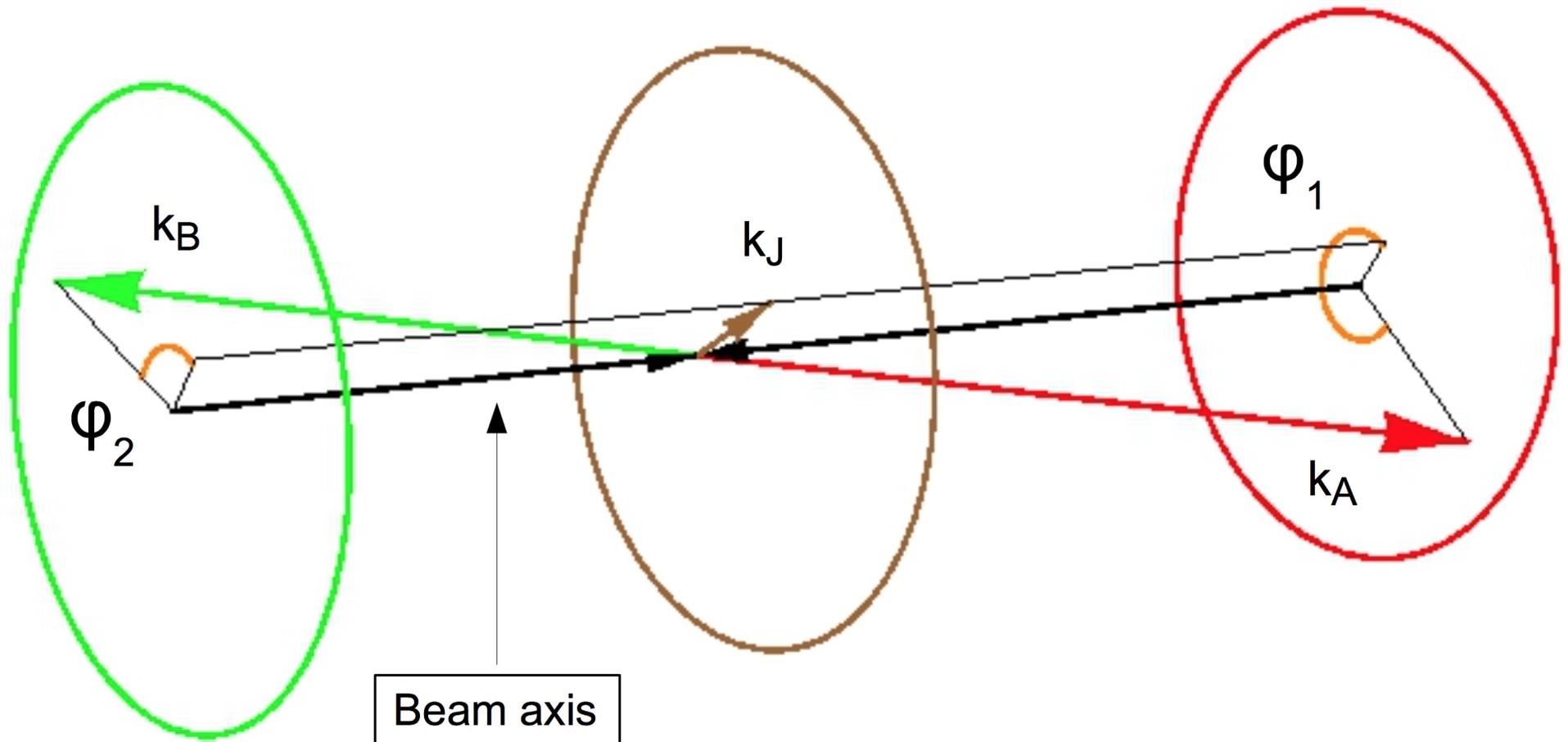
Now, let us move to events with three tagged jets



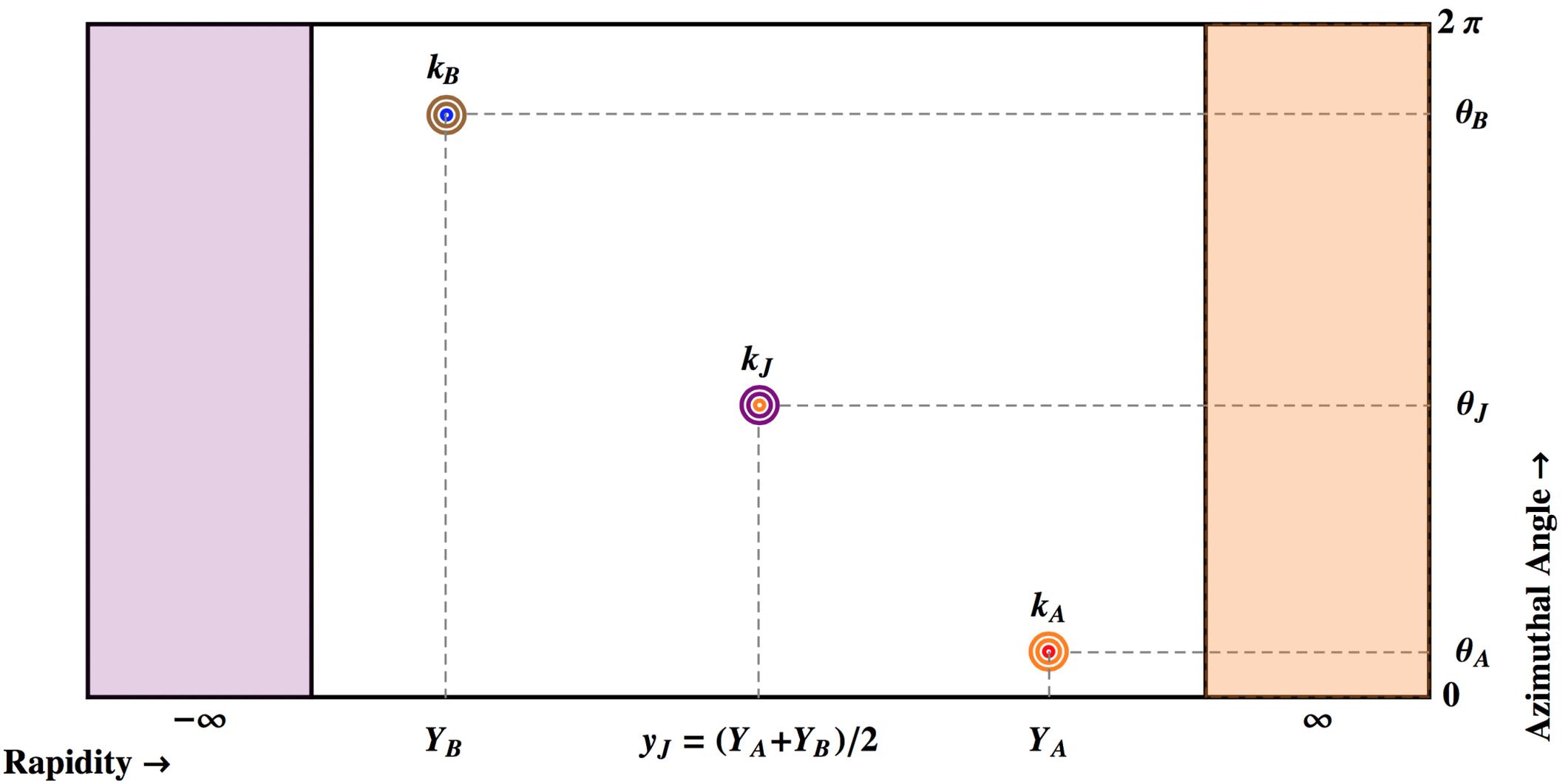


$$f(k_A, k_B, Y_A - Y_B) = \int d^2 q f(k_A, q, Y_A - y) f(q, k_B, y - Y_B)$$

An event with three tagged jets (detector view)

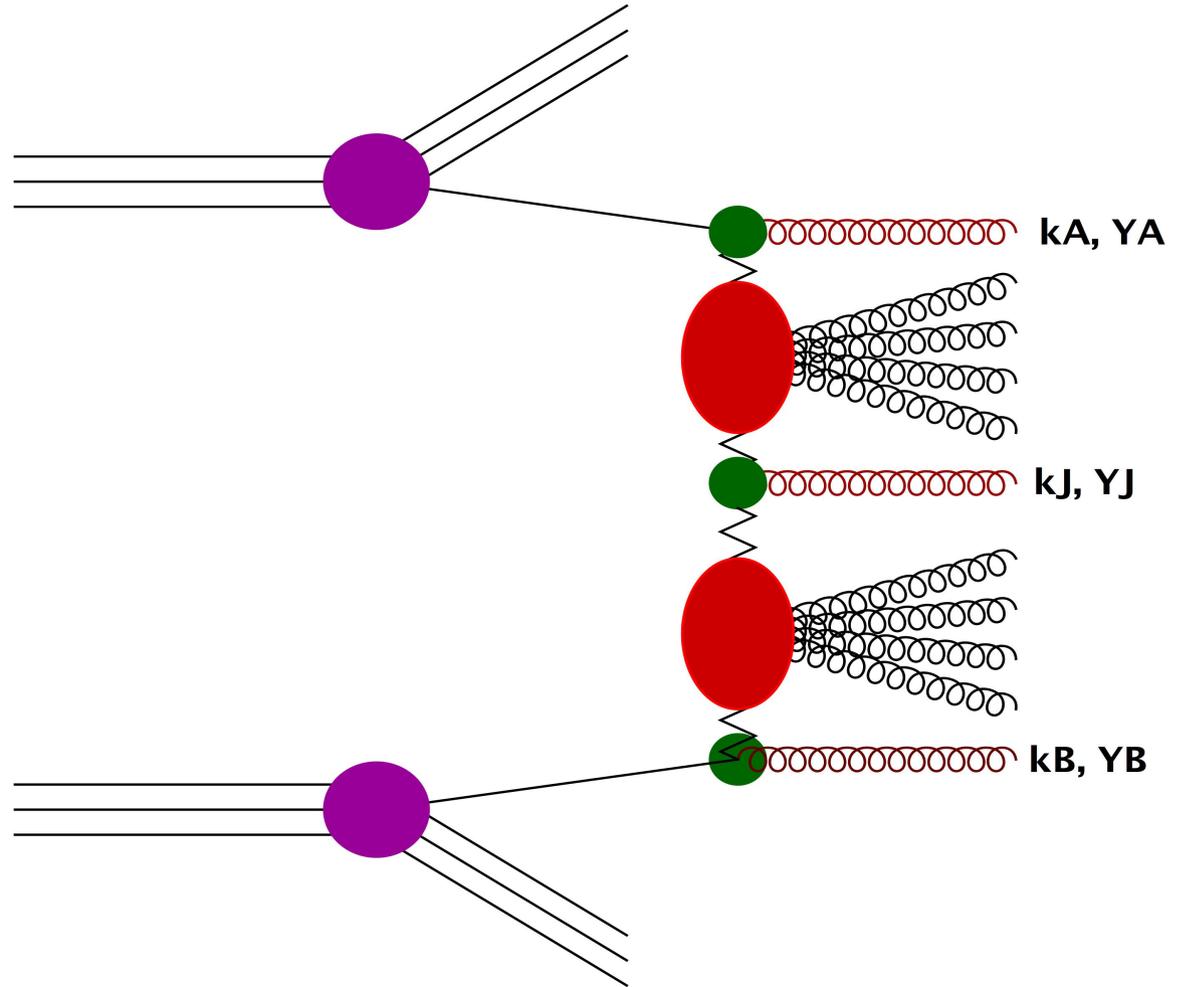


A primitive lego-plot (3-jets)



3-jets partonic cross section

Assuming that $Y_A > y_J > Y_B$ and also that k_A and k_B are fixed we can write for the differential cross section:



$$\frac{d^3 \sigma^{3\text{-jet}}}{d^2 \vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2 \vec{p}_A \int d^2 \vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B)$$

Starting point...
THEN:

The main idea is to integrate over all angles after using the **projections** on the **two azimuthal angle differences** between the central jet and k_A and k_B respectively

Integrate over all angles after using projections

$$\frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2\vec{p}_A \int d^2\vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B)$$

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \\ \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} \\ = \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \\ \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta)}^N} \\ \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta, p_B^2, y_J - Y_B)$$

Integrate over all angles after using projections

$$\begin{aligned}
 & \int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \\
 & \quad \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} \\
 & = \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \\
 & \quad \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)}^N} \\
 & \quad \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, p_B^2, y_J - Y_B)
 \end{aligned}$$

$$\begin{aligned}
 & \langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle \\
 & = \frac{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}
 \end{aligned}$$

... so that you can define new
(partonic level) observables:

$$\mathcal{R}_{P,Q}^{M,N} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

How would an experimentalist measure this*?

$$\mathcal{R}_{P,Q}^{M,N} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

* Coming from theorists, this would appear to be more of a cooking recipe, apologies to our experimental colleagues in advance for any naivety here.

How would an experimentalist measure this?

1. For 7 (and 8) TeV energies, just pick up the data that were used for Mueller-Navelet studies. For 13 TeV, one may work in parallel.
2. From these data, isolate those events that have in addition a very central jet.
3. Choose integers M, N, P, Q, e.g. M=1, N=3, P=1, Q=2
4. For each event, measure the azimuthal angle difference between the forward-central jets, $\Delta\theta_1 = (\theta_A - \theta_J - \pi)$, and the backward-central jets, $\Delta\theta_2 = (\theta_J - \theta_B - \pi)$.
5. For each event calculate two quantities:
num = $\text{Cos}(1 * \Delta\theta_1) * \text{Cos}^*(3 * \Delta\theta_2)$ and denom = $\text{Cos}(1 * \Delta\theta_1) * \text{Cos}^*(2 * \Delta\theta_2)$.
6. Calculate the average of num ($\langle \text{num} \rangle$) and denom ($\langle \text{denom} \rangle$) over all the events. Divide $\langle \text{num} \rangle$ over $\langle \text{denom} \rangle$ to have the quantity below:

$$\left\{ \mathcal{R}_{P,Q}^{M,N} \right. = \frac{\langle \cos (M (\theta_A - \theta_J - \pi)) \cos (N (\theta_J - \theta_B - \pi)) \rangle}{\langle \cos (P (\theta_A - \theta_J - \pi)) \cos (Q (\theta_J - \theta_B - \pi)) \rangle}$$

Now introduce PDF's and running of the strong coupling to get theoretical predictions on a hadronic level for various kinematical cuts

We have used two kinematical cuts

$$k_A^{\min} = 35 \text{ GeV}, k_B^{\min} = 35 \text{ GeV}, k_A^{\max} = k_B^{\max} = 60 \text{ GeV} \text{ (symmetric)}$$

Here we will focus on the asymmetric one:

$$k_A^{\min} = 35 \text{ GeV}, k_B^{\min} = 50 \text{ GeV}, k_A^{\max} = k_B^{\max} = 60 \text{ GeV} \text{ (asymmetric)}$$

Introduce higher order corrections and check stability

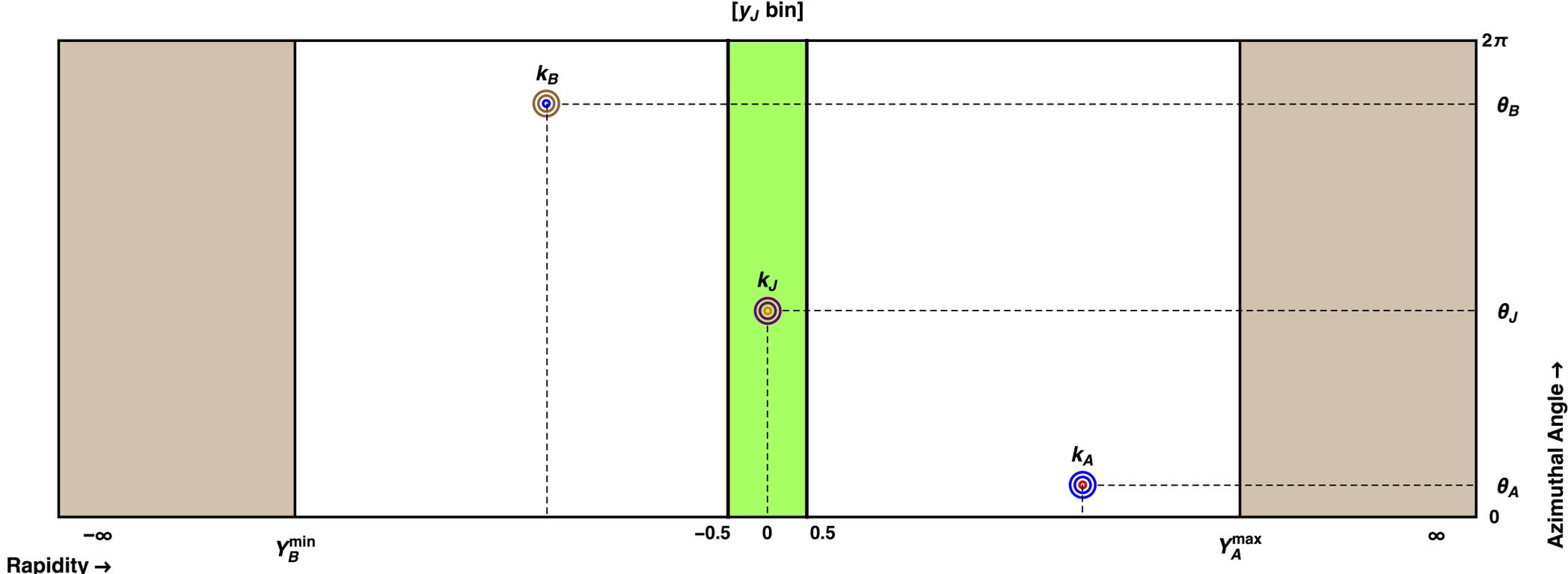
- For the partonic cross section change from LLA GGF to NLLA GGF
- Jet vertex corrections are missing and need to be included
- Use BLM scheme
[S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. D 28, 228 (1983)]
- Consider three cases for the p_T of the central jet

$$20 \text{ GeV} < k_J < 35 \text{ GeV} \text{ (bin-1)}$$

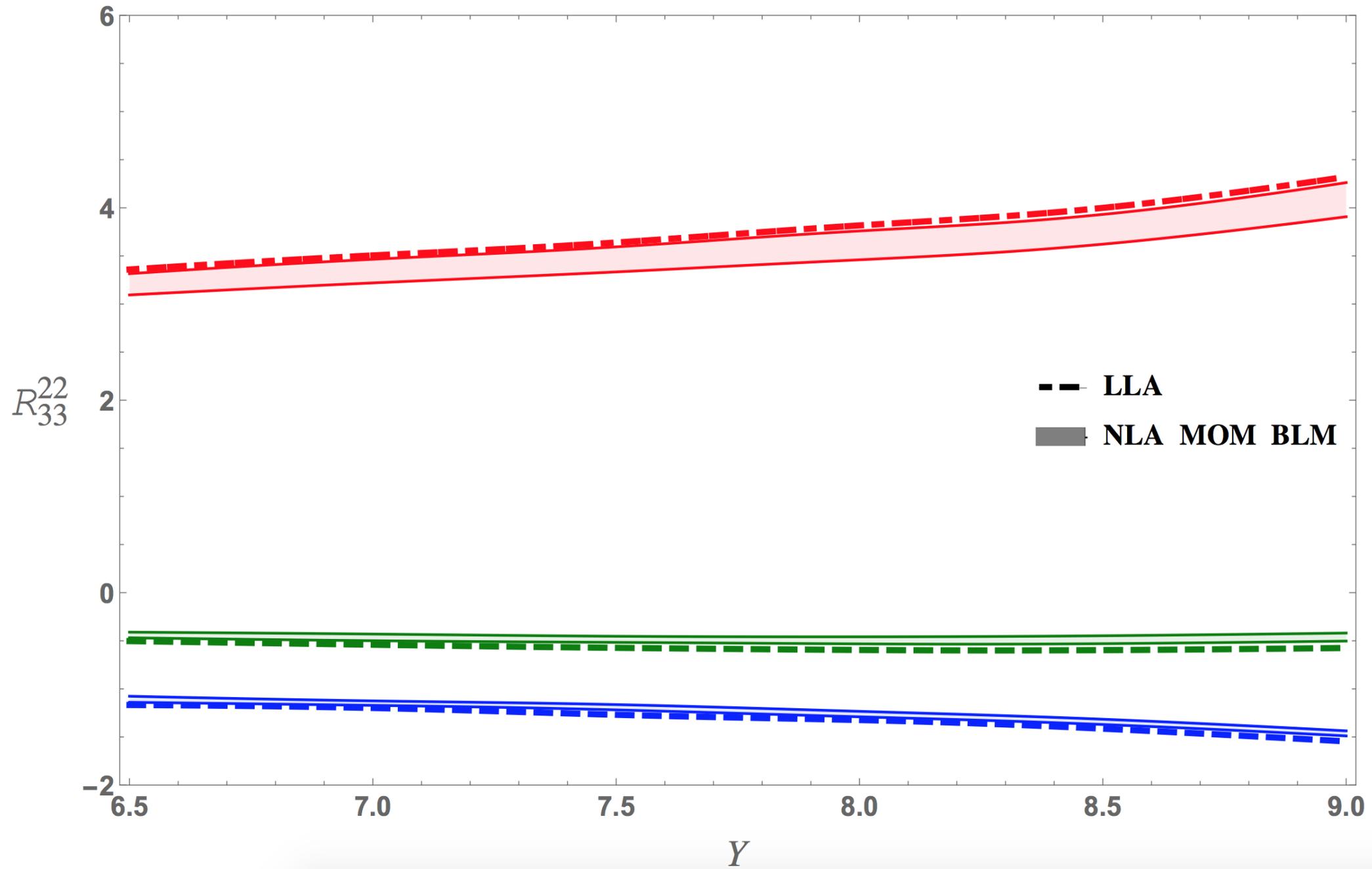
$$35 \text{ GeV} < k_J < 60 \text{ GeV} \text{ (bin-2)}$$

$$60 \text{ GeV} < k_J < 120 \text{ GeV} \text{ (bin-3)}$$

Integrate over a central rapidity bin

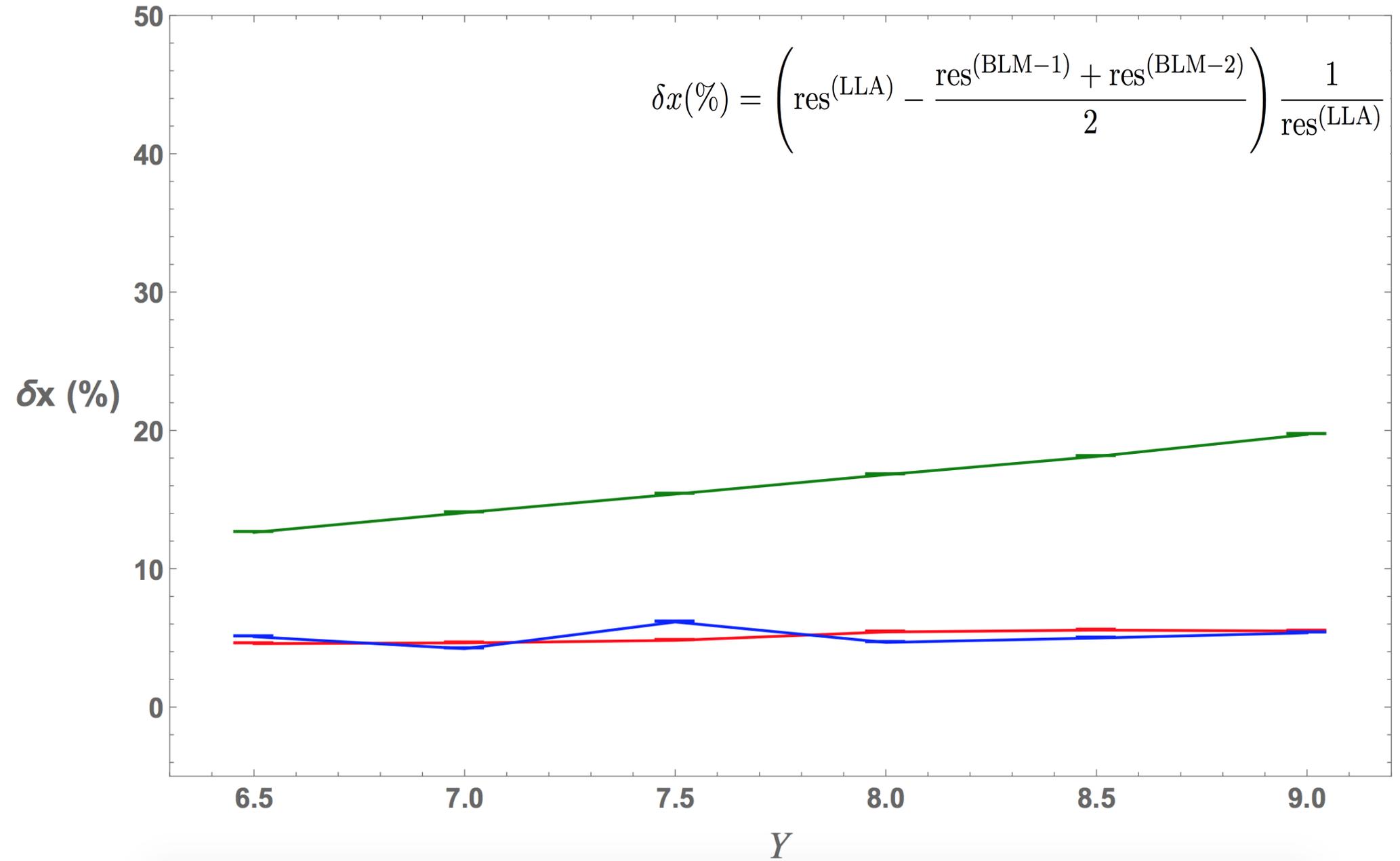


$\sqrt{s} = 7 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_J \in \text{bin-1 (red), bin-2 (green), bin-3 (blue)}$

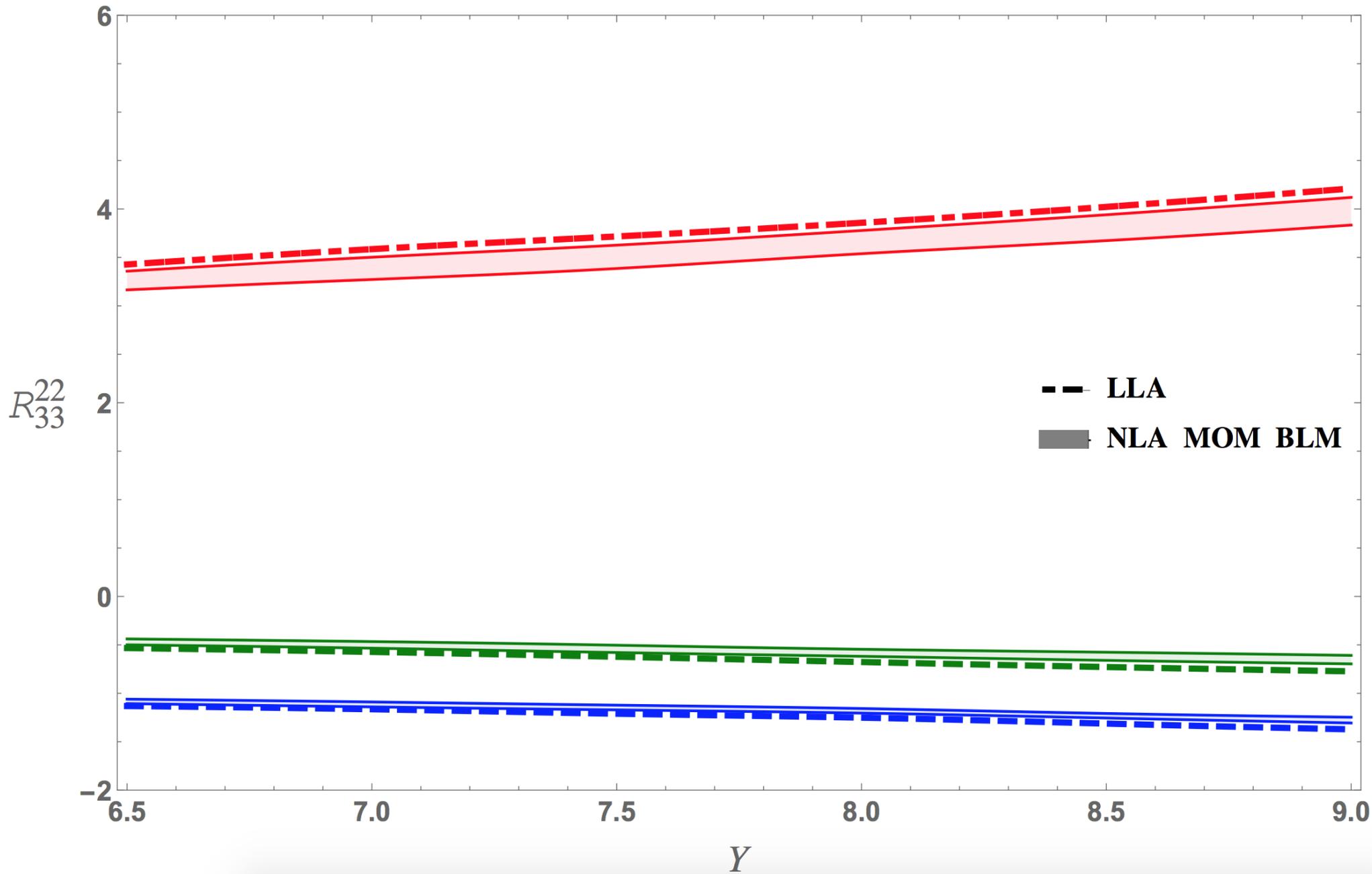


$\sqrt{s} = 7 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_J \in \text{bin-1 (red), bin-2 (green), bin-3 (blue)}$

$$\delta x(\%) = \left(\text{res}^{(\text{LLA})} - \frac{\text{res}^{(\text{BLM-1})} + \text{res}^{(\text{BLM-2})}}{2} \right) \frac{1}{\text{res}^{(\text{LLA})}}$$



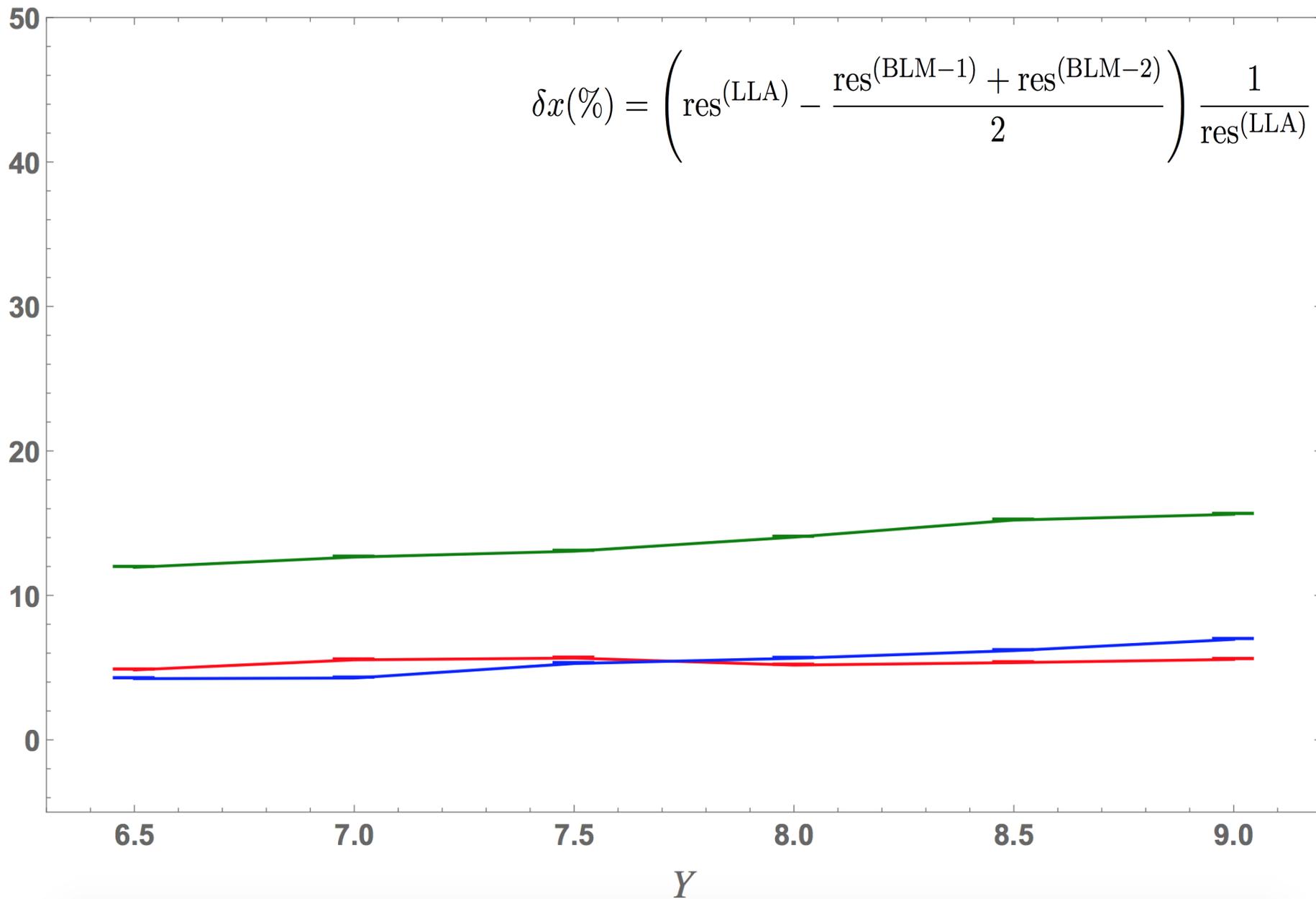
$\sqrt{s} = 13 \text{ TeV}; \quad k_B^{\text{min}} = 50 \text{ GeV}; \quad k_J \in \text{bin-1 (red), bin-2 (green), bin-3 (blue)}$



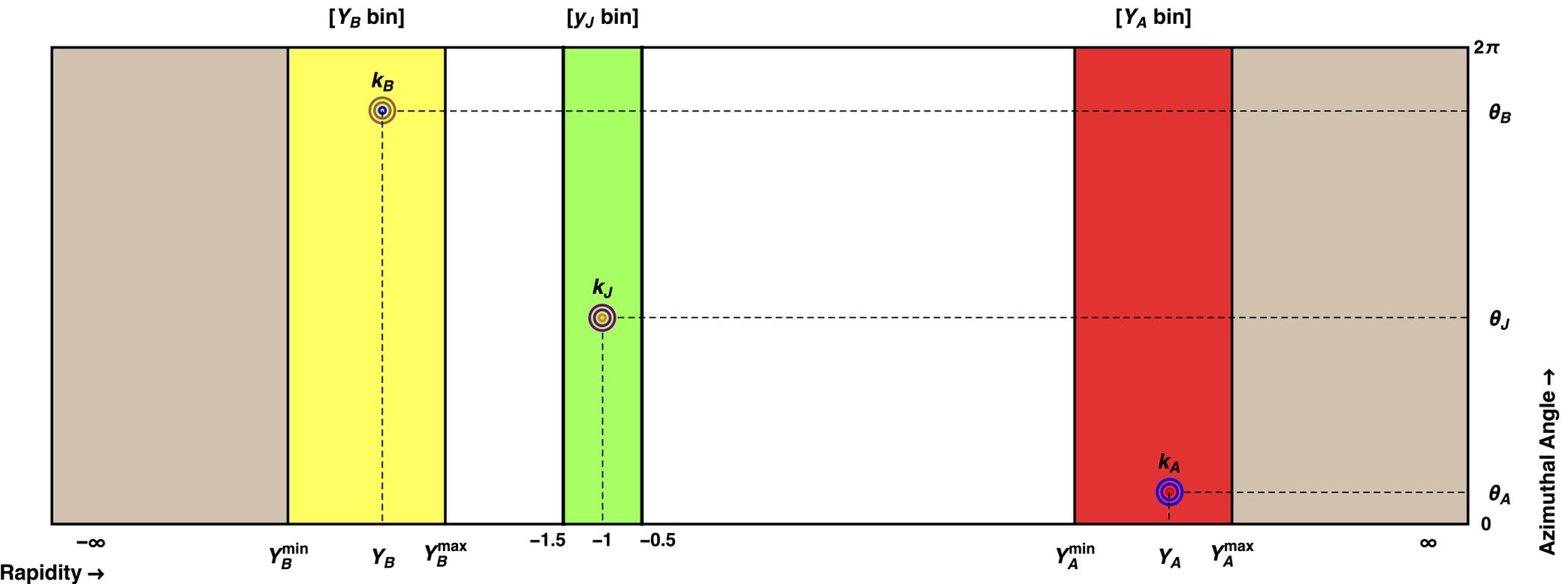
$\sqrt{s} = 13 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_J \in \text{bin-1 (red), bin-2 (green), bin-3 (blue)}$

$$\delta x(\%) = \left(\text{res}^{(\text{LLA})} - \frac{\text{res}^{(\text{BLM-1})} + \text{res}^{(\text{BLM-2})}}{2} \right) \frac{1}{\text{res}^{(\text{LLA})}}$$

$\delta x (\%)$



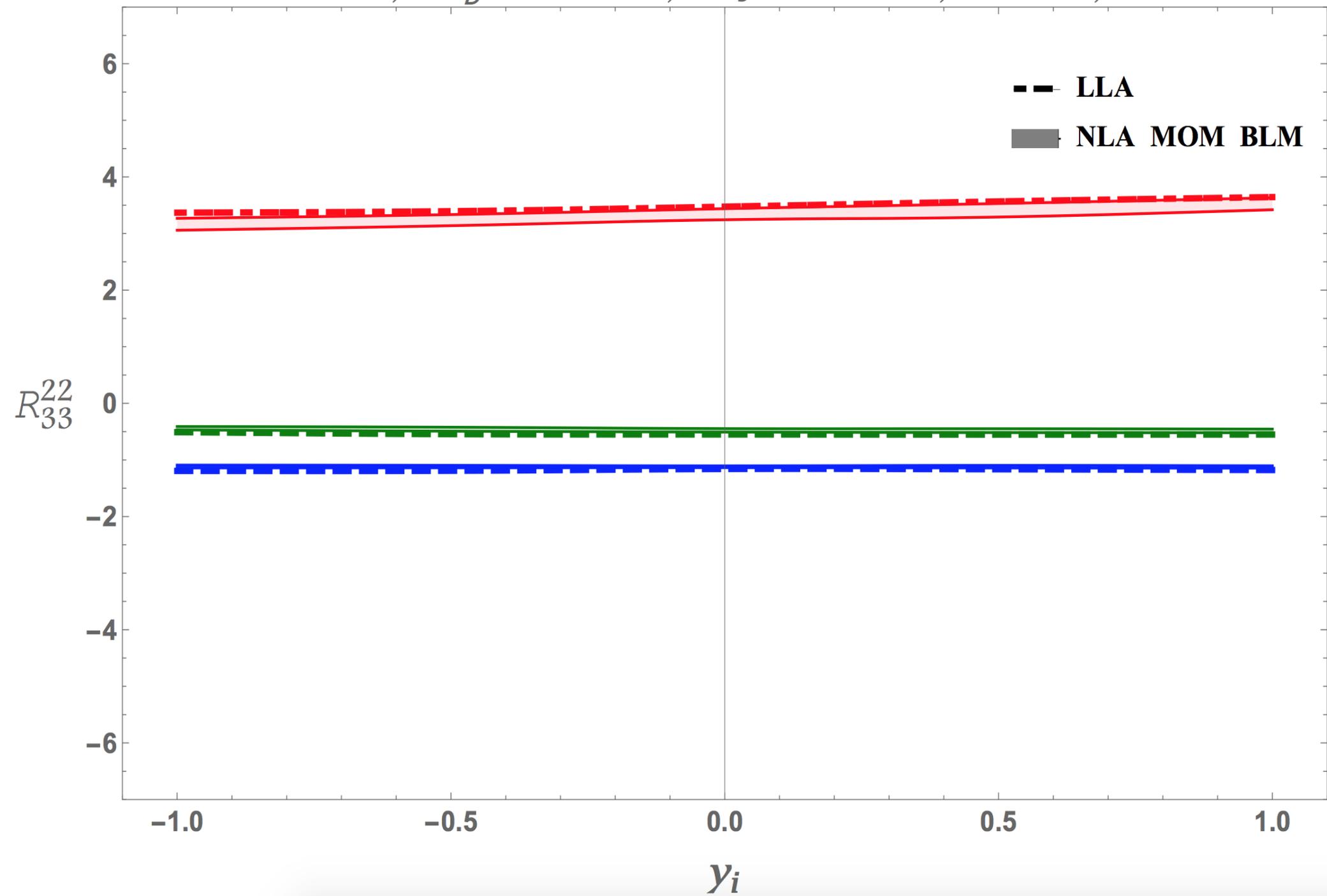
Integrate over a forward, backward and central rapidity bin



$$(Y_A^{\min} = 3) < Y_A < (Y_A^{\max} = 4.7)$$

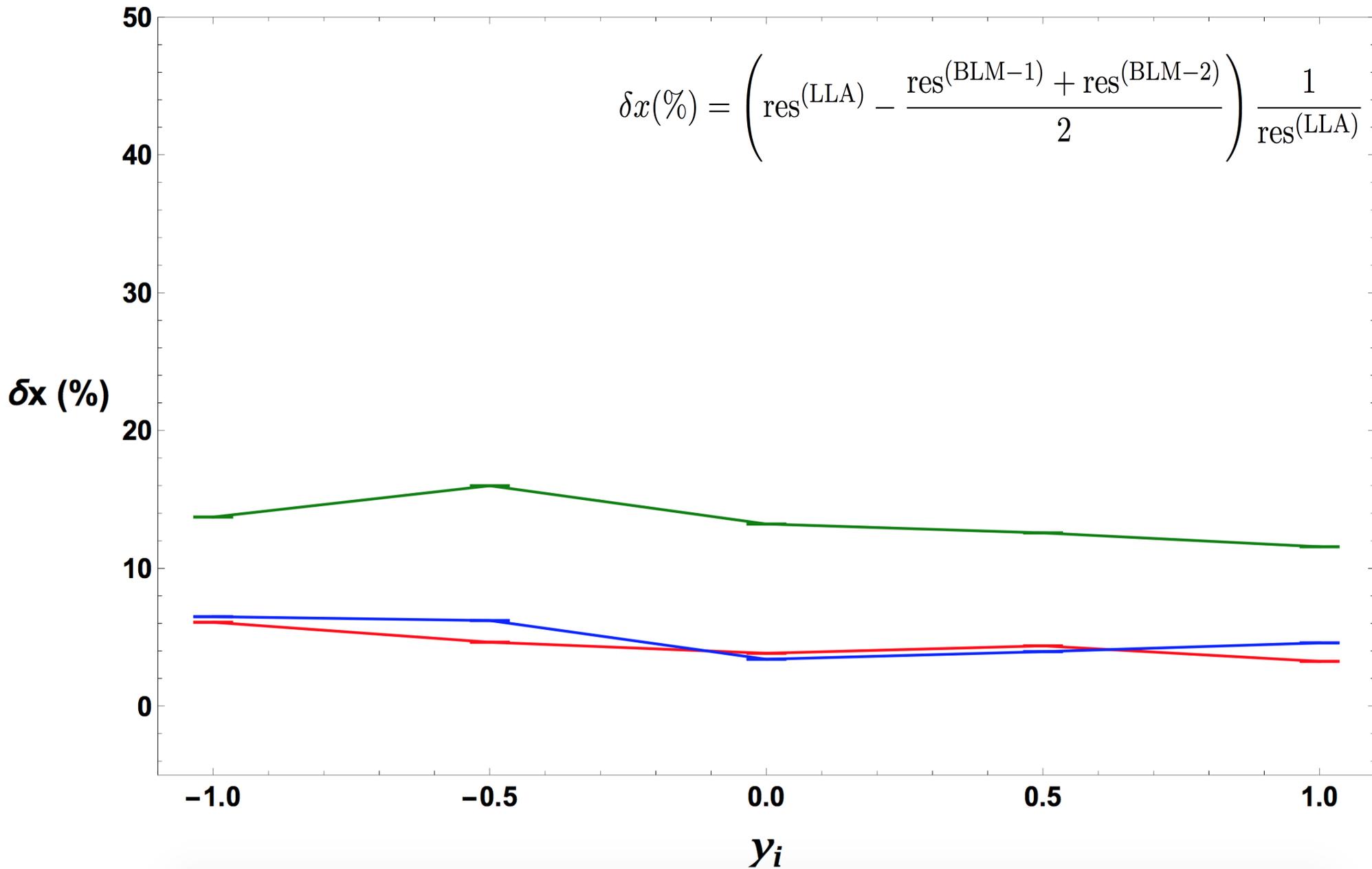
$$(Y_B^{\min} = -4.7) < Y_B < (Y_B^{\max} = -3)$$

$\sqrt{s} = 7 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_j \in \text{bin-1, bin-2, bin-3}$

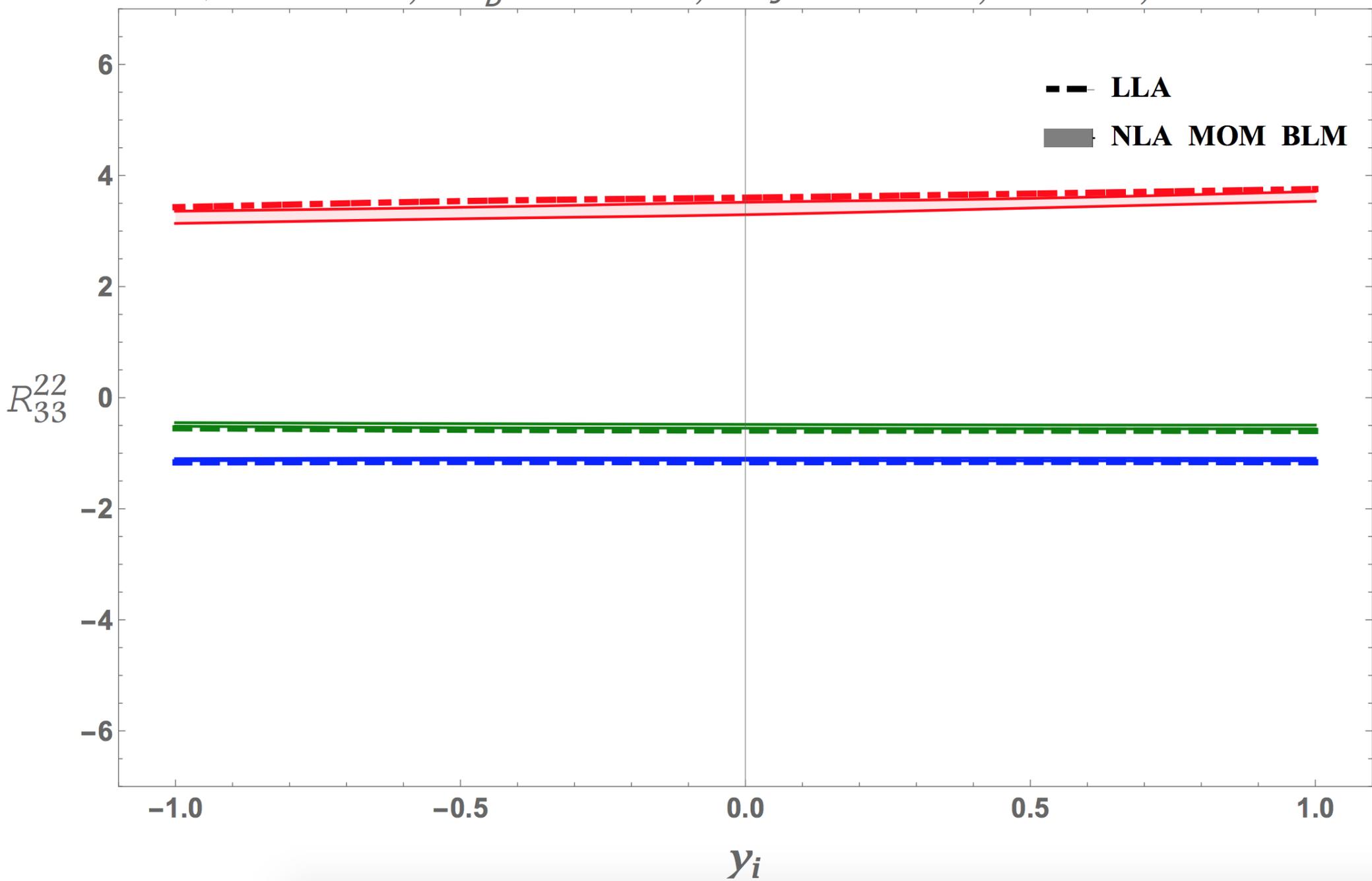


$\sqrt{s} = 7 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_J \in \text{bin-1, bin-2, bin-3}$

$$\delta x(\%) = \left(\text{res}^{(\text{LLA})} - \frac{\text{res}^{(\text{BLM-1})} + \text{res}^{(\text{BLM-2})}}{2} \right) \frac{1}{\text{res}^{(\text{LLA})}}$$



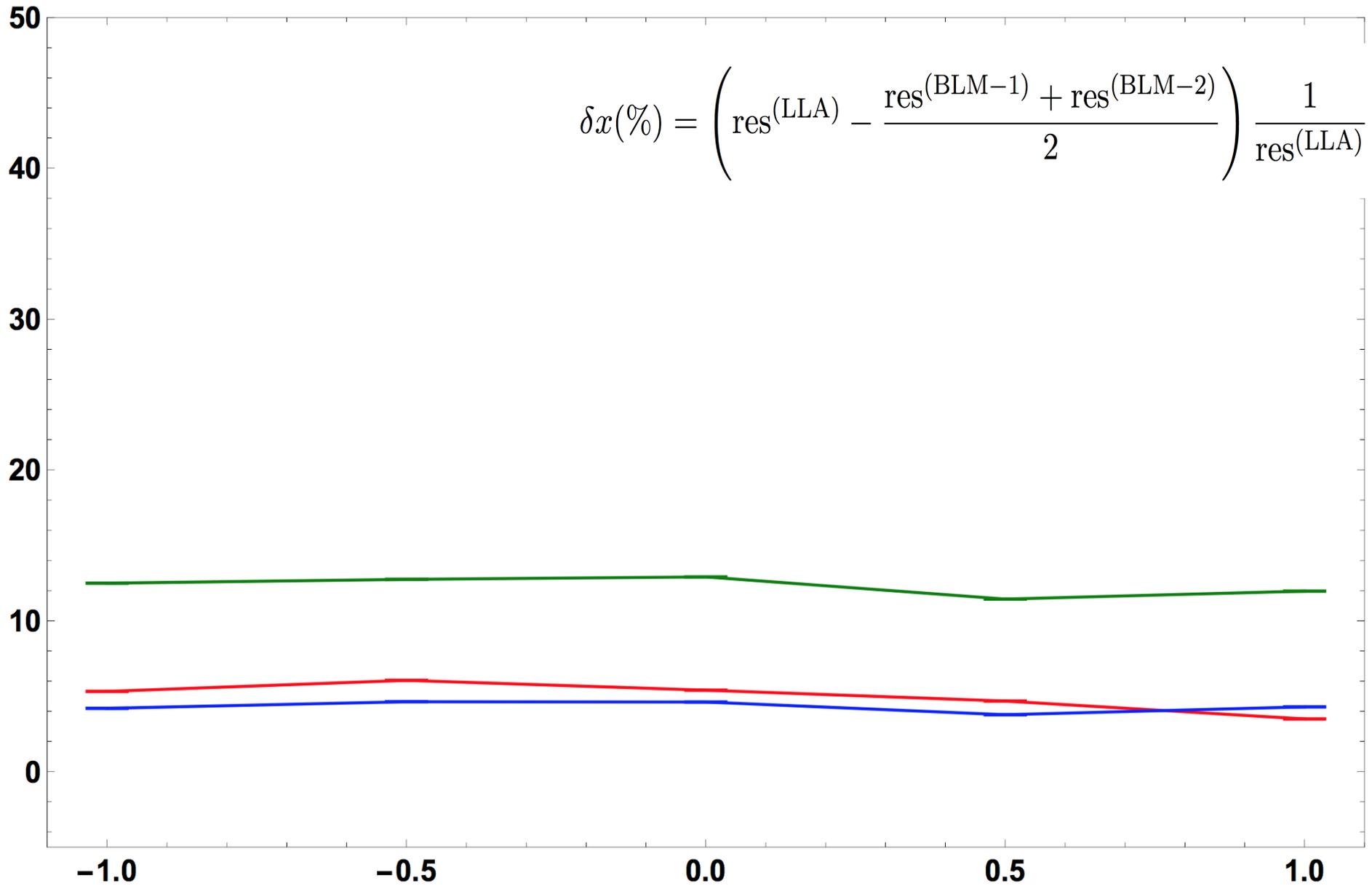
$\sqrt{s} = 13 \text{ TeV}; \quad k_B^{\min} = 50 \text{ GeV}; \quad k_j \in \text{bin-1 (red), bin-2 (green), bin-3 (blue)}$



$\sqrt{s} = 13$ TeV; $k_B^{\min} = 50$ GeV; $k_j \in$ ● bin-1, ● bin-2, ● bin-3

$$\delta x(\%) = \left(\text{res}^{(\text{LLA})} - \frac{\text{res}^{(\text{BLM-1})} + \text{res}^{(\text{BLM-2})}}{2} \right) \frac{1}{\text{res}^{(\text{LLA})}}$$

δx (%)



y_i

Conclusions & Outlook

- We use events with three tagged jets to propose **new observables** with a distinct signal of BFKL dynamics.
- We use **ratios** of correlation functions to **minimize the influence of higher order effects**.
- The **stability** of the azimuthal observables after introducing higher order corrections is very good.
- The measurement of the observables proposed here **poses no intricacies** in the analysis of the data. Any new input from the experimental side would be extremely valuable